Week 8

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Qual Study

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Solution 2018.12. a. The perturbation is

$$H_1 = cx$$

so the first order correction to the energy is

$$E_n^1 = \langle n \,|\, H_1 \,|\, n \rangle = 0$$

since H_1 is odd and $|\psi_n(x)|^2$ is even. The second order correction to the energy is

$$\begin{aligned}
E_n^2 &= \sum_{m \neq n} \frac{|\langle m \,|\, H_1 \,|\, n \rangle|^2}{E_n - E_m} \\
\langle m \,|\, H_1 \,|\, n \rangle &= c \langle m \,|\, x \,|\, n \rangle \\
&= c \left\langle m \,\middle|\, \sqrt{\frac{\hbar}{2m\omega}} (a^\dagger + a) \,\middle|\, n \right\rangle \\
&= c \sqrt{\frac{\hbar}{2m\omega}} \left[\sqrt{n+1} \langle m \,|\, n+1 \rangle + \sqrt{n} \langle m \,|\, n-1 \rangle \right] \\
E_n^2 &= c^2 \frac{\hbar}{2m\omega} \left[\frac{n+1}{E_n - E_{n+1}} + \frac{n}{E_n - E_{n-1}} \right] \\
&= -\frac{c^2}{2m\omega^2}
\end{aligned}$$

b. The exact eigenvalues are found by completing the square:

$$V(x) = \frac{1}{2} k x^2 + cx + \frac{c^2}{2k} - \frac{c^2}{2k}$$

$$= \left(\sqrt{\frac{k}{2}}x + \frac{c}{\sqrt{2k}}\right)^2 - \frac{c^2}{2k}$$
$$= \frac{k}{2}\left(x + \frac{c}{k}\right)^2 - \frac{c^2}{2k}$$

Therefore, the perturbed eigenstates are $\psi_n(x) = \psi_n^{(0)}(x + c/k)$ and the perturbed eigenvalues are $E_n = E_n^{(0)} - \frac{c^2}{2k}$. This agrees exactly with what was found in (a).

c. As $k \to 0$, the shift in energies from the perturbation dominates.

Solution 2018.14. a. The Hamiltonian can be written using

$$J^2 = (\mathbf{J}_a + \mathbf{J}_b) \cdot (\mathbf{J}_a + \mathbf{J}_b)$$
$$= J_a^2 + J_b^2 + 2\mathbf{J}_a \cdot \mathbf{J}_b$$
$$H = \frac{A}{2}(J^2 - J_a^2 - J_b^2)$$
$$= \frac{A}{2}\left(J^2 - \frac{3}{4}\hbar^2 - \frac{15}{4}\hbar^2\right)$$

Thus, we see that the eigenstates of H are the total spin eigenstates. A spin 1/2 and a spin 3/2 can add to create total spin 2 or 1. Thus, the energy eigenvalues are $\frac{A\hbar^2}{2}(6 - 9/2)$ and $\frac{A\hbar^2}{2}(2 - 9/2)$.

b. Their degeneracies are 5 and 3 respectively.

Solution 2017.11.

Solution 2017.13.

Solution 2016.12.

Solution 2016.14.

Solution 2015.11.

Solution 2015.13.

Solution 2014.12.

Solution 2014.14.

Solution 2013.11.

Solution 2013.13.