

MATH 307: Group Homework X

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05/07/21, Dr. Guo

Problem 1

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Problem 2

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

A^T would obviously be:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Calculating P :

$$\begin{aligned} P &= A(A^T A)^{-1} A^T = A \left(\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \right)^{-1} A^T \\ &= A \left(\begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix} \right)^{-1} A^T = A \left(\frac{1}{2} \begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix} \right) A^T = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1.5 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0.5 \\ 1 & -1 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = P = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \end{bmatrix} \end{aligned}$$

Calculating Pb :

$$Pb = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

Problem 3

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