Exploring high-frequency intraday volatility dynamics through E-mini S&P 500 Futures

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Abstract

This paper discusses intraday volatility in public markets, focusing on its modeling and practical implications. The first section provides a review of existing literature on intraday volatility modeling, emphasizing key theoretical frameworks and empirical findings. Building on this foundation, the paper applies a simplified model to aggregated high-frequency data of S&P 500 futures, reporting its results and capturing key characteristics of intraday price dynamics. Furthermore, this paper discusses potential clustering effects and their implications for market behavior.

1. Introduction

The modeling and prediction of intraday volatility play a crucial role in modern financial research and practice. As trading volumes increase and markets become more dynamic, understanding the nuanced patterns of high-frequency financial data has gained prominence. Intraday volatility, characterized by time-varying fluctuations within a trading day, influences a wide array of decisions from algorithmic trading to risk management and portfolio optimization.

Early studies in this field, such as Andersen and Bollerslev (1997, 1998), identified the pervasive diurnal periodicity of intraday volatility, emphasizing its U-shaped profile driven by trading activity and market structure. Subsequent advancements introduced multiplicative component frameworks to account for daily, periodic, and stochastic components of intraday returns (Engle & Sokalska, 2012). These models have been applied to various asset classes, including equities, bonds, and foreign exchange, showcasing their adaptability and predictive power. Notably, innovations such as the Fourier flexible form and non-parametric filtering have enhanced the accuracy of seasonality and trend estimations (Vatter et al., 2015; Zhang & Dufour, 2019).

The literature reflects a progression toward integrating granular data into robust econometric models. Techniques such as GARCH variants, EWMA for daily volatility scal-

ing, and cross-sectional pooling have emerged to address the challenges posed by high-frequency data. These approaches have demonstrated superior forecasting capabilities compared to traditional models, particularly for capturing volatility spillovers and market dynamics (Engle & Sokalska, 2012; Bollerslev et al., 2018). Moreover, the role of macroeconomic announcements and liquidity effects has been examined to refine intraday volatility predictions further.

This paper seeks to consolidate the key advancements in intraday volatility modeling, critically assessing their methodologies and applications across diverse financial contexts. By bridging theoretical insights and empirical findings, this review aims to provide a comprehensive perspective on the state-of-the-art techniques and identify potential avenues for future research in this evolving domain.

2. Intraday Volatility Models

2.1 GARCH Models

Compared to intraday volatility forecasting, there exists a much larger amount of literature about volatility forecasting on a daily scale. Conventional models applied to daily volatility forecasting are generally variations based on a simple GARCH(1,1) model. The model can be expressed as:

$$y_t = \mu + \epsilon_t,$$

$$\epsilon_t = \sigma_t z_t,$$

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2,$$

where:

- y_t : Observed time series,
- ϵ_t : Error term or innovation,
- σ_t^2 : Conditional variance,
- z_t : i.i.d. standard normal random variable,
- $\omega > 0$, $\alpha \ge 0$, $\beta \ge 0$: Model parameters.

The GARCH(1,1) model captures periods of persistent high or low volatility, which commonly occur in daily return time series. The model also captures the heavy-tailed nature of daily return data, as well as the long run variance.

However, the GARCH(1,1) alone proves insufficient for modeling volatility of intraday returns, especially at higher frequencies. For instance, the estimation of a MA(1)-GARCH(1,1) model for intraday returns yielded results that were inconsistent with time aggregated GARCH models (Andersen and Bollerslev, 1997). This discrepancy can be attributed to distinct intraday volatility patterns and trading activity, and any effective intraday volatility model should account for these factors in its forecast.

2.2 Multiplicative Models

To address the inadequacies of the GARCH(1,1) various multiplicative models were developed specifically to capture the intraday volatility patterns. For example, Giot (2005) models the intraday patterns with cubic splines, which was then multiplied on a GARCH model.

Various other parameters are also included in the multiplicative models to account for abnormal trading activity

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This is an example of fourth level head - paragraph head

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3. Example of First Level Head

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3.1.1 This is an example of third level head - subsubsection head

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4. How to Include Equations

Equations in LaTeX can either be inline or set as display equations. For inline equations use the \$...\$ commands. Eg: the equation $H\psi = E\psi$ is written via the command \$H \psi = E \psi\$.

For display equations (with auto generated equation numbers) one can use the equation or equarray environments:

$$\|\tilde{X}(k)\|^{2} \leq \frac{\sum_{i=1}^{p} \|\tilde{Y}_{i}(k)\|^{2} + \sum_{j=1}^{q} \|\tilde{Z}_{j}(k)\|^{2}}{p+q},$$
(1)

where

$$D_{\mu} = \partial_{\mu} - ig \frac{\lambda^{a}}{2} A^{a}_{\mu}$$

$$F^{a}_{\mu\nu} = \partial_{\mu} A^{a}_{\nu} - \partial_{\nu} A^{a}_{\mu} + g f^{abc} A^{b}_{\mu} A^{a}_{\nu}$$

$$(2)$$

Notice the use of \nonumber in the align environment at the end of each line, except the last, so as not to produce equation numbers on lines where no equation numbers are required. The \label{} command should only be used at the last line of an align environment where \nonumber is not used.

$$Y_{\infty} = \left(\frac{m}{\text{GeV}}\right)^{-3} \left[1 + \frac{3\ln(m/\text{GeV})}{15} + \frac{\ln(c_2/5)}{15}\right]$$
 (3)

The class file also supports the use of \mathbf{R} , \mathbf{R} and \mathbf{R} produces \mathbf{R} , and \mathbf{R} respectively

Equations must be provided as editable text, either in a Word or LaTeX source file. They should be numbered consecutively through the manuscript as shown in Equations 1, 2 and 3. In APA style, when discussing numbered equations in the text, write out the word "Equation" and give the number. For example, you would write "see Equation 1." Use no punctuation after the equation if it appears at the end of a sentence; however, it is permissible (and may even be necessary) to place some form of punctuation after it (a comma or semi-colon, for example) if it appears in the middle of the sentence and is followed by text. In any case, maintain the coherence of all sentences with equations in them.

5. How to Include Tables

Use the table and tabular environments for basic tables — see Tables 1 and 2, for example. Table 1 is an sample figure including table footnotes. For more information, please see this help article on tables.

column 1	column 2	column 3	column 4
row 1	data 1	data 2	data 3
row 2	data 4	data 5^1	data 6
row 3	data 7	data 8	data 9^2

Table 1: Sample table with footnotes

Source: This is an example of table footnote. This is an example of table footnote.

Table 2: Example of a lengthy table which is set to full textwidth.

	Element 1 ¹		Element 2^2			
Project	Energy	$\sigma_{ m calc}$	$\sigma_{ m expt}$	Energy	$\sigma_{ m calc}$	$\sigma_{ m expt}$
Element 3 Element 4	990 A 500 A	1168 961	1547 ± 12 922 ± 10	780 A 900 A	1166 1268	

Note: This is an example of table footnote this is an example of table footnote this is an example of table footnote this is an example of table footnote.

¹ Example of a first table footnote.

² Example of a second table footnote.

¹ Example of a first table footnote.

² Example of a second table footnote.

6. How to Include Figures

First you have to upload the image file from your computer using the upload link in the file-tree menu. Then use the includegraphics command to include it in your document. Use the figure environment and the caption command to add a number and a caption to your figure. See the code for Figure 1 in this section for an example. As shown in Figures 1 and 2, the images should be single-page documents.

Note that your figure will automatically be placed in the most appropriate place for it, given the surrounding text and taking into account other figures or tables that may be close by. You can find out more about adding images to your documents in this help article on including images on Overleaf.



Figure 1: This cat picture is located at the 'figures' folder.

6.1 More information about figures

As per display LATEX standards one has to use eps images for latex compilation and pdf/jpg/png images for pdflatex compilation. This is one of the major differences between latex and pdflatex. The images should be single-page documents. The command for inserting images for latex and pdflatex can be generalized. The package used to insert images in latex/pdflatex is the graphicx package. Figures can be inserted via the normal figure environment as shown in the below example:

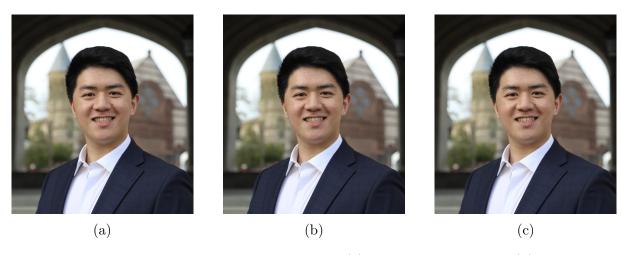


Figure 2: Overall caption for the three figures: (a) caption for figure a, (b) caption for figure b, and (c) caption for figure c.

7. How to Include Algorithms, Program Codes, and Listings

Packages algorithm, algorithmicx, and algorithms in latex. For this, one has to use the below format:

```
\begin{algorithm}
\caption{<alg-caption>}\label{<alg-label>}
\begin{algorithmic}[1]
. . .
\end{algorithmic}
\end{algorithm}
```

You may need to refer to the above-listed package documentation for more details before setting an algorithm environment. To set program codes, one has to use the program package. We need to use the \begin{program} . . . \end{program} environment to set program codes.

Similarly, for listings, one has to use the listings package. To set environments similar to the verbatim environment, the \begin{lstlisting} ... \end{lstlisting} environment is used. Refer to the lstlisting package documentation for more details on this.

```
for i:=maxint to 0 do
begin
{ do nothing }
end;
Write('Case_insensitive_');
Write('Pascal_keywords.');
```

Algorithm 1 Calculate $y = x^n$

```
Require: n \ge 0 \lor x \ne 0
Ensure: y = x^n
 1: y \Leftarrow 1
 2: if n < 0 then
          X \Leftarrow 1/x
          N \Leftarrow -n
 5: else
 6:
          X \Leftarrow x
          N \Leftarrow n
 7:
 8: end if
 9: while N \neq 0 do
         if N is even then
10:
              X \Leftarrow X \times X
11:
              N \Leftarrow N/2
12:
         else[N \text{ is odd}]
13:
              y \Leftarrow y \times X
14:
              N \Leftarrow N - 1
15:
         end if
16:
17: end while
```

8. How to Include Lists

List in LaTeX can be of three types: numbered, bulleted, and unnumbered. The "enumerate" environment produces a numbered list, the "itemize" environment produces a bulleted list, and the "unlist" environment produces an unnumbered list. In each environment, a new entry is added via the \item command.

- 1. This is the 1st item
- 2. Enumerate creates numbered lists, itemize creates bulleted lists, and unnumerate creates unnumbered lists.
 - a. Second level numbered list. Enumerate creates numbered lists, itemize creates bulleted lists, and description creates unnumbered lists.
 - b. Second level numbered list. Enumerate creates numbered lists, itemize creates bulleted lists, and description creates unnumbered lists.
 - (i) Third level numbered list. Enumerate creates numbered lists, itemize creates bulleted lists, and description creates unnumbered lists.
 - (ii) Third level numbered list. Enumerate creates numbered lists.
 - c. Second level numbered list. Enumerate creates numbered lists, itemize creates bulleted lists, and description creates unnumbered lists.
- 3. Numbered lists continue.

Lists in LaTeX can be of three types: enumerate, itemize, and description. In each environment, a new entry is added via the \item command.

- First level bulleted list. This is the 1st item
- First level bulleted list. Itemize creates bulleted lists, and description creates unnumbered lists.
 - Second level dashed list. Itemize creates bulleted lists, and description creates unnumbered lists.
 - Second level dashed list. Itemize creates bulleted lists, and description creates unnumbered lists.
- First level bulleted list. Bullet lists continue.

Example of unnumbered list items:

Sample unnumberd list text. Sample unnumberd list text. Sample unnumberd list text. Sample unnumberd list text.

Sample unnumberd list text. Sample unnumberd list text. Sample unnumberd list text.

Sample unnumberd list text. Sample unnumberd list text. Sample unnumberd list text. Sample unnumberd list text.

9. How to Add Citations and a References List

You can simply upload a .bib file containing your BibTeX entries, created with a tool such as JabRef. You can then cite entries from it, like this: **greenwade93**. Just remember to specify a bibliography style, as well as the filename of the .bib. You can find a video tutorial here to learn more about BibTeX.

Here is an example citation when you want an author name like **collins2011a** to appear in the text. And here's how to do a parenthetic citation, when you want to mention a reference at the end of a sentence or part of a sentence (**collins2013**). It is possible to cite multiple references at the same time (**collins2011b**; **collins2016**; **lunn2007a**; **lunn2007b**; **ross2006**; **shannon1948**).

If you have an upgraded account, you can also import your Mendeley or Zotero library directly as a .bib file, via the upload menu in the file-tree.

9.1 Citation in text

Please ensure that every reference cited in the text is also present in the reference list (and vice versa). Citations in the text should follow the referencing style used by the American Psychological Association. You are referred to the Publication Manual of the

American Psychological Association (APA), Seventh Edition, ISBN 978-1-4338-3215-4, copies of which may be ordered online. References in the Abstract should be avoided, but if essential, then cite the author(s) and year(s). Unpublished results and personal communications are not recommended in the reference list but may be mentioned in the text. If these references are included in the reference list, they should follow the standard reference style of the journal and should include a substitution of the publication date with either 'Unpublished results' or 'Personal communication'. The citation of a reference as 'in press' implies that the item has been accepted for publication.

An APA in-text citation includes only three items: the last name(s) of the author(s), the year the source was published, and sometimes the page or location of the information. More than one reference from the same author(s) in the same year must be identified by the letters 'a', 'b', 'c', etc., placed after the year of publication. The following paragraph shows examples of APA style of citations.

Here is an example citation when you want an author name like **collins2011a** to appear in the text. And here's how to do a parenthetic citation when you want to mention a reference at the end of a sentence or part of a sentence (**collins2013**). It is possible to cite multiple references at the same time (**collins2011b**; **collins2016**; **lunn2007a**; **lunn2007b**; **ross2006**; **shannon1948**).

The followings are examples of \textcite{...}: rahman2019centroidb, krizhevsky2012image. horvath2018dna, and lecun2015deep; zhang2018fine; ravi2016deep. Another example of \parencite{...}: (bahdanau2014neural; imboden2018cardiorespiratory; motiian2017unified; murphy2012machine; ji20123d).

9.2 References

The Reference Section, also called the Reference List or Cited Works List, is a list of the full-text details of the in-text citations that have been used in the main text. It includes information such as the name of the author(s), the year the source was published, the full title of the source, and the URL or page range. The Reference Section allows the reader to find the text easily and can be considered as the long-hand format of the in-text citation. It is found at the end of the piece of writing. The works in a reference section should be arranged first alphabetically and then further sorted chronologically if necessary.

9.2.1 Web references

As a minimum, the full URL and the date when the reference was last accessed should be given. Any further information, if known (DOI, author names, dates, reference to a source publication, etc.), should also be given. Web references can be listed separately (e.g., after the reference list) under a different heading if desired or can be included in the reference list. With standard numerical .bst files, only numerical citations are possible. With an author-year .bst file, both numerical and author-year citations are possible.

9.2.2 Examples of reference style

You can find information about the examples of APA-style references to various sources at the following site:

https://apastyle.apa.org/style-grammar-guidelines/references/examples.

10. Conclusions

Some conclusions here.

Conflicts of Interest

The authors must declare conflicts of interest or state "The authors declare no conflict of interest." Authors must identify and declare any personal circumstances or interests that may be perceived as inappropriately influencing the representation or interpretation of reported research results. A detailed definition of conflicts of interest is available at the following site: https://academic.oup.com/journals/pages/authors/preparing_your_manuscript/ethics#conflict.

Author Contributions

The authors must specify the individual contributions of all authors, identified by full names, according to NISO CrediT (Contributer Roles Taxonomy) described at the following site: https://credit.niso.org/. An example statement is as follows:

Kunwoo Lee: Conceptualization, Methodology, Software. Shuming Gao: Data curation, Writing—original draft. Sang Hun Lee: Visualization, Investigation. Jami J. Shah: Supervision. Hiromasa Suzuki: Software, Validation. Myung-Il Roh: Writing—review & editing.

Funding

Cite all funding for your research, providing the grant number and the funder name. An example statement is as follows: This work is supported in part by funds from the National Science Foundation (NSF: # 1636933 and # 1920920).

If the funder is listed in the Crossref funder registry (https://www.crossref.org/services/funder-registry/), the funder name should appear exactly as it does in that database. Where grants were received by specific members of the author group, they should be identified by initials.

More information on funding agency requirements is available at https://academic.oup.com/pages/open-research/open-access/complying-with-funder-policies.

Data Availability

The data availability statement should provide information on where and under what conditions the data directly supporting the publication can be accessed. Sample data availability statements are available at the following site: https://academic.oup.com/pages/open-research/research-data#Data%20Availability%20Statements.

Acknowledgments

The authors thank those people or institutions that have helped you in the preparation of the manuscript.

Appendix

A. Some Notation

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$$Y_{\infty} = \left(\frac{m}{\text{GeV}}\right)^{-3} \left[1 + \frac{3\ln(m/\text{GeV})}{15} + \frac{\ln(c_2/5)}{15}\right]$$
 (A1)

A.2 Appendix subsection title here

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Table A1: Sample table with three parts and five columns

column 1	column 2	column 3	column 4	column 5
row 1 row 2	data 0 data 4	data 1 data 5	data 2 data 6	data 3 data 7
row 3	data 8	data 9	data 10	data 11

B. Some More Notation

As shown in Figure B1, the section number is inserted in the figure number. Aliquam lectus. Vivamus leo. Quisque ornare tellus ullamcorper nulla. Mauris porttitor pharetra tortor. Sed fringilla justo sed mauris. Mauris tellus. Sed non leo. Nullam elementum, magna in cursus sodales, augue est scelerisque sapien, venenatis congue nulla arcu et pede. Ut suscipit enim vel sapien. Donec congue. Maecenas urna mi, suscipit in, placerat ut, vestibulum ut, massa. Fusce ultrices nulla et nisl.

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B.1 Appendix subsection title here

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Figure B1: This cat picture is located at the 'figures' folder.