Long Exercise No. 1

Answer Key

Total Score: 100

Note: Less 1 point for every wrong term

1.) $\int (\sqrt{\sin x} + \cos x)^2 dx$

Solution:

$\int (\sin x + 2\sqrt{\sin x}\cos x + \cos^2 x) dx$ 2

$\int \sin x\, dx + 2\int \sin^{1/2} x \cos x\, dx + \int \cos^2 x\, dx$

$-\cos x + \dfrac{2\sin^{3/2} x}{3/2} + \int \dfrac{1+\cos 2x}{2} dx$

$-\cos x + \frac{4}{3}\sin^{3/2} x + \frac{1}{2}\int (1+\cos 2x) dx$

$-\cos x + \frac{4}{3}\sin^{3/2} x + \frac{1}{2}\int dx + \frac{1}{2}\int \cos 2x\, dx$

$$\boxed{-\cos x + \frac{4\sin^{3/2} x}{3} + \frac{x}{2} + \frac{\sin 2x}{4} + C}$$

2 2 2 2

2.) $\int \sin^7 x \cos^3 x\, dx$

Solution:

$\int \sin^7 x \cos^2 x \cos x\, dx$ 2

$\int \sin^7 x (1 - \sin^2 x) \cos x\, dx$ 2

$\int \sin^7 x \cos x\, dx - \int \sin^9 x \cos x\, dx$ 2

let $u = \sin x$

$du = \cos x\, dx$

so,

$$\boxed{\frac{\sin^8 x}{8} - \frac{\sin^{10} x}{10} + C}$$

2 2

3.) $\int \sqrt{\tan x} \sec^6 x\, dx$

Solution:

$\int \tan^{1/2} x \sec^6 x\, dx$

$\int \tan^{1/2} x \sec^4 x \sec^2 x\, dx$ 2

$\int \tan^{1/2} x (\sec^2 x)^2 \sec^2 x\, dx$

$\int \tan^{1/2} x (1 + \tan^2 x)^2 \sec^2 x\, dx$ 2

$\int \tan^{1/2} x (1 + 2\tan^2 x + \tan^4 x) \sec^2 x\, dx$ 2

$\int \tan^{1/2} x \sec^2 x\, dx + 2\int \tan^{5/2} x \sec^2 x\, dx$

$+ \int \tan^{9/2} x \sec^2 x\, dx$ 1

let $u = \tan x$ $\quad du = \sec^2 x\, dx$

so, $\dfrac{\tan^{3/2} x}{3/2} + \dfrac{2\tan^{7/2} x}{7/2} + \dfrac{\tan^{11/2} x}{11/2} + C$

$$\therefore \boxed{\frac{2\tan^{3/2} x}{3} + \frac{4\tan^{7/2} x}{7} + \frac{2\tan^{11/2} x}{11} + C}$$

1 1 1

4.) $\int \tan^7 7x\,dx$

Solution:

$\int \tan^5 7x \tan^2 7x\,dx$

$\int \tan^5 7x(\sec^2 7x - 1)dx$ 2

$\int \tan^5 7x \sec^2 7x\,dx - \int \tan^5 7x\,dx$ 1

let $u = \tan 7x$ n.f. $= 1/7$

$du = 7\sec^2 7x\,dx$

so, $\frac{1}{7}\frac{\tan^6 7x}{6} - \int \tan^3 7x \tan^2 7x\,dx$

$\frac{\tan^6 7x}{42} - \int \tan^3 7x(\sec^2 7x - 1)dx$ 1

$\frac{\tan^6 7x}{42} - \int \tan^3 7x \sec^2 7x\,dx$
$+ \int \tan^3 7x\,dx$

$\frac{\tan^6 7x}{42} - \frac{1}{7}\frac{\tan^4 7x}{4} + \int \tan 7x \tan^2 7x\,dx$

$\frac{\tan^6 7x}{42} - \frac{\tan^4 7x}{28} + \int \tan 7x(\sec^2 7x - 1)dx$ 1

$\frac{\tan^6 7x}{42} - \frac{\tan^4 7x}{28} + \int \tan 7x \sec^2 7x\,dx$
$- \int \tan 7x\,dx$ 1

let $u = \tan 7x$ n.f. $= 1/7$

$du = 7\sec^2 7x\,dx$

Now,

$\frac{\tan^6 7x}{42} - \frac{\tan^4 7x}{28} + \frac{1}{7}\frac{\tan^2 7x}{2} + \frac{1}{7}\ln|\cos 7x| + C$

$$\boxed{\frac{\tan^6 7x}{42} - \frac{\tan^4 7x}{28} + \frac{\tan^2 7x}{14} + \frac{\ln|\cos 7x|}{7} + C}$$

or $\frac{\tan^6 7x}{42} - \frac{\tan^4 7x}{28} + \frac{\tan^2 7x}{14} - \frac{\ln|\sec 7x|}{7} + C$

1 1 1 1

5.) $\int \csc^6 6x\,dx$

Solution:

$\int (\csc^4 6x)(\csc^2 6x)\,dx$ 2

$\int (\csc^2 6x)^2 \csc^2 6x\,dx$

$\int (1 + \cot^2 6x)^2 \csc^2 6x\,dx$ 2

$\int (1 + 2\cot^2 6x + \cot^4 6x)\csc^2 6x\,dx$ 2

$\int \csc^2 6x\,dx + 2\int \cot^2 6x \csc^2 6x\,dx$
$+ \int \cot^4 6x \csc^2 6x\,dx$ 1

let $u = \cot 6x$ n.f. $= -1/6$

$du = -6\csc^2 6x\,dx$

$-\frac{\cot 6x}{6} + 2\left(\frac{-1}{6}\right)\frac{\cot^3 6x}{3} + \left(\frac{-1}{6}\right)\frac{\cot^5 6x}{5} + C$

$$\therefore \boxed{-\frac{\cot 6x}{6} - \frac{\cot^3 6x}{9} - \frac{\cot^5 6x}{30} + C}$$

1 1 1

6.) $\int \sqrt{\cot 3x}\,\csc^4 3x\,dx$

Solution: $\int \cot^{1/2} 3x \csc^4 3x\,dx$

$\int \cot^{1/2} 3x \csc^2 3x \csc^2 3x\,dx$ 2

$\int \cot^{1/2} 3x(1 + \cot^2 3x)\csc^2 3x\,dx$ 2

$\int \cot^{1/2} 3x \csc^2 3x\,dx + \int \cot^{5/2} 3x \csc^2 3x\,dx$ 2

let $u = \cot 3x$ n.f. $= -1/3$

$du = -3\csc^2 3x\,dx$

so, $-\frac{1}{3}\frac{\cot^{3/2} 3x}{3/2} + \frac{-1}{3}\frac{\cot^{7/2} 3x}{7/2} + C$ 1 1

$$\therefore \boxed{-\frac{2\cot^{3/2} 3x}{9} - \frac{2\cot^{7/2} 3x}{21} + C}$$

1 1

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Note: Less 1 point for every wrong term

7.) $\displaystyle\int \frac{\operatorname{sech}^2(\ln x)}{x}\,dx$

Solution: $\displaystyle\int \operatorname{sech}^2(\ln x)\,\frac{dx}{x}$

let $u = \ln x$ 3

$du = \dfrac{dx}{x}$ 3

so, $\boxed{\tanh(\ln x) + c}$ 4

8.) $\displaystyle\int \coth 2x \ln(\sinh 2x)\,dx$

Solution:

$\displaystyle\int \ln(\sinh 2x)\ \coth 2x\,dx$

let $u = \ln(\sinh 2x)$ 2

$du = \dfrac{1}{\sinh 2x}\,d(\sinh 2x)$

$du = \dfrac{1}{\sinh 2x}(\cosh 2x)\,2\,dx$

$du = 2\coth 2x\,dx$ 2

n.f. = $1/2$ 2

So, applying power formula

$\dfrac{1}{2}\,\dfrac{\ln^2(\sinh 2x)}{2} + C$ 2

$\therefore$ $\boxed{\dfrac{\ln^2(\sinh 2x)}{4} + C}$

2

9.) $\displaystyle\int \frac{dx}{\sqrt{5-4x-x^2}}$

Solution: By completing the $\square$

$5-4x-x^2 = -x^2-4x+5$

$= -(x^2+4x)+5$

$= -(x^2+4x+4)+5+4$

$= -(x^2+4x+4)+9$

$= -(x+2)^2+9$

$= 9-(x+2)^2$

3

So, $\displaystyle\int \frac{dx}{\sqrt{9-(x+2)^2}}$ 2

let $a^2 = 9$ $\quad u^2 = (x+2)^2$

$a = 3$ 1 $\quad u = x+2$ 1

$du = dx$ 1

thus, apply $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + c$

$\therefore$ $\boxed{\arcsin\dfrac{x+2}{3} + C}$

2

10.) $\int \frac{4x+9}{x^2-4x+20}\,dx$

Solution: By completing the $\square$

$x^2-4x+20 = (x^2-4x+4)+16$

$= (x-2)^2+16$ 2

let

$u^2 = (x-2)^2 \qquad a^2 = 16$

$u = x-2 \qquad a = 4$

$du = dx$

let

$u = x^2-4x+20$

$du = (2x-4)\,dx$

So, $\int \frac{2(2x-4)+17}{x^2-4x+20}\,dx$ 2

$\int \frac{2(2x-4)}{x^2-4x+20}\,dx + \int \frac{17\,dx}{x^2-4x+20}$

$2\int \frac{2x-4}{x^2-4x+20}\,dx + 17\int \frac{dx}{(x-2)^2+16}$ 1

let $u = x^2-4x+20$

$du = (2x-4)\,dx$ 1

let $u^2 = (x-2)^2 \qquad a^2 = 16$

$u = x-2 \qquad a = 4$ 1

$du = dx$ 1

Now,

$2\ln|x^2-4x+20| + 17\left(\frac{1}{4}\right)\arctan\frac{x-2}{4} + c$

$$\boxed{2\ln|x^2-4x+20| + \frac{17}{4}\arctan\frac{x-2}{4} + c}$$

1 1

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