MATH 307: Group Homework 6

John Mays, Henry Zhong, Zhitao Chen, Huajin Xin

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Problem 1

Reflection:

For a reflection across the y-axis, the x-component of a vector must become -x
$A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix}$
If $A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, then $A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1(x)+0(x) \\ 0(y)+1(y) \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix}$
Therefore A is the reflection matrix.

Rotation:

To obtain the rotation matrix, we can compute the default Cartesian rotation matrix with $\theta = \frac{\pi}{4}$
$B = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ Therefore B is the rotation matrix.

Entire Operation:

The matrix for the entire operation is the reflection matrix times the rotation matrix,
$AB = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} (-1)(\frac{1}{\sqrt{2}})+(0)(\frac{1}{\sqrt{2}}) & (-1)(-\frac{1}{\sqrt{2}})+(0)(\frac{1}{\sqrt{2}}) \\ (0)(\frac{1}{\sqrt{2}})+(1)(\frac{1}{\sqrt{2}}) & (0)(-\frac{1}{\sqrt{2}})+(1)(\frac{1}{\sqrt{2}}) \end{pmatrix} =$
$\begin{pmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$

Problem 2

Equation in question: $(A+B)^2 = A^2 + 2AB + B^2$ for two square matrices of the same size, A and B.

LHS:

The ij-th entry of $(A+B)$ is $a_{ij} + b_{ij}$
The ij-th entry of $(A+B)^2 = (A+B)(A+B)$ is $\sum_{k=1}^{n}(a_{ik}+b_{ik})(a_{kj}+b_{kj}) = \sum_{k=1}^{n} a_{ik}a_{kj} + a_{ik}b_{kj} + b_{ik}a_{kj} + b_{ik}b_{kj}$

RHS:

The ij-th entry of A^2 is $\sum_{k=1}^{n} a_{ik}a_{kj}$
It follows that the ij-th entry of B^2 is $\sum_{k=1}^{n} b_{ik}b_{kj}$
The ij-th entry of AB is $\sum_{k=1}^{n} a_{ik}b_{kj}$
Therefore the ij-th entry of $A^2 + 2AB + B^2$ is $\sum_{k=1}^{n} a_{ik}a_{kj} + 2a_{ik}b_{kj} + b_{ik}b_{kj}$

Conclusion:

The respective ij-th entry of the LHS and the RHS of the equation are not equivalent, therefore $(A+B)^2 = A^2 + 2AB + B^2$ is not true for two square matrices, A and B, of the same size.