MATH 307: Individual Homework 13

John Mays

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Problem 1

Q is an arbitrary 3×3 unitary matrix, $x = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, y = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$,
and $\theta =$ the angle between Qx and Qy.

By the property of a unitary matrix, matrix multiplication of two vectors by a unitary matrix will not change the angle between the two matrices.

$$
\begin{aligned}
cos(\theta) &= \frac{\langle x, y \rangle}{||x||||y||} \\
&= \frac{(1)(-3) + (2)(0) + (3)(1)}{\sqrt{(1)^2 + (2)^2 + (3)^2}\sqrt{(-3)^2 + (0)^2 + (1)^2}} \\
&= 0
\end{aligned}
$$

Therefore $\theta = 90°$.

Problem 2

The null space of $A \in F^{m \times n}$ is a subspace of F^n because it satisfies the three conditions of a subspace:

null$(A) = \{x \in F^n | Ax = 0\}$, also assume $y \in F^n$ and $\lambda \in F$

1. Closure under addition:
$A(x + y) = Ax + Ay = 0 + 0 = 0 \implies x + y \in$ null(A).
2. Closure under scalar multiplication:
$A(\lambda x) = \lambda Ax = \lambda 0 \implies \lambda x \in$ null(A).
3. 0 belongs to range(A):
$A0 = 0 \in$ null(A).

Therefore the null(A) is a subspace of F^n.

Problem 3

$\text{range}(A) = \text{span}(\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \end{pmatrix}) = \text{span}(\begin{pmatrix} 1 \\ 1 \end{pmatrix})$.
Therefore a basis for A is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Problem 4

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