MATH 307: Group Homework 8

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Problem 1

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Problem 2

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Problem 3

$A \in F^{n \times n} = U\Sigma V^*$ where U, V are unitary and Σ is diagonal.

$$\begin{aligned} A^*A &= (U\Sigma V^*)^*(U\Sigma V^*) \\ &= (V\Sigma^* U^*)(U\Sigma V^*) \\ &= V\Sigma^* U^* U\Sigma V^* \\ &= V\Sigma^* I\Sigma V^* \\ &= V\Sigma^*\Sigma V^* \\ &= V(\Sigma^*\Sigma)V^* \end{aligned}$$

Since V is a unitary matrix, it is also invertible, and $V^* = V^{-1}$. Therefore,

$$V(\Sigma^*\Sigma)V^* = V(\Sigma^*\Sigma)V^{-1}$$

Furthermore, the resultant matrix of $\Sigma^*\Sigma$ will also be a diagonal matrix = $\text{diag}(|\sigma_1|^2, |\sigma_2|^2, \ldots, |\sigma_n|^2)$

Since V is invertible, $V^* = V^{-1}$, and $\Sigma^*\Sigma$ is a diagonal matrix, $V(\Sigma^*\Sigma)V^*$ is an eigendecomposition of A^*A.

Problem 4

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