CHAPTER 1: MATHEMATICS IN OUR WORLD

LEARNING OBJECTIVE

By the end of this lesson, the student will be able to:

1. Define Mathematics and express its importance in one's life.
2. Recognize and identify the occurrence of Fibonacci sequence in nature.
3. Recognize patterns in nature.
4. Become more aware of the natural beauty that surrounds them.
5. Express appreciation for mathematics as a human endeavor.

CONTENT

I. Mathematics

II. Patterns and Numbers in Nature

III. Mathematics and the Real World

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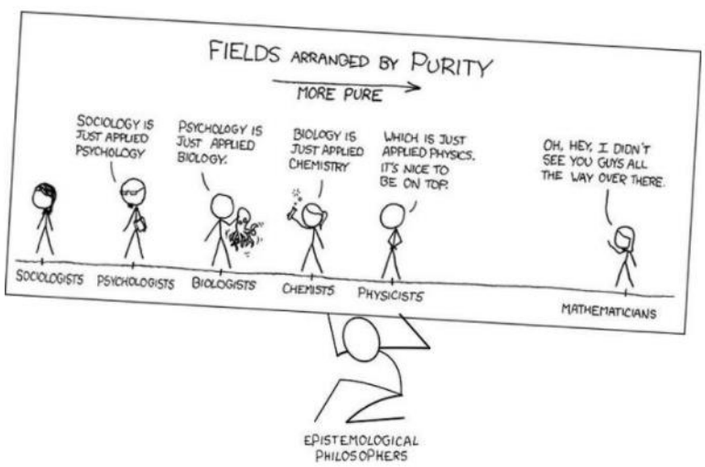
pure math

Quite true! A Math student will understand this at the university level and beyond, where Math has no more numbers and is full of symbols and jargon! Although even the most abstract Math has applications, the applications are only discovered years later, hence Pure Math is indeed one of the most pure subjects around.



A webcomic of romance, sarcasm, math, and language – By Randall Munroe.

What is Mathematics?

It is just about Arithmetic.
It is just about the study of numbers?
It is just a body of formulas and rules for solving equations?
A useless obstacle course in school?

Many people consider Mathematics to be a boring and formal Science. But any good work in Mathematics always has in it:

Beauty, Simplicity, Structure, Imagination, Crazy ideas

Just like Music and Poetry!

Human culture has developed a formal system of thought for recognizing, classifying, and making sense of patterns.

We call it Mathematics.

I.0 MATHEMATICS

Mathematics is defined in many ways.

- Mathematics is a branch of science, which deals with numbers and their operations. It involves calculation, computation, solving of problems etc.
- Its dictionary meaning states that, ‘Mathematics is the science of numbers and space’.
- Mathematics is the science of measurement, quantity and magnitude’. It is exact, precise, systematic and a logical subject.
- It is a set of problem-solving tools, a language, a process of thinking, and a study of patterns, among others.

Whatever point of view is taken, there is no denying the reality that mathematics is everywhere. Individuals from around the world use math in their daily lives. Mathematics has various applications in the world.

Mathematics is a useful way to think about nature and our world. It is not confined in classroom. We live in a world of mathematical patterns.

Mathematics helps us to organize and systemize our ideas about patterns that it can also be used to infer some of the underlying principles that govern the world of nature.

This chapter is dedicated to nature of mathematics, patterns and numbers in nature and the world and the uses of mathematics.

1.1 PATTERNS IN NATURE AND THE WORLD

Patterns in nature are visible regularities of form found in the natural world. These patterns recur in different contexts and can sometimes be modeled mathematically. Natural patterns include symmetries, fractals, spirals, meanders, waves, foams, tessellations, cracks, flow, tiling, spots and stripes.

Sequences or designs that are orderly and that repeat, anything that is not random.

Symmetries

- Symmetry is variously defined as "proportion," "perfect, or harmonious proportions," and "a structure that allows an object to be divided into parts of an equal shape and size."

Classification of Symmetry

1. Bilateral (mirror) symmetry is a symmetrical with respect to its reflection. This means that you can make an axis of symmetry straight down the center.

2. Radial (rotational) symmetry is where similar parts are regularly arranged around a central axis and the pattern looks the same after certain amount of rotation. It can either be cyclic or dihedral. Flowers most often exhibit this type of symmetry.

3. Translational symmetry, such a repeating tile or wallpaper patterns, means that a particular translation of an object to another location does not change its pattern.

4. Scaling symmetry which is the property of a pattern where each part of which is identical to the whole as seen at different magnifications. This is commonly called self-similarity-a property that characterizes a fractal shape.

5. Time symmetry, is periodic behavior which involves changes in time. Symmetry can also be a description of non-geometric forms such as time and space.

SOME EXAMPLES OF SYMMETRY IN NATURE

Peacock is a beautiful bird with bright colors, large size, an attractive symmetrical tail and repeated patterns in feathers. Also, an example of bilateral symmetry.

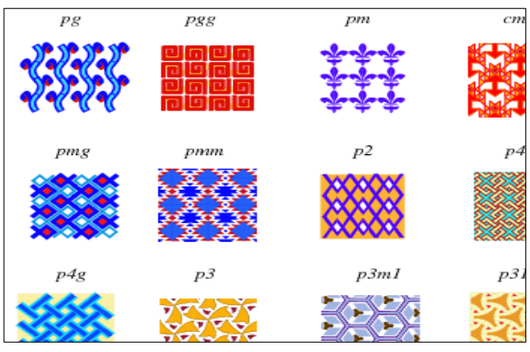
Honeycomb is a perfect hexagonal figure made by bees. It is a wallpaper symmetry, where a repeated pattern covers a plane.

Sunflower exhibits radial symmetry and numerical symmetry which is called as Fibonacci sequence.

Dragonfly has bilateral symmetry or reflective symmetry.

Some scientists theorize that orb webs are built for strength, with radial symmetry helping to evenly distribute the force of impact when a spider's prey contacts the web.

Acacia tree, a large tree that looks like a collection of many smaller trees of various sizes. The repetition of branching that forms the tree also generates the tree's self-similarity, is called scaling symmetry.

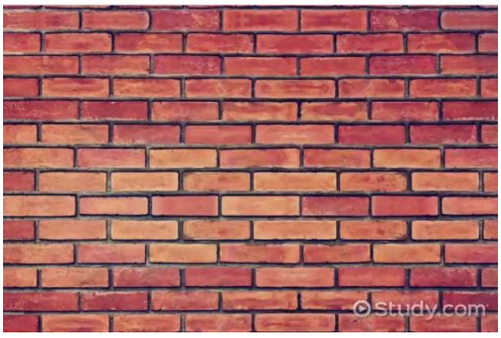


Beautiful wallpaper patterns can be created by repeating geometric and artistic motifs have translational symmetry.

Swiss Alps

Mountains provide a breathtaking example of self-similarity when viewed from above. .

Rotational symmetry is found at different scales among non-living things including the crown-shaped splash pattern formed when a drop falls into a pond.



Bricks at the wall of a building is an example of translational symmetry.

FRACTALS

In mathematics, fractal is any class of complex geometric shapes that commonly have fractional dimension. It derived from Latin word ***fractus*** which means fragmented of broken. It is described as a self-similar object; one whose component parts resemble the whole and remains invariant under changes of scale. It has scaling symmetry.

Fractal-like patterns occur widely in nature, in phenomena as diverse as clouds, river networks, geologic fault lines, mountains, coastlines, animal coloration, snowflakes, crystals, blood vessel branching, lightning, ferns and ocean waves.

The fern is one of many flora that are fractal; it's an especially good example.

Lightning's terrifying power is both awesome and beautiful. The fractals created by lightning are fascinatingly arbitrary and irregular.

Pineapple skin is a fractal pattern

Romanesco, showing its self-similar form

Both chemically formed crystals and ice and frost crystals are breathtaking examples of fractals in nature.

From the macro view of a leaf to the span of a tree's branches, fractals turn up frequently.

TESSELATIONS

A Tessellation (or Tiling) is when we cover a surface with a pattern of flat shapes so that there are no overlaps or gaps. A pattern of shapes that fit perfectly together.

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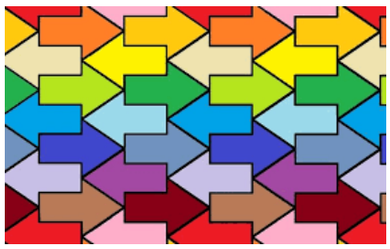

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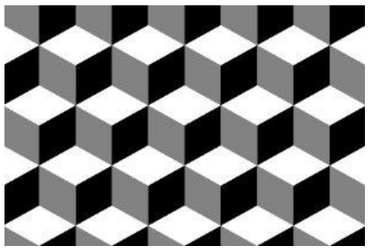


Cube Tessellation
www.quora.com

Snake skin is tessellation in nature

Zellige terracotta tiles in Marrakech, forming edge-to-edge, tessellations. Ian Alexander - Own work.

Soccer ball designs as tesselation.

The **Giant's Causeway** is an area of about 40,000 interlocking basalt columns, the result of an ancient volcanic fissure eruption, located in County Antrim on the north coast of Northern Ireland, about three miles (4.8 km) northeast of the town of Bushmills. It is a tessellation in nature.

SPIRALS

Spiral is a plane curve that, in general, winds around a point while moving even father from that point.

A spiral is a curved pattern that focuses on a center point and a series of circular shapes that revolve around it.

Examples of spirals are pinecones, pineapples, hurricanes. A spiral shape cause plants to condense themselves while trying to grow and not take up much space, causing it to be stronger, more durable against element and secured.

Weather pattern over Iceland

The Philippine golden snail

In addition to having a near - perfect **mirror image/symmetry**, the Milky Way has another incredible design similar to nautilus shells and sunflowers where each “arm” of the galaxy represents a logarithmic spiral beginning at the center of the galaxy and expanding outwards.

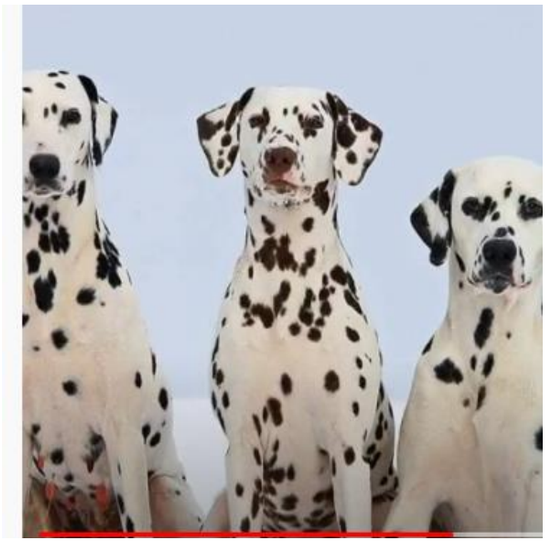
The Great Mosque of Samarra, Iraq, with spiraling minaret

SPOTS, STRIPES and ROSETTES

Beauty comes in all forms in the animal world. Stripes, patterns, bands, dots, colors, and Spots! Nature has bestowed some animals with its precious marks in the form of bold but beautiful patterns, and course spots. Various hypotheses have been suggested to provide a reason as to why some animals have spots on their bodies. Whether it is to repel insects, to provide camouflage or optical illusion, to confuse predators, to reduce body temperature, for mating purposes or to help the animals recognize each other- spots have always been an interesting and attractive feature of some animals.

Zebras are fascinating animals, in large part because of their beautiful coats. Though the three species of zebras have their own basic patterns, no two zebras share the same stripe pattern. Their s tripe stripes are as unique as our fingerprints and have an important role in their survival in the wild.

The **leopard**, one of the “big cats” living in the world today is found across parts of Asia and a wide range in sub-Saharan Africa. Compared to other wild cats, the leopard has relatively short legs and a long body with a large skull. Its fur is marked with spots called rosettes. They are jagged black circles resembling roses that are smaller and grouped closer together.



Dalmatians are the medium-sized dogs popular for their unique black-spotted white coat, which may either black or brown in colour, that even inside their mouth have spots.

Chital or spotted deer is also known as the axis deer. The species has a golden brown body with white spots mostly located on the upper part of the body.

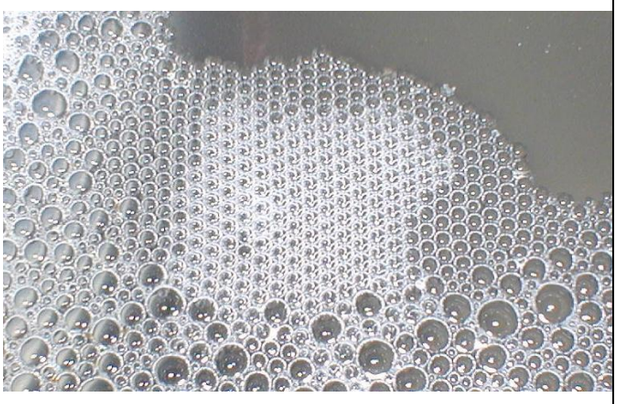
Giraffes are the tallest animal on planet earth has spots look similar to the human finger print. Study found that roundness and smoothness of a giraffe's spots were strongly linked between mother and offspring and likely to be heritable.

The **Holstein-Friesian** is the largest of the dairy breeds and the largest producer of milk. These cows have a white coat with distinct and big black patches. It is the most popular breed and has its origin in the northern provinces of North Holland and West Friesland of the Netherlands.

BUBBLES AND FOAMS

A soap bubble forms a sphere, a surface with minimal area — the smallest possible surface area for the volume enclosed. Two bubbles together form a more complex shape: the outer surfaces of both bubbles are spherical; these surfaces are joined by a third spherical surface as the smaller bubble bulges slightly into the larger one.

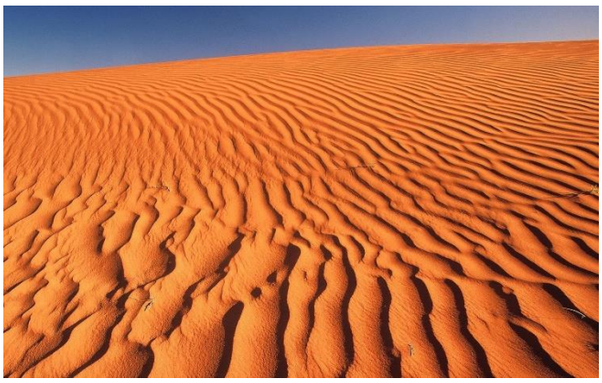
A foam is a mass of bubbles; foams of different materials occur in nature. Foams composed of soap films obey Plateau's laws,



Soap bubbles are physical examples of the complex mathematical problem of minimal surface. They will assume the shape of least surface area possible containing a given volume.

WAVES and DUNES

Wave is created by a disturbance and travels through a medium, away from its origin. Waves transfer an amount of energy given by the initial shock through the medium it is into. Waves can be observed everywhere, in music with sound waves, in light with light waves and more.

	
Mentawai Islands, Indonesia	Colorado Sand Dunes

More patterns: 1. Rosette Patterns
2. Frieze Pattern
3. Cracks

1.2 FIBONACCI SEQUENCE AND THE GOLDEN RATIO

Leonardo Pisano *was born late in the twelfth century in Pisa, Italy: Pisano in Italian indicated that he was from Pisa. His father was a merchant called Guglielmo Bonaccio. When scholars were studying the handwritten copies of Liber Abaci (as it was published before printing was invented), they misinterpreted part of the title – "filius Bonacci" meaning* ***"son of Bonaccio"*** *– as his surname, and so the name Fibonacci was born.*

He studied calculation with Arab Master, later went to Egypt, Syria, Greece, Sicily,

In his Liber Abaci the Hindu-Arabic numerals were introduced to European through the translations of writings of the 9th century Arab Mathematician Al- Khwarizmi. By addressing the applications of both commercial tradesmen and mathematicians, it promoted the superiority of the system, and the use of these glyphs. It is a very thorough treatise on algebraic methods and problems in which the use of Hindu-Arabic numerals is strongly advocated.

Many consider this book the greatest arithmetic text of middle ages, for he was the first mathematician to demonstrate the superiority of Hindu-Arabic Numeral System versus the Roman system.

He presented the Indeterminate second-degree equations which was developed by Greek Mathematician Diophantus and third-degree equations which he solved by method of approximation. He produced Practica Geometriae (Practice of Geometry) which included 8 chapters of theorems based on Euclid's Elements and On Divisions.

His other book Liber Quadratorum (Book of square Numbers,1225) was considered his masterpiece.

His name is known to modern mathematicians mainly because of the ***Fibonacci***

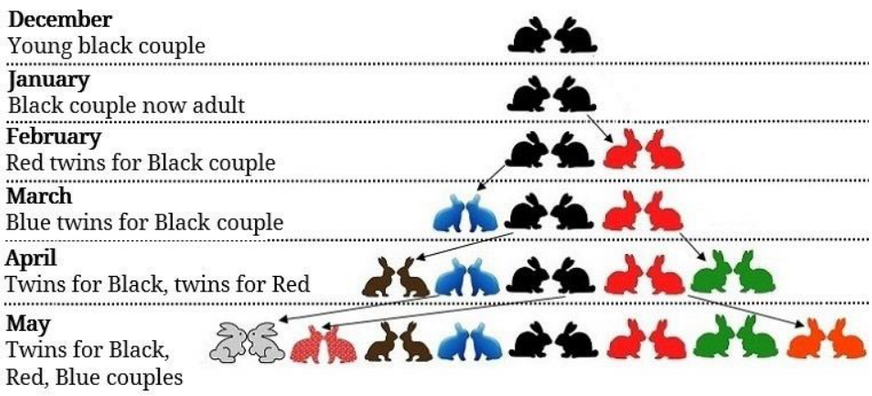
and Provence where he studied numerical systems and methods of calculation. *He wrote the book Liber Abaci,(1202, "Book of Abacus"), the first European work on Indian and Arabian Mathematics.*	***sequence** derived from a problem in the* Liber abaci. *https://www.britannica.com/biography/Fibonacci*

THE RABBIT PROBLEM

One of the mathematical problems Fibonacci investigated in Liber Abaci was about how fast rabbits could breed in ideal circumstances.

Suppose a newly born pair of rabbits, one male, one female, are put in a field. Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits. Suppose that no rabbits will die, and that female rabbit always produces one new pair (one male, one female) every month from the second month on. The puzzle that Fibonacci posed was... How many pairs will there be in one year?

ILLUSTRATION:



So by the end of NOVEMBER (12TH MONTH), there will be _____ pairs of rabbits.

Findings:

- The resulting number sequence, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55,… (Fibonacci himself omitted the first term), in which each number is the sum of the two preceding numbers, is the first recursive number sequence (in which the relation between two or more successive terms can be expressed by a formula) known in Europe.

  Though real rabbits don't breed as Fibonacci hypothesized, but his sequence still appears frequently in nature.

- The sequence is infinite.

- The Fibonacci sequence is all about growth; This is a very simple way of generating growth quickly and explains why the Fibonacci numbers appear in nature so often. The sequence is applicable to the growth of all living things, from a single plant cell to a honeybee's family tree; nature relies on simple operations to build immensely complex, often beautiful, structures, and the Fibonacci sequence reflects this."

- The rule for generating the Fibonacci numbers: add the last two to get the next.

  where $F_0 = 0$ and $F_1 = 1$

Fibonacci relationship

$F_1 = 1$
$F_2 = 1$
$F_3 = 1 + 1 = 2$
$F_4 = 2 + 1 = 3$
$F_5 = 3 + 2 = 5$

In general :

$F_n = F_{n-1} + F_{n-2}$

or

$F_{n+1} = F_n + F_{n-1}$

Each number in the sequence is the sum of the two numbers that precede it. Fibonacci sequence is one of the most famous formulas in mathematics.

- Terms in sequence were stated in above formula by French born Mathematician Albert Girard in 1634.

- The sequence is also closely related to a famous number called the *golden ratio*.

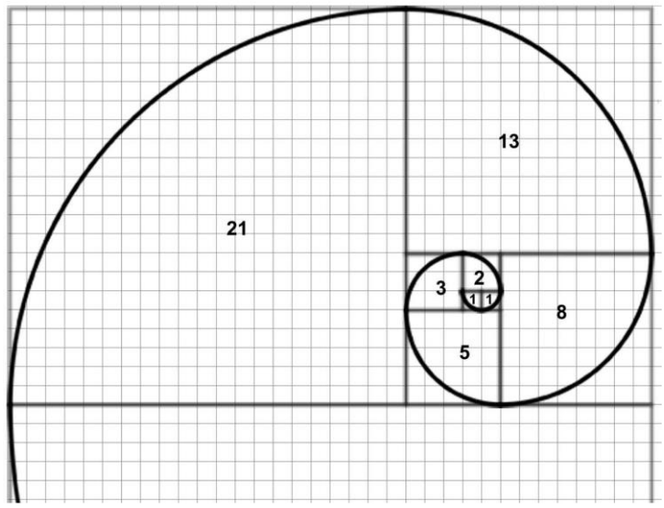
  It was noted by mathematician Robert Simson in 1753 that as the numbers increased in magnitude the ratio between succeeding numbers approached the number called golden ratio, ***φ* (phi)** in which value is 1.618034…, ($\frac{1+\sqrt{5}}{2}$).

- In the 19th century, French mathematician Edouard Lucas coined the term Fibonacci sequence and scientists began to discover such sequences in nature; for example, in the spirals of sunflower heads, in pinecones, in the regular descent (genealogy) of the male bee, in the related logarithmic (equiangular) spiral in snail shells, in the arrangement of leaf buds on a stem, and in animal horns.

- Fibonacci sequence is God's fingerprint.

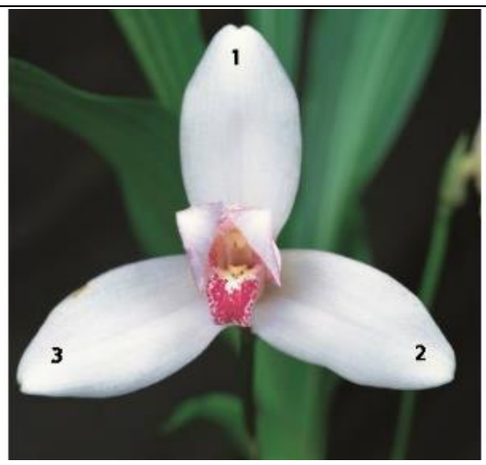
- It's been called "nature's secret code," and "nature's universal rule." It is said to govern the dimensions of everything from the Great Pyramid at Giza, to the iconic seashell that likely graced the cover of your school math textbook.

1.3 FIBONACCI SPIRAL

The **Fibonacci spiral** also known as *golden spiral* has an association with the *golden mean*, and it is based on the Fibonacci sequence. Fibonacci spiral is also referred to as golden spiral. In logarithm, it means a logarithmic spiral which gets wider by a factor of ϕ after making a quarter turn. A Fibonacci spiral having an initial radius of 1 has a polar equation like that of other logarithmic spirals.	

1.4 SOME EXAMPLES OF FIBONACCI NUMBERS IN NATURE

FLOWERS



Lily with 3 petals

Flax with 5 petals

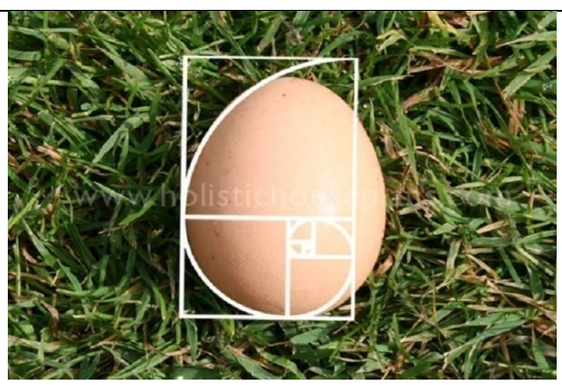
Cosmos with 8 petals

Sunflower with 21 petals

Other flowers found to have Fibonacci number of petals:

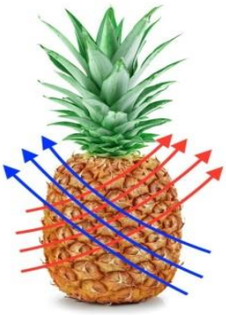
Flowers	Number of Petals
Lilies, Iris	3
Buttercups, Pinks, Some delphinium	5
Marigold	13
Some Asters, black-eyed susan	21
Daisy	21, 34, 55 and 89

MORE FIBONACCI NUMBERS IN NATURE

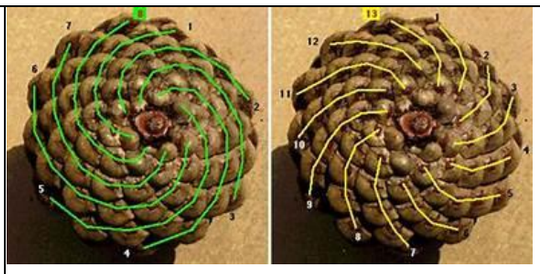


Fibonacci is even connected to the shape of **Chicken egg**

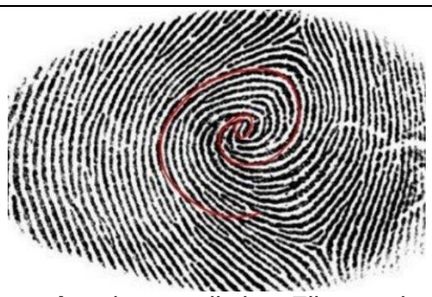
Spiral aloe. Numerous cactus display the Fibonacci spiral, each set of leaves spiral outward.



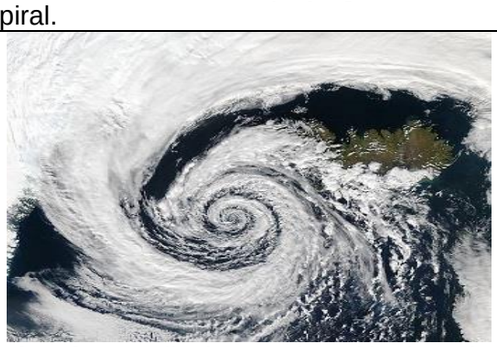
The spirals in the **skin of pineapple**.



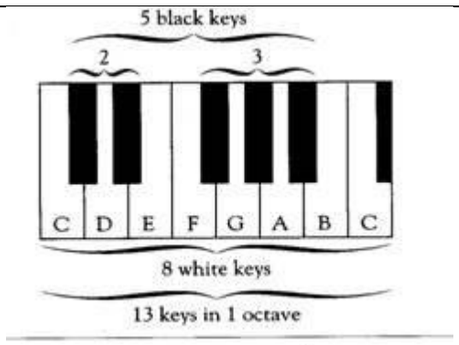
All pinecones display a fibonacci sequence. The umbo on **pinecones** increase in size as you move outward, displaying a Fibonacci spiral.



Fingerprints have a distinct Fibonacci spiral.

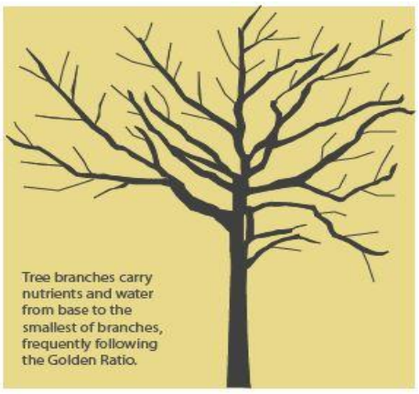


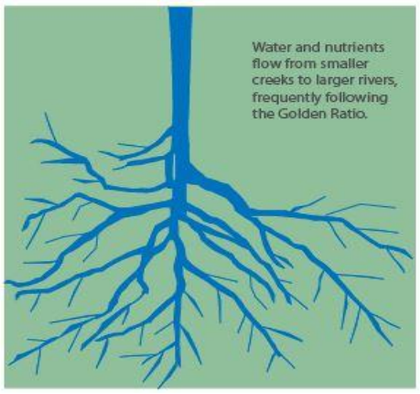
A **hurricane** displays a logarithmic spiral, one that gets smaller as it goes. Fibonacci Spiral.



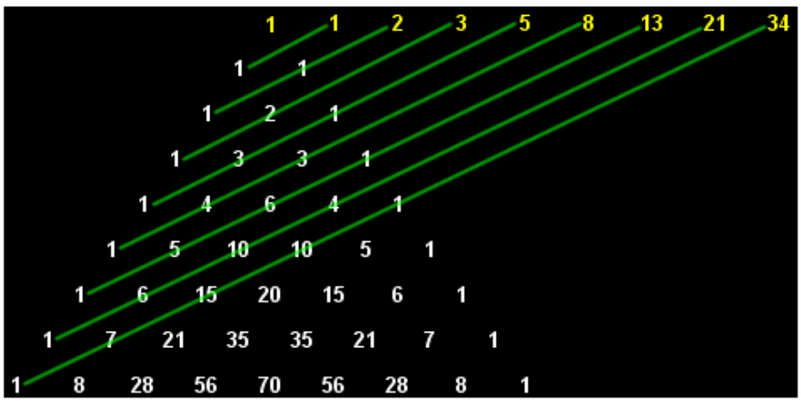
The Piano

The piano has an interesting connection to the Fibonacci Sequence. In each octave, there are 13 keys, the 7th number in the Fibonacci Sequence. From the 13 keys in each octave, there are a set of 2 black keys, a set of 3 black keys, and 8 white keys, the 3rd, 4th, and 6th numbers of the Fibonacci Sequence. Additionally, the group of 2 black keys are surrounded by 3 white keys, while the group of 3 black keys are surrounded by the remaining 5 white keys, also both numbers in the Fibonacci Sequence.





Branches and root of trees



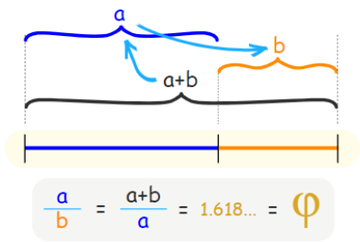
The number on the diagonals of Pascal's triangle add to the Fibonacci series

THE GOLDEN RATIO

- The Golden Ratio is also called the golden section, golden mean, golden number, divine proportion, divine section, and golden proportion.

- We find the golden ratio when we divide a line into two parts so that the whole length divided by the long part is equal to the long part divided by the short part.

- It is a ratio of line segments when a line is divided into two parts (a and b), such that the ratio of 'a' to 'b' is the same as the ratio of (a + b) to 'a'. This ratio is called the golden ratio and is signified by the Greek letter phi (Φ). Its mathematical value is 1.61803398…

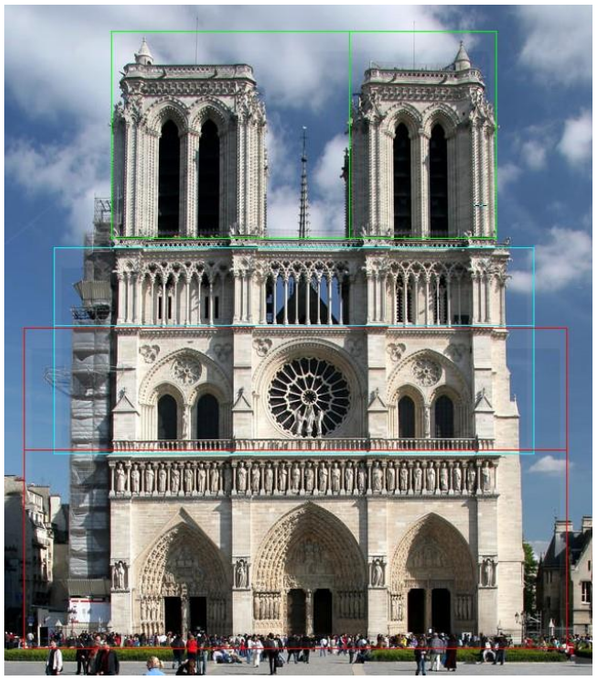


- The ratio of consecutive Fibonacci approaches $\boldsymbol{\varphi}$**.** It is also an irrational number.

F(n)	F(n-1)	F(n)/F(n-1)
1	1	1
2	1	2
3	2	1.5
5	3	1.666666667
8	5	1.6
13	8	1.625
21	13	1.615384615
34	21	1.619047619
55	34	1.617647059
89	55	1.618181818

- It appears many times in geometry, art, architecture, and other areas.
- Many buildings and artworks have the Golden Ratio in them, such as the Parthenon in Greece, but it is not really known if it was designed that way.
- Some artists and architects believe the Golden Ratio makes the most pleasing and beautiful shape.

The CN Tower in Toronto, the tallest tower and freestanding structure in the world, has contains the golden ratio in its design. The ratio of observation deck at 342 meters to the total height of 553.33 is 0.618 or phi, the reciprocal of Phi.



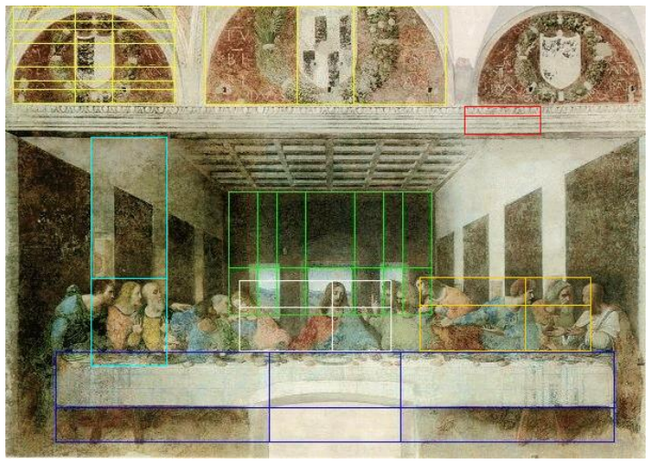
Notre Dame in Paris, which was built between 1163 and 1250. In architecture, the golden ratio is visible in any shape composed by a square and a rectangle whose combined dimensions roughly correspond to a 1:1.61 ratio. This ratio is known to be a dimension of perfection in art.

The golden ratio lines of the green, blue and red rectangles conform closely to the major architectural lines, which represent:

Red – Vertical height of base at ground level top of first level, top of second floor

Blue – Vertical height of base of second level , top of second level, Top of third level

Green – Horizontal width of outside of left top section, inside of top right section, iOutside of top right section:

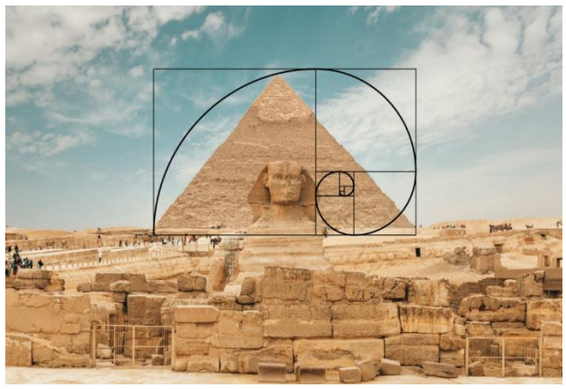


Leonardo da Vinci used golden ratio in his creation of the “Last Supper”, it was known during the Renaissance period as the **Divine Proportion**.

Same mathematical principle applied to fine cellos, violas, and stringed bass instruments created by the master maker and even in the designs of high-quality speaker to produce harmonious and pleasing to the ear’s music.

The proportions of a violin follow the Fibonacci Sequence perfectly. Antonio Stradivarius, who lived from 1644 to 1737, was a violin maker who was able to perfect the violin’s proportions, using the first 7 numbers of the Fibonacci Sequence. This proportion is the best for sound quality as well as sound projection, which are both necessary for an optimal performance.

The violinmaker ensured the proportion of the neck, pegbox and scroll to the body of the violin (upper bout, waist, and lower bout) achieves the ratio. Also, subdivisions of the instrument – waist to upper bout, waist, and upper bout to those sections plus the neck – meet the 1.6 ratio as well.

 **The Great Pyramid of Giza**	The Great Pyramid of Giza (2570 BC) considered one of the seven wonders of Ancient World, it was also considered tallest man-made structure, which took 10 years of preparation and 20 years of building. It was said to be one of the most ancient and the best examples of the use of the Phi. The ratio of the height of the pyramid to either of the sides of its base approximately equals to Phi.

III. MATHEMATICS AND THE REAL WORLD

The following are some applications of Mathematics:

- Mathematics helps organize patterns and regularities in nature.
- Mathematics helps predict the behavior of nature and many phenomena.
- Mathematics helps control nature and occurrences in the world for our own good.
- Mathematics has applications in many human endeavors.

"Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of the world."

Roger Bacon (1214-1294), English

Activity: The following are your assignments.

1. Watch in You tube “The Story of Maths. The Language of the Universe”

https://www.youtube.com/watch?v=pb0MSMGSIeY&t=984s

After Watching the above video, make your reaction/reflection consider the following:

- Share your impression and experiences.
- Did the video interest you or bother you? Why or why not?
- What did you realize after watching?

Then submit your answer to Ms Teams assignment in general channel.

2. You may also watch the following video, (optional) for additional information.

Additional video to watch, Science Documentary: The Math Mystery, 2016.

References and e-books:

- Smith, Karl J. The Nature of Mathematics. 12ed. Cengage Learning. 2012
- Angel, Abbott, Runde. Survey of Mathematics with Applications. 10ed. Pearson. 2016
- Lippman, David. Mathematics in Society. 2ed. 2017
- Thomas, Christopher. Schaum’s Outline of Mathematics for the Liberal Arts. McGrawHill. 2009

Online References:

https://www.coursehero.com/file/67488732/Mathematics-in-the-Modern-World28pdf/
https://en.wikipedia.org/wiki/Taklamakan_Desert#/media/File:Taklamakan_desert.jpg
https://animaldestination.blogspot.com/2011/06/kuhol.html
https://www.momtastic.com/webecoist/2008/09/07/17-amazing-examples-of-fractals-in-nature/
https://www.fi.edu/sites/default/files/General_Fractals_1280_720.gif
https://www.bbc.co.uk/bitesize/clips/zpn6n39
https://www.youtube.com/watch?v=xmstuly4zqU

https://en.wikipedia.org/wiki/Holstein_Friesian_cattle
https://www.fi.edu/sites/default/files/General_Voronoi_1280_720.gif
https://en.wikipedia.org/wiki/Tessellation#/media/File:Ceramic_Tile_Tessellations_in_Marrakech.jpg
https://www.sciencedirect.com/topics/pharmacology-toxicology-and-pharmaceutical-science/holstein-friesian-cattle