Q1.

(a) 10! = 3628800

This is because the substitution cypher will have 10 options first pick, then 9 options second pick and so on... so 10 x 9 x 8 ... 1 which is 10 factorial

(b) 9! x 2 = 725760

When the letter E and F must be beside each other I treated them as if they were one letter, so 9 factorial, but as E could either come before or after F I multiplied the result by 2.

(c) 6! / 3! / 2! = 60

Similar reasoning as part (a), there are 6 letters so 6 factorial, but as there are 3 A's and 2 N's, which are not unique we divided the total number by their factorial.

(d) $5C3 = (5 \times 4 \times 3) / 3! = 10$

5 letters total and we draw (choose) 3

Q2.

(a) $6 \times 6 \times 6 \times 6 = 1296$

Theres are 6 outcomes per roll, and as each roll has no effect on the others that number of possible outcomes is 6⁴

(b) $4C2 \times 5 \times 5 = 150$

There are 4 rolls of which we choose the outcome of 2, this must then be multiplied by 5 (6 sided dice minus the chance it lands on 3) twice for the remaining 2 rolls to give us the amount of possible permutations

(c) $(4C2 \times 5 \times 5) + (4C3 \times 5) + (4C4) = 171$

Same reasoning as (b) but since this is at least two 3's we must add on the permutations where we get exactly 3 3's and exactly 4 3's, hence the addition above.

Q3.

(a) $8! / 2!^4 = 8! / 2 / 2 / 2 / 2 = 2520$

There are 8 cards so the total number of ways is 8 factorial, but as the duplicate cards are indistinguishable from each other we must remove these permutations, so we divide the total number of ways by each of the card pairs.

(b) 4C2 = 6

As the order of the cards does not matter it is 4 (the number of distinct cards) choose 2

(c) $4C2/2! = (4 \times 3)/(2! + 2!) = 3$

There are 4 good cards and we get two, so its 4C2, but as the suit pairs are indistinguishable we remove those ways by divided the total number by 2 factorial (the number of ways two cards can be ordered)