INSTRUCTIONAL MATERIALS IN
MATH 20053 CALCULUS 2

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CHAPTER 1
BASIC INTEGRATION FORMULA

1. 1 Integration

Definition:

The inverse process of determining a function whose derivative is known is called ***antidifferentiation* or *integration*** and the required function is called an ***antiderivative*** or ***integral*** of the given function.

Illustration:

$$d(x^3) = 3x^2$$
$$d(x^3+1) = 3x^2$$
$$d(x^3+2) = 3x^2 \quad \longrightarrow \quad \int 3x^2dx = x^3 + c$$, where c is any constant.
$$\vdots$$
$$d(x^3+c) = 3x^2$$

Definition:

$$\int \mathbf{f(x)dx} = \mathbf{F(x)} + \mathbf{c}$$

where:

$\int$ is the integral symbol or sign

$\mathbf{f(x)}$ is the integrand

$\mathbf{F(x)}$ is the particular integral

c is the constant of integration

$\mathbf{F(x)} + \mathbf{c}$ is the indefinite integral of $f(x)$

Integration Formulas (Indefinite Integral)

1. $\int du = u + c$
2. $\int (u \pm v)dx = \int udx \pm \int vdx$
3. $\int cudx = c\int udx$ where c is any constant
4. $\int u^n du = \dfrac{u^{n+1}}{n+1} + c\, ,\ n \neq -1$ (Power Formula)

Example: **Evaluate the following:**

1. $\int(10x^4-8x^3-6x+7)dx$
2. $\int(6x-7)(4x+5)dx$
3. $\int(4x+7)^2dx$
4. $\int\sqrt{x}(\sqrt[3]{x}+7\sqrt[5]{x})dx$
5. $\int\frac{x^3+27}{x+3}dx$
6. $\int\sqrt{x^4+2x^3+x^2}dx$

Activity / Exercise No. 1
I. INTEGRATION BY POWER FORMULA

I. Evaluate the following:

1. $\int(18x^5-16x^3-12x^2+8x)dx$
2. $\int(8x-11)(3x-4)dx$
3. $\int(2x-3)^3dx$
4. $\int\frac{\sqrt[4]{x}-5\sqrt[6]{x}}{\sqrt{x}}dx$
5. $\int\frac{2x^3-5x^2+5x-6}{x-2}dx$
6. $\int\left(e^3-\frac{5}{x^2}+\frac{7}{\sqrt[3]{x}}-\frac{9}{\sqrt[7]{x}}\right)dx$

1. 2 Integration by Power Formula (Substitution)

$$\int u^n du=\frac{u^{n+1}}{n+1}+c,\ n\neq-1$$

Example: **Evaluate the following:**

1. $\int(x^3-7)^2x^2dx$
2. $\int(x^3-4x)^4(9x^2-12)dx$
3. $\int\frac{(t+3)}{\sqrt[4]{t^2+6t+7}}dt$
4. $\int\frac{5e^{3t}}{\sqrt[3]{\sqrt{2-3e^{3t}}}}dt$
5. $\int\frac{(\ln 4x)^5}{x}dx$
6. $\int(6x^3+x)\sqrt[3]{6x^2+1}\,dx$

Activity / Exercise No. 2
II. INTEGRATION BY POWER FORMULA (SUBSTITUTION)

I. Evaluate each of the following indefinite integrals:

1. $\int \left(x^5 - 2x^2 + 3x\right)^4 \left(5x^4 - 4x + 3\right) dx$
2. $\int \sqrt{6x^4 + 7x^2 - 5x}\left(48x^3 + 28x - 10\right) dx$
3. $\int \frac{3e^{2t}}{\sqrt[3]{1 + e^{2t}}} dt$
4. $\int \frac{2x - 1}{\sqrt{\sqrt[4]{2x^2 - 2x + 5}}} dx$
5. $\int \frac{3\ln^6 7x}{7x} dx$
6. $\int \left(5x^5 + 3x^2\right) \sqrt[4]{5x^3 + 3}\, dx$

1. 3 Integrals Leading to Logarithms

$$\int \frac{du}{u} = \ln|u| + c$$

$\boxed{\textbf{Example:}}$ **Evaluate each of the following indefinite integrals:**

1. $\int \frac{dx}{2x + 1}$
2. $\int \frac{x^2}{x^3 - 7} dx$
3. $\int \frac{t + 3}{t^2 + 6t + 7} dt$
4. $\int \frac{t + 7}{t + 9} dt$
5. $\int \frac{x^2 + 3x - 7}{x + 5} dx$
6. $\int \frac{dx}{x(7 + 3\ln x)}$

Activity / Exercise No. 3
INTEGRALS LEADING TO LOGARITHMS

I. Evaluate each of the following indefinite integrals:

1. $\int \frac{dx}{5 - 7x}$
2. $\int \frac{5x^4 - 4x + 3}{x^5 - 2x^2 + 3x} dx$
3. $\int \frac{3e^{2t}}{1 + e^{2t}} dt$
4. $\int \frac{t - 8}{t - 11} dt$
5. $\int \frac{x^5 - 3x^3 - 3x}{x^2 - 1} dx$
6. $\int \frac{x^3 dx}{(x^4 - 7)\ln(x^4 - 7)}$

1. 4 Integration of Exponential Functions

$$\int e^u du = e^u + c$$

$$\int a^u du = \frac{a^u}{\ln a} + c \text{, where } a > 0,\ a \neq 1$$

Example: **Evaluate the following:**

1. $\int 3e^{2t} dt$
2. $\int x^3 e^{x^4} dx$
3. $\int \frac{e^{\sqrt[3]{x}} dx}{\sqrt[3]{x^2}}$
4. $\int (e^{\frac{1}{4}x-7} + e + x^e + e^x) dx$
5. $\int 5^{7x} dx$
6. $\int 7^x e^x dx$

Activity / Exercise No. 4
INTEGRATION OF EXPONENTIAL FUNCTIONS

I. Evaluate the following:

1. $\int \frac{dx}{e^{4x}}$
2. $\int \frac{e^{\frac{1}{x^2}} dx}{x^3}$
3. $\int \frac{7e^{\sqrt{y}} dy}{\sqrt{y}}$
4. $\int (e^x + e^{-x})^2 dx$
5. $\int 3^{-x^2} x dx$
6. $\int 5^{2x} e^x dx$

1. 5 Integration of Trigonometric Functions

1. $\int \sin u \ du = -\cos u + c$
2. $\int \cos u \ du = \sin u + c$
3. $\int \tan u \ du = -\ln|\cos u| + c$
4. $\int \cot u \ du = \ln|\sin u| + c$
5. $\int \sec u \ du = \ln|\sec u + \tan u| + c$
6. $\int \csc u \ du = -\ln|\csc u + \cot u| + c$
   $= \ln|\csc u - \cot u| + c$
7. $\int \sec u \tan u \ du = \sec u + c$
8. $\int \csc u \cot u \ du = -\csc u + c$
9. $\int \sec^2 u du = \tan u + c$
10. $\int \csc^2 u du = -\cot u + c$

Example: **Evaluate the following:**

A.

1. $\int \sin 3x \; dx$
2. $\int x^3 \cos 3x^4 \; dx$
3. $\int (1+\csc x)^2 \; dx$
4. $\int \dfrac{\sin^3 x}{1-\cos x} dx$
5. $\int 3x^2 \sec x^3 \tan x^3 \, dx$
6. $\int \dfrac{dx}{\sin 2x \tan 2x}$
7. $\int (\tan 2x + \cot 2x)^2 dx$
8. $\int \dfrac{\cos x + \sin x}{\cos^2 x} dx$
9. $\int \dfrac{\sin 2y}{2 \sin y \cos^2 y} dy$
10. $\int \cos x \quad \cos(\sin x)\, dx$

B.

1. $\int \tan^3 x \sec^2 x \, dx$
2. $\int e^{\cos 3x} \sin 3x \, dx$
3. $\int \dfrac{dy}{\sin y \cos y}$

Activity / Exercise No. 5
INTEGRATION OF TRIGONOMETRIC FUNCTIONS

I. Evaluate the following:

A.

1. $\int \cos \frac{1}{2} x \, dx$
2. $\int e^x \sin(e^x) dx$
3. $\int (1+\sec 2x)^2 \; dx$
4. $\int \dfrac{1+\cos 4x}{\sin 4x} dx$
5. $\int x^4 \csc x^5 \cot x^5 \, dx$
6. $\int \dfrac{dx}{\sin 3x \cot 3x}$
7. $\int (\cot 3x - \tan 3x)^2 \, dx$
8. $\int \dfrac{\sin 2x + \cos 2x}{\sin^2 2x} dx$
9. $\int \dfrac{\cos 2x}{\cos x} dx$
10. $\int \sec^2 x \quad \sec^2(\tan x)\, dx$

B.

1. $\int \cot^4 x \csc^2 x \; dx$
2. $\int e^{\sin 2x} \cos 2x dx$
3. $\int \dfrac{dx}{1-\sin x}$

1. 6 Integrals Leading to Inverse Trigonometric Functions

1. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + c$

2. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + c$

3. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{u}{a} + c$

Example: **Evaluate the following:**

1. $\int \frac{dy}{\sqrt{25 - 16y^2}}$

2. $\int \frac{\sec^2\theta}{\sqrt{5 - \tan^2\theta}}d\theta$

3. $\int \frac{dt}{t(1 + 4\ln^2 t)}$

4. $\int \frac{dx}{x^2 + 4x + 5}$

5. $\int \frac{x^4 dx}{x^2 + 1}$

6. $\int \frac{dx}{\sqrt{2x - x^2}}$

7. $\int \frac{dt}{t\sqrt{4t^2 - 1}}$

8. $\int \frac{xdx}{x^2 - x + 2}$

Activity / Exercise No. 6
INTEGRALS LEADING TO INVERSE TRIGONOMETRIC FUNCTIONS

I. Evaluate the following:

1. $\int \frac{dy}{\sqrt{9 - y^2}}$

2. $\int \frac{e^{3x} dx}{9 + 4e^{6x}}$

3. $\int \frac{x\,dx}{25 + 16x^4}$

4. $\int \frac{dx}{x^2 - 3x + 4}$

5. $\int \frac{2x + 3}{x^2 + 9}dx$

6. $\int \frac{dx}{\sqrt{5 - 4x - x^2}}$

7. $\int \frac{dx}{x\sqrt{x^2 - 1}}$

8. $\int \frac{4x + 9}{x^2 - 4x + 20}dx$

1. 7 Integration of Hyperbolic Functions

A. Fundamental Identities / Properties of Hyperbolic Functions

1. $\tanh x = \dfrac{\sinh x}{\cosh x}$
2. $\operatorname{sech} x = \dfrac{1}{\cosh x}$
3. $\operatorname{csch} x = \dfrac{1}{\sinh x}$
4. $\coth x = \dfrac{1}{\tanh x} = \dfrac{\cosh x}{\sinh x}$
5. $\cosh^2 x - \sinh^2 x = 1$
6. $\tanh^2 x + \operatorname{sech}^2 x = 1$
7. $\coth^2 x - \operatorname{csch}^2 x = 1$
8. $\sinh 2x = 2\sinh x \cosh x$
9. $\cosh 2x = \cosh^2 x + \sinh^2 x$
10. $\cosh x - \sinh x = e^{-x}$
11. $\cosh x + \sinh x = e^{x}$

B. Integration of Hyperbolic Functions

1. $\int \cosh u\, du = \sinh u + c$
2. $\int \sinh u\, du = \cosh u + c$
3. $\int \operatorname{sech}^2 u\, du = \tanh u + c$
4. $\int \operatorname{csch}^2 u\, du = -\coth u + c$
5. $\int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + c$
6. $\int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + c$
7. $\int \tanh u\, du = \ln\left|\cosh u\right| + c$
8. $\int \coth u\, du = \ln\left|\sinh u\right| + c$

Example: **Evaluate the following:**

1. $\int \operatorname{sech} 3x \tanh 3x \, dx$
2. $\int \dfrac{\sinh\sqrt{x}}{\sqrt{x}}\, dx$
3. $\int \dfrac{\coth^2 \ln x}{x} dx$
4. $\int e^t \cosh\left(e^t\right)\sinh\left(e^t\right) dt$
5. $\int \operatorname{csch}^2 6x \coth^4 6x \, dx$
6. $\int \tanh^2 5y\, dy$
7. $\int \dfrac{\ln x \sinh \ln^2 x}{x} dx$
8. $\int \dfrac{\operatorname{sech}^2(\ln x)}{x} dx$

Activity / Exercise No. 7
INTEGRATION OF HYPERBOLIC FUNCTIONS

I. Evaluate the following:

1. $\int \cosh 7x dx$
2. $\int e^{-t} \operatorname{csch}^2 (e^{-t}) dt$
3. $\int \tanh x \ln(\cosh x) \, dx$
4. $\int \sinh^3 x \cosh x \, dx$
5. $\int \operatorname{sech}^3 3x \tanh 3x \, dx$
6. $\int \operatorname{sech}^4 x \, dx$
7. $\int e^{\sec x} \sec x \tan x \operatorname{sech}(e^{\sec x}) \tanh(e^{\sec x}) \, dx$
8. $\int \coth 2x \ \ln(\sinh 2x) \, dx$