MATH 307: Individual Homework 7

John Mays

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Problem 1

$$\sum_i |x_i| \leq \sqrt{n}\sqrt{\sum_i |x_i|^2}, \text{ for all } x \in \mathbb{C}^n$$

Proof:

Begin with Cauchy-Schwartz Inequality:
Let $n = 1$.

$$|\langle X, 1\rangle| \leq |X||1|$$
$$\sum_i |x_i||1| \leq |X||\sqrt{n}|$$
$$\sum_i |x_i| \leq \sqrt{n}\sqrt{\langle X, X\rangle}$$
$$\sum_i |x_i| \leq \sqrt{n}\sqrt{\sum_i |x_i|^2}$$

Thus the inequality is proven for $n = 1$.
If $n \neq 1$, $n > 1$, and therefore $\sqrt{n} > 1$, which means that the inequality will still hold, because the RHS is being multiplied by a scalar > 1 (and both sides are nonnegative values):
In other words: if $|a| \leq |b|$ and $\alpha > 1$, then $|a| \leq \alpha|b|$.
Therefore,

$$\sum_i |x_i| \leq \sqrt{n}\sqrt{\sum_i |x_i|^2}, \text{ for all } x \in \mathbb{C}^n$$

Problem 2

$$\sum_i \sum_j |a_{ij}|^2 |a_{ij}|^3 \leq \sqrt{\sum_i \sum_j |a_{ij}|^4} \sqrt{\sum_i \sum_j |b_{ij}|^6}, \text{ for all } A, B \in \mathbb{C}^{m \times n}.$$

Proof:

Begin with Cauchy-Schwartz Inequality:

$$\begin{aligned}
< A^2, B^3 > &\leq |A^2||B^3| \\
A^2 \cdot B^3 &\leq \sqrt{\langle A^2, A^2 \rangle} \sqrt{\langle B^3, B^3 \rangle} \\
\sum_i \sum_j |a_{ij}|^2 |b_{ij}|^3 &\leq \sqrt{\sum_i \sum_j |a_{ij}|^2 |a_{ij}|^2} \sqrt{\sum_i \sum_j |b_{ij}|^3 |b_{ij}|^3} \\
\sum_i \sum_j |a_{ij}|^2 |b_{ij}|^3 &\leq \sqrt{\sum_i \sum_j |a_{ij}|^4} \sqrt{\sum_i \sum_j |b_{ij}|^6}
\end{aligned}$$