MATH 307: Individual Homework 17

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Problem 1

Because U and V are orthogonal, $U^* = U^{-1}$ and $V^* = V^{-1}$. Therefore,

$$\begin{aligned}
A^{-1} &= (U\Sigma V^*)^{-1} \\
&= (V^*)^{-1}\Sigma^{-1}U^{-1} \\
&= (V^{-1})^{-1}\Sigma^{-1}U^{-1} \\
A^{-1} &= V\Sigma^{-1}U^*
\end{aligned}$$

Problem 2

Inner Product Case:

$$\begin{aligned}
A^*A &= (U\Sigma V^*)^*(U\Sigma V^*) \\
&= (V^*)^*\Sigma^*U^*U\Sigma V^* \\
&= V\Sigma^*U^*U\Sigma V^* \\
&= V\Sigma^*(U^*U)\Sigma V^* \\
&= V\Sigma^*I\Sigma V^* \\
A^*A &= V\Sigma^*\Sigma V^*
\end{aligned}$$

And the new singular matrix, $(\Sigma^*\Sigma)$, will look like this:

$$(\Sigma^*\Sigma) = \begin{bmatrix} \sigma_1 & & & \dots & 0 \\ & \ddots & & \dots & 0 \\ & & \sigma_n & \dots & 0 \end{bmatrix}^{n\times m} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_n \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix}^{m\times n} = \begin{bmatrix} |\sigma_1|^2 & & \\ & \ddots & \\ & & |\sigma_n|^2 \end{bmatrix}^{n\times n}$$

The last diagonal entry will be equal to $|\sigma_n|^2$, a positive value, A^*A is invertible.

Outer Product Case:

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