TRINITY COLLEGE DUBLIN

School of Computer Science and Statistics

Week 2 Questions

ST3009: Statistical Methods for Computer Science

For each problem, explain/justify how you obtained your answer in order to obtain full credit. In fact, most of the credit for each problem will be given for the derivation/model used as opposed to the final answer.

Question 1. A 6-sided die is rolled three times.

- (a) How many elements are there in the sample space?
- (b) Out of the possible sets of outcomes, calculate in how many at least one 2 is rolled. Using this, calculate what the probability is that at least one 2 is rolled.
- (c) Write a small matlab simulation of this experiment and confirm that the observed probability that at least one 2 is rolled matches your calculation in (a).
 - (d) What is the probability that the sum of the die rolls is 17?
- (e) What is the probability that the sum of the three die rolls is 12 given that the first roll was a 1?

Question 2. I roll a 6-sided die. If it comes up a 1 then I throw a six-sided die and otherwise a 20-sided die.

- (a) What is the probability that the second throw comes up a 5?
- (b) What is the probability that the second throw comes up a 15? Hint: use marginalisation

Question 3. At a certain stage of a criminal investigation, the inspector in charge is 60 percent convinced of the guilt of a certain suspect. Suppose, however, that a new piece of evidence which shows that the criminal has a certain characteristic (such as left-handedness, baldness, or brown hair) is uncovered. If 20 percent of the population possesses this characteristic, use Bayes Rule to calculate how certain of the guilt of the suspect should the inspector now be if it turns out that the suspect has the characteristic.

Question 4. Your cell phone is constantly trying to keep track of where you are. At any given point in time, for all nearby locations, your phone stores a probability that you are in that location. Right now your phone believes that you are in one of 16 different locations arranged in a grid with the following probabilities (see the figure on the left):

Prior Belief of Location

0.05 0.10 0.05 0.05 0.05 0.10 0.05 0.05 0.05 0.05 0.10 0.05 0.10 0.05 0.05 0.05

P(Observe two bars of signal | Location)

| 0.75 | 0.95 | 0.75 | 0.05 |
|------|------|------|------|
| 0.05 | 0.75 | 0.95 | 0.75 |
| 0.01 | 0.05 | 0.75 | 0.95 |
| 0.01 | 0.01 | 0.05 | 0.75 |
| | | | |

Your phone connects to a known cell tower and records two bars of signal. For each grid location L you know the probability of observing two bars from this particular tower, given that the cell phone is in location L (see the figure on the right). Example: the highlighted cell on the left figure means that you believed there was a 0.05 probability that the user was in the bottom right grid cell prior to observing the cell tower signal. The highlighted

cell on the right figure means that you think the probability of observing two bars, given the user was in the bottom right grid cell, is 0.75.

For each of the 16 location positions, calculate the new probability that the user is in each location given the cell tower observation. Write a program to calculate the probabilities. Report the probabilities of all 16 cells and write a short explanation of your program.