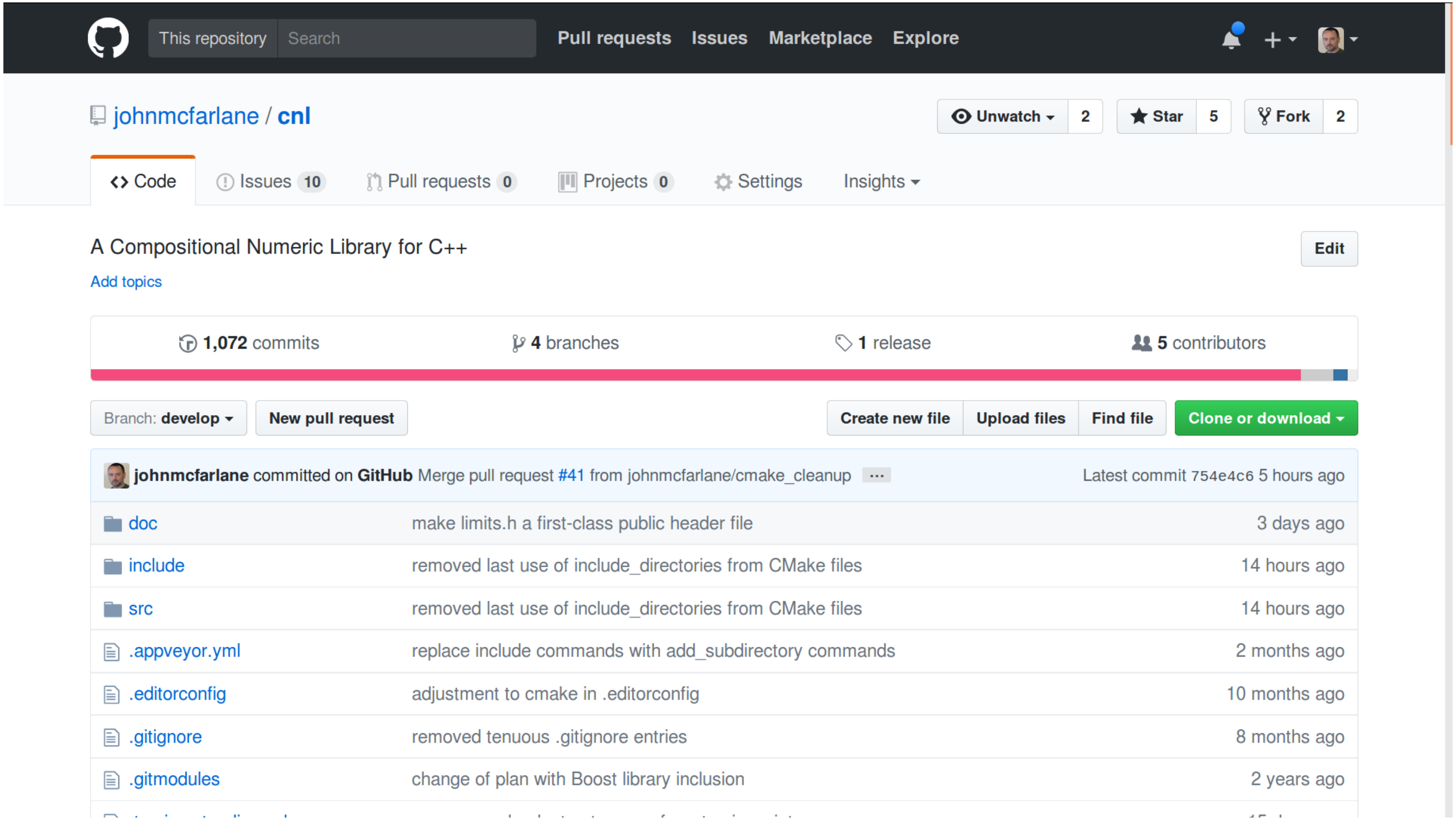
Better Numeric Types in C++

John McFarlane

A9.com

Background

Background



johnmcfarlane / cnl

A Compositional Numeric Library for C++

1,072 commits · 4 branches · 1 release · 5 contributors

johnmcfarlane committed on GitHub Merge pull request #41 from johnmcfarlane/cmake_cleanup … Latest commit 754e4c6 5 hours ago

doc	make limits.h a first-class public header file	3 days ago
include	removed last use of include_directories from CMake files	14 hours ago
src	removed last use of include_directories from CMake files	14 hours ago
.appveyor.yml	replace include commands with add_subdirectory commands	2 months ago
.editorconfig	adjustment to cmake in .editorconfig	10 months ago
.gitignore	removed tenuous .gitignore entries	8 months ago
.gitmodules	change of plan with Boost library inclusion	2 years ago

The Problem with Integers

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1. Low resolution

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1. Low resolution
2. Limited range

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1. Low resolution
2. Limited range
3. 'Interesting' behavior of arithmetic operations

The Problem with Floating-Point

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    - ± 1.significand * 2^{exponent}
    - special values, denormalized values, -0

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    - ± 1.significand * 2^{exponent}
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2. Occasional weirdness can surprise:
    - determinism, associativity, commutativity and ordering
3. `<cmath>` functions lack `constexpr`
4. Variable resolution
5. Costly in energy and silicon

Analysis

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- But we can do a lot better.
- so …

Goal of CNL

"Do for `int` what the STL did for `[]`."

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- Provide zero-cost abstractions over language-level features:

```
std::array<T, N> a;  // T a[N]
std::array<T, N>::iterator i = std::begin(a); // T* i
```

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- Maintain a familiar interface:

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  auto const& third = a[2];
  for (auto const& element : a) { /* ... */ }
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  ```

- Allow users to opt in to positive-cost functionality:

  ```
  std::array<T, N> a;
  auto const& bad_element = a.at(N);  // throws std::out_of_range!
  ```

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"Do for `int` what the STL did for `[]`."

- Provide zero-cost abstractions over language-level features:

```
std::array<T, N> a;  // T a[N]
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- Allow users to opt in to positive-cost functionality:

```
std::array<T, N> a;
auto const& bad_element = a.at(N);  // throws std::out_of_range!
```

- And most importantly...

Goal of CNL

"Do for `int` what the STL did for `[]`."

- Compose!

```
using fs_cache = unordered_map<filesystem::path, vector<byte>>;
```

Non-Goal

"Don't do for `int` what STL doesn't do for `[]`."

Non-Goal

"**Don't do for `int` what STL doesn't do for `[]`.**"

- Don't make the user pay for what they don't use.

Fixed-Point Arithmetic

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```
// cnl/fixed_point.h
namespace cnl {

    template<typename Rep = int, int Exponent = 0>
    class fixed_point {
        // ...
    private:
        Rep r;
    };

}
```

Fixed-Point Arithmetic

```
// cnl/fixed_point.h
namespace cnl {

    template<typename Rep = int, int Exponent = 0>
    class fixed_point {
        // ...
    private:
        Rep r;
    };

}
```

Example usage:

```
using cnl::fixed_point;

void f() {
    auto n = fixed_point<int, -8>{ 0.25 };
    std::cout << n * 5; // prints "1.25"
}
```

The Good

The Good

```cpp
// what the programmer writes
bool foo(float f) {
    auto fixed = fixed_point<int, -16>{f};
    auto fixed_plus_one = fixed + 1;
    return fixed_plus_one > fixed;
}
```

The Good

```
// what the programmer writes
bool foo(float f) {
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```
// what the compiler sees
bool foo(float) {
    return true;
}
```

The Good

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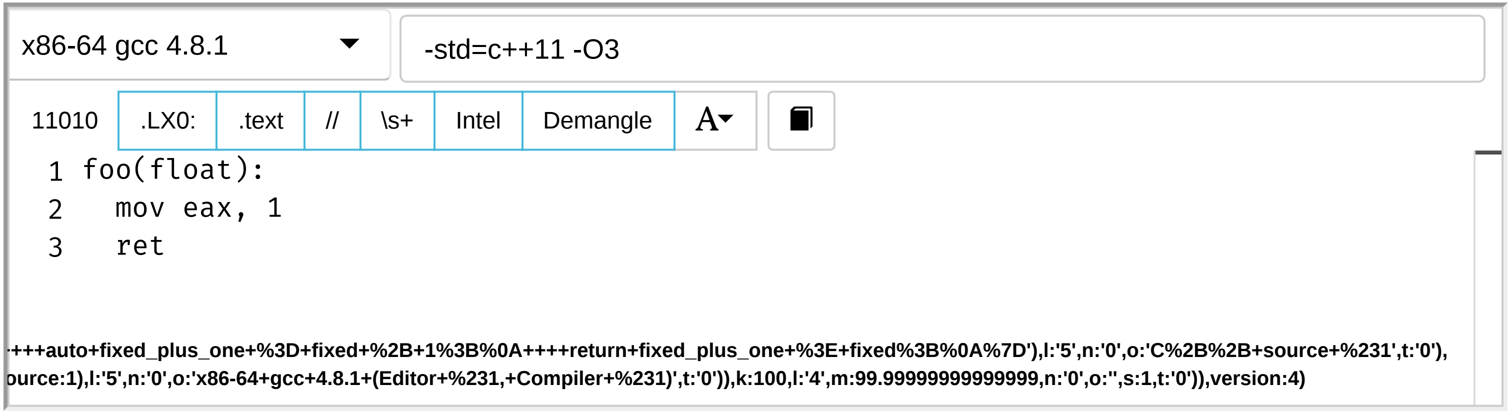


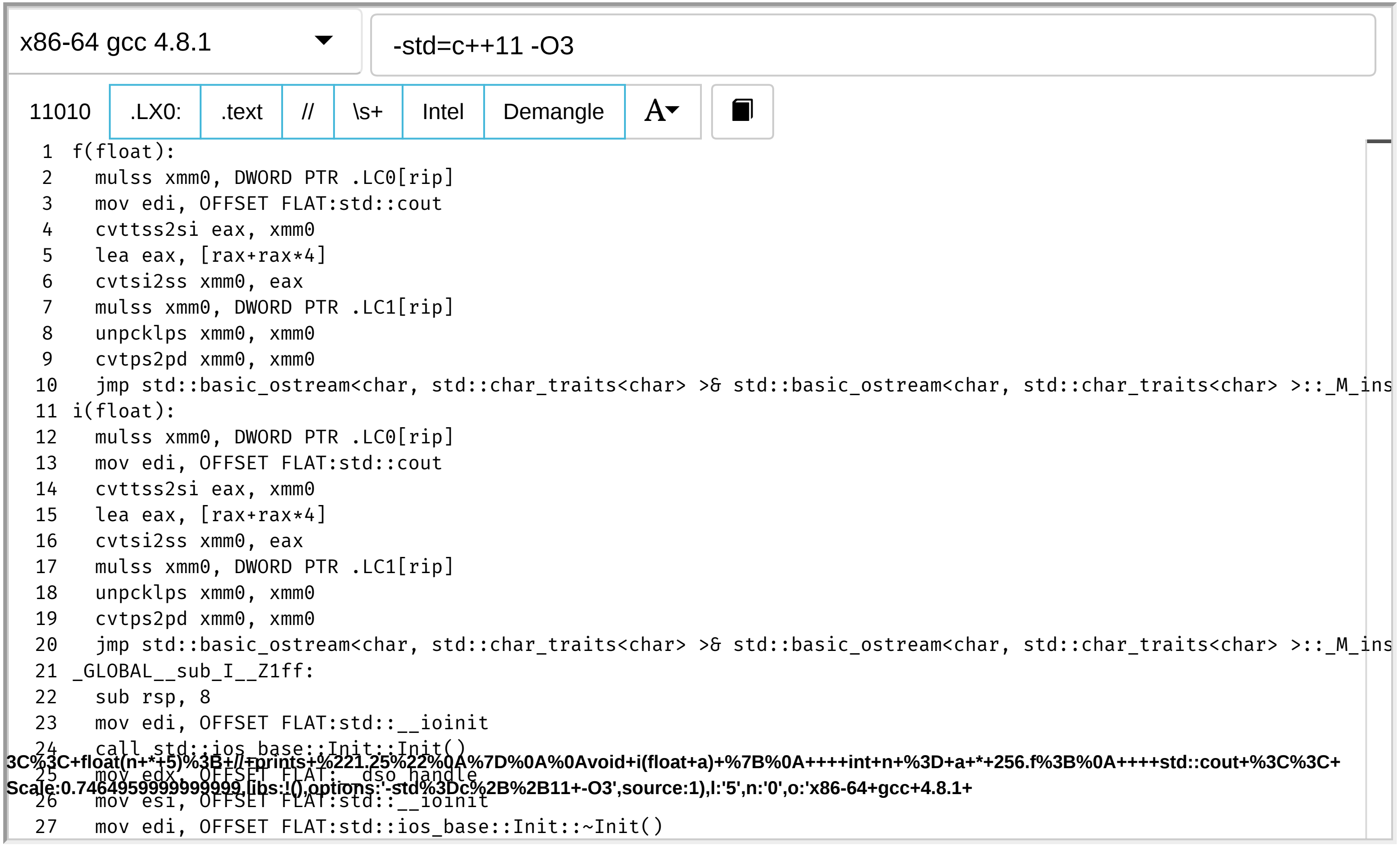
x86-64 gcc 4.8.1

-std=c++11 -O3

```
foo(float):
  mov eax, 1
  ret
```

++auto+fixed_plus_one+%3D+fixed+%2B+1%3B%0A++++return+fixed_plus_one+%3E+fixed%3B%0A%7D'),l:'5',n:'0',o:'C%2B%2B+source+%231',t:'0'),
ource:1),l:'5',n:'0',o:'x86-64+gcc+4.8.1+(Editor+%231,+Compiler+%231)',t:'0')),k:100,l:'4',m:99.99999999999999,n:'0',o:'',s:1,t:'0')),version:4)

The Good



x86-64 gcc 4.8.1

-std=c++11 -O3

```
f(float):
  mulss xmm0, DWORD PTR .LC0[rip]
  mov edi, OFFSET FLAT:std::cout
  cvttss2si eax, xmm0
  lea eax, [rax+rax*4]
  cvtsi2ss xmm0, eax
  mulss xmm0, DWORD PTR .LC1[rip]
  unpcklps xmm0, xmm0
  cvtps2pd xmm0, xmm0
  jmp std::basic_ostream<char, std::char_traits<char> >& std::basic_ostream<char, std::char_traits<char> >::_M_ins
i(float):
  mulss xmm0, DWORD PTR .LC0[rip]
  mov edi, OFFSET FLAT:std::cout
  cvttss2si eax, xmm0
  lea eax, [rax+rax*4]
  cvtsi2ss xmm0, eax
  mulss xmm0, DWORD PTR .LC1[rip]
  unpcklps xmm0, xmm0
  cvtps2pd xmm0, xmm0
  jmp std::basic_ostream<char, std::char_traits<char> >& std::basic_ostream<char, std::char_traits<char> >::_M_ins
_GLOBAL__sub_I__Z1ff:
  sub rsp, 8
  mov edi, OFFSET FLAT:std::__ioinit
  call std::ios_base::Init::Init()
  mov edx, OFFSET FLAT:__dso_handle
  mov esi, OFFSET FLAT:std::__ioinit
  mov edi, OFFSET FLAT:std::ios_base::Init::~Init()
```

3C%3C+float(n+*+5)%3B+//+prints+%221.25%22%0A%7D%0A%0Avoid+i(float+a)+%7B%0A++++int+n+%3D+a+*+256.f%3B%0A++++std::cout+%3C%3C+
Scale:0.7464959999999999,libs:!(),options:'-std%3Dc%2B%2B11+-O3',source:1),l:'5',n:'0',o:'x86-64+gcc+4.8.1+

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```
// range exceeded! (undefined behavior)
auto a = numeric_limits<int>::max() + 1;
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auto a = numeric_limits<int>::max() + 1;
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```
// also undefined behavior
auto b = numeric_limits<fixed_point<int, -16>>::max() + 1;
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```
// compiles
static_assert(1 == 1, "this does compile");

// error: static assertion failed: this does not compile
static_assert(1 != 1, "this does not compile");

// error: left shift count >= width of type
static_assert(1 << 1000, "this does not compile");
```

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// error: left shift count >= width of type
static_assert(1 << 1000, "this does not compile");
```

```
// compiles
static_assert(numeric_limits<fixed_point<int, -16>>::max() - 1, "this compiles");

// fatal error: static_assert expression is not an integral constant expression
static_assert(numeric_limits<fixed_point<int, -16>>::max() + 1, "this does not!");
```

The Bad

```
// wat?!?
static_assert(unsigned{1} < signed>{-1}, "OK(!)");
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// wat now?!?
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```

```
// wat now?!?
static_assert(fixed_point<unsigned>{1} < fixed_point<signed>{-1}, "OK(!)");
```

```
// C++17 feature: class template deduction
static_assert(fixed_point{1u} < fixed_point{-1});
```

The Ugly

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```
// multiplication
auto n = fixed_point<int, -8>{1.5};
auto nn = n * n;    // fixed_point<int, -16>;
```

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// multiplication
auto n = fixed_point<int, -8>{1.5};
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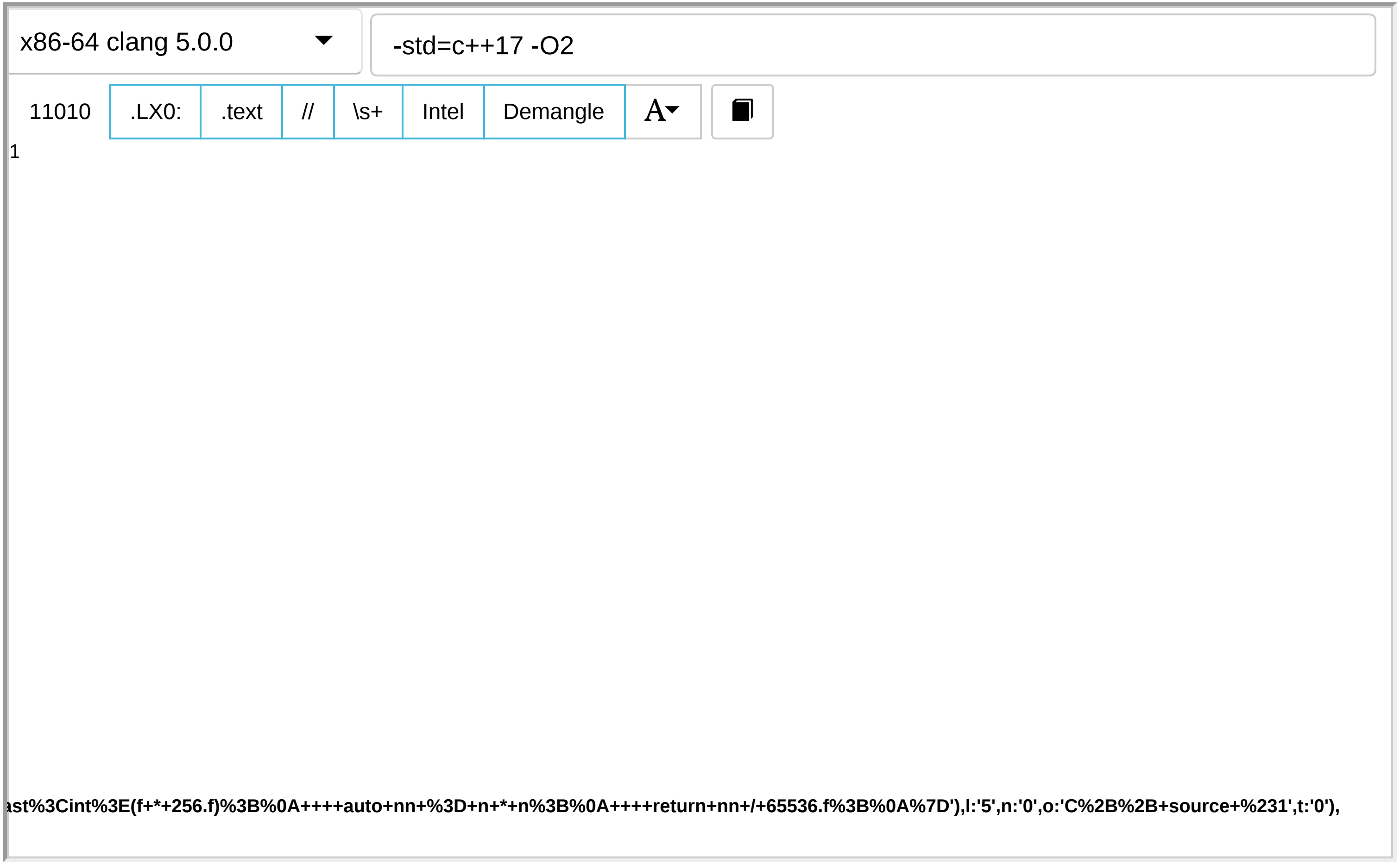
```

```
// (-8) + (-8) = -16
static_assert(std::is_same_v<decltype(nn), fixed_point<int, -16>>);
```

The Ugly



x86-64 clang 5.0.0

```
-std=c++17 -O2
```

```
ast%3Cint%3E(f+*+256.f)%3B%0A++++auto+nn+%3D+n+*+n%3B%0A++++return+nn+/+65536.f%3B%0A%7D'),l:'5',n:'0',o:'C%2B%2B+source+%231',t:'0'),
```

The Ugly

```
constexpr auto n = fixed_point<int, -8>{1.5};
constexpr auto d = fixed_point<int, -8>{2.25};
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constexpr auto n = fixed_point<int, -8>{1.5};
constexpr auto d = fixed_point<int, -8>{2.25};
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```
// (-8) - (-8) = 0
constexpr auto q = n / d;    // fixed_point<int, 0>;
```

The Ugly

```
constexpr auto n = fixed_point<int, -8>{1.5};
constexpr auto d = fixed_point<int, -8>{2.25};
```

```
// (-8) - (-8) = 0
constexpr auto q = n / d;    // fixed_point<int, 0>;
```

```
// (-8) - (23)
constexpr auto q = cnl::divide(n, d);    // fixed_point<long, -31>;
```

How Do You Solve a Problem Like Division?

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```
5.5 * 5.5 = 30.25
55. * .55 = 30.25
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1 / 100 = 0.01
10 / 5.5 = 1.818181818181...
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template<typename Integer> class fraction { Integer numerator, denominator; ... };
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AAA.BBBBB * CCCCCC.DD = AAACCCCCC.BBBBBDD
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AAA.BBBBB / CCCCCC.DD = AAADD.BBBBBCCCCCC
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template<typename Integer> class fraction { Integer numerator, denominator; ... };
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AAA.BBBBB * CCCCCC.DD = AAACCCCCC.BBBBBDD
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AAA.BBBBB / CCCCCC.DD = AAADD.BBBBBCCCCCC
```

```
constexpr auto n = fixed_point<int, -8>{1.5};
constexpr auto d = fixed_point<int, -8>{2.25};
constexpr auto q = cnl::divide(n, d);    // fixed_point<long, -31>;
```

Elasticity

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```
auto n = fixed_point<uint8_t, -8>{0.99609375};
auto nn = n * n;
```

Elasticity

```
auto n = fixed_point<uint8_t, -8>{0.99609375};
auto nn = n * n;    // fixed_point<int, -16>{0.9922027587890625};
```

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```
auto n = fixed_point<uint8_t, -8>{0.99609375};
auto nn = n * n;    // fixed_point<int, -16>{0.9922027587890625};
```

```
auto n = fixed_point<int, -31>{0.99609375};
auto nn = n * n;    // fixed_point<int, -62>{?!?!?!?!?!?!};
```

Elasticity

```
template<int Digits, class Narrowest = int>
class elastic_integer { WideEnoughInteger r; /* other stuff */ };
```

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template<int Digits, class Narrowest = int>
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```
#include <cnl/elastic_integer.h>
using cnl::elastic_integer;
auto e = elastic_integer<31>{0x7FFFFFFF};   // r has 31 or more digits
```

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template<int Digits, class Narrowest = int>
class elastic_integer { WideEnoughInteger r; /* other stuff */ };
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```
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auto e = elastic_integer<31>{0x7FFFFFFF};   // r has 31 or more digits
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```
auto ee = e * e;   // elastic_integer<62>{INT64_C(0x3FFFFFFF00000001)}
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```

```
auto ee = e * e;   // elastic_integer<62>{INT64_C(0x3FFFFFFF00000001)}
```

```
auto _2ee = ee + ee;   // elastic_integer<63>{INT64_C(0x7FFFFFFE00000002)}
```

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template<int Digits, class Narrowest = int>
class elastic_integer { WideEnoughInteger r; /* other stuff */ };
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auto _2ee = ee + ee;   // elastic_integer<63>{INT64_C(0x7FFFFFFE00000002)}
```

```
auto fpe = fixed_point<elastic_integer<31>, -31>{0.99609375};
```

Elasticity

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template<int Digits, class Narrowest = int>
class elastic_integer { WideEnoughInteger r; /* other stuff */ };
```

```
#include <cnl/elastic_integer.h>
using cnl::elastic_integer;
auto e = elastic_integer<31>{0x7FFFFFFF};   // r has 31 or more digits
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auto ee = e * e;   // elastic_integer<62>{INT64_C(0x3FFFFFFF00000001)}
```

```
auto _2ee = ee + ee;   // elastic_integer<63>{INT64_C(0x7FFFFFFE00000002)}
```

```
auto fpe = fixed_point<elastic_integer<31>, -31>{0.99609375};
```

```
auto sq = fpe * fpe;  // fixed_point<elastic_integer<62>, -62>{0.9922027587890625}
```

Elasticity

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template<int Digits, class Narrowest = int>
class elastic_integer { WideEnoughInteger r; /* other stuff */ };
```

```
#include <cnl/elastic_integer.h>
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auto e = elastic_integer<31>{0x7FFFFFFF};   // r has 31 or more digits
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auto ee = e * e;   // elastic_integer<62>{INT64_C(0x3FFFFFFF00000001)}
```

```
auto _2ee = ee + ee;   // elastic_integer<63>{INT64_C(0x7FFFFFFE00000002)}
```

```
auto fpe = fixed_point<elastic_integer<31>, -31>{0.99609375};
```

```
auto sq = fpe * fpe;  // fixed_point<elastic_integer<62>, -62>{0.9922027587890625}
```

```
#include <cnl/auxiliary/elastic_fixed_point.h>
auto q = sq / sq; // fixed_point<elastic_integer<124>, -62>{1}, q), "");
```

Run-time Safety

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```
#include <cnl/safe_integer.h>
using cnl::safe_integer;
auto i = safe_integer<uint8_t>{255};
```

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using cnl::safe_integer;
auto i = safe_integer<uint8_t>{255};
```

```
auto j = i + 1;
```

Run-time Safety

```
#include <cnl/safe_integer.h>
using cnl::safe_integer;
auto i = safe_integer<uint8_t>{255};
```

```
auto j = i + 1; // safe_integer<int>{256}
```

Run-time Safety

```
#include <cnl/safe_integer.h>
using cnl::safe_integer;
auto i = safe_integer<uint8_t>{255};
```

```
auto j = i + 1; // safe_integer<int>{256}
```

```
safe_integer<uint8_t> k = i + 1;  // throw std::overflow_error
```

Run-time Safety

```
#include <cnl/safe_integer.h>
using cnl::safe_integer;
auto i = safe_integer<uint8_t>{255};
```

```
auto j = i + 1; // safe_integer<int>{256}
```

```
safe_integer<uint8_t> k = i + 1;  // throw std::overflow_error
```

```
constexpr safe_integer<uint8_t> k = i + 1;
static_assert(cnl::_impl::identical(safe_integer<int>{256}, k));
```

```
[ 29%] Building CXX object src/test/CMakeFiles/fp_test.dir/cppcon2017.cpp.o
/home/john/cnl/src/test/cppcon2017.cpp:151:37: fatal error: constexpr variable 'k' must be i
    constexpr safe_integer<uint8_t> k = i + 1;
                                    ^~~~~~~~
/home/john/cnl/include/cnl/overflow.h:52:40: note: subexpression not valid in a constant exp
            return condition ? value : throw std::overflow_error("");
                                       ^
```

and so on...

Deduction and UDLs

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```
auto x = fixed_point{42ul}; // fixed_point<unsigned long, 0>{42}
```

Deduction and UDLs

```
auto x = fixed_point{42ul}; // fixed_point<unsigned long, 0>{42}
```

```
auto y = fixed_point{128};  // fixed_point<int, 0>{1}
```

Deduction and UDLs

```
auto x = fixed_point{42ul}; // fixed_point<unsigned long, 0>{42}
```

```
auto y = fixed_point{128};  // fixed_point<int, 0>{1}
```

```
using cnl::literals;
auto z = fixed_point{128_c};  // fixed_point<int, 7>{128}
```

Deduction and UDLs

```
auto x = fixed_point{42ul}; // fixed_point<unsigned long, 0>{42}
```

```
auto y = fixed_point{128};  // fixed_point<int, 0>{1}
```

```
using cnl::literals;
auto z = fixed_point{128_c};  // fixed_point<int, 7>{128}
```

```
auto a = fixed_point{0b10000000000000000000000000000000000000000_c};
// a === fixed_point<int, 40>{0b10000000000000000000000000000000000000000l}
```

Deduction and UDLs

```
auto x = fixed_point{42ul}; // fixed_point<unsigned long, 0>{42}
```

```
auto y = fixed_point{128};  // fixed_point<int, 0>{1}
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using cnl::literals;
auto z = fixed_point{128_c};  // fixed_point<int, 7>{128}
```

```
auto a = fixed_point{0b100000000000000000000000000000000000000000_c};
// a === fixed_point<int, 40>{0b100000000000000000000000000000000000000000l}
```

```
auto b = fixed_point{0b11111111111111111111111111111111111111111_c};
// b === fixed_point<long, 0>{0b11111111111111111111111111111111111111111l}
```

Deduction and UDLs

Deduction and UDLs

```
auto c = elastic_integer{2017_c}; // elastic_integer<11>{2017}
```

Deduction and UDLs

```
auto c = elastic_integer{2017_c}; // elastic_integer<11>{2017}
```

```
auto e = 0x7f000_elastic; // fixed_point<elastic_integer<7>, 12>{0x7f000}
```

Interoperability - Boost.Multiprecision

```
#include <cnl/auxiliary/boost.multiprecision.h>
using namespace boost::multiprecision;

template<int NumBits, int Exponent = 0>
using mp_fixed_point = cnl::fixed_point<
        number<cpp_int_backend<NumBits, NumBits, signed_magnitude, unchecked, void
        Exponent>;
```

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- googol (10^{100}) ✓
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- googolplex (10^{googol}) ✗

Interoperability - Boost.SIMD

```
#include <cnl/auxiliary/boost.simd.h>
using boost::simd::pack;

template<class T, std::size_t N, int Exponent>
using fixed_point_pack = fixed_point<pack<T, N>, Exponent>;
```

Interoperability - Boost.SIMD

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#include <cnl/auxiliary/boost.simd.h>
using boost::simd::pack;

template<class T, std::size_t N, int Exponent>
using fixed_point_pack = fixed_point<pack<T, N>, Exponent>;
```

```
using fpp = fixed_point_pack<int, 4, -16>;
using initializer = initializer<fpp>;

auto expected = fpp{initializer{7.9375+-1, -8.+.125, 0+-5, 3.5+-3.5}};
auto augend = fpp{initializer{7.9375, -8., 0, 3.5}};
auto addend = fpp{initializer{-1, .125, -5, -3.5}};
auto sum = augend + addend;
```

Interoperability - Boost.SIMD

```
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             !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
          !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
        !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
      !!!!!!""""""""""""""""""""!!!!!!!!!!!!!!!!!!!!!
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   !!""""""""""""#####$$%'*a2'%$###""""!!!!!!!!!!!!!!!!!
    !!!!""""""""""""####$$&(%%###"""!!!!!!!!!!!!!!!!!!!
     !!!!!!!""""""""""""""""""""!!!!!!!!!!!!!!!!!!!!!!
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          !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
             !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
                  !!!!!!!!!!!!!!!!!!!!!!!
```

CNL Today and Tomorrow

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- Arbitrary width

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- Full Support for Rounding and Overflow

```
template<class Rep = int, class RoundingTag = closest_rounding_tag>
class precise_integer;
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template<class Rep = int, class RoundingTag = closest_rounding_tag>
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- Full complement of operators for `safe_integer` and `precise_integer`
- Full complement of free functions

```
add(saturated_overflow, UINT32_C(0xFFFFFFFF), UINT32_C(0x12345678))
divide(closest_rounding_tag, 2, 3);
```

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add(saturated_overflow, UINT32_C(0xFFFFFFFF), UINT32_C(0x12345678))
divide(closest_rounding_tag, 2, 3);
```

- Better Literals

```
auto a = 0b1111.1111_elastic;  // fixed_point<elastic_integer<8>, -4>
```

Thank You!

Now go to GitHub and try it out! `github.com/johnmcfarlane/cnl`

```
git clone https://github.com/johnmcfarlane/cnl.git
```

Bonus Slides - Beyond Exponents

`std::ratio` scales things statically:

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template<int Num, int Denom = 1> class ratio;
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What if `Exponent` was replaced with a type parameter?

```
// equivalent to fixed_point<int, -8>
using a = fixed_point<int, ratio<1, 256>>;
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What if `Exponent` was replaced with a type parameter?

```
// equivalent to fixed_point<int, -8>
using a = fixed_point<int, ratio<1, 256>>;
```

This should be possible: simply separate `fixed_point`'s two concerns:

```
// a type which stores an integer and scales it
template<typename Rep, typename Scale>
class scaled_integer;

// a type which scales integers
template<int Base, int Power> class exponent;
```

Bonus Slides - Beyond Exponents

Now `fixed_point` is just one of many `scaled_integer` types.

```
template<typename Rep, int Power>
using fixed_point = scaled_integer<Rep, exponent<2, Power>>

// (Decimal fixed-point is already on the to-do list.)
template<typename Rep, int Power>
using decimal_fixed_point = scaled_integer<Rep, exponent<10, Power>>
```

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// (Decimal fixed-point is already on the to-do list.)
template<typename Rep, int Power>
using decimal_fixed_point = scaled_integer<Rep, exponent<10, Power>>
```

And a souped-up `std::ratio` is another way to scale integers.

```
template<typename Rep, int Power>
using dollar = scaled_integer<Rep, ratio<1, 100>>;

template<typename Rep, int Power>
using angle = scaled_integer<Rep, ratio<1, 360>>;

// units in the range [0, 1]
template<typename Rep>
using unit = scaled_integer<Rep, ratio<1, std::numeric_limits<Rep>::max()>>;
```

Bonus Slides - Beyond Exponents

And scaling of integers is just the beginning…

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```
// type-safety prevents units from being confused
struct length_tag {};
struct time_tag {};

// individual quantities
template<typename BaseTag, int Exponent>
struct base_quantity;

// is quantity a scale, like exponent or std::ratio?
template<typename ... BaseQuantities>
struct quantity;

using time = quantity<base_quantity<time_tag, 1>>;
using length = quantity<base_quantity<length_tag, 1>>;

template<typename ... BaseQuantitiesA, typename ... BaseQuantitiesB>
auto operator/(quantity<BaseQuantitiesA...>, quantity<BaseQuantitiesB...>)

// quantity<base_quantity<length_tag, 1>, base_quantity<time_tag, -1>>
auto speed = length{} / time{};
```

Disclaimer: none of this compiles ... yet!

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