Notes on HMS_optics

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Abstract

This document covers the technical details behind the procedure for optimization of the reconstruction matrix in the HMS_optics.

1 Introduction

The original code for the HMS_optics was written by Brash [1]. The rewritten code is leaner but follows same underlying principles and it can be obtained here [2].

The program can be thought of as being composed of roughly four parts: reconstructing events by using an unoptimized reconstruction matrix, determining the "true" initial values, setting up an SVD problem, and calculating the "new" reconstruction matrix. The following section will go through each part and detail the key features.

Currently, the HMS_optics is only used for optimization of the Y, Y', and X' reconstruction coefficients. The optimization of D elements still needs to be added.

2 Coordinate systems

Two coordinate systems are used to describe an event, the target or vertex coordinate system, and the spectrometer or transport coordinate system (see Figure 1). In the vertex system, the z-axis is oriented along the beamline, the x-axis is pointing beam-left, and the y-axis is towards the ceiling of the hall. In the spectrometer system, the z-axis is oriented along the central ray going into the spectrometer, the x-axis is marking the dispersive direction with $+x$ pointing towards hall's floors, and the y-axis is oriented along the nondispersive direction with $+y$ being to beam-left.

Note that the Hall C BPMs have $+x$ beam right and $+y$ towards the ceiling.

To ease the description, three fixed coordinate systems are defined: target coordinate system, which is fixed on the target center, sieve slit system, which is fixed on the sieve slit, and the focal plane coordinate system, which is fixed on the focal plane of the spectrometer. These three systems follow the global spectrometer coordinate system, just having a different fixed origin.

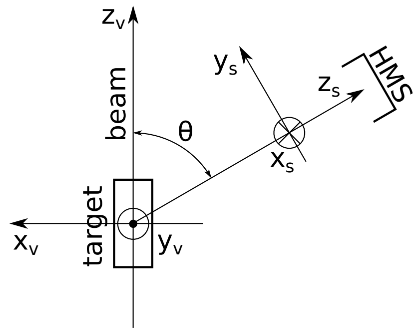

Figure 1: Vertex coordinate system is fixed on the target, while the spectrometer coordinate system is following the spectrometer's central ray.

3 Event reconstruction

The event reconstruction is a process of calculating event coordinates in the target system from coordinates measured in the focal plane system and fast raster information for focus corrections. The main part of the reconstruction is done by a set of polynomial functions, one for each reconstructed variables:

$$x'_{\rm tar} = \sum_{i,j,k,l,m} X'_{i,j,k,l,m} \cdot x^i_{\rm fp}\, x'^j_{\rm fp}\, y^k_{\rm fp}\, y'^l_{\rm fp}\, x^m_{\rm tar} \tag{1a}$$

$$y^{\rm rec}_{\rm tar} = \sum_{i,j,k,l,m} Y_{i,j,k,l,m} \cdot x^i_{\rm fp}\, x'^j_{\rm fp}\, y^k_{\rm fp}\, y'^l_{\rm fp}\, x^m_{\rm tar} \tag{1b}$$

$$y'_{\rm tar} = \sum_{i,j,k,l,m} Y'_{i,j,k,l,m} \cdot x^i_{\rm fp}\, x'^j_{\rm fp}\, y^k_{\rm fp}\, y'^l_{\rm fp}\, x^m_{\rm tar} \tag{1c}$$

$$\delta_{\rm tar} = \sum_{i,j,k,l,m} D_{i,j,k,l,m} \cdot x^i_{\rm fp}\, x'^j_{\rm fp}\, y^k_{\rm fp}\, y'^l_{\rm fp}\, x^m_{\rm tar} \tag{1d}$$

Note that the $y^{\rm rec}_{\rm tar}$ here is not the y coordinate of the vertex in the target spectrometer coordinate system (see Figure 2). Also, the x' and y' are not angles, but derivatives or tangents of the angles:

$$x' = \frac{{\rm d}x}{{\rm d}z} \tag{2a}$$

$$y' = \frac{{\rm d}y}{{\rm d}z} \tag{2b}$$

However, these quantities are usually small, so they are approximately equal to their respective angles in radians.

The coefficients X', Y, Y', D and the exponents i, j, k, l, m are stored on disk in the so called reconstruction matrices with the following structure:

$$\begin{array}{lllll} \vdots \\ X'_{i,j,k,l,m} & Y_{i,j,k,l,m} & Y'_{i,j,k,l,m} & D_{i,j,k,l,m} & ijklmn \\ \vdots \end{array} \tag{3}$$

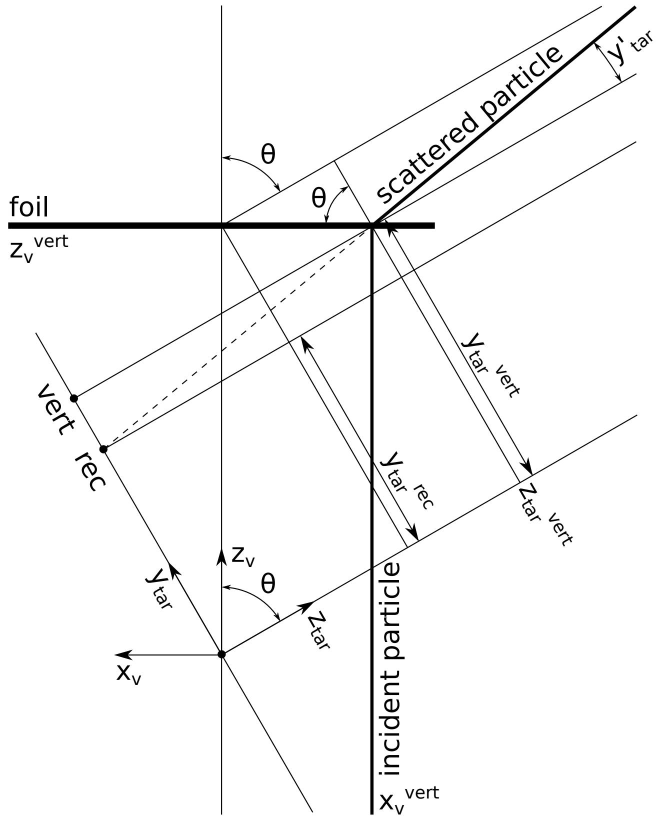

Figure 2: Detailed drawing of event coordinates. The subscript "v" denotes the vertex coordinate system while "tar" stands for target. The "vert" point marks the vertex projection of the interaction vertex onto the target coordinate system. On the other hand, the "rec" point is the reconstructed point as given by Equation 1. The vertex projection must be calculated from the reconstructed point.

The event reconstruction is done using Equation 1 with an iterative approach, where the value of x_{tar} is updated in each iteration.

For the first iteration, the x_{tar} is taken as:

$$\begin{aligned} x_{\text{tar}}^{\text{vertex}} &= -y_{\text{v}}^{\text{vertex}} - x^{\text{off}} \\ &= -(y_0^{\text{beam}} - y_{\text{fr}}) - x^{\text{off}} \end{aligned} \tag{4}$$

where subscript "fr" denotes the fast raster information and superscript "off" stands for offset or mispointing. Using this value and the focal plane coordinates, a reconstruction of target variables is made.

The reconstructed quantities are then corrected for the offsets and mispointings:

$$y_{\text{tar}}^{\text{rec}} = \tilde{y}_{\text{tar}}^{\text{rec}} + y^{\text{off}} \tag{5a}$$

$$x'_{\text{tar}} = \tilde{x}'_{\text{tar}} + \phi^{\text{off}} \tag{5b}$$

$$y'_{\text{tar}} = \tilde{y}'_{\text{tar}} + \theta^{\text{off}} \tag{5c}$$

where the tilde variables come from Equation 1. All the other variables can be calculated from here:

$$y_{\text{tar}}^{\text{rec}} = y_{\text{tar}}^{\text{vert}} - z_{\text{tar}}^{\text{vert}}\, y'_{\text{tar}} \tag{6a}$$

$$x_{\text{tar}}^{\text{vert}} = -y^{\text{beam}} = -(y_0^{\text{beam}} - y_{\text{fr}}) \tag{6b}$$
$$y_{\text{tar}}^{\text{vert}} = z_{\text{v}}^{\text{vert}} \sin(\theta) + x_{\text{x}}^{\text{vert}} \cos(\theta) \tag{6c}$$
$$z_{\text{tar}}^{\text{vert}} = z_{\text{v}}^{\text{vert}} \cos(\theta) - x_{\text{x}}^{\text{vert}} \sin(\theta) \tag{6d}$$

$$x_{\text{v}}^{\text{vert}} = -x^{\text{beam}} = -(x_0^{\text{beam}} - x_{\text{fp}}) \tag{6e}$$
$$y_{\text{v}}^{\text{vert}} = y^{\text{beam}} = y_0^{\text{beam}} - y_{\text{fp}} \tag{6f}$$
$$\Rightarrow z_{\text{v}}^{\text{vert}} = \frac{y_{\text{tar}}^{\text{rec}} + x^{\text{beam}}(\cos(\theta) + y'_{\text{tar}} \sin(\theta))}{\sin(\theta) - y'_{\text{tar}} \cos(\theta)} \tag{6g}$$

$$x_{\text{tar}}^{\text{rec}} = x_{\text{tar}}^{\text{vert}} - z_{\text{tar}}^{\text{vert}}\, x'_{\text{tar}} \quad (-x^{\text{off}}) \tag{6h}$$

As can be seen from Equation 6h, the first iteration produces a new estimate for the true value of x_{tar}, i.e., $x_{\text{tar}}^{\text{rec}}$. Iterating this procedure should yield an increasingly convergent estimates. At least one iteration after the initial one is needed to achieve good accuracy.

4 Measurements

To optimize the reconstruction matrix elements, a specific set of measurements needs to be taken. For angular positional reconstruction terms, sieve slit measurements with extended optics target are needed. The extended optics target is made of carbon foils positioned at specific z-coordinates which define the interaction point along the beam direction. The holes in the sieve slit define beams of electrons going at specific angles from the interaction point.

The `HMS_optics` does not currently try to optimize the δ reconstruction coefficients. Nonetheless, to also optimize these terms, a momentum scan data would be needed. That is measurements of elastic scattered electrons at different momentum settings of the spectrometer.

5 True initial values

In order to optimize the reconstruction matrix, the reconstructed event variables need to be compared to the "true" initial values. In practice, this means determining from which target foil the particle has scattered and through which sieve slit hole it went. Two passes over the reconstructed events are needed to achieve this.

The first pass over the data is used to determine from which target foil each of the scattered particles originated. This is done by filling the $z_{\text{v}}^{\text{vert}}$ and $y_{\text{tar}}^{\text{vert}}$ into their respective histograms and fitting the peaks (see Figure 3). The mean of the $z_{\text{v}}^{\text{vert}}$ for each peak is the true value for that foil.

In the second pass, the fitted peaks from previous step are used to assign events to their respective foils. A cut on the events is made to select only the ones within $2 - 3\,\sigma$ of the foil. For each foil,

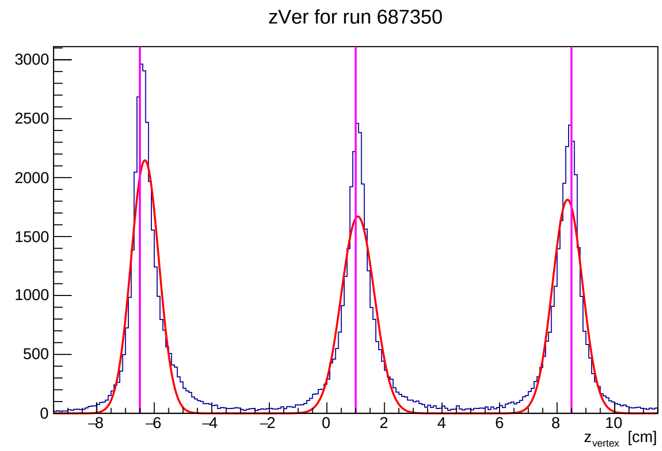

Figure 3: Fit of the target foils with $z_{\rm v}^{\rm vert}$ histogram. The peaks of the red line are the "true" initial values and the purple lines represent the real position of the foils.

the particles are transported to the sieve slit and histogrammed in $x_{\rm sieve}$ vs. $y_{\rm sieve}$ (see Figure 4). The histogram is then fitted for the position of each sieve hole. The means again represent the true values.

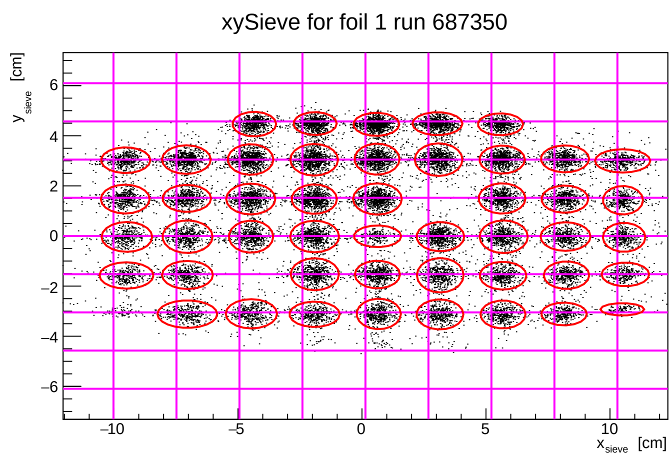

Figure 4: Fit of the sieve slit holes with $x_{\rm sieve}$ vs. $y_{\rm sieve}$ histogram. The centers of the red ellipses present the "true" initial values while the purple grid shows the real centers of the holes.

When setting the SVD problem in the next pass, all events are again sorted by their respective target foil and sieve slit hole using the above fits and cutting on $2-3\,\sigma$ around the mean.

In order to do the matrix optimization, the fitted foil positions and sieve holes need to be matched with the physical foils and holes. These are then used to calculate the so called "real" quantities, which are used to do the optimization. "Real" quantitites are denoted with a hat.

$$\hat{x}_{\text{tar}}^{\text{vert}} = -y^{\text{beam}} \tag{7a}$$
$$\hat{y}_{\text{tar}}^{\text{vert}} = z^{\text{foil}} \sin(\theta) + x^{\text{beam}} \cos(\theta) \tag{7b}$$
$$\hat{z}_{\text{tar}}^{\text{vert}} = z^{\text{foil}} \cos(\theta) + x^{\text{beam}} \sin(\theta) \tag{7c}$$

$$\hat{x'}_{\text{tar}} = \frac{x^{\text{hole}} - \hat{x}_{\text{tar}}^{\text{vert}}}{z^{\text{sieve}} - \hat{z}_{\text{tar}}^{\text{vert}}} \tag{7d}$$
$$\hat{y'}_{\text{tar}} = \frac{y^{\text{hole}} - \hat{y}_{\text{tar}}^{\text{vert}}}{z^{\text{sieve}} - \hat{z}_{\text{tar}}^{\text{vert}}} \tag{7e}$$

$$\hat{x}_{\text{tar}}^{\text{rec}} = \hat{x}_{\text{tar}}^{\text{vert}} - \hat{z}_{\text{tar}}^{\text{vert}}\, \hat{x'}_{\text{tar}} - x^{\text{off}} \tag{7f}$$
$$\hat{y}_{\text{tar}}^{\text{rec}} = \hat{y}_{\text{tar}}^{\text{vert}} - \hat{z}_{\text{tar}}^{\text{vert}}\, \hat{y'}_{\text{tar}} - y^{\text{off}} \tag{7g}$$

6 Setting up an SVD problem

A singular value decomposition or SVD has a multitude of uses [3], e.g., χ^2 minimization, solving homogeneus set of linear equations, ... In the `HMS_optics` it is used to do the actual optics optimization.

The usual setup is to treat each event as an separate entry for the χ^2 minimization. This produces an overdetermined system, which is then solved using SVD. However, in the `HMS_optics` the problem is set a bit differently. Basically, an inhomogeneous system of linear equations is set up and then solved using the SVD [1].

The χ^2 of the reconstruction can be defined as:

$$\begin{aligned}\chi_o^2 &= \sum_{e \in \text{events}} \left(\hat{o}_{\text{tar}} - o_{\text{tar}}^{\text{rec}}\right)^2 \\ &= \sum_e \left(\hat{o}_{\text{tar}} - \sum_{i,j,k,l,m} O_{i,j,k,l,m} \cdot x_{\text{fp}}^i \, {x'}_{\text{fp}}^j \, y_{\text{fp}}^k \, {y'}_{\text{fp}}^l \, x_{\text{tar}}^m\right)\end{aligned} \tag{8}$$

Where o stands for observable, i.e., x', y, or y', and $O_{i,j,k,l,m}$ stands for the corresponding reconstruction coefficient, i.e., X', Y, and Y'. At the χ^2 minimum, the derivatives of χ^2 over matrix elements is 0:

$$\frac{\mathrm{d}\chi_o^2}{\mathrm{d}O_{\bar{i},\bar{j},\bar{k},\bar{l},\bar{m}}} = -2 \sum_e \left(\hat{o}_{\text{tar}} \sum_{i,j,k,l,m} O_{i,j,k,l,m} \cdot x_{\text{fp}}^i \, {x'}_{\text{fp}}^j \, y_{\text{fp}}^k \, {y'}_{\text{fp}}^l \, x_{\text{tar}}^m\right) \cdot x_{\text{fp}}^{\bar{i}} \, {x'}_{\text{fp}}^{\bar{j}} \, y_{\text{fp}}^{\bar{k}} \, {y'}_{\text{fp}}^{\bar{l}} \, x_{\text{tar}}^{\bar{m}} \tag{9a}$$
$$\left.\frac{\mathrm{d}\chi_o^2}{\mathrm{d}O_{\bar{i},\bar{j},\bar{k},\bar{l},\bar{m}}}\right|_{\chi^2_{o,min}} = 0 \tag{9b}$$

[1] SVD is not ideally suited for solving an $n \times n$ system of homogeneus equations, but it works. However, more efficient methods exist for this problem.

where the derivative is taken over a coefficient with specific indices $\bar{i}$, $\bar{j}$, $\bar{k}$, $\bar{l}$, $\bar{m}$, which are repeated in the last term. The sum, however, goes over all indices. The notation is simplified by using:

$$n \Rightarrow (i, j, k, l, m) \tag{10a}$$
$$\lambda_n \Rightarrow x_{\mathrm{fp}}^{i}\, x'^{j}_{\mathrm{fp}}\, y_{\mathrm{fp}}^{k}\, y'^{l}_{\mathrm{fp}}\, x_{\mathrm{tar}}^{m} \tag{10b}$$
$$\Lambda \Rightarrow \sum_{i,j,k,l,m} O_{i,j,k,l,m} \cdot \lambda_n^e \tag{10c}$$

From Equations 9 and 10 now follows:

$$\sum_e \left(\hat{o}_{\mathrm{tar}} - \Lambda^e\right) \cdot \lambda_{\bar{n}} = 0 \tag{11a}$$
$$\begin{aligned}\sum_e \hat{o}_{\mathrm{tar}}\, \lambda_{\bar{n}} &= \sum_e \Lambda^e\, \lambda_{\bar{n}} \\ &= \sum_e \sum_n O_n\, \lambda_n\, \lambda_{\bar{n}} \\ &= \sum_n O_n \sum_e \lambda_n\, \lambda_{\bar{n}} \end{aligned} \tag{11b}$$

If all terms of the reconstruction matrix were included in the Λ^e calculation, then the Λ^e equals to the $o_{\mathrm{tar}}^{e,\,\mathrm{rec}}$. But the terms with non-zero powers of x_{tar} $(m \neq 0)$ or x_{tar}-dependent terms need to be treated differently than the terms with zeroth powers of x_{tar} $(m = 0)$ or x_{tar}-independent terms. The reasons for this are detailed in the `README` file of [1]. The short version is that not enough information is available to fit those parameters, since the problem is under-constrained due to the lack of appropriate measurements. Because of this, the x_{tar}-dependent terms are taken as-is from `COSY` calculations and only the x_{tar}-independent terms are optimized. However, x_{tar}-dependent terms still need to be taken into account during event reconstruction since they provide important corrections.

This can be written by splitting the Λ^e into a x_{tar}-dependent part Λ^e_{dep} and a x_{tar}-independent part $\Lambda^e_{\mathrm{indep}}$:

$$\Lambda^e = \Lambda^e_{\mathrm{indep}} + \Lambda^e_{\mathrm{dep}} \tag{12a}$$
$$\Lambda^e_{\mathrm{indep}} = \sum_{n' \in \mathrm{indep}} O_{n'}\, \lambda_{n'} \tag{12b}$$
$$\Lambda^e_{\mathrm{dep}} = \sum_{n'' \in \mathrm{dep}} O_{n''}\, \lambda_{n''} \tag{12c}$$

Combining Equations 11 and 12 yields:

$$\sum_e \left(\hat{o}_{\mathrm{tar}} - \Lambda^e\right) \lambda_{\bar{n}} = \sum_e \left(\hat{o}_{\mathrm{tar}} - \Lambda^e_{\mathrm{dep}} - \Lambda^e_{\mathrm{indep}}\right) \lambda_{\bar{n}} = 0 \tag{13a}$$
$$\sum_e \left(\hat{o}_{\mathrm{tar}} - \Lambda^e_{\mathrm{dep}}\right) \lambda_{\bar{n}} = \sum_e \Lambda^e_{\mathrm{indep}}\, \lambda_{\bar{n}} \tag{13b}$$
$$\sum_e \left(\hat{o}_{\mathrm{tar}} - \Lambda^e_{\mathrm{dep}}\right) \lambda_{\bar{n}} = \sum_{n' \in \mathrm{indep}} O_{n'} \sum_e \lambda_{n'}\, \lambda_{\bar{n}} \tag{13c}$$

There is one such equation for each term of the reconstruction matrix that needs to be optimized, i.e., for each $\bar{n} \in$ indep:

$$\sum_{n' \in \text{indep}} O_{n'} \sum_e \lambda_{n'} \lambda_0 = \sum_e \left(\hat{o}_{\text{tar}} - \Lambda^e_{\text{dep}}\right) \lambda_0 \tag{14a}$$

$$\sum_{n' \in \text{indep}} O_{n'} \sum_e \lambda_{n'} \lambda_1 = \sum_e \left(\hat{o}_{\text{tar}} - \Lambda^e_{\text{dep}}\right) \lambda_1 \tag{14b}$$

$$\vdots$$

$$\sum_{n' \in \text{indep}} O_{n'} \sum_e \lambda_{n'} \lambda_{N'} = \sum_e \left(\hat{o}_{\text{tar}} - \Lambda^e_{\text{dep}}\right) \lambda_{N'} \tag{14c}$$

This set of equations can be rewritten as a matrix equation:

$$\underline{A}^o \cdot \vec{x}^o = \vec{b}^o \tag{15}$$

with matrix elements $a^o_{i,j}$, and vector elements x^o_i and b^o_i being:

$$a^o_{i,j} = \sum_e \lambda_i \lambda_j \tag{16a}$$

$$x^o_i = O_i \tag{16b}$$

$$b^o_i = \sum_e \left(\hat{o}_{\text{tar}} - \Lambda^e_{\text{dep}}\right) \lambda_i \tag{16c}$$

Looking back at Equation 14, there is a sum over events on both the left and right side. When solving the system for the optimization of the reconstruction parameters, not all recorded events are used here. Because of cross section and acceptance effects, not all target foils and sieve holes have equal number of associated events. This could introduce a bias into the final reconstruction matrix, since more populated points would have more weight in the optimization. To avoid this, there is an upper limit of 50 events (currently) for each foil-hole combination. Any extra events are ignored while setting the SVD problem.

7 Determining new reconstruction matrix

Equation 15 can be written for each of the observables, i.e., x'_{tar}, y_{tar}, and y'_{tar}. The optimal reconstruction parameters are obtained by solving these equations for $\vec{x}_o$. This vector presents, along with the x_{tar}-dependent terms from `COSY` a column of the reconstruction matrix as shown in Equation 3. The solution for $\vec{x}_o$ is obtained by:

$$\vec{x}_o = \underline{A}_o^{-1} \cdot \vec{b}_o \tag{17}$$

which can be done by the SVD.

Again, the problem in Equation 15 must be set to optimize only the x_{tar}-independent coefficients, since there are no appropriate measurements to constrain the x_{tar}-dependent coefficients.

8 Splitting the reconstruction matrix

As was demonstrated in previous sections, the x_{tar}-independent and x_{tar}-dependent reconstruction coefficients are treated differently. This is why it makes sense to split the reconstruction matrix into two parts, one for x_{tar}-independent and another for x_{tar}-dependent terms. This can either be done internally within the program or externally by storing two files like it is done in the current version of the `HMS_optics`.

Splitting the matrix creates a natural separation between the two sets of terms which leads to a cleaner code and offers some easy optimizations. Like it is mentioned in Section 3, multiple iterations over Equation 1 are made with increasingly more accurate value of x_{tar}. This equation can be rewritten as:

$$o_{\text{tar}} = \sum_{i,j,k,l;m=0} O_{i,j,k,l,m} \cdot x_{\text{fp}}^{i}\, {x'}_{\text{fp}}^{j}\, y_{\text{fp}}^{k}\, {y'}_{\text{fp}}^{l} + \sum_{i,j,k,l;m\neq 0} O_{i,j,k,l,m} \cdot x_{\text{fp}}^{i}\, {x'}_{\text{fp}}^{j}\, y_{\text{fp}}^{k}\, {y'}_{\text{fp}}^{l}\, x_{\text{tar}}^{m} \tag{18}$$

where only the second term changes during each iteration. The first term needs to be calculated only once.

Another example is in setting the SVD problem. The calculation of Λ_{dep}^{e} from Equation 14 is cleanly done by just looping over the x_{tar}-dependent matrix.

Also the whole SVD problem needs to be set such, that only the x_{tar}-independent coefficients are optimized. In the original code [1], the matrix system is initially set for optimization of all terms and is then post-processed to exclude the x_{tar}-dependent from optimization. This is done by setting the off-diagonal matrix elements of $\bar{A}$ and the values of $\vec{b}$ for x_{tar}-dependent terms to 0. The corresponding diagonal elements of $\bar{A}$ are set to 1. Then SVD is used to do the optimization.

However, the current version of the code [2] makes an explicit reduction of the SVD problem to only cover the x_{tar}-independent terms. This is done by making the coefficients i and j in the Equation 16 go only over the x_{tar}-independent terms, thus producing a smaller matrix $\bar{A}$, and vectors $\vec{x}$ and $\vec{b}$. This has beneficial effects on both the run time and accuracy, since the smaller problem is faster to setup and solve, and has better numerical stability.

References

[1] `https://github.com/brash99/HMS_optics` (22 March 2017)

[2] `https://github.com/JureBericic/HMS_optics` (22 March 2017)

[3] `https://en.wikipedia.org/wiki/Singular_value_decomposition` (22 March 2017)