Comp Maths Assignment 1

Q1 It should be $[2 \ 2 \ -6] + [1 \ 0 \ 2 \ 4]$

$\therefore$ the answer is E.

Q2 I entered the commands in Matlab and the resulting matrix was B.

Q3 After running the script the answer outputted to the console was 1 3, $\therefore$ C

Q4 $f(x) = 3 - 17x^3 \qquad x = 2.5$

$f(2.5) = -262.625$

Taylor series - degree two about the point $x_0 = 2$

$$f(x) = 3 - 17x^2 \qquad f(2) = -133$$
$$f'(x) = -51x^2 \quad \Rightarrow \quad f'(2) = -204$$
$$f''(x) = -102x \qquad f''(2) = -204$$
$$a = 2 \qquad x = 2.5$$

$$P(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2$$

$$= -133 + (-204)(2.5-2) + \frac{(-204)}{2!}(2.5-2)^2$$

$$= -260.5$$

Truncation error $= -262.625 - (-260.5)$

$= -2.125$

$\therefore$ answer is E, none of the above

Q5 $f(x) = 16x^5 - 73x^2 - 133$

$x_0 = 3$

$x_1 = 2.5$

$$x_2 = x_0 - f(x_0) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

Using these functions we can fill the table:

Iteration	$f(x_1)$	x_2
1	973.25	2.270973
2	456.96685	2.068259
3	160.270987	1.958756
4	48.25973	1.9115769
5	8.642332	1.901285
6	0.6308032	1.900475

$\epsilon = x_n - x_{n-1} < 0.001$, $\therefore$ the root of the function is 1.900475.

$\therefore$ the answer is C

Q6 $f(x) = x^6 - x - 1$

$f'(x) = 6x^5 - 1$

$x_n = x_{n-1} - \left(\dfrac{f(x_{n-1})}{f'(x_{n-1})} \right)$

Iteration	x_0	x_1
1	1.5	1.30049
2	1.30049	1.181488
3	1.18148	1.139455
4	1.139455	1.134778
5	1.134778	1.134724

After the 5^{th} iteration ~~the~~ $x_n - x_{n-1} <$ 0.001

$\therefore$ the Answers is A = 1.134778

Q7 $f(x,y) = x^2 + xy - 10 = 0$ 22

$g(x,y) = 3xy^2 + y - 57 = 0$

Newton's method states the following equations:

$x_{n+1} = x_n + \Delta x$

$y_{n+1} = y_n + \Delta y$

$f'(x) = 2x + y$

$g'(x) = 3y^2$

$f'(y) = x$

$g'(y) = 6xy + 1$

$$\underset{(f,g)}{\text{Jacobian}} = \det\begin{bmatrix} f'(x) & f'(y) \\ g'(x) & g'(y) \end{bmatrix}$$

$$= \det\begin{bmatrix} (2x+y) & x \\ 3y^2 & (6xy+1) \end{bmatrix}$$

$$\Delta x = \frac{-f(x_n,y_n)\left.\frac{\partial g}{\partial y}\right|_{x_n,y_n} + g(x_n,y_n)\left.\frac{\partial f}{\partial y}\right|_{x_n,y_n}}{\text{Jacobian}(f,g)}$$

$$\Delta y = \frac{-g(x_n,y_n)\left.\frac{\partial f}{\partial x}\right|_{x_n,y_n} + f(x_n,y_n)\left.\frac{\partial g}{\partial x}\right|_{x_n,y_n}}{\text{Jacobian}(f,g)}$$

Iteration	x	y
0	1.5	3.5
~~2~~ 1	2.036	3.0527
~~3~~ 2	2.001	3.0006
3	2	3

$x = 2$, $y = 3$

$\therefore$ Answer is B

Q8

$$\left(\begin{array}{ccc|ccc} 0 & -3 & -2 & 1 & 0 & 0 \\ 1 & -4 & -2 & 0 & 1 & 0 \\ -3 & 4 & 1 & 0 & 0 & 1 \end{array}\right)$$ Swap R1 and R2

$$\left(\begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & 0 & 0 \\ -3 & 4 & 1 & 0 & 0 & 1 \end{array}\right)$$ $R2 \div -3 = R2$

$3R1 + R3 \to R3$

$$\left(\begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & 1 & 2/3 & -1/3 & 0 & 0 \\ 0 & -8 & -5 & 0 & 3 & 1 \end{array}\right)$$ $4R2 + R1 \to R1$

$8R2 + R3 \to R3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2/3 & -4/3 & 1 & 0 \\ 0 & 1 & 2/3 & -1/3 & 0 & 0 \\ 0 & 0 & 1/3 & -8/3 & 3 & 1 \end{array}\right)$$ $R3 \times 3 \to R3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2/3 & -4/3 & 1 & 0 \\ 0 & 1 & 2/3 & -1/3 & 0 & 0 \\ 0 & 0 & 1 & -8 & 9 & 3 \end{array}\right)$$ $-2/3\,R3 + R1 \to R1$

$-2/3\,R3 + R2 \to R2$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & -5 & -2 \\ 0 & 1 & 0 & 5 & -6 & -2 \\ 0 & 0 & 1 & -8 & 9 & 3 \end{array}\right)$$

∴ Answer is D

Q9 $12x_1 + 7x_2 + 3x_3 = 2$

$x_1 + 5x_2 + x_3 = -5$

$2x_1 + 7x_2 - 11x_3 = 6$

$$[A] = \begin{bmatrix} 12 & 7 & 3 \\ 1 & 5 & 1 \\ 2 & 7 & -11 \end{bmatrix} \quad [B] = \begin{bmatrix} 2 \\ -5 \\ 6 \end{bmatrix}$$

Initial guess $x_1 = 1$, $x_2 = 3$, $x_3 = 5$

1st Iteration:

$x_1 = (b_1 - (a_{12}x_2 + a_{13}x_3))/a_{11}$

$= (2 - (7x_2 + 3x_3))/12 = -2.833$

$x_2 = (b_2 - (a_{21}x_1 + a_{23}x_3))/a_{22}$

$= (-5 - (x_1 + x_3))/5 = -1.4334$

$x_3 = (b_3 - (a_{31}x_1 + a_{32}x_2))/a_{33}$

$= (6 - (2x_1 + 7x_2))/-11 = -1.9727$

2nd Iteration:

$x_1 = 1.49599$

$x_2 = -0.994658$

$x_3 = -0.84915$

3rd Iteration:

$x_1 = 0.90667$

$x_2 = -1.012$

$x_3 = -1.0246$

$\therefore$ the answer is C

$x_1 = 0.90666$

$x_2 = -1.0115$

$x_3 = -1.0243$

Q10 $[a] = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}$ $[b] = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix}$

1. Calculate 1st column of $[L]$:
   for $i = 1, 2, 3$ $\quad L_{i1} = a_{i1}$
2. Set diagonal of $[u] = 1$.
3. for $j = 2, 3$ $\quad u_{1j} = a_{1j} / L_{11}$

   $u_{12} = 2/1 = 2$

   $u_{13} = 4/1 = 4$ $\quad [u] = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & \\ 0 & 0 & 1 \end{bmatrix}$

4. $L_{22} = a_{22} - L_{21} u_{12} = 8 - 3(2) = 2$

   $[L] = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 2 & & \end{bmatrix}$

5. $u_{23} = \dfrac{(a_{23} - (L_{21} u_{13}))}{L_{22}} = \dfrac{14 - (3 \times 4)}{2} = 1$

   $[u] = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

6. $L_{32} = a_{32} - L_{31} u_{12} = 6 - 2(2) = 2$

   $L_{33} = a_{33} - (L_{31} u_{13} + L_{32} u_{23})$

   $= 13 - (2(4) + 2(1)) = 3$

   $[L] = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 2 & 2 & 3 \end{bmatrix}$

$$[L][Y] = [b]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix}$$

$Y_1 = 3$

$Y_2 = (13 - 3Y_1)/2 = 2$

$Y_3 = (4 - 2Y_2 - 2Y_1)/3 = -2$

$$[U][X] = [Y]$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$$

$x_3 = -2$

$x_2 = 2 - x_3 = 4$

$x_1 = 3 - 2x_2 - 4x_3 = 3$

$$[x] = \begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix}$$

$\therefore$ The answer is <u>B</u>