

## I. Fundamental or Basic Identities

### A. Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta} \qquad \sec \theta = \frac{1}{\cos \theta} \qquad \cot \theta = \frac{1}{\tan \theta}$$

### B. Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \qquad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

### C. Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1 \qquad \tan^2 \theta + 1 = \sec^2 \theta \qquad \cot^2 \theta + 1 = \csc^2 \theta$$

## II. Identities for Negatives

$$\sin(-\theta) = -\sin \theta \qquad \cos(-\theta) = \cos \theta \qquad \tan(-\theta) = -\tan \theta$$

## III. Co - function Identities

$$\begin{aligned} \sin(90^\circ \pm \theta) &= \cos \theta & \sin(\theta \pm 90^\circ) &= \pm \cos \theta \\ \cos(90^\circ \pm \theta) &= \mp \sin \theta & \cos(\theta \pm 90^\circ) &= \mp \sin \theta \end{aligned}$$

## IV. Sum and Difference Identities

$$\begin{aligned} \sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta & \tan(\alpha \pm \beta) &= \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \end{aligned}$$

## V. Double - Angle Identities

$$\begin{aligned} \sin 2\theta &= 2 \sin \theta \cos \theta \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta = 2 \cos^2 \theta - 1 \\ \tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ \sin^2 \theta &= \frac{1 - \cos 2\theta}{2} & \cos^2 \theta &= \frac{1 + \cos 2\theta}{2} \end{aligned}$$

## VI. Half - Angle Identities

$$\begin{aligned} \sin \frac{1}{2} \theta &= \pm \sqrt{\frac{1 - \cos \theta}{2}} & \cos \frac{1}{2} \theta &= \pm \sqrt{\frac{1 + \cos \theta}{2}} \\ \tan \frac{1}{2} \theta &= \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta} \end{aligned}$$

## *Summary of Formula*

### Derivative of a Function by Formula / Rules for Differentiation

- |                                     |  |
|-------------------------------------|--|
| 1. The Constant Rule                | $\frac{d}{dx}(c) = 0$  |
| 2. The Identity Function Rule       | $\frac{d}{dx}(x) = 1$  |
| 3. The Constant & a Function Rule   | $\frac{d}{dx}(cu) = c \frac{du}{dx}$   |
| 4. The Sum / Difference Rule        | $d(u + v) = du + dv \quad d(u - v) = du - dv$  |
| 5. The Product Rule                 | $d(uv) = u dv + v du$  |
| 6. The Quotient Rule                | $d\left(\frac{u}{v}\right) = \frac{v du - u dv}{v^2}$  |
| 7. The Power Rule / Formula         | $d(u^n) = nu^{n-1} du$   |
| 8. Chain Rule                       | $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$  |
| 9. Inverse Function Rule            | $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}, \text{ where } \frac{dx}{dy} \neq 0$                         |
| 10. Parametric Function Rule        | $\frac{dy}{dx} = \frac{\frac{dy}{du}}{\frac{dx}{du}}, \text{ where } \frac{dx}{du} \neq 0$             |
| 11. Higher Parametric Function Rule | $\frac{d^2y}{dx^2} = \frac{1}{\frac{dx}{du}} \left[ \frac{d}{du} \left( \frac{dy}{dx} \right) \right]$ |

### Derivative of Trigonometric Functions

- |  |  |
|--|--|
| 1. $\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$   | 4. $\frac{d}{dx}(\sec u) = \sec u \tan u \frac{du}{dx}$  |
| 2. $\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$  | 5. $\frac{d}{dx}(\csc u) = -\csc u \cot u \frac{du}{dx}$ |
| 3. $\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx}$ | 6. $\frac{d}{dx}(\cot u) = -\csc^2 u \frac{du}{dx}$      |

### Derivative of Inverse Trigonometric Functions

1.  $\frac{d}{dx}(\arcsin u) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$
2.  $\frac{d}{dx}(\arccos u) = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$
3.  $\frac{d}{dx}(\arctan u) = \frac{1}{1+u^2} \frac{du}{dx}$
4.  $\frac{d}{dx}(\operatorname{arccot} u) = -\frac{1}{1+u^2} \frac{du}{dx}$
5.  $\frac{d}{dx}(\operatorname{arcsec} u) = \frac{1}{u\sqrt{u^2-1}} \frac{du}{dx}$
6.  $\frac{d}{dx}(\operatorname{arccsc} u) = -\frac{1}{u\sqrt{u^2-1}} \frac{du}{dx}$

### Derivative of Exponential and Logarithmic Functions

1.  $\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}$
2.  $\frac{d}{dx}(a^u) = a^u \ln a \frac{du}{dx}$
3.  $\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$

### Derivative of Hyperbolic Functions

1.  $\frac{d}{dx}(\sinh u) = \cosh u \frac{du}{dx}$
2.  $\frac{d}{dx}(\cosh u) = \sinh u \frac{du}{dx}$
3.  $\frac{d}{dx}(\tanh u) = \operatorname{sech}^2 u \frac{du}{dx}$
4.  $\frac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u \tanh u \frac{du}{dx}$
5.  $\frac{d}{dx}(\operatorname{csch} u) = -\operatorname{csch} u \coth u \frac{du}{dx}$
6.  $\frac{d}{dx}(\coth u) = -\operatorname{csch}^2 u \frac{du}{dx}$

### Derivative of Inverse Hyperbolic Functions

1.  $\frac{d}{dx}(\sinh^{-1} u) = \frac{\frac{du}{dx}}{\sqrt{1+u^2}}$
2.  $\frac{d}{dx}(\cosh^{-1} u) = \frac{\frac{du}{dx}}{\sqrt{u^2-1}}$
3.  $\frac{d}{dx}(\operatorname{sech}^{-1} u) = \frac{\frac{du}{dx}}{u\sqrt{1-u^2}}$
4.  $\frac{d}{dx}(\operatorname{csch}^{-1} u) = \frac{\frac{du}{dx}}{u\sqrt{1+u^2}}$
5.  $\frac{d}{dx}(\tanh^{-1} u) = \frac{\frac{du}{dx}}{1-u^2}$  where  $-1 < u < 1$
6.  $\frac{d}{dx}(\coth^{-1} u) = \frac{\frac{du}{dx}}{1-u^2}$  where  $u > 1$  or  $u < -1$