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Comp Maths Assignment 2

Q1 The result of these commands is

C : 2.43

Q2 After this program executes I found

$$A = \begin{pmatrix} 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & -1 & 3 & 1 \\ -1 & -1 & 1 & 3 \end{pmatrix}$$

So the answer is A

Q3 C : You simultaneously plot each of those data series using hold on.

Q4 $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$ $\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{pmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{pmatrix}$$

$$\det(A-\lambda I) = (1-\lambda)(-20 + 5\lambda - 4\lambda + \lambda^2 + 18) + 3(12 - 3\lambda - 18) + 3(-18 + 30 + 6\lambda)$$

$$\det(A-\lambda I) = (1-\lambda)(-2 + \lambda + \lambda^2) + 3(-6 - 3\lambda) + 3(12 + 6\lambda)$$

$$\det(A-\lambda I) = \lambda^3 - 12\lambda - 16 = 0$$

$\det(A-\lambda I) = 0$ when $\lambda = 4$

$$\lambda - 4 \;\overline{)\; \lambda^3 - 0\lambda^2 - 12\lambda - 16}$$

$$\lambda^2 + 4\lambda + 4$$

$$-\lambda^3 + 4\lambda^2$$

$$4\lambda^2 - 12\lambda$$

$$-4\lambda^2 + 16\lambda$$

$$4\lambda - 16$$

$$-4\lambda + 16$$

$$0$$

$$(\lambda - 4)(\lambda^2 + 4\lambda + 4)$$

$a = 1 \quad b = 4 \quad c = 4$

$$\lambda = \frac{-4 \pm \sqrt{4^2 - 4(1)(4)}}{2(1)}$$

$= -2$ and -2

So C is the answer, $\lambda = 4, -2$

Q5 $A = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 4 & -3 \\ 0 & -1 & 1 \end{pmatrix}$ $x^{(0)} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$x_1 = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 4 & -3 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} = -1 \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

$$x_2 = A \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -7 \\ 8 \\ -1 \end{pmatrix} = 8 \begin{pmatrix} -7/8 \\ 1 \\ -1/8 \end{pmatrix}$$

$$x_3 = A \begin{pmatrix} -7/8 \\ 1 \\ -1/8 \end{pmatrix} = \begin{pmatrix} -29/8 \\ 49/8 \\ -9/8 \end{pmatrix} = 49/8 \begin{pmatrix} -29/49 \\ 1 \\ -9/49 \end{pmatrix}$$

$$x_4 = A \begin{pmatrix} -29/49 \\ 1 \\ -9/49 \end{pmatrix} = \begin{pmatrix} -136/49 \\ 281/49 \\ -58/49 \end{pmatrix} = 281/49 \begin{pmatrix} -136/281 \\ 1 \\ -58/281 \end{pmatrix}$$

$$x_5 = A \begin{pmatrix} -136/281 \\ 1 \\ -58/281 \end{pmatrix} = \begin{pmatrix} -689/281 \\ 1570/281 \\ -339/281 \end{pmatrix} = 5.59 \begin{pmatrix} -0.44 \\ 1 \\ -0.22 \end{pmatrix}$$

This answer matches none of the options so the answer is E

Q6 – B

Q6	x Weight	y Length	x^2	xy
1	30	70	900	2100
2	40	90	1600	3600
3	40	100	1600	4000
4	50	120	2500	6000
5	50	130	2500	6500
6	50	150	2500	7500
7	60	160	3600	9600
8	70	190	4900	13300
9	70	200	4900	14000
10	80	200	6400	16000
11	80	220	6400	17600
12	80	230	6400	18400
Σ	700	1860	44200	118600

N = 12

$$m = \frac{N\Sigma(xy) - \Sigma x \Sigma y}{N\Sigma(x^2) - (\Sigma x)^2}$$

$$= \frac{12(118600) - (700)(1860)}{12(44200) - (700)(700)}$$

$$= 3$$

Get Y-intercept:

$$b = \frac{\Sigma y - m\Sigma x}{N} = \frac{1860 - 3(700)}{12}$$

$$= -20$$

$$y = 3x - 20$$

(i) $x = 35$, $y = 3(35) - 20 = 85m$

(ii) $x = 85$, $y = 3(85) - 20 = 235m$

(iii) $x = 100$, $y = 3(100) - 20 = 280m$

So the answer is B.

Q7 $\rho = k_1 e^{-k_2 h}$

we know $k_2 = 0.1315$

@ sea level

$h = 0$

so $\rho = k_1$

@ top

$\rho = k_1 e^{-k_2 h}$

but we know $\rho = 1/1000$ when @ sea level

so

$$\frac{k_1}{1000} = k_1 e^{-k_2 h}$$

$$\frac{1}{1000} = e^{-k_2 h}$$

$$-k_2 h = \ln(1/1000)$$

$$h = \frac{\ln(1/1000)}{-k_2}$$

but we know $k_2 = 0.1315$

$h = 52.53$ km

∴ answer is D

Q8 Take 3 closest points:
(15, 24) (18, 37) (22, 25)

$y = ax^2 + bx + c$

① $24 = a(15)^2 + b(15) + c$

② $37 = a(18)^2 + b(18) + c$

③ $25 = a(22)^2 + b(22) + c$

from ①:

$c = 24 - 225a - 15b$

sub ① into ②:

$18b = 37 - 324a - c$

$18b = 37 - 324a - (24 - 225a - 15b)$

$18b = 37 - 324a - 24 + 225a + 15b$

$3b = -99a + 13$

$b = -33a + 13/3$

sub ② into ③:

$484a = 25 - 22b - c$

$484a = 25 - 22(-33a + 13/3) - (24 - 225a - 15b)$

$484a = 25 - 22(-33a + 13/3) - (24 - 225a - 15(-33a + 13/3))$

$484a = 25 + 726a - 286/3 - (24 - 225a + 495a - 65)$

$28a = -88/3$

$a = -1.0476$

$b = -33a + 13/3 = 38.9041$

$c = 24 - 225a - 15b = -323.8515$

$v = a(16)^2 + b(16) + c$

$= 30.4285$

$= 30.43 \text{ ms}^{-1}$

∴ The answer is C

Q9 $(0, 0)$ $(0.5, 19.32)$ $(1, 90.62)$
$(1.5, 175.71)$ $(2, 407.11)$

find N when $V = 2.5$
↳ y ↳ x

Lagrange interpolation formula:

$$P(x) = \sum_{i=1}^{n} P_i(x)\, y_i$$

$$= \sum_{i=1}^{n} \left(y_i \cdot \prod_{j \neq i}^{n} \frac{(x - x_j)}{(x_i - x_j)} \right)$$

$$P(x) = 113.807x^4 - 392.34x^3 + 493.308x^2 - 124.155x$$

$P(V) = N$, when $V = 2.5$

$N = 1088.061$

$= 1088\ N$

So, the answer is $\underline{\underline{C}}$

Q10. $f(x) = \sqrt{x}$;

$x_0 = 2 \quad x_1 = 3 \quad x_2 = 7$

find poly @ $x = 2.5$

@ $x = 2$, $f(x) = \sqrt{2}$

@ $x = 3$, $f(x) = \sqrt{3}$

@ $x = 7$, $f(x) = \sqrt{7}$

$$\frac{\sqrt{3} - \sqrt{2}}{3 - 2} = 0.3178$$

$$\frac{\sqrt{7} - \sqrt{3}}{7 - 3} = 0.2284$$

$$\frac{0.2284 - 0.3178}{7 - 2} = -0.01788$$

So the answer is D