Statistics Assignment 2 - John Sinclair - 16325734

Q1.

(a) 6 x 6 x 6 = 216

A single dice being rolled three times is the same as rolling three dice at once, so with each dice there are 6 permutations, so with two dice the total number of elements in the sample space is 6 x 6 = 36, roll the third and we multiply by 6 again = 216 elements in the sample space.

(b) (3C1 * 5 * 5) + (3C2 * 5) + 3C3 = 91 => 91 / 216 = 0.421

To get the number of permutations we must add the probability of getting one 2 plus two 2’s plus three 2’s.

To get the probability of getting one 2: 3C1 * 5 * 5 = 75 -this is because choose gives us the number of ways the 2 can be positioned in the sequence, and the other two dice can be any number excluding 2 (5 options each)

Using the same logic we get:

The probability of getting two 2s: 3C2 * 5 = 15

The probability of getting three 2s: 3C3 = 1

Then divide the number of permutations by the total number of elements in the sample space, hence 91 / 216.

(c) Code

```
rolls = 10000000;
count = 0;

for i = 1:rolls
    first = randi(6);
    second = randi(6);
    third = randi(6);
    if (first == 2 || second == 2 || third == 2)
        count = count + 1;
    end
end

prob = count/rolls
```

Output

```
>> statsWeek2

prob =

    0.4212
```

Explanation

I first initialize two variables, the rolls variable determines how many iterations of the for loop below are performed, and the count variable acts as a running tally of how many rolls have at least one 2.
Within the for loop I initialise three variables first, second and third, each using the randi() function, by passing 6 to this function it returns a random integer between 1 and 6. I then check if any of the three variables are a 2 and if so I increment the count.
Once the loop is finished I divide the count by the total number of rolls to get the probability a roll had at least one 2. As you can see the output matches my answer to part (b).

(d) {6, 6, 5} x 3 = 3 / 216 = 0.0138
There is only one sum of three dice that sums to 17: two 6s and a 5. As there are three positions each digit can be in, our one set of digits is multiplied by 3, giving us 3 elements in the sample space. We then divide this by the total number of elements in the sample space to get the probability.

(e) 1 X X => 6 x 6 = 36 => {1, 5, 6} or {1, 6, 5} => 2 / 36 = 0.055
Given we know the outcome of the first dice roll, that leaves two dice uncertain, with 6 options each we get 6 * 6 = 36 total permutations. In order for the total sum to be 12 the two dice with unknown outcomes must add to 11, there are only two ways this can occur, with a 5 and a 6 or vice versa. This gives us 2 permutations out of a possible 36, 2 / 36 = 0.055 .

Q2.

(a) (5 / 6 x 1 / 20) + (1 / 6 x 1 / 6) = 0.0694
As the second dice outcome is 5 we cannot tell what the outcome of the first throw was so we must account for both eventualities.
The first case is when the first throw results in a 1, a 1 in 6 chance, in this case the second throw is with another six sided dice, so for it to result in a 5 has a 1 in 6 chance. In total for this case we have 1 / 6 * 1 / 6.
The second case, where the first throw does not result in a 1, means we roll a 20 sided dice for the second throw. Therefore the probability of it resulting in a 5 is 5 / 6 * 1 / 20.
To account for both eventualities we sum the probability of both outcomes.

(b) (5 / 6) x (1 / 20) = 0.0416
As the outcome of the second throw is greater than 6 we can eliminate the scenario where the first throw is a 1. Meaning the probability of rolling a 15 is 5 / 6 * 1 / 20.

Q3.
From Bayes Rule: $P(E|F) = P(F|E)P(E) / P(F)$
Where:

$P(E)$ = the prior, the probability a person is guilty
$P(F)$ = the probability a person has a characteristic
$P(F|E)$ = the likelihood, the probability a person has a curtain characteristic given that they are guilty
$P(E|F)$ = the posterior, the probability a person is guilty given that they have a certain characteristic - this is what we are looking for.

From the question we know:

$P(E) = 60\% = 0.6$

The new piece of evidence tells us the guilty person has a certain characteristic and so

$P(F|E) = 1$

Since we know 20% of the population has a certain characteristic, we know the probability of a person having the characteristic, given they are not guilty is 20%

$P(F|E^C) = 0.2$

We now have the three required variables to determine P(F) using the formula:

$P(F) = P(F|E)P(E) + P(F|E^C)(P(F|E)-P(E))$
$P(F) = 1(0.6) + 0.2(1 - 0.6) = 0.68$

Then using Bayes Rule:

$P(E|F) = P(F|E)P(E) / P(F)$
$P(E|F) = (1 * 0.6) / 0.68 = 0.882$

Q4. Code

```
PL = [0.05 0.1 0.05 0.05; 0.05 0.1 0.05 0.05; 0.05 0.05 0.1
0.05; 0.05 0.05 0.1 0.05];
PBL = [0.75 0.95 0.75 0.05; 0.05 0.75 0.95 0.75; 0.01 0.05 0.75
0.95; 0.01 0.01 0.05 0.75];

PB = [1 1 1 1; 1 1 1 1; 1 1 1 1; 1 1 1 1];

PLB = (PL.*PBL)./PB
```

Output

```
>> statsWeek2

PLB =

    0.0375    0.0950    0.0375    0.0025
    0.0025    0.0750    0.0475    0.0375
    0.0005    0.0025    0.0750    0.0475
    0.0005    0.0005    0.0050    0.0375
```

Explanation
PL = matrix of probability of being at a certain location.
PBL = matrix of probability of having 2 bars of service given a certain location.
Since we are given the cell tower observation, we know the probability of having 2 bars at any location is 1, so
PB = matrix of probability of having 2 bars at a location.
Using Bayes Rule we can calculate PLB = (PL * PBL) / PB.