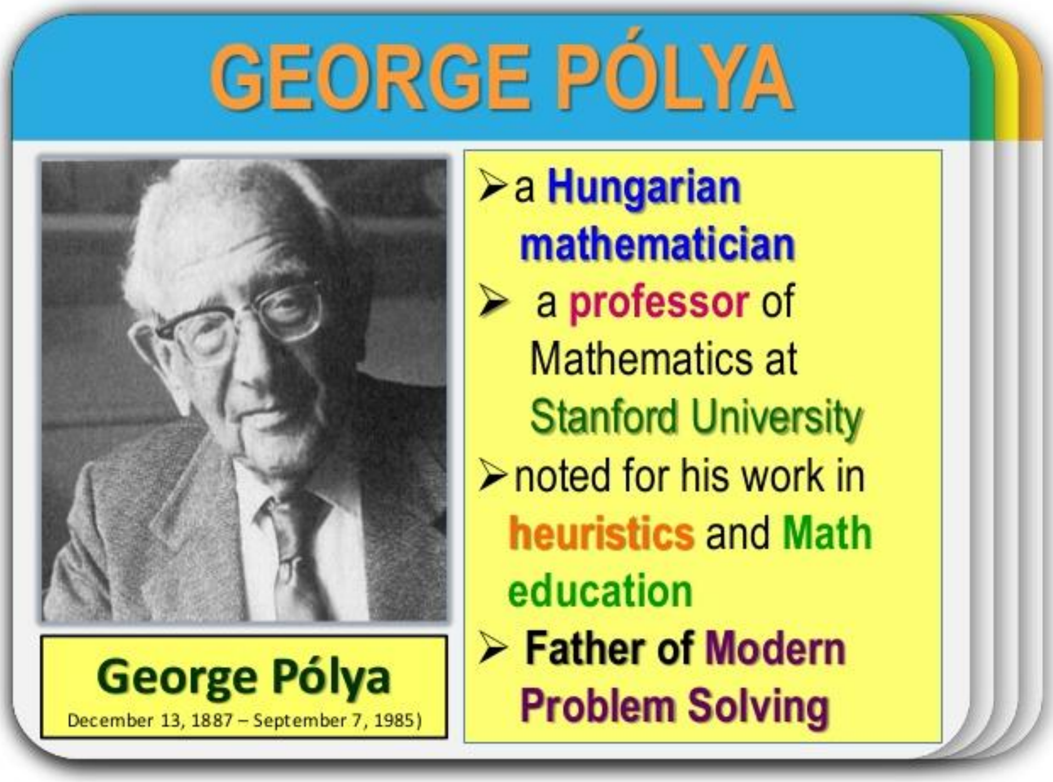
CHAPTER 3: PROBLEM SOLVING

LEARNING OBJECTIVES

At the end of this chapter the student should be able to:

1.To learn to solve problems using Polya's Four-Step Problem Solving Strategy.
2.To differentiate deductive and inductive reasoning.
3. Use inductive reasoning to make conjecture.
4. Determine counterexamples.
5. To solve inductive and deductive reasoning types of problems.
6. To become a better problem solver.



❖ **Heuristics** is the study of the methods and rules of discovery and invention.

George Polya, a Professor of Mathematics at Stanford University, born in Hungary in 1887 and received his Ph. D. at the University of Budapest. He is the author of the famous book "**How to Solve it**" (Princeton University Press, 1973) where he identifies the four basic principles of problem solving. His book has been translated into 17 languages, sold more than 1 million copies making it his most prized publication. He died at age 98 in 1985.

Pólya's book contains problem solving model as well as a treasure trove of strategy, know-how, rules of thumb, good advice, anecdotes, history, and problems at all levels of mathematics. He outlined a strategy for solving problems from virtually any discipline.

3.1 POLYA'S GUIDELINES FOR PROBLEM SOLVING

1. Preparation: **Understand the problem**. Ask questions, experiment, or otherwise rephrase the question in your own words.

Some suggested questions to asked:

- Do you understand all the words used in stating the problem?
- What are you asked to find or show?
- Can you restate the problem in your own words?
- Can you think of a picture or diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?

2. Thinking Time: **Devise a plan**

Once the problem is understood, set the problem aside for a while. Your subconscious mind may keep working on it. Moving on to think about other things may help you stay relaxed, flexible, and creative rather than becoming tense, frustrated, and forced in your efforts to solve the problem.

Some possible strategies:

- Draw a picture.
- Choose helpful names for variables or unknowns.
- Be systematic.
- Solve a simpler version of the problem.
- Guess and check. Trial and error. Guess and test. (Guessing is OK.)
- Look for a pattern or patterns.
- Make a list.
- Use a formula and solve an equation.

3. Insight: **Carry out the plan**

This step is usually easier than devising the plan. In general, all you need is care and patience, given that you have the necessary skills. Follow through with the plan that you have chosen. If it continues not to work discard it and try another approach. Do not be misled, this is how mathematics is done, even by professionals. The key is to keep trying until something works.

4. Verification: **Look back**

Polya mentioned that much can be gained by taking the time to reflect and look back at what you have done, what worked, and what did not. Doing this will enable you to predict what strategy to use to solve future problems.

Once you have a potential solution, check to see if it works.

- Check your answer, were you able to answer the question?
- Check if all the conditions stated in the problem are satisfied.
- Try fixing any error or mistakes in the computation.
- Try another solution which may be simpler.
- Can the problem or method be generalized to be useful for future problems?

Summarizing the above steps.

▼ Polya's Four-Step Problem-Solving Strategy

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Review the solution.

Example 1: Source: Nature of Mathematics, 12th Edition, Karl J. Smith, page 11

A jokester tells you that he has a group of cows and chickens and that he counted 13 heads and 36 feet. How many cows and chickens does he have?

Solution: Let us use Polya's problem-solving guidelines.

Understand the Problem: A good way to make sure you understand a problem is to attempt to phrase it in a simpler setting as shown below,

One chicken and one cow: 2 heads and 6 feet (chickens have two feet; cows have four)
Two chickens and one cow: 3 heads and 8 feet
One chicken and two cows: 3 heads and 10 feet

Devise a Plan: Organizing materials is often important in problem solving.

Let us organize the information into a table.

No. of chickens	No. of cows	No. of heads	No. of feet
0	13	13	13(4) = 52

We started with the above table showing that we must have13 heads. There are other possible starting places (13 chicken and 0 cows, for example) but an important aspect of the problem solving is to start with some plan.

Table 3.0

No. of chickens	No. of cows	No. of heads	No. of feet
1	12	13	(1 x 2) + (12 x 4) = 50
2	11	13	(2 x 2) + (11 x 4) = 48
3	10	13	(3 x 2) + (10 x 4) = 46
4	9	13	(4 x 2) + (9 x 4) = 44

Carry out the plan: Now look for patterns. *Take note that number of cows decreases by one and the number of chickens increases by one, the number of feet must decrease by two*. Does this make sense to you? Remember, step 1 requires that you not just push numbers around, but that you understand what you are doing. Since we need 36 feet for the solution to this problem, we see,

$$44 – 36 = 8$$

This means that the number of chickens must increase by an additional four, which means **8 chickens and 5 cows.**

Look back:

No. of chickens	No. of cows	No. of heads	No. of feet
8	5	13	36

Check:

8 chickens have 16 feet, and 5 cows have 20 feet, so the total number of heads is

$$8 + 5 = 13$$

Example 2: Twice the difference of a number and 1 is 4 more than that number. Find the number.

Solution: Following Polya's problem-solving guidelines.

Understand the problem: Read the problem carefully, since we are looking for a number,

Let x – be the number.

Devise a plan: Translate the problem into mathematical statement,

Difference of a number and 1 = $x – 1$

Twice the difference of a number and 1 = $2 (x -1)$

4 more than that number = $x + 4$

Twice the difference of a number and 1 is 4 more than that number,

$$\mathbf{2(x – 1) = x + 4}$$

Carry out the plan: The above equation can now be solved using algebraic procedures,

$$2(x – 1) = x + 4$$

Using distributive properties of multiplication over addition,

$$2x - 2 = x + 4$$

Collecting all terms with x at the left side of the equation and all constants to the right side of the equation by transposition,

$$2x - x = 4 + 2$$

$$\mathbf{X = 6}$$

Look back: Checking by substituting the value of x = 6 to the original equation,

$$2(x - 1) = x + 4$$
$$2\,(6 - 1) = 6 + 4$$
$$2(5) = 10$$
$$10 = 10$$

The final answer is x = 6.

Example 3: The length of a rectangular lot is 1 meter more than 3 times the width. Find the dimensions if the perimeter is to be 26 meters.

Solution: Following Polya's problem-solving guidelines.

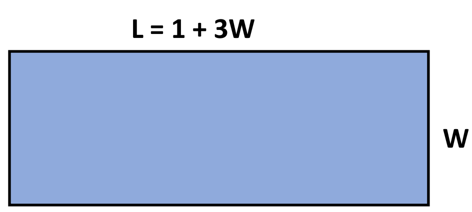
Understand the Problem: Read the problems carefully. Since the dimensions of the rectangle is required, it may be represented by the following variables:

Let L – be the length of the rectangular lot
W – be the width of the rectangular lot
P – be the perimeter of the rectangular lot

The problem states that the of length of a rectangular lot is 1 meter more than 3 times the width,

$$L = 1 + 3W$$

Devise a plan: This time we translate the whole problems into a mathematical symbols and equations,



Substituting the value of perimeter and the given data for length and simplifying equations as follows,

$$P = 2\,(L + W)$$

$$26 = 2\,(1 + 3W + W)$$

Carry out the plan: Solving for the length and width using algebraic procedures,

$$26 = 2\,(1 + 4W)$$
$$26 = 2 + 8W$$
$$8W = 26 - 2$$
$$8W = 24$$
$$W = 3 \text{ meters}$$

Since L = 1 + 3W, then L = 1 + 3 (3) = 10 meters

Lookback: Checking by substituting the value obtained for length and width to the perimeter,

$$P = 2\,(10 + 3) = 2\,(13) = 26 \text{ meters}$$

The dimension of the rectangular lot is length = 10 meters and its width = 3 meter.

Example 4: Mary and Maiko sold 12 show tickets altogether. Mary sold 2 more tickets than Maiko. How many tickets did each girl sell?

Solution: Using Polya's problem-solving guidelines.

Understand the problem: You need to know that a total of 12 tickets were sold and that Mary sold 2 more tickets than Maiko.

Device a plan: We may try guessing and tabulating data as follows,

Table 3.1

Number of Tickets Sold by Maiko	Number of Tickets Sold by Mary	Total Number of Tickets Sold
2	4	6
3	5	8
4	6	10
5	7	12
6	8	14

Carry out the plan:

Shown in table 3.1 the several ways of guessing the number of tickets sold by Mary and Maiko, and from the highlighted row (3rd row) of the table,

$$5 + 7 = 12 \text{ tickets sold}$$

Lookback: Checking the answers,

$7 + 5 = 12$
$7 – 5 = 2$ (indicates that Mary sold 2 more tickets than Maiko)

Therefore, Mary sold 7 tickets and Maiko sold 5 tickets, totals 12 tickets.

Example 5: A true-false quiz contains five questions. In how many ways can a student answer the questions if the student answers two of the questions with "false" and the other three with "true"?

Solution: Use Polya's problem-solving guidelines.

Understand the Problem:

There are several ways to answer the questions so that two answers are "false" and three answers are "true." Some ways are as follows,

T T T F F
F F T T T

Devise a Plan: Make an organized list. Try the strategy of listing a T unless doing so will produce too many Ts or a duplicate of one of the previous orders in list.

Carry Out the Plan: Starting with 3 Ts in a row.

1	T	T	T		
2	T	T		T	
3	T	T			T
4	T		T	T	
5	T		T		T
6	T			T	T
7		T	T	T	
8		T	T		T
9		T		T	T
10			T	T	T

Then filling in all blank

1	T	T	T	F	F
2	T	T	F	T	F
3	T	T	F	F	T
4	T	F	T	T	F
5	T	F	T	F	T
6	T	F	F	T	T
7	F	T	T	T	F
8	F	T	T	F	T
9	F	T	F	T	T
10	F	F	T	T	T

Lookback: Reviewing the solution, each entry in the list has two Fs and three Ts. Since the list is complete and has no duplications, we know that **there are 10 ways for a student to mark two questions with "false" and the other three with "true."**

Example 6. In a seminar, 30 students were present. During their meet-and-greet activity, they were asked to have a handshake with everyone in the room. If each did handshake with everyone, how many handshakes took place?

Understand the Problem:

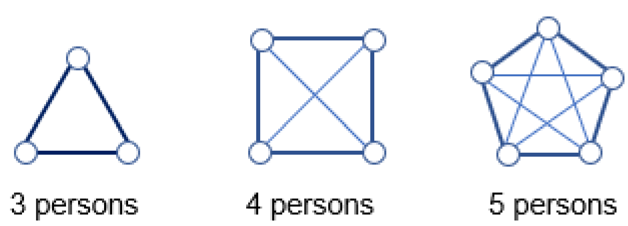
There were 30 attendees. A simple handshake means letting a distinct pair be recognized. Moreover, if A shakes hands with B, then B shakes hands with A as well.

Devise a plan:

We start with solving simper cases, say 3, 4, and 5 persons. We can draw a diagram where a person is represented by nodes while handshakes by arcs connecting the nodes. From here, we try to find a pattern.

Carry out the plan:

The following figures represent the handshakes that took place among 3, 4 and 5 persons.



A group of 3 persons makes 3 handshakes, a group of 4 persons makes 6 handshakes and a group of 5 persons makes 10 handshakes. Now for each case with *k* persons, each of these persons has to have a handshake with other $k-1$ person, so the product $k(k-1)$ is the number of all handshakes from individual perspective. Note *that if A shakes hands with B, then B shakes hands with A as well* Thus only half of $k(k-1)$ represents the total number of handshakes. Hence, a pattern is generalized by $\frac{k(k-1)}{2}$ will lead to the number of handshakes that took place in a group of *k persons.* Therefore, there were a total of

$$\frac{k(k-1)}{2} = \frac{30(30-1)}{2} = 435\ handshakes$$

Look back:

Every person will be shaking hands with 29 others. Thus, 870 handshakes are noted for individual perspective. Half of which is 435.

REVIEW EXERCISES 3.0

Name _________________________________ Score ___________ Date ________________
Course, Year & Section _____________Student no. ________Professor _______________

Use Polya's 4-Step Methods for problem solving to solve the following problems.

1. Suppose that thirty-two students signed up for classes during an orientation session. If exactly twenty of them signed up for Mathematics and exactly sixteen of them signed up for Physics, how many of them signed up for both Mathematics and Physics?

2. A young lady has eight shirts and four pairs of pants. How many different outfits can she make by combining one shirt with one pair of pants?

3. In a class of 25 students, 17 lived with both parents, 21 lived with their mothers, and 20 lived with their fathers. How many lived with neither parent?

4. Emily is thinking of a number. If you multiply her number by 93, add 6, and divide by 3, you obtain 436. What is her number?

5. A family has five children. How many different gender combinations are possible, assuming that order matters? (For example, having four boys and then a girl is distinct from having a girl and then four boys).

6. Rica was given a piggy bank on her seventh birthday, and she put it to use immediately. Each time she puts one or more coins into the piggy bank, she keeps track of the number of coins she has collected to date and the accumulated value of her collection. Janice collects only ₱5.00, ₱10.00, and ₱1.00. Six months after her seventh birthday, Janice looked at her record and ascertained that she had collected 500 coins, which were worth ₱3,350.00. (a) How many combinations of coins are possible in Dennis' collection? (b) Dennis counted 100 one-peso coins in her savings. How many ten-peso and five-peso coins are in her collection?

7. Dennise is transferring his favorites pet animals, ducks and carabaos to her new farm. She counts 20 heads and 58 feet. How many ducks and carabaos are in his new farm?

8. How many fenceposts are needed to support a 100-meter-long fence, if a fencepost must be used every 2 meters?

9. Siblings Margareth and Dennise were taught by their parents to save money in the bank. Initially they have the same amount of savings. Margareth already spent ₱ 3,450.00 while Dennise spent ₱800.00. At the end of the year, Dennise savings is six times as much as Margareth's money in the bank. How much savings the siblings has?

10. A rich man died and left the following instructions for his will and properties, half to his wife; $\frac{1}{7}$ of what was left went to his son; $\frac{2}{3}$ of what was left went to his butler; the man's pet pig got the remaining ₱100,000. How much money did the man leave behind altogether?

REEASONING

- It is a process of thinking during which the individual is aware of a problem identifies, evaluates, and decides upon a solution.

- Reasoning is the term applied to highly purposeful, controlled, and selective thinking. - Gates.

- Reasoning is the word used to describe the mental recognition of cause-and-effect relationships, it may be the prediction of an event from an observed cause or the inference of a cause from an observed event. - Skinner.

- Highly specialized thinking which helps an individual to explore mentally the cause-and-effect relationship of an event or solution of a problem by adopting some well-organized systematic steps based on previous experience combined with present observation.

3.3 TYPES OF REASONING

I. INDUCTIVE REASONING

- It is a type of reasoning done first by observing patterns and then predicting answers for more complicated problems.
- It is the process of reaching a general conclusion by examining specific examples. The conclusion formed by using inductive reasoning is often called **conjecture** since it may or may not be correct.
- Its limitations, even if all the premises are true in a statement, inductive reasoning allows for the conclusion to be false.

3.4 CONJECTURE

It is a mathematical statement which appears likely to be true but has not been formally proven to be true under the rules of mathematical logic.

To verify a conjecture is true, you need to prove that it is true in all cases. To prove a conjecture is false you need to provide one single *counterexample.*

Example 1: Observe the conjecture of the following statements.

a.) Dogs A and B have fleas.
All observed dogs have fleas.
Therefore, all dogs have fleas.

b.) All dolphins are mammals.
All mammals have kidneys.
Therefore, all dolphins have kidneys.

c. Most A are B
This C is A,
Therefore, C is likely B.

d. Rodrigo is mortal.
Leny is mortal.
Donald is mortal.
Therefore, all human beings are mortal.

e. Harold is a grandfather.
Harold is bald.
Therefore, all grandfathers are bald.

Example 2: Use inductive reasoning to predict the next number in the following:

a. 5, 10, 15, 20, 25, . . .? b. 1, 3, 6, 10, 15, …?

Solution:

a. It can be seen from the given 5, 10, 15, 20, 25, . . .? that each number has a difference of 5, so we predict the next number to be **30.**

b. From the given 1, 3, 6, 10, 15, …? take note that the difference between the first and second number is 2, and the difference between 3 and 6 is 3, between 6 and 10 is 4. It appears that the difference between any two numbers is always 1 more than the preceding difference. Since the difference between 10 and 15 is 5 so we predict that the next number in the list will be 6 larger than 15 which is **21.**

Example 3: Consider the following procedure:

Pick a number.

Multiply the number by 9, add 15 to the product, divide the sum by 3, and subtract 5.

Complete the above procedure for several different numbers. Use inductive reasoning to make a conjecture about the relationship between the size of the resulting number and the size of the original number.

Solution: The first number I picked is 5, following the procedure stated above:

Pick a number:	5
Multiply by 9:	5 x 9 = 45
Add 15:	45 + 15 = 60
Divide by 3:	60 ÷ 3 = 20
Subtract 5:	20 – 5 = 15

The second number I picked is 7, following the same procedure,

Pick a number:	7
Multiply by 9:	7 x 9 = 63
Add 15:	63 + 15 = 78
Divide by 3:	78 ÷ 3 = 26
Subtract 5:	26 – 5 = 21

Conjecture: After applying the procedures stated in the problem to the picked numbers 5 and 7, the resulting numbers always yield a number which is 3 times the original (picked) number.

Example 4: A tsunami is a sea wave produced by an underwater earthquake. The height of a tsunami as it approaches land depends on the velocity of the tsunami. Use the table the below and inductive reasoning to answer the following questions.

a. What happens to the height of a tsunami when its velocity is doubled?
b. What should be the height of a tsunami if its velocity is 30 feet per second?

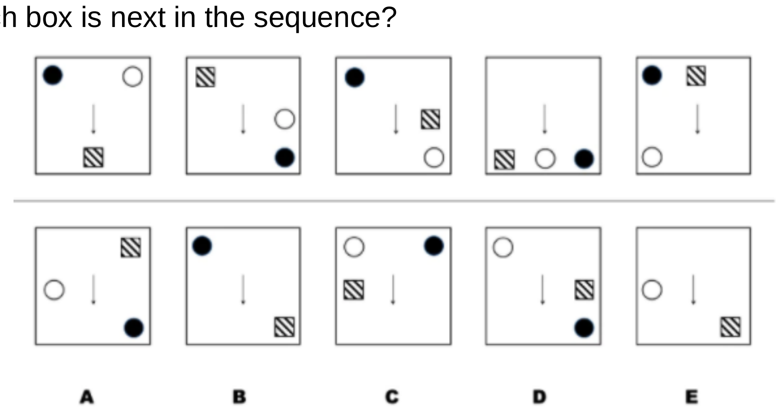
Velocity of Tsunami (feet per second)	Height of Tsunami (feet)
6	4
9	9
12	16
15	25
18	36
21	49
24	64

Solution:

a. The given table shows that when the velocity of a tsunami is doubled, its height is quadrupled.

b. Tsunami with a velocity of 30 feet per second will have a height that is four times that of a tsunami with a speed of 15 feet per second. Thus, we predict a height of 4 x 25 = 100 feet for a tsunami with a velocity of 30 feet per second.

Example 5: Which box is next in the sequence?



Answer: Observe that the black is moving backwards and forwards between the top-left corner and the bottom-left corner. The white circle is moving anti-clockwise around the box half a side at a time and the stiped square is moving around the box 1 ½ sides each time. The arrow is not moving so it is irrelevant. **The final answer is E.**

3.5 COUNTER EXAMPLES

A counterexample to an argument or a proposition is a situation which shows that the argument can have true premises and a false conclusion. It is an example that proves a conjecture is false. All it takes is one example to disprove the conjecture.

To verify a conjecture is true, you need to prove that it is true in all cases. To prove a conjecture is false you need to provide one single *counterexample.*

Example 6: All prime numbers are odd.
The Prime number 2 is a counterexample to the statement.

Example 7: If God exists, then life has meaning.
But there is no God.
Therefore, life is meaningless.

Example 8: Identify which of the following numbers is a counter example for the conjecture:

The domain ***f*** is the set of all positive real numbers, $f(x) = \frac{1}{\sqrt{x-3}}$

a. x = 5 b. x = 6 c. x = 3

Answer: Since we are looking for a counterexample, we want to find a positive real number that does not work in that function since that contradicts the original conjecture. Substituting each value of x, we find that in x = 3, the denominator is which makes the function undefined
the function undefined at that point. **Therefore, x = 3 is a positive real number NOT in the domain, which makes x = 3 the counterexample**.

Example 9: Find a counterexample that disproves the conjecture,

If $x^2 = 4$, then x = 2 is the solution.

Answer: **We see that another number -2, that when squared equals 4.**

Example 10: Determine if the following conjecture is true if false give counterexample.

Two supplementary angles are not congruent**.**

Answer: False, 90° and 90° are congruent angles.

Note: Note that one counterexample is enough to prove that a line of reasoning is false, but one positive example is never enough to prove that it is true.

REVIEW EXERCISES 3.1

Name ________________________________ **Score** __________ **Date** _______________
Course, Year & Section ____________**Student no.** _______**Professor** ______________

1. Use inductive reasoning to make a conjecture about the arithmetic problem.

Consider the following procedure: Pick a number. Multiply the number by 8, add 6 to the product. Divide the sum by 2 and subtract 3 from the result.

2. Write a conjecture that describes the pattern shown. Then use conjecture to find the next figure in the sequence.

3. Write a conjecture that describes the pattern in the sequence, then use your conjecture to find the next item in the sequence,

$$1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$$

4. Water makes up about sixty percent of the human body. Keeping well hydrated, then, is an essential part of remaining healthy. Having enough water each day is an essential part of flushing the body free of pathogens and other irritants. It also keeps tissues properly hydrated and working correctly. In order to be properly hydrated, an individual should drink between half an ounce to an ounce of water per pound they weigh. If you weigh 150 lbs, you should drink somewhere from 75-150 ounces of water a day. Question: Individuals should drink between half an ounce to an ounce of water per pound of body weight.

a. True b. False c. Cannot say

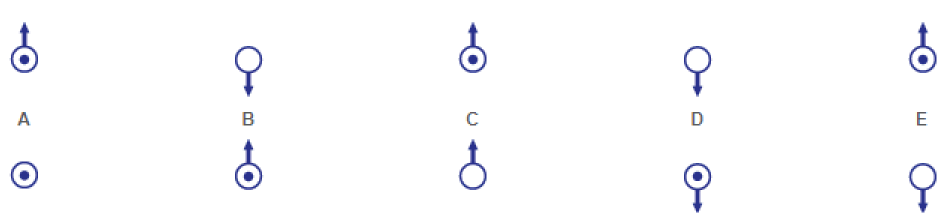
5. Deanna, Annie, Clayton, Billy, and Elise are all standing in line for the roller coaster. Deanna is either before Annie or after Elise. The first two spots are filled by boys. Elise is at the end of the line. If Annie is in front of Elise, then Billy is behind Clayton Who is behind Billy?

a. Annie b. Clayton c. Deanna d. Elise

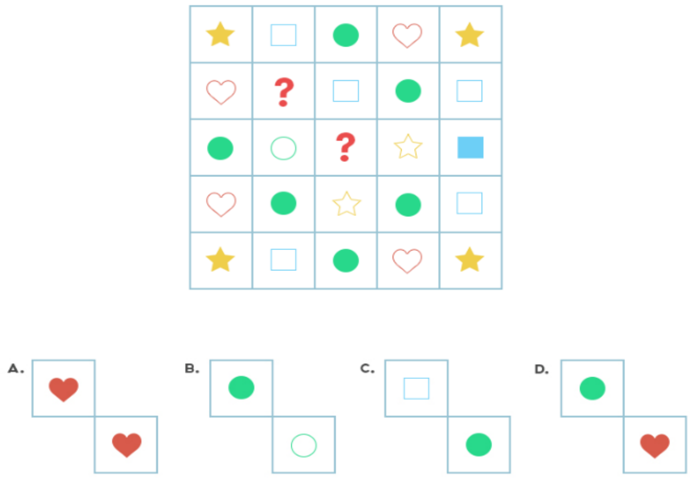
6. Choose the pattern that best completes the sequence of matrix.

a.

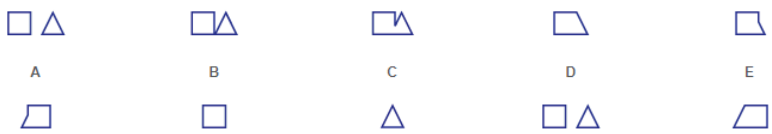
b.



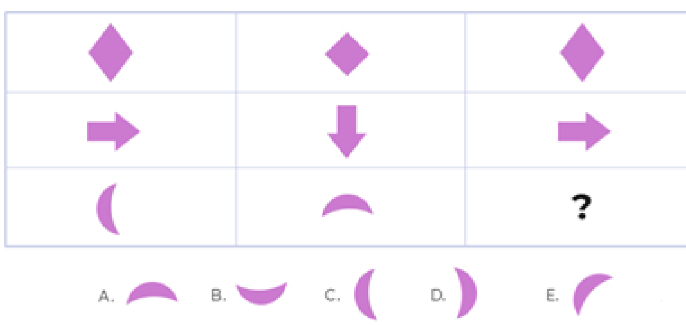
c.



d.



e.



7. Identify a pattern in each of the following sequences of numbers, then use the established pattern to find the next term in the sequence.

a. 4, 9, 14, 19, _____

b. 2, 6, 18, 54, _____

c. 5, 6, 8, 11, _____

8. Which of the following numbers can be used to show that if the product of two integers is positive, their sum is not always positive?

a. 5 and 5   b. 5 and -5   c. 1 and 25

d. -1 and 25   e. -5 and -5

9. Which of the following expressions can be used to show that the sum of two numbers is not always greater than both numbers?

Choose all answers that apply:

a. $\frac{3}{2} + \frac{1}{2}$ b. 0 + 4   c. -2 +1

10. Jack made the following conjecture about the information in the table.

If the first number is negative
And the second number is positive,
The sum is always negative.

Addends		Sum
-5	3	-2
-3	2	-1
-8	4	-4
-10	6	-4

Find a counterexample for his conjecture.

3.5 DEDUCTIVE REASONING

- Deductive reasoning is a basic form of valid reasoning. It starts out with a general statement, or hypothesis, and examines the possibilities to reach a specific, logical conclusion.

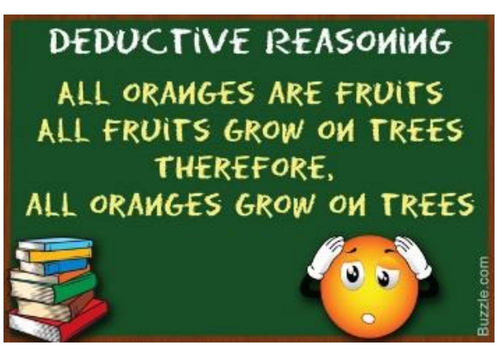
- It follows the following steps. First, there is a premise, then a second premise, and finally an inference. A common form of deductive reasoning is the *syllogism*, in which two statements — a **major premise** (more general premise) and a **minor premise** (more specific premise) — reach a logical **conclusion**.

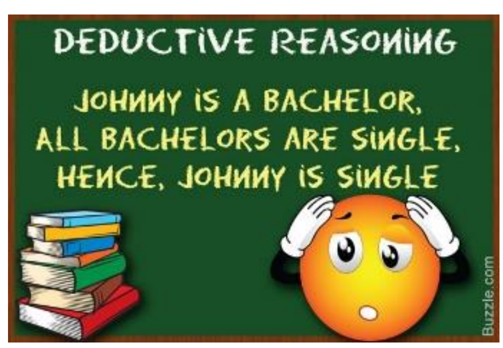
- Its limitation, the conclusions of deductive reasoning can only be true if all the premises set in the inductive study are true and the terms are clear.

- Deductive arguments can only be **valid or invalid**, **sound or unsound** or **true or false**, because the statement that is derived may be true, but in the context of the derived conclusion, it may not be valid.

- A deductive argument can be **valid**, only if the conclusion necessarily follows from the premises given, that is when both the premises are true and the conclusion that is derived from them cannot be false. If the conclusion is **incorrect**, then one of the premises must be faulty or true only for a specific situation, which is not applicable to all situations. A **valid deductive** argument with true premises is said to be **sound,** whereas a deductive argument which is **invalid** or has one or more false premises or both, is said to be **unsound**.

EXAMPLE 1: The following examples of valid and sound deductive reasoning.





Example 2: The following examples are valid, but unsound argument.

a. All flight attendants know how to swim
   Ralph knows how to swim
   Hence, Ralph is a flight attendant.

The above conclusion is untrue, because it is not necessary that only flight attendants know how to swim. Absolutely any swimmer can swim.

b. If Superman exists, then at least one superhero is real.
   Superman exists.
   At least one superhero is real.

The above reasoning, the first premise is true, but the second premise is false--there is no Superman. So the above argument is valid but unsound.

c. If the moon is made of green cheese, squirrels are mammals.
The moon is made of green cheese.
Therefore, squirrels are mammals.

The above argument is valid. It is not sound, because its second premise is false.

Example 3: Consider the following illustrations.

Premise 1: Noble gases are stable.
Premise 2: Neon is a noble gas.
Conclusion: Neon is stable.

Major Premise: All mammals are animals.
Minor Premise: All elephants are mammals.
Conclusion: Therefore, all elephants are animals.

All birds have feathers.
Ducks are birds.
Therefore, ducks have feathers.

All bachelors are unmarried,
Ted is a Bachelor.
Therefore, Ted is unmarried
All *A* are *B*
And all *C* are *A*
Therefore, all *C* must be *B*.

All human beings are mortal.
You are a human being.
Therefore, you are mortal.

3.6 SYLLOGISM

Syllogism is a form of deductive reasoning where you arrive at a specific conclusion by examining two other premises or idea (hypotheses). The word syllogism is derived from the Greek word *syllogismos,* meaning conclusion or inference.

It has three parts: two premises, or hypotheses, and a conclusion. The premises give us information from which we form a conclusion. **If the conclusion follows from the premises, it is called a valid syllogism; if not, it is called invalid.**

Consider the following examples:

	Valid Forms of Reasoning	**Invalid Forms of Reasoning**
Premise	All Chevrolets are automobiles.	Some People are nice.
Premise	All automobiles have four wheels.	Some people are broke.
Conclusion	All Chevrolets have four wheels.	There are some nice broke people.
Premise	All teachers are crazy.	All dodos are extinct.
Premise	Juan Dela Cruz is a teacher.	No dinosaurs are dodos.
Conclusion	Juan Dela Cruz is crazy.	All dinosaurs are extinct.

Parts of Syllogism

1. Major Term – the predicate term of the conclusion
2. Minor Term – the subject term of the conclusion
3. Middle Term – the term repeated in the premises, but not in the conclusion.
4. Major Premise – the premise that contains the major term
5. Minor Premise – the premise that contains the minor term

Major premise: All mammals are **animals with hearts.**

Minor Premise**:** All **dogs** are mammals.

Conclusion: All **dogs** are **animals with hearts**

Minor term Major Term

Middle Term

Most common types of syllogisms

I. Universal Syllogisms

Universal syllogisms are called "universal" because they use words that apply completely and totally, such as **"no"** and **"none"** or **"all"** and **"only.**

Universal Affirmative, (A)

All A are B.
All C are A.
So, all C are B.

Note: The subject A is distributed

Example 4: All mammals are animals.
All elephants are mammals.
Therefore, all elephants are animals.

Universal Negative, (E), - the exact opposite of A

No A are B.
All C are A.
So, no C are B.

Note: Both the subject A and predicate B are distributed.

Example 5: No mammals are frogs.
All elephants are mammals.
Therefore, no elephants are frogs.

II. Particular Syllogisms

Particular syllogisms use words like "**some**" or "**most**" instead of "all" or "none." Within this category, there are two main types:

Particular Affirmative, (I)

All A are B.
Some C are A.
Therefore, some C are B."

Note: Neither the subject nor the predicate are distributed.

Example 6: All elephants have big ears.
Some animals are elephants.
Therefore, some animals have big ears.

Particular Negative, (O)

No A are B.
Some C are A.
Therefore, some C are not B."

Note: The predicate B is distributed.

Example 7: No doctors are children.
Some immature people are doctors.
Therefore, some immature people are not children.

Note: In Some A are not B, the predicate B is distributed.

III. Categorical syllogisms

Categorical syllogisms is the most commonly used type of syllogisms.

The basic for this syllogism type is: If A is a part of C, then B is a part of C
(A and B are members of C).

Example 8: Consider the following illustrations.

Premise one: A left-handed person is the killer
(Killer is the major term--*predicate of the conclusion*)

Premise two: Ms. Red is the only left-handed person
(Left-handed is the middle term--*found in both premises*)

Conclusion: Ms. Red is the killer.
(Ms. Red is the minor term--*subject of the conclusion*)

Example 9: Major premise: All men are mortal.
Minor premise: Socrates is a man.
Conclusion: Socrates is mortal.

Note: Both premises are known to be valid, by observation or historical facts. Because the two premises are valid, the conclusion must be valid as well. Be aware that this conclusion is based on logical reasoning and thus it does not have to represent the "truth" always.

Example 10: Consider the following illustrations.

All New Yorkers are happy.
Some people live in New York.
Some people are happy.

All cars have wheels.
I drive a car.
Therefore, my car has wheels.

All insects frighten me.
That is an insect.
Therefore, I am frightened.

Note: The major term is the predicate term of the conclusion. The minor term is the subject term of the conclusion. The middle term is the term that appears twice in the premises.

Definition of Some Words used in this chapter.

Argument consists of two or more propositions offered as evidence for another proposition.

It is important to note that when one gives an argument, one does not necessarily attack or criticize the other. In this way, an argument can also be viewed as a support of someone's viewpoint.

Valid deductive argument is one in which it is impossible for the conclusion to be false if the premises are true.

Invalid argument is one in which it is possible for the conclusion to be false if the premises are true.

A **sound** argument is valid, and its premises are actually true. All invalid arguments are, by definition, **unsound**.

Valid + True Premises = Sound
Valid + At Least One False Premise = Unsound
Invalid = Unsound

Premise is a proposition that is used as evidence in an argument.

Conclusion is a proposition for which evidence is offered.

REVIEW EXERCISES 3.2

Name _________________________________ Score ___________ Date _______________
Course, Year & Section _____________Student no. ________Professor _______________

1. Use deductive reasoning to show that the given procedure always produces the number 5.

    Pick a number. Add 4 to the number and multiply the sum by 3, subtract 7 and then decrease this difference by the triple of the original number.

2. If the first two statements are true, the conclusion must be:

    Some rectangles are squares,
    Some parallelograms are rectangles
    All squares are rectangles.

    a. True b. False c. Uncertain

3. The following conclusion is based on which assumption:

    Gregory had studied all night for his examination.
    Gregory received a bad grade on his test.
    Gregory didn't study enough.

    a. Studying more is the only way to improve your score on examination.
    b. Gregory didn't get enough sleep.
    c. Gregory was paying attention in class.
    d. Gregory studied the right material.

3. Joyce came home late one night to find her neighbor murdered in the kitchen. Which two statements deductively prove that Joyce is innocent of murder?

    i. Joyce loves to have coffee with her neighbor on Sunday morning.
    ii. If Joyce works late, she picks up pizza on her way home.
    iii. All murderers have been to jail.
    iv. If Joyce picks up pizza after work, then she sleeps at her boyfriend's house.
    v. Joyce has never seen a prison cell.

    a. i and ii b. iii and iv c. iii and v d. ii and iv

4. Mark finished ahead of Dinar. Dennis and Emmanuel both finished before Medric. Owen did not finish last. Who was the last to finish?

    a. Owen b. Emmanuel c. Medric d. Dinar e. Dennis

5. Rowena, Ruth, Cathy and MM are sitting in a row (left to right) in the school auditorium. Ruth and MM exchange seats, then MM and Rowena decided to exchange places as well. Where is Ruth?

    Rowena Ruth Cathy MM
    Rowena MM Cathy Ruth
    MM Rowena Cathy Ruth

    Answer: Ruth is at the rightmost seat

6. Identify the following as inductive or deductive.

__________1. If you brush and floss your teeth daily then you will have fewer cavities. Marie brushes and flosses her teeth daily. Thus, she will have fewer cavities.

__________ 2. It has been observed that earthquakes precede the eruption of volcanoes. Thus, earthquakes cause volcanoes to erupt.

__________ 3. Most people are pro-choice on abortion. At least most of the people I know.

__________ 4. Lorenze likes playing basketball with Julius. Lorenze is not playing today. Thus Julius will not play.

__________ 5. All students go to school. You are a student. Therefore you go to school.

__________ 6. Most labrador retrievers are friendly. Kimber is a Labrador retriever. Therefore, Kimber is friendly.

__________ 7. If a figure is a rectangle, then it is a parallelogram. Figure A is a rectangle. Therefore, Figure A is a parallelogram.

__________ 8. You have 50 pens in front of you. Upon checking the first 10 pens, 5 are red and 5 are green. Therefore, Half of the pens are red and half are green.

__________ 9. The left-handed people I know use left-handed scissors; therefore, all left-handed people use left-handed scissors.

__________ 10. Sarah leaves the house at 7:30am and she arrives late for work. Sarah concludes that every time she leaves the house at 7:30 am, she will be late for work.

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