

MATH 307: Individual Homework 20

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Problem 1

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 6 & 6 & 3 \\ 3 & 0 & -1 \end{bmatrix}$$

Finding U:

Row 2: subtract 2 times row 1,

Row 3: subtract row 1:

$$L_2 A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ 6 & 6 & 3 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & -2 & -2 \end{bmatrix}$$

Row 3 in new matrix: add row 2

$$L_1(L_2 A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & -2 & -2 \end{bmatrix} = U = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

Finding L:

$$L_1 L_2 A = U = L^{-1} A \implies$$

$$L^{-1} = L_1 L_2 \implies$$

$$L = (L_1 L_2)^{-1} = L_2^{-1} L_1^{-1}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

Conclusion:

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 6 & 6 & 3 \\ 3 & 0 & -1 \end{bmatrix} = LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$