

Better Numeric Types in C++

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ACCU Bay Area - 2018-05-09

Background

Background

The screenshot shows the GitHub repository page for `johnmcfarlane/cnl`. The repository is titled "A Compositional Numeric Library for C++". It has 1,317 commits, 13 branches, 1 release, and 5 contributors. The repository is currently on the `develop` branch. The file list includes `.idea/codeStyles`, `doc`, `include`, `src`, `appveyor.yml`, `editorconfig`, `gitignore`, `gitmodules`, `travis-setup-linux.sh`, `travis.yml`, `CMakeLists.txt`, `LICENSE_1_0.txt`, and `README.md`. The `README.md` file is selected and displayed below the file list. It contains the title "CNL: A Compositional Numeric Library for C++", build status indicators, a description of the library, and a list of features.

johnmcfarlane / **cnl**

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A Compositional Numeric Library for C++

1,317 commits 13 branches 1 release 5 contributors

Branch: develop New pull request Create new file Upload files Find file Clone or download

File	Commit	Time
<code>.idea/codeStyles</code>	adds CLion formatting config file	4 months ago
<code>doc</code>	update gh-pages submodule reference w. fractional docs	10 days ago
<code>include</code>	Vc::sind interop tests (#147)	7 days ago
<code>src</code>	Vc::sind interop tests (#147)	7 days ago
<code>appveyor.yml</code>	adds support for ctest command	2 months ago
<code>editorconfig</code>	adjustment to cmake in .editorconfig	a year ago
<code>gitignore</code>	removed tenuous .gitignore entries	a year ago
<code>gitmodules</code>	updates gh-pages submodule reference and url	3 months ago
<code>travis-setup-linux.sh</code>	removes redundant apt source from travis script	7 months ago
<code>travis.yml</code>	adds support for ctest command	2 months ago
<code>CMakeLists.txt</code>	adds support for ctest command	2 months ago
<code>LICENSE_1_0.txt</code>	adds boost license	2 years ago
<code>README.md</code>	minor clarification to README.md	10 days ago

README.md

CNL: A Compositional Numeric Library for C++

Build passing Build passing

CNL is a numerics library written in C++ which aims to do for `float` what the STL does for arrays. Its main roles are to help:

- approximate real numbers using fixed-point arithmetic;
- detect and prevent overflow and
- increase precision through alternative rounding modes.

A compositional design promotes seamless interoperability between numeric types. And by providing the thinnest wrappers over the widest range of numeric types, CNL aims to assist the development of:

- large-scale simulations including video games and scientific models;
- resource-constrained applications on embedded and mobile devices and

← github.com/johnmcfarlane/cnl

The Problem with Integers

The Problem with Integers

1. Fixed resolution

The Problem with Integers

1. Fixed resolution
2. Limited range

The Problem with Integers

1. Fixed resolution
2. Limited range
3. 'Interesting' arithmetic behavior

The Problem with Floating-Point

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- $\pm 1.\text{significand} * 2^{\text{exponent}}$
- special values, denormalized values, -0

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3. `<cmath>` functions lack `constexpr`

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4. Variable resolution

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2. Occasional weirdness can surprise:

- determinism, associativity, commutativity and ordering

3. `<cmath>` functions lack `constexpr`

4. Variable resolution

5. Costly in energy and silicon

Analysis

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- Floating-point problems are not so bad.

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- Floating-point problems are not *so* bad.
- Integers are a powerful abstraction over registers.
- But we can do a lot better.
- so ...

Goal of CNL

"Do for `int` what the STL did for `[]`."

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- Provide zero-cost abstractions over language-level features:

```
std::array<T, N> a; // T a[N]  
std::array<T, N>::iterator i = std::begin(a); // T* i
```

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- Maintain a familiar interface:

```
auto const& third = a[2];  
for (auto const& element : a) { /* ... */ }
```

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- Allow users to opt in to positive-cost functionality:

```
std::array<T, N> a;  
auto const& bad_element = a.at(N); // throws std::out_of_range!
```

Goal of CNL

"Do for `int` what the STL did for `[]`."

- Provide zero-cost abstractions over language-level features:

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std::array<T, N> a; // T a[N]  
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auto const& third = a[2];  
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- Allow users to opt in to positive-cost functionality:

```
std::array<T, N> a;  
auto const& bad_element = a.at(N); // throws std::out_of_range!
```

- And most importantly...

Goal of CNL

"Do for `int` what the STL did for `[]`."

- Compose!

```
using fs_cache = unordered_map<filesystem::path, vector<byte>>;
```


Non-Goal

"Don't do for `int` what STL doesn't do for `[]`."

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"Don't do for `int` what STL doesn't do for `[]`."

- Don't make the user pay for what they don't use.

Fixed-Point Arithmetic

Fixed-Point Arithmetic

```
namespace cnl {  
    template<typename Rep=int, int Exponent=0, int Radix=2>  
    class fixed_point {  
        // ...  
    private:  
        Rep r;  
    };  
}
```

Fixed-Point Arithmetic

```
namespace cnl {  
  
    template<typename Rep=int, int Exponent=0, int Radix=2>  
    class fixed_point {  
        // ...  
    private:  
        Rep r;  
    };  
  
}
```

Example usage:

```
#include <cnl/fixed_point.h>  
  
void f() {  
    auto n = cnl::fixed_point<int, -8>{ 0.25 };  
    std::cout << n * 5; // prints "1.25"  
}
```

The Good

The Good

```
// what the programmer writes
bool foo(float f) {
    auto fixed = fixed_point<int, -16>{f};
    auto fixed_plus_one = fixed + 1;
    return fixed_plus_one > fixed;
}
```

The Good

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// what the programmer writes
bool foo(float f) {
    auto fixed = fixed_point<int, -16>{f};
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```
// what the compiler sees
bool foo(float) {
    return true;
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The Good

```
// what the programmer writes
bool foo(float f) {
    auto fixed = fixed_point<int, -16>{f};
    auto fixed_plus_one = fixed + 1;
    return fixed_plus_one > fixed;
}
```

```
// what the compiler sees
bool foo(float) {
    return true;
}
```

```
+auto+fixed_plus_one+%3D+fixed+%2B+1%3B%0A++++return+fixed_plus_one+%3E+fixed%3B%0A%7D'),l:'5',n:'0',o:'C%2B%2B+source+%231',t:'0'),
'5',n:'0',o:'x86-64+gcc+4.8.1+(Editor+%231,+Compiler+%231)+C%2B%2B',t:'0')),k:100,l:'4',m:99.99999999999999,n:'0',o:',s:1,t:'0')),version:4)
```

The Good

A

Save/Load

C++

x86-64 gcc 4.8.1

-std=c++11 -O3

11010

.LX0:

.text

//

\s+

Intel

Demangle

A

Libraries

```

1  #include <https://gist.githubusercontent.com/johnmcfarlane/11010>
2  #include <iostream>
3
4  using cnl::fixed_point;
5
6  void f(float a) {
7      auto n = fixed_point<int, -8>{ a };
8      std::cout << float(n * 5); // prints "1.25"
9  }
10
11 void i(float a) {
12     int n = a * 256.f;
13     std::cout << (n * 5) / 256.f; // prints "1.25"
14 }

```

```

1 f(float):
2     mulss xmm0, DWORD PTR .LC0[rip]
3     mov edi, OFFSET FLAT:std::cout
4     cvttss2si eax, xmm0
5     lea eax, [rax+rax*4]
6     cvtsi2ss xmm0, eax
7     mulss xmm0, DWORD PTR .LC1[rip]
8     unpcklps xmm0, xmm0
9     cvtps2pd xmm0, xmm0
10    jmp std::basic_ostream<char, std::char_traits<char>>::<
11 i(float):
12     mulss xmm0, DWORD PTR .LC0[rip]
13     mov edi, OFFSET FLAT:std::cout
14     cvttss2si eax, xmm0
15     lea eax, [rax+rax*4]
16     cvtsi2ss xmm0, eax
17     mulss xmm0, DWORD PTR .LC1[rip]
18     unpcklps xmm0, xmm0
19     cvtps2pd xmm0, xmm0
20    jmp std::basic_ostream<char, std::char_traits<char>>::<
21     sub rsp, 8
22     mov edi, OFFSET FLAT:std::ioinit

```

The Bad

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```
// range exceeded! (undefined behavior)  
auto a = numeric_limits<int>::max() + 1;
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auto a = numeric_limits<int>::max() + 1;
```

```
// also undefined behavior
auto b = numeric_limits<fixed_point<int, -16>>::max() + 1;
```

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auto b = numeric_limits<fixed_point<int, -16>>::max() + 1;
```

```
// compiles
static_assert(1 == 1, "this does compile");

// error: static assertion failed: this does not compile
static_assert(1 != 1, "this does not compile");

// error: left shift count >= width of type
static_assert(1 << 1000, "this does not compile");
```

The Bad

```
// range exceeded! (undefined behavior)
auto a = numeric_limits<int>::max() + 1;
```

```
// also undefined behavior
auto b = numeric_limits<fixed_point<int, -16>>::max() + 1;
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// compiles
static_assert(1 == 1, "this does compile");

// error: static assertion failed: this does not compile
static_assert(1 != 1, "this does not compile");

// error: left shift count >= width of type
static_assert(1 << 1000, "this does not compile");
```

```
// compiles
static_assert(numeric_limits<fixed_point<int, -16>>::max() - 1, "this compiles");

// fatal error: static_assert expression is not an integral constant expression
static_assert(numeric_limits<fixed_point<int, -16>>::max() + 1, "this does not!");
```

The Bad

```
// compiles  
static_assert(unsigned{1} < signed{-1}, "wat?");
```


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// compiles  
static_assert(unsigned{1} < signed{-1}, "wat?");
```

```
// compiles  
static_assert(fixed_point<unsigned>{1} < fixed_point<signed>{-1}, "huh?");
```

The Bad

```
// compiles  
static_assert(unsigned{1} < signed{-1}, "wat?");
```

```
// compiles  
static_assert(fixed_point<unsigned>{1} < fixed_point<signed>{-1}, "huh?");
```

```
// compiles (C++17)  
static_assert(fixed_point{1U} < fixed_point{-1});
```

The Ugly

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```
// multiplication  
auto n = cnl::fixed_point<int, -8>{1.5};  
auto nn = n * n;    // type?
```

The Ugly

```
// multiplication
auto n = cnl::fixed_point<int, -8>{1.5};
auto nn = n * n;    // type?
```

```
// (-8) + (-8) = -16
static_assert(std::is_same_v<decltype(nn), cnl::fixed_point<int, -16>>);
```

The Ugly

```
t%3Cint%3E(f!*+256.f)%3B%0A++++auto+nn+%3D+n*+n%3B%0A++++return+nn+/+65536.f%3B%0A%7D'),l:'5',n:'0',o:'C%2B%2B+source+%231',t:'0'),
```

The Ugly

```
constexpr auto n = cnl::fixed_point<int, -8>{1.5};  
constexpr auto d = cnl::fixed_point<int, -8>{2.25};
```

The Ugly

```
constexpr auto n = cnl::fixed_point<int, -8>{1.5};  
constexpr auto d = cnl::fixed_point<int, -8>{2.25};
```

```
// (-8) - (-8) = 0  
constexpr auto q = n / d;    // cnl::fixed_point<int, 0>;
```


The Ugly

```
constexpr auto n = cnl::fixed_point<int, -8>{1.5};  
constexpr auto d = cnl::fixed_point<int, -8>{2.25};
```

```
// (-8) - (-8) = 0  
constexpr auto q = n / d;    // cnl::fixed_point<int, 0>;
```

```
constexpr auto r = n % d;    // cnl::fixed_point<int, -8>;
```

The Ugly

```
constexpr auto n = cnl::fixed_point<int, -8>{1.5};  
constexpr auto d = cnl::fixed_point<int, -8>{2.25};
```

```
// (-8) - (-8) = 0  
constexpr auto q = n / d;    // cnl::fixed_point<int, 0>;
```

```
constexpr auto r = n % d;    // cnl::fixed_point<int, -8>;
```

```
// (-8) - (23)  
constexpr auto q = cnl::divide(n, d);    // cnl::fixed_point<long, -31>;
```

How Do You Solve a Problem Like Division?

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$$5.5 * 5.5 = 30.25$$

$$55. * .55 = 30.25$$

$$5.5 * 55. = 302.5$$

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$$AAA.BBBBB * CCCCC.DD = AAACCCCC.BBBBDD$$

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$$10 / 100 = 00.01$$

$$1 / 1000 = 0.001$$

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$$10 / 100 = 00.01$$

$$1 / 1000 = 0.001$$

$$AAA.BBBBB / CCCCC.DD = AAADD.BBBBBCCCCC$$

$$1 / 3 = 1.33333333\dots$$

How Do You Solve a Problem Like Division?

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```
template<typename Numerator, typename Denominator>
struct fractional {
    Numerator numerator;
    Denominator denominator;
};
```

How Do You Solve a Problem Like Division?

```
template<typename Numerator, typename Denominator>
struct fractional {
    Numerator numerator;
    Denominator denominator;
};
```

```
constexpr auto n = fixed_point<int16_t, -8>{1.5};
constexpr auto d = fixed_point<int16_t, -8>{2.25};
constexpr auto f = fractional{n, d};
constexpr auto q = fixed_point{f}; // fixed_point<int32_t, -15>{.66666667}
```

Elasticity

Elasticity

```
auto n = fixed_point<uint8_t, -8>{0.99609375};  
auto nn = n * n;
```

Elasticity

```
auto n = fixed_point<uint8_t, -8>{0.99609375};  
auto nn = n * n;    // fixed_point<int, -16>{0.9922027587890625};
```

Elasticity

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auto n = fixed_point<uint8_t, -8>{0.99609375};  
auto nn = n * n;    // fixed_point<int, -16>{0.9922027587890625};
```

```
auto n = fixed_point<int, -31>{0.99609375};  
auto nn = n * n;    // fixed_point<int, -62>{?!?!?!?!?!?!};
```

Elasticity

```
template<int Digits, class Narrowest = int>  
class elastic_integer { WideEnoughInteger r; /* other stuff */ };
```


Elasticity

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template<int Digits, class Narrowest = int>  
class elastic_integer { WideEnoughInteger r; /* other stuff */ };
```

```
#include <cnl/elastic_integer.h>  
using cnl::elastic_integer;  
auto e = elastic_integer<31>{0x7FFFFFFF}; // r has 31 or more digits
```

Elasticity

```
template<int Digits, class Narrowest = int>  
class elastic_integer { WideEnoughInteger r; /* other stuff */ };
```

```
#include <cnl/elastic_integer.h>  
using cnl::elastic_integer;  
auto e = elastic_integer<31>{0x7FFFFFFF}; // r has 31 or more digits
```

```
auto ee = e * e; // elastic_integer<62>{INT64_C(0x3FFFFFFFF00000001)}
```

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template<int Digits, class Narrowest = int>  
class elastic_integer { WideEnoughInteger r; /* other stuff */ };
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#include <cnl/elastic_integer.h>  
using cnl::elastic_integer;  
auto e = elastic_integer<31>{0x7FFFFFFF}; // r has 31 or more digits
```

```
auto ee = e * e; // elastic_integer<62>{INT64_C(0x3FFFFFFF00000001)}
```

```
auto _2ee = ee + ee; // elastic_integer<63>{INT64_C(0x7FFFFFFE00000002)}
```

Elasticity

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template<int Digits, class Narrowest = int>
class elastic_integer { WideEnoughInteger r; /* other stuff */ };
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#include <cnl/elastic_integer.h>
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```
auto ee = e * e; // elastic_integer<62>{INT64_C(0x3FFFFFFF00000001)}
```

```
auto _2ee = ee + ee; // elastic_integer<63>{INT64_C(0x7FFFFFFE00000002)}
```

```
auto fpe = fixed_point<elastic_integer<31>, -31>{0.99609375};
```

Elasticity

```
template<int Digits, class Narrowest = int>
class elastic_integer { WideEnoughInteger r; /* other stuff */ };
```

```
#include <cnl/elastic_integer.h>
using cnl::elastic_integer;
auto e = elastic_integer<31>{0x7FFFFFFF}; // r has 31 or more digits
```

```
auto ee = e * e; // elastic_integer<62>{INT64_C(0x3FFFFFFF00000001)}
```

```
auto _2ee = ee + ee; // elastic_integer<63>{INT64_C(0x7FFFFFFE00000002)}
```

```
auto fpe = fixed_point<elastic_integer<31>, -31>{0.99609375};
```

```
auto sq = fpe * fpe; // fixed_point<elastic_integer<62>, -62>{0.9922027587890625}
```

Elasticity

```
template<int Digits, class Narrowest = int>
class elastic_integer { WideEnoughInteger r; /* other stuff */ };
```

```
#include <cnl/elastic_integer.h>
using cnl::elastic_integer;
auto e = elastic_integer<31>{0x7FFFFFFF}; // r has 31 or more digits
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auto ee = e * e; // elastic_integer<62>{INT64_C(0x3FFFFFFF00000001)}
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auto _2ee = ee + ee; // elastic_integer<63>{INT64_C(0x7FFFFFFE00000002)}
```

```
auto fpe = fixed_point<elastic_integer<31>, -31>{0.99609375};
```

```
auto sq = fpe * fpe; // fixed_point<elastic_integer<62>, -62>{0.9922027587890625}
```

```
auto q = fixed_point{fractional{sq, sq}};
// fixed_point<elastic_integer<124>, -62>{1}
```

Run-time Safety

Run-time Safety

```
#include <cnl/safe_integer.h>
using cnl::safe_integer;
auto i = safe_integer<uint8_t>{255};
```


Run-time Safety

```
#include <cnl/safe_integer.h>
using cnl::safe_integer;
auto i = safe_integer<uint8_t>{255};
```

```
auto j = i + 1;
```

Run-time Safety

```
#include <cnl/safe_integer.h>
using cnl::safe_integer;
auto i = safe_integer<uint8_t>{255};
```

```
auto j = i + 1; // safe_integer<int>{256}
```

Run-time Safety

```
#include <cnl/safe_integer.h>  
using cnl::safe_integer;  
auto i = safe_integer<uint8_t>{255};
```

```
auto j = i + 1; // safe_integer<int>{256}
```

```
safe_integer<uint8_t> k = i + 1; // throw std::overflow_error
```

Run-time Safety

```
#include <cnl/safe_integer.h>
using cnl::safe_integer;
auto i = safe_integer<uint8_t>{255};
```

```
auto j = i + 1; // safe_integer<int>{256}
```

```
safe_integer<uint8_t> k = i + 1; // throw std::overflow_error
```

```
constexpr safe_integer<uint8_t> k = i + 1;
static_assert(cnl::_impl::identical(safe_integer<int>{256}, k));
```

```
[ 29%] Building CXX object src/test/CMakeFiles/fp_test.dir/cppcon2017.cpp.o
/home/john/cnl/src/test/cppcon2017.cpp:151:37: fatal error: constexpr variable 'k' must be initialized
constexpr safe_integer<uint8_t> k = i + 1;
                                   ^~~~~~
/home/john/cnl/include/cnl/overflow.h:52:40: note: subexpression not valid in a constant expression
return condition ? value : throw std::overflow_error("");
                        ^
```

and so on...

Deduction and UDLs

Deduction and UDLs

```
auto x = fixed_point{42UL}; // fixed_point<unsigned long, 0>{42}
```

Deduction and UDLs

```
auto x = fixed_point{42UL}; // fixed_point<unsigned long, 0>{42}
```

```
auto y = fixed_point{128}; // fixed_point<int, 0>{128}
```

Deduction and UDLs

```
auto x = fixed_point{42UL}; // fixed_point<unsigned long, 0>{42}
```

```
auto y = fixed_point{128}; // fixed_point<int, 0>{128}
```

```
using cnl::literals;  
auto z = fixed_point{128_c}; // fixed_point<int, 7>{128}
```


Deduction and UDLs

```
auto x = fixed_point{42UL}; // fixed_point<unsigned long, 0>{42}
```

```
auto y = fixed_point{128}; // fixed_point<int, 0>{128}
```

```
using cnl::literals;
auto z = fixed_point{128_c}; // fixed_point<int, 7>{128}
```

```
auto a = fixed_point{0b100000000000000000000000000000000_c};  
// a == fixed_point<int, 40>{0b100000000000000000000000000000000L}
```

Deduction and UDLs

```
auto x = fixed_point{42UL}; // fixed_point<unsigned long, 0>{42}
```

```
auto y = fixed_point{128}; // fixed_point<int, 0>{128}
```

```
using cnl::literals;
auto z = fixed_point{128_c}; // fixed_point<int, 7>{128}
```

```
auto a = fixed_point{0b100000000000000000000000000000000_c};  
// a == fixed_point<int, 40>{0b100000000000000000000000000000000L}
```

```
auto b = fixed_point{0b11111111111111111111111111111111_c};  
// b == fixed_point<long, 0>{0b11111111111111111111111111111111}
```

Deduction and UDLs

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```
auto c = elastic_integer{2018_c}; // elastic_integer<11>{2018}
```

Deduction and UDLs

```
auto c = elastic_integer{2018_c}; // elastic_integer<11>{2018}
```

```
auto e = 0x7f000_elastic; // fixed_point<elastic_integer<7>, 12>{0x7f000}
```

Deduction and UDLs

```
auto c = elastic_integer{2018_c}; // elastic_integer<11>{2018}
```

```
auto e = 0x7f000_elastic; // fixed_point<elastic_integer<7>, 12>{0x7f000}
```

```
auto s = e >> 1_c; // fixed_point<elastic_integer<7>, 11>{0x3f800}
```

Interoperability - Boost.Multiprecision

```
#include <cnl/auxiliary/boost.multiprecision.h>
using namespace boost::multiprecision;

template<int NumBits, int Exponent = 0>
using mp_fixed_point = cnl::fixed_point<
    number<cpp_int_backend<NumBits, NumBits, signed_magnitude, unchecked, void
    Exponent>;
```

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Fixed-Point + Boost.Multiprecision:

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Fixed-Point + Boost.Multiprecision:

- googol (10^{100}) ✓

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- googol (10^{100}) ✓
- googolth ($1 / \text{googol}$) ✓
- googolplex (10^{googol}) ✗

Interoperability - Boost.SIMD

```
#include <cnl/auxiliary/boost.simd.h>
using boost::simd::pack;

template<class T, std::size_t N, int Exponent>
using fixed_point_pack = fixed_point<pack<T, N>, Exponent>;
```

Interoperability - Boost.SIMD

```
#include <cnl/auxiliary/boost.simd.h>
using boost::simd::pack;

template<class T, std::size_t N, int Exponent>
using fixed_point_pack = fixed_point<pack<T, N>, Exponent>;
```

```
using fpp = fixed_point_pack<int, 4, -16>;
using initializer = initializer<fpp>;

auto expected = fpp{initializer{7.9375+-1, -8.+.125, 0+-5, 3.5+-3.5}};
auto augend = fpp{initializer{7.9375, -8., 0, 3.5}};
auto addend = fpp{initializer{-1, .125, -5, -3.5}};
auto sum = augend + addend;
```

Interoperability - Boost.SIMD

```

#####
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```

CNL Today and Tomorrow

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- Arbitrary width

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- Arbitrary width
- Full Support for Rounding and Overflow

```
template<class Rep = int, class RoundingTag = closest_rounding_tag>  
class precise_integer;
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template<class Rep = int, class RoundingTag = closest_rounding_tag>  
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template<class Rep = int, class RoundingTag = closest_rounding_tag>  
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```

- Full complement of operators for `safe_integer` and `precise_integer`
- Full complement of free functions

```
add(saturated_overflow, UINT32_C(0xFFFFFFFF), UINT32_C(0x12345678))  
divide(closest_rounding_tag, 2, 3);
```

CNL Today and Tomorrow

- Arbitrary width
- Full Support for Rounding and Overflow

```
template<class Rep = int, class RoundingTag = closest_rounding_tag>  
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- Full complement of operators for `safe_integer` and `precise_integer`
- Full complement of free functions

```
add(saturated_overflow, UINT32_C(0xFFFFFFFF), UINT32_C(0x12345678))  
divide(closest_rounding_tag, 2, 3);
```

- Better Literals

```
auto a = 0b1111.1111_elastic; // fixed_point<elastic_integer<8>, -4>
```

Thank You!

Now go to GitHub and try it out! github.com/johnmcfarlane/cnl

```
git clone https://github.com/johnmcfarlane/cnl.git
```

Bonus Slides - Beyond Exponents

`std::ratio` scales things statically:

```
template<int Num, int Denom = 1> class ratio;
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What if Exponent was replaced with a type parameter?

```
// equivalent to fixed_point<int, -8, 2>  
using a = fixed_point<int, ratio<1, 256>>;
```

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What if Exponent was replaced with a type parameter?

```
// equivalent to fixed_point<int, -8, 2>  
using a = fixed_point<int, ratio<1, 256>>;
```

This should be possible: simply separate `fixed_point`'s two concerns:

```
// a type which stores an integer and scales it  
template<typename Rep, typename Scale>  
class scaled_integer;  
  
// a type which scales integers  
template<int Exponent, int Radix> class power;
```


Bonus Slides - Beyond Exponents

Now `fixed_point` is just one of many `scaled_integer` types.

```
template<typename Rep, int Exponent, int Radix>
using fixed_point = scaled_integer<Rep, power<Exponent, Radix>>

// (Decimal fixed-point is already on the to-do list.)
template<typename Rep, int Exponent>
using decimal_fixed_point = scaled_integer<Rep, power<Exponent, 10>>
```

Bonus Slides - Beyond Exponents

Now `fixed_point` is just one of many `scaled_integer` types.

```
template<typename Rep, int Exponent, int Radix>
using fixed_point = scaled_integer<Rep, power<Exponent, Radix>>

// (Decimal fixed-point is already on the to-do list.)
template<typename Rep, int Exponent>
using decimal_fixed_point = scaled_integer<Rep, power<Exponent, 10>>
```

And a souped-up `std::ratio` is another way to scale integers.

```
template<typename Rep, int Power>
using dollar = scaled_integer<Rep, ratio<1, 100>>;

template<typename Rep, int Power>
using angle = scaled_integer<Rep, ratio<1, 360>>;

// units in the range [0, 1]
template<typename Rep>
using unit = scaled_integer<Rep, ratio<1, std::numeric_limits<Rep>::max()>>;
```

Bonus Slides - Beyond Exponents

And scaling of integers is just the beginning...

Bonus Slides - Beyond Exponents

And scaling of integers is just the beginning...

```
// type-safety prevents units from being confused
struct length_tag {};
struct time_tag {};

// individual quantities
template<typename BaseTag, int Power>
struct base_quantity;

// is quantity a scale, like exponent or std::ratio?
template<typename ... BaseQuantities>
struct quantity;

using time = quantity<base_quantity<time_tag, 1>>;
using length = quantity<base_quantity<length_tag, 1>>;

template<typename ... BaseQuantitiesA, typename ... BaseQuantitiesB>
auto operator/(quantity<BaseQuantitiesA...>, quantity<BaseQuantitiesB...>)

// quantity<base_quantity<length_tag, 1>, base_quantity<time_tag, -1>>
auto speed = length{} / time{};
```

Disclaimer: none of this compiles ... yet!

Thank You! - @JSAMcFarlane

Now go to GitHub and try it out!

```
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```