John Sinclair 16325734

Comp Maths Assignment 3

Q1 Answer : B, this is because the backslash ('\') allows B to be the numerator. However, when run in MatLab the output will be Nan, this is because A is an uninvertable matrix.

Q2 Answer is B, matlab outputs an error since we are altering the size of A each time.

Q3 Answer: A, this is because plot3 is used to plot a three dimensional graph

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Q4 $t$ | 0 | 0.2 | 0.4 | 0.6 |
| $x$ | 1 | 1.2 | 1.3 | 1.25 |
| $y$ | 2 | 2.1 | 2.3 | 2.6 |

@ $t = 0.5$ find $(x, y)$

find $L_{1-4}$:

$$L_1 = \frac{(x - x_2)(x - x_3)(x - x_4)}{(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)}$$

$$L_2 = \frac{(x - x_1)(x - x_3)(x - x_4)}{(x_2 - x_1)(x_2 - x_3)(x_2 - x_4)}$$

$L_3 = \ldots$

$$L_1 = \frac{(x - 0.2)(x - 0.4)(x - 0.6)}{(0 - 0.2)(0 - 0.4)(0 - 0.6)}$$

$$= \frac{x^2 - 0.6x + 0.08\,(x - 0.6)}{-0.048}$$

$$= \frac{x^3 - 0.6x^2 + 0.08x - 0.6x^2 + 0.36x - 0.048}{-0.048}$$

$$= \frac{x^3 - 1.2x^2 + 0.44x - 0.048}{-0.048}$$

@ $x = 0.5$

$L_1 = 0.0625$

$L_2 = -0.3125$

$L_3 = 0.9375$

$L_4 = 0.3125$

@ $t = 0.5$

$x = 0.0625(1) + (-0.3125)(1.2) + 0.9375(1.3)$
$+ 0.3125(1.25)$
$= 1.296875$

$y = 0.0625(2) - 0.3125(2.1) + 0.9375(2.3)$
$+ 0.3125(2.6)$
$= 2.4375$ B $(1.3, 2.44)$

Q5

| t (s) | 10 | 15 | 20 | 22 |
| --- | --- | --- | --- | --- |
| v ($ms^{-1}$) | 22 | 36 | 57 | 10 |

@ 17 sec

① $f(15) = a + 15b + 225c = 36$
② $f(20) = a + 20b + 400c = 57$
③ $f(22) = a + 22b + 484c = 10$

① $a = 36 - 15b - 225c$
① → ② $(36 - 15b - 225c) + 20b + 400c = 57$
$5b + 175c = 21$
$b = 4.2 - 35c$
② → ① $a = 36 - 15(4.2 - 35c) - 225c$
$a = 36 + 525c - 63 - 225c$
$a = 300c - 27$
①,② → ③ $a + 22b + 484c = 10$

$300c - 27 + 22(4.2 - 35c) + 484c = 10$
$300c - 27 + 92.4 - 770c + 484c = 10$
$14c = -55.4$
$c = -3.95714$
$b = 142.6999$
$a = -1214.142$

$f(x) = -1214.142 + 142.6999x - 3.95714x^2$
$f'(x) = 142.699 - 7.91428x$
$f'(17) = 8.15624\ ms^{-2}$
$\therefore$ Answer is C 8.157

Q6 $f(-1) = 9$ (i) Lagrange form
$f(1) = 5$ (ii) Newton's Poly
$f(2) = 12$

(i) $(-1, 9)$ $(1, 5)$ $(2, 12)$
$x_1 \; y_1$ $x_2 \; y_2$ $x_3 \; y_3$

$$P(x) = \sum_{1}^{3} P_i(x) y_i$$

$$P(x) = \underbrace{\left[\frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)}\right]}_{L_1} y_1$$

$$+ \underbrace{\left[\frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)}\right]}_{L_2} y_2$$

$$+ \underbrace{\left[\frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)}\right]}_{L_3} y_3$$

$$= \frac{(x-1)(x-2)}{(-1-1)(-1-2)} 9 + \frac{(x+1)(x-2)}{(1+1)(1-2)} 5 +$$

$$\frac{(x+1)(x-1)}{(2+1)(2-1)} 12$$

$$= 3/2(x^2 - 3x + 2) - 5/2(x^2 - x - 2) + 4(x^2 - 1)$$

$$= 3(x^2 - 3x + 2) - 5(x^2 - x - 2) + 8(x^2 - 1)$$
$$= 3x^2 - 9x + 6 - 5x^2 + 5x + 10 + 8x^2 - 8$$
$$= 6x^2 - 4x + 8 = \underline{3x^2 - 2x + 4}$$

(ii) Newton $(x_i ; f_i)$

$$P_n(x_i) = f_i \,, \quad i = 0, 1, \ldots n-1$$

$$y_{21} = \frac{y_2 - y_1}{x_2 - x_1} \qquad\qquad y_{321} = \frac{y_{32} - y_{21}}{x_3 - x_1}$$

| | | | |
|---|---|---|---|
| $-1$ | $9$ | | |
| | | $y_{21} = \frac{5-9}{1+1} = -2$ | |
| $1$ | $5$ | | $y_{321} = \frac{7+2}{2+1} = 3$ |
| | | $y_{32} = \frac{12-5}{2-1} = 7$ | |
| $2$ | $12$ | | |

$$\begin{aligned}
P(x) &= y_1 + y_{21}(x - x_1) + y_{321}(x - x_1)(x - x_2) \\
&= 9 + (-2)(x+1) + 3(x+1)(x-1) \\
&= 9 - 2x - 2 + 3x^2 - 3 \\
&= \underline{3x^2 - 2x + 4}
\end{aligned}$$

Answer is D

Q7

$$P_2(x) = a_1 + a_2(x - x_{-1}) + a_3(x - x_1)(x - x_0)$$

@ $x = x_{-1}$

$$a_1 = f(x_{-1})$$

@ $x = x_0$

$$a_2 = \frac{f(x_0) - f(x_{-1})}{h_1}$$

@ $x = x_1$

$$a_3 = \frac{f(x_0) - a_1 - a_2(h_2 + h_1)}{(h_2 + h_1)(h_2)}$$

$$a_3 = \frac{f(x_0) - f(x_{-1}) - \left(\frac{f(x_0) - f(x_{-1})}{h_1}\right)(h_2 + h_1)}{(h_2 + h_1)(h_2)}$$

$$P_2'(x_0) = a_2 + a_3 h_1 \qquad \downarrow$$

$$= \frac{f(x_0) - f(x_{-1})}{h_1} + a_3 h_1$$

we know $h_1 = h_2 = h$

$$P_2'(x_0) = \frac{f(x_0) - f(x_{-1})}{h} + a_3 h$$

$$P_2'(x_0) = \frac{f(x_1) - f(x_{-1})}{h}$$

Answer is B

Q8

| $x_j$ | 1 | 2 | 3 | 5 |
|---|---|---|---|---|
| $f_j$ | 0.1 | 0.24 | 0.67 | 0.91 |

$h_1 = 2-1 = 1 \qquad h_2 = 3-2 = 1 \qquad h_3 = 5-3 = 2$

$$\underbrace{h_1 S''(x_1)} + 2(h_1+h_2)S''(x_2) + h_2 S''(x_3)$$

$$= 6\left(\frac{f_3-f_2}{h_2} - \frac{f_2-f_1}{h_1}\right) = 6\left(\frac{0.67-0.24}{1} - \frac{0.21-0.1}{1}\right) = 1.74$$

$$h_2 S''(x_2) + 2(h_2+h_3)S''(x_3) + \underbrace{h_3 S''(x_4)}$$

$$= 6\left(\frac{f_4-f_3}{h_3} - \frac{f_3-f_2}{h_2}\right) = 6\left(\frac{0.24}{2} - \frac{0.43}{1}\right) = -1.86$$

Natural cubic spline $\Rightarrow$ $S'' = 0$ at both end points

$$2(1+1)S''(x_2) + S''(x_3) = 1.74$$
$$S''(x_2) + 2(1+2)S''(x_3) = -1.86$$

$$\begin{pmatrix} 4 & 1 \\ 1 & 6 \end{pmatrix}\begin{pmatrix} S''(x_2) \\ S''(x_3) \end{pmatrix} = \begin{pmatrix} 1.74 \\ -1.86 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 1 \\ 1 & 6 \end{pmatrix}^{-1}\begin{pmatrix} 1.74 \\ -1.86 \end{pmatrix} = \begin{pmatrix} S''(x_2) \\ S''(x_3) \end{pmatrix}$$

$$\begin{pmatrix} 6/23 & -1/23 \\ -1/23 & 4/23 \end{pmatrix}\begin{pmatrix} 1.74 \\ -1.86 \end{pmatrix} = \begin{pmatrix} S''(x_2) \\ S''(x_3) \end{pmatrix}$$

$$S''(x_2) = 0.53478$$
$$S''(x_3) = -0.39913$$

$$a_i = \frac{s''(x_i + 1) - s''(x_i)}{6h_i}$$

$$a_3 = \frac{s''(x_4) - s''(x_3)}{6h_3} = \frac{0 - (-0.39913)}{6(2)}$$

$$= 0.033261$$

$$b_i = \frac{s''(x_i)}{2} \qquad b_3 = \frac{s''(x_3)}{2}$$

$$= -0.199565$$

$$c_3 = \frac{f_4 - f_3}{h_3} - \left(\frac{s''(x_4) + 2s''(x_3)}{6}\right) h_3$$

$$= \frac{0.91 - 0.67}{2} - \frac{0 + 2(-0.39913)}{6}(2)$$

$$= 0.12 - (-0.133043)(2)$$

$$= 0.386087$$

<u>Answer is B</u>

Q9 $f(x_0+h) + f(x_0-h) = 2f(x_0)$

$$+ 2\left(\frac{f''(x_0)}{2!}\right)h^2 + \left(\frac{f^4(x_0)}{4!}\right)(h^4) + \left(\frac{f^4(x_0)}{4!}\right)h^4$$

$$f''(x_0) = \frac{f(x_0-h) - 2f(x_0) + f(x_0+h)}{h^2}$$

$$- \frac{2f^4(x_0)\ h^4}{4!\ h^2}$$

$$= \frac{f(x_0-h) - 2f(x_0) + f(x_0+h)}{h^2} - \frac{1}{12} f^4(x_0)\, h^2$$

Answer is $C = -1/12\ f^{iv}(x_0)$

Q10 $\Delta x = \frac{b-a}{n} = \frac{2-0}{4} = 1/2$

$x_i = \{0, 1/2, 1, 3/2, 2\}$

$f(x_0) = f(0) = \cosh(0) = 1$
$f(1/2) = 1.128$
$f(1) = 1.543$
$f(3/2) = 2.352$
$f(2) = 3.762$

$\int_0^2 \cosh(x)\,dx \approx$

$\frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$

$= 0.5/3\,[1 + 4(1.128) + 2(1.543) + 4(2.352) + 3.762]$

$= 3.628$

Answer is B, 3.63