MATH 307: Individual Homework 9

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Problem 1

Default inner product in $\mathbb{C}^3$:

$$\forall u, v \in \mathbb{C}^3, \langle u, v\rangle = |u_1||v_1| + |u_2||v_2| + |u_3||v_3|$$

The induced norm is defined as:

$$\begin{aligned} ||u|| &= \sqrt{\langle u, u\rangle} \\ &= \sqrt{|u_1||u_1| + |u_2||u_2| + |u_3||u_3|} \\ &= \sqrt{|u_1|^2 + |u_2|^2 + |u_3|^2} \end{aligned}$$

The 2-norm is defined as:

$$\begin{aligned} ||u||_2 &= \left(\sum_{i=1}^{3} |u_i|^2\right)^{\frac{1}{2}} \\ &= \sqrt{\sum_{i=1}^{3} |u_i|^2} \\ &= \sqrt{|u_1|^2 + |u_2|^2 + |u_3|^2} \end{aligned}$$

Therefore the induced norm is equal to the 2-norm:

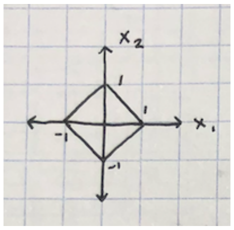
$$\forall u \in \mathbb{C}^3, ||u||_{ind} = ||u||_2$$

$||u|| = \sqrt{|1+i|^2 + |2i|^2 + |3+i|^2} = \sqrt{\sqrt{2}^2 + 2^2 + \sqrt{10}^2}$
$= \sqrt{2+4+10} = \sqrt{16} = 4$
$d(u, v) = ||u - v|| = ||(i, -3+3i, 3+2i)|| = \sqrt{|i|^2 + |-3+3i|^2 + |3+2i|^2} =$
$\sqrt{1+18+13} = \sqrt{32} = 4\sqrt{2}$

Problem 2

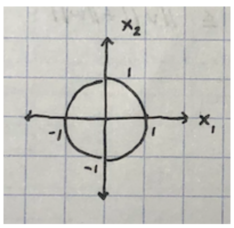
1-norm:

$||x||_1 = |x_1| + |x_2| = 1$



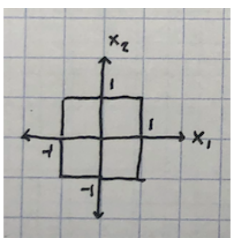
2-norm:

$||x||_2 = \sqrt{|x_1|^2 + |x_2|^2} = 1$
$||x||_2 = |x_1|^2 + |x_2|^2 = 1$



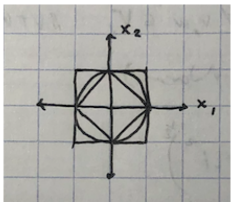
∞-norm:

$||x||_\infty = \max{(x_1, x_2)} = 1$



1 on all norms:

Composing the three norms on top of each other,



we can see that four points on the x_1, x_2 plane qualify as 1 for all norms: $(0,1),(1,0),(0,-1),(-1,0)$.