

MATH 307: Individual Homework 15

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Problem 1

First Inequality:

$$\|A\| = \sup_{x \neq 0} \frac{\|Ax\|}{\|x\|}$$

Because $\|A\|$ is the minimum upper bound of the fraction, we can say:

$$\frac{\|Ax\|}{\|x\|} \leq \|A\| \implies \|Ax\| \leq \|A\|\|x\|$$

Second Inequality:

$$\|AB\| \leq \|A\|\|B\|$$

$$\|AB\|_1 \leq \|A\|_1\|B\|_1$$

$$\sup_{\|x\|_1=1} \|ABx\|_1 \leq \sup_{x \neq 0} \frac{\|Ax\|_1}{\|x\|} \sup_{x \neq 0} \frac{\|Bx\|_1}{\|x\|}$$

sup

...

Problem 2

$$A = \begin{pmatrix} 2 & 4 & -11 & 4 & -9 \\ 4 & -1 & 6 & 2 & -1 \\ 1 & 5 & 6 & -7 & -8 \\ -20 & 20 & 2 & -2 & 0 \end{pmatrix}$$

Infinity Norm:

$$\|A\|_\infty = \max_i \sum_{j=1}^n |a_{ij}| = |-20| + |20| + |2| + |-2| + |0| = 44$$

1 Norm:

$$\|A\|_1 = \sup_{\|x\|_1=1} \|Ax\|_1 = \max_j \sum_{i=1}^m |a_{ij}| = |4| + |-1| + |5| + |20| = 30$$

Frobenius Norm:

$$\|A\|_F = \sqrt{\sum_{i=1}^m \sum_{j=1}^n |a_{ij}|^2} = (4 + 16 + 121 + 16 + 81 + 16 + 1 + 36 + 4 + 1 + 1 + 25 + 36 + 49 + 64 + 400 + 400 + 4 + 4 + 0)^{\frac{1}{2}} = \sqrt{1279}$$

Problem 3

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Problem 4

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