MATH 307: Group Homework 9

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Problem 1

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Problem 2

...

Problem 3

$A \in \mathbb{C}^{m \times n}$

SVD:

$A = U\Sigma V^*$ and $A^* = V\Sigma^* U^*$, where
$\text{range}(A) = \text{span}(u_1, u_2, \ldots, u_r) \subset \mathbb{C}^m$ and
$\text{null}(A^*) = \text{span}(u_{r+1}, u_{r+2}, \ldots, u_m) \subset \mathbb{C}^m$

Therefore,

$$\text{range}(A) + \text{null}(A^*) = \text{span}(u_1, u_2, \ldots, u_r) + \text{span}(u_{r+1}, u_{r+2}, \ldots, u_m)$$
$$\text{range}(A) + \text{null}(A^*) = \text{span}(u_1, u_2, \ldots, u_m)$$
$$\text{range}(A) + \text{null}(A^*) = \mathbb{C}^m$$

And by the properties of SVD, we know that U is an orthogonal matrix, therefore all the columns of U, u_i, are mutually orthogonal $\implies$ that the two collections of u_i, $\text{range}(A)$ and $\text{null}(A^*)$, are orthogonal subspaces of $\mathbb{C}^m$.

Therefore $\text{range}(A) \oplus \text{null}(A^*) = \mathbb{C}^m$