Statistics Assignment 3 - John Sinclair - 16325734

Q1.

(a) $(1 / 6)^6 = 0.0000214$

As the events are independent the chance of any event occurring on a dice roll is one in six, so any 6 events occurring is $(1 / 6)^6$

(b) $((6C4) \times 5 \times 5) / 6^6 = 0.008$

6 rolls, we are choosing the outcome of 4, then the remaining two rolls cannot land on 3 (hence 5 not 6) so we multiply by 5 twice. This gets us the number of permutations where we have exactly four 3s, now we divide by the total number of possible permutations, 6 sides rolled 6 times, so 6 x 6 x 6 x 6 x 6 x 6 or 6^6 to get the probability

(c) $((6C1) \times 5^5) / 6^6 = 0.4019$

Same logic as above, choosing the outcome of one roll, 5 options for the remaining five rolls, divided by the total number of permutations.

(d) $((6C1 \times 5^5) + (6C2 \times 5^4) + (6C3 \times 5^3) + (6C4 \times 5^2) + (6C5 \times 5) + (6C6)) / 6^6 = 0.6651$

To calculate the probability we roll one or more 1s we have to account for all the possible ways this can happen.

(probability we roll just one 1) + (prob we roll two 1s) + (prob we roll three 1s) + (prob we roll four 1s) + (prob we roll five 1s) + (prob we roll six 1s). This gets us the number of permutations we are looking for, then we divide by the total number of permutations.

Q2.

A = the event that the first die comes up 1

B = the event that the sum of the two die is 2 - two possible outcomes $\{(1^1, 1^2),(1^2, 1^1)\}$

It is said two events are independent if the order in which they occur doesn't matter.

If two events are independent then P(AnB) = P(A)P(B).

In our case:

P(A) = 1 / 6 and P(B) = $2 / 6^2$ = 1 / 18 so P(A)P(B) = 1 / 108

P(AnB) = P(A|B)P(B), to calculate P(AnB) we must calculate P(A|B), if we are given B has occurred, we know the die had to result in a 1, therefore P(A|B) = 1, so P(AnB) = P(B)

Since P(AnB) != P(A)P(B) we know these events are dependent.

Q3.

(a) 1 / (n - (k - 1))

The formula can be split into two sections, the fail cases and the success case. The fail cases are represented by (n - k) / n - (k - 1), this is because the probability you fail on the first choice is (n - 1) / n, on the second it is (n - 2) / (n - 1), from this pattern we can derive (n - k) / (n - (k - 1)). To account for the success case it is 1 / n - k. When simplified we get 1 / (n - (k - 1)).

(b) (5 / 6) x (4 / 5) x (1 / 4) = 0.16

In order for her to get the correct password on the third try she must have gotten the first two guesses wrong. The probability of getting the incorrect password the first time is 5 / 6, the incorrect choice was then removed leaving five. The probability of getting the second choice wrong is 4 / 5 as there are only 5 passwords remaining, this choice is removed, 4 passwords remaining. The third attempt is successful, with only one correct password and 4 options the probability of this happening in 1 / 4.

(c) $(1 / n) * ((n - 1) / n)^{k-1}$

My formula can be broken into two sections, the first accounts for the correct attempt and the second accounts for the incorrect attempts. To get the probability of the correct attempt it is 1 / n, this is because there are n passwords to choose from. For the incorrect attempts it is $(n - 1) / (n)^{k-1}$, broken down, the n - 1 is because we can choose from any of the passwords bar the one correct one, we then divide by the number of passwords. Since the hacker gets the correct password on the kth attempt, they made k - 1 incorrect attempts hence why we multiply the probability by itself k - 1 times.

(d) (5 / 6) x (5 / 6) x (1 / 6) = 0.1157

Same logic as part (b) but this time the hacker is not removing incorrect choices from the password options

Q4.

(a) $(0.7) + (0.7 \times 0.3) + (0.7 \times 0.3^2) = 0.973$

With three tests there are three points at which a robot can be flagged, to find the probability we must find the probability the robot gets flagged at each point and add them together. The first case is where the robot gets caught on the first test so the probability is 1 - 0.3. The second case the robot gets caught on the second test, therefore must've passed the first test so the probability is 0.7 x 0.3. The third scenario is where the robot gets caught at the third test, meaning it passed the first two so 0.3 x 0.3 x 0.7

(b) $(0.05) + (0.05 \times 0.95) + (0.05 \times 0.95^2) = 0.142625$

Same logic as part (a), we take into account the probability the human gets flagged at each of the three tests and sum their probabilities

(c) 0.973 / (0.973 + 9(0.142625)) = 0.43117

Using bayes rule:

Rule = P(E|F) = P(F|E)P(E) / P(F)

P(E) = prob they are a robot = 0.1

P(F) = prob they get flagged = ? - we will calculate

P(F|E) = prob they get flagged given they are a robot = 0.973

$P(F|E^C)$ = prob they get flagged given they are not a robot (human) = 0.142625

The probability they get flagged depends on whether they are a human or a robot and so we must take both cases into account.

P(F) = (0.9)(0.142625) + (0.1)(0.973) = 0.2256625

Using the formula from bayes rule we can determine the probability a flagged user is a robot:

$P(E|F) = (0.973 \times 0.1) / 0.2256625 = 0.43117$