Chapter 2

SET THEORY

LEARNING OBJECTIVE

1. Define sets and identify the types/kinds of sets.

2. Demonstrate the operations on sets and relate them to Venn Diagram.

3. Solve practical problems involving sets and its operations.

2.0 SET

It is a well-defined collection of distinct objects. It is denoted by capital letters of the English alphabet.

Example 1: A = {2, 4, 6, 8, 10}

B = {1, 2, 3, …}

C = {colors of the rainbow}

D = {l, o, v, e}

E = {x ∈ R | x ≥ 10 }

Read: **"The set of all x's that are a member of the Real Numbers, such that x is greater than or equal to 10"**

Or "**All Real Numbers from 3 upwards**"

Members/Element of a set are objects that composed a set. It is represented by the symbol ∈.

Example 2: From sets A, B and C in the previous examples,

8 ∈ A,	read	"8 is an element of set A"
100 ∈ B,	read	"100 is a member of set B"
0 ∉ B,	read	"0 is not an element of set B"
Amber ∉ C	read	"Amber is not an element of set C

Note: The elements of sets A, B and C are well-defined because we could easily identity the elements of each set. Also, each set have distinct objects because each member of a set is different from one another and therefore cannot be written more than once in a given set.

2.1 WAYS OF WRITING A SET

1. **The Tabular or roster form** is a method of writing a set by enumerating its elements within a pair of braces and each element is separated by a comma.

Example 3: A = {2, 4, 6, 8, 10}

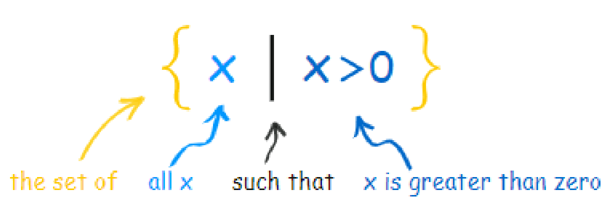
B = {1, 2, 3, …}

2. **The Rule form** is a method of writing a set by describing the elements of a set.

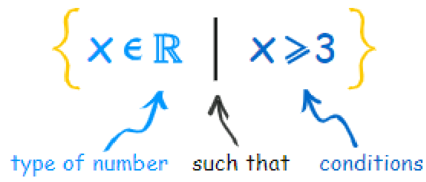
Example 4: A = {set of all even numbers less than 11}

B = {set of positive integers}

SET-BUILDER NOTATION is a method of writing the rule form into symbolic form. It uses the following symbols:



Or sometimes it shows the type of number the variable x represents.



The following are commonly used symbols to express some type of numbers:

1. Whole numbers are numbers 0, 1,2,3 …

2. **N**- Counting numbers/Natural numbers are whole numbers without the zero, such as 1,2, 3,…

3. **Z** - Integers are whole numbers but includes the negative numbers, such as …, -2, -1, 0, 1, …

4. **Q** – Rational numbers are numbers which can be expressed as ratio of integers such as ½, 5/3, 100/23…

5. **P** – Irrational numbers are numbers which can't be expressed as ration of integers such as $\sqrt{2}$, π, e,…

6. **R –** Real numbers are composed of both rational and irrational numbers, such 2/3, 3.5, 10, $\sqrt{2}$, π, …

7. **I –** Imaginary numbers such as 2i, 2-3i, $\sqrt{-10}$,…

Example 5: Write the previous sets A and B to Set-Builder Notation form.

Answer: Rule Form: A = {2, 4, 6, 8, 10}

Set-Builder Form: A = $\{x|x \text{ } is \text{ } an \text{ } even \text{ } integer \text{ } between \text{ } 1 \text{ } and \text{ } 11\}$

or A = $\{x|x \text{ } is \text{ } an \text{ } even \text{ } integer \text{ } less \text{ } than \text{ } 11\}$

Rule Form: B = {1, 2, 3, …}

Set-Builder Form: B = {x / x ∈ N}

2.2 Cardinality of a set

It is the total number of elements in each set. It may be denoted by symbols C(S) = N, where S represent the name of the set and N is the total number of elements in the given set.

Example 6: A = {2, 4, 6, 8, 10} B = ϕ

C(A) = 5 C(B) =0

2.3 TYPES OF SETS

1. **Equal Sets**

Sets A and B are equal denoted by A = B if A and B have the same elements.

Example 7: A = {a, l, g, e, b, r}

B = {r, b, a, e, g, l}

A = B

2. **Equivalent Sets**

Sets A and B are equivalent, denoted by A ~ B, if A and B have the same number of elements.

Example 8: M = {red, blue, yellow}

N = {blue, yellow, red}

M ~ N

Take note: Sets A and B in example 7 are also equivalent sets.

3. **Finite Sets**

Finite sets are sets having finite or countable number of members.

Example 9: The above sets A, B, M and N are finite sets

4. **Infinite Sets**

A set which contains an infinite number of elements.

Example 10: C = { 0,1,2,3,…}

D = {stars in the sky}

E = {Fibonacci numbers}

5. **Universal Sets**

A **universal set** is all the elements, or members, of any group under consideration, denoted by capital letter **U.**

Example 11: U = {set of natural numbers}

U = {set of basic geometric shapes}

6**. Joint Sets**

Sets that have common elements.

Example 12: F = {5, 10, 15, 20, 25}

G = {10, 20, 30 ,40, 50}

Sets F ang G have common elements 10and 20, therefore they are joint sets.

7. **Disjoint Sets**

Sets that have no common element.

Example 13: R = {triangle, square, quadrilateral}

S = {hexagon, pentagon, octagon}

Sets R and S have no common element; therefore, they are disjoint sets.

8. **Null Set/Empty Set**

It is a set with no element, denoted by { } or the symbol ϕ.

9. **Singleton Set**

A set which contains a single element.

H = { moon}

2.4 RELATIONS BETWEEN SETS

1. **Subsets**

Set A is a subset of B, "$\mathrm{A} \subseteq \mathrm{B}$" if every element of A belongs to B.

Example 14: $\mathrm{M} = \{1,2,3,4,5,6,7,8,9,10\}$ and $\mathrm{K} = \{1,3,5,7,9\}$

$\mathrm{K} \subseteq \mathrm{M} \qquad \mathrm{K} \subseteq \mathrm{K} \qquad \mathrm{M} \subseteq \mathrm{M}$
$\emptyset \subseteq \mathrm{M} \qquad \emptyset \subseteq \mathrm{K}$

Take Note: 1. That an empty set is a subset of any set.
2. That every set is a subset of itself.

2. **Proper Subse**ts

Set A is a proper subset, B "$A \subset B$" if every element of A belongs to B and B has at least one element not found in A.

Example: Given $M = \{1,2,3,4,5,6,7,8,9,10\}$ and $K = \{1,3,5,7,9\}$

$K \subset M$ $\quad M \not\subset M$ $\quad K \not\subset K$
$\emptyset \subset K$ $\quad \emptyset \subset M$

Take Note: 1. The empty set is a proper set of all sets except null set.

Power set, P(A) – is the set of all subsets of a given set and can be found by the formula: $\mathbf{2^n}$ where n is the number of elements in the given set.

Example 15: Set A = {1,2,3} has 8 subsets, because $2^3 = 8$ subsets

The subsets are as follows: {1, 2, 3}; {1, 2}; {1, 3}; {2, 3}; {1}; {2;} {3}; { }

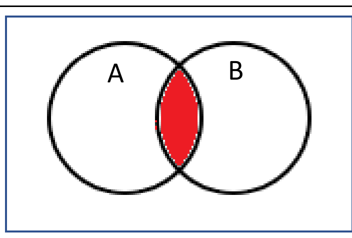
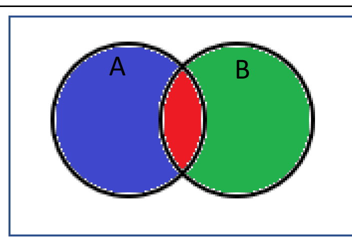
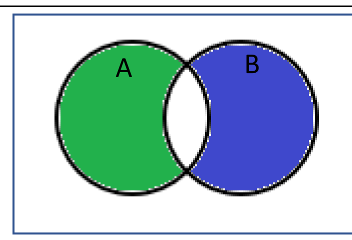
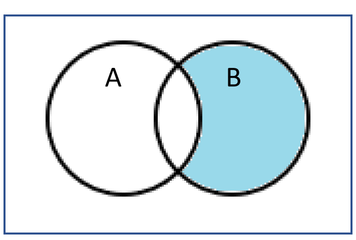
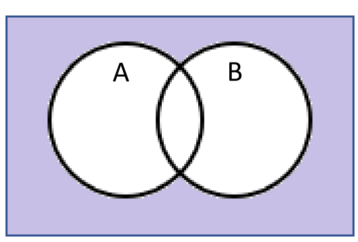
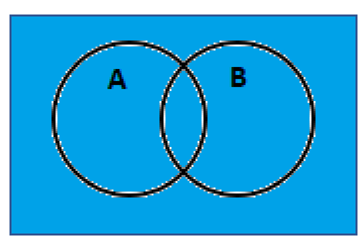
2.5 VENN DIAGRAM

A **Venn diagram** is a pictorial representation of the relationships between sets. It may also refer to as primary diagram, logic diagram or set diagram.

It was popularized by English Logician John Venn in 1880. It was invented for use in Set Theory, another branch of Mathematics.

Geometric shapes such rectangles and circles may represent universal set and subsets of universal set, respectively.

COMPONENTS OF VENN DIAGRAMS

 **Intersection of Sets** Overlapping circles $A \cap B$	 **Union of Sets** Combined circles $A \cup B$	 **Symmetric Difference** The shaded part of two sets are the parts that are different from each other. $\mathbf{A \Delta B}$
 **Relative Complement** The shaded section of an individual circle that is not included into the intersection. **B - A**	 **Absolute Complement** The shaded section of the diagram and does not include the circles. **(A U B)'**	 **Universal Set** The shaded portion of the rectangles including the circles. **U**

2.6 OPERATIONS ON SETS

I. **Union of Sets,**

$A \cup B$, is the set of all elements found in A or B or both.

2**. Intersection of Sets,**

$A \cap B$, is the set of all elements common to both A and B.

3**. Complement of A**

A' is the set of all element in the universal set but not found in A.

4. **Difference of sets A and B (Relative complement)**

A – B, is the set of all elements found in A but not in B.

B – A, is the set of all elements found in B but not in A.

5. **Symmetric difference** of two sets also known as the **disjunctive union**, is the set of elements which are in either of the sets, but not in their intersection. It is denoted by the symbol Δ.

2.7 LAWS OF ALGEBRA OF SETS

1. **COMMUTATIVE LAWS:** The order in which the sets are taken does not affect the result.

A U B = B U A; **A ∩ B = B ∩ A**

2**. ASSOCIATIVE LAWS:** The grouping in which three or more sets are taken does not affect the result.

A U (B U C) = (A U B) U C; **A ∩ (B ∩ C) = (A ∩ B) ∩ C**

3**. IDENTITY LAWS:** For any subset A of the universal set U the following identities hold:

1. **A** ∪ ∅ **= A** **3. A ∩** ∅ **=** ∅

2. A ∩ U = A **4. A** ∪ **U = U**

4. INVERSE OR COMPLEMENT LAWS: For any subset A of the universal set U the following identities hold:

1. **A U A' = U;** **2. A ∩ A' =** ∅

5. DISTRIBUTIVE LAWS: For any given three sets involving two different operations.

1. A ∪ **(B ∩ C) = (A** ∪ **B) ∩ (A** ∪ **C)**

2. A ∩ (B ∪ **C) = (A ∩ B)** ∪ **(A ∩ C)**

6. DE MORGAN'S LAWS: This relate the intersection and union of sets through complements**.**

1. (A ∪ **B)' = A' ∩ B'**

2. (A ∩ B)' = A' ∪ **B'**

Example 14: A survey was conducted on the Television Channels the residents of one barangay in Metro Manila have watched last during the last quarter of the year. There are 150 respondents, and the result of the survey is as follows:

75 residents watched channel X
80 residents watched channel Y
95 residents watched channel Z
32 watched channels X and Y
48 watched channels X and Z
39 watched channels Y and Z
15 watched channels X, Y and Z

Illustrate the above information using a Venn diagram and answer the following questions.

1. How many residents watched channels X and Y only?
2. How many residents watched channels X and Z only?
3. How many residents watched channels Y and Z only?
4. How many residents did not watch any of the said channels?

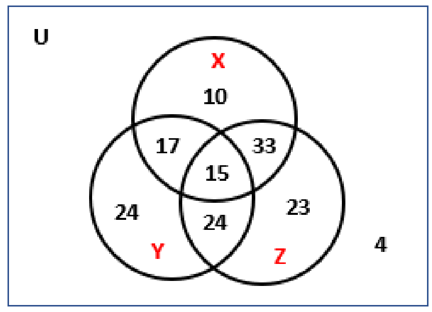
Solution: First draw the Venn diagram as follows. Enter 15 to the intersection of the 3 channels and see the computation below of other entries.

For the intersection of X and Y: 32 - 15 = 17

For the intersection of X and Z: 48 – 15 = 33

For the intersection of Y and Z: 39 – 15 = 24

For those residents that did not watch any channels: 150 - (10+17+15+33+24+24+23) = 4



So, the answer to the following questions are:

1. How many residents watched channels X and Y only? **17**
2. How many residents watched channels X and Z only? **33**
3. How many residents watched channels Y and Z only? **24**
4. How many residents did not watch any of the said channels? 4

Example 15: Angel, Bobby, Claire, Daniel, Eric, Fred, George, Hero, Imee and John are best friends. They've been friends for a long time and shared many common things such as the kind of sports they like and play.

Angel, Claire, Daniel and Hero play soccer
Claire, Daniel and John play Tennis
Daniel, George and John play Volleyball

Draw the Venn diagram showing the above information. Give a summary of your findings in the Venn diagram.

Solution:

The diagram shows the ff. information about the ten best friends' kind of sports:

- Daniel plays soccer, volleyball and tennis
  - Claire plays Soccer and tennis
- John plays tennis and volleyball
- George plays volleyball only

Example 16: Given U = { 1,2,3,…10};

A = { 1, 2, 5, 6, 9}; B = { 3, 4, 5, 7, 8, 10} and C = { 4, 7, 10}.

List the elements of the following.

1. A U B	2. A U B U C
3. A' ∩ B' ∩ C'	4. (A ∩ B)'
5. B – C	6. C – A
7. A ∩ B ∩ C	8. (A ∩ B ∩ C)'

Solution:

1. A U B = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
2. A U B U C = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

3. A’ ∩ B’ ∩ C’ = { }

A’ = {3, 4, 7, 8, 10}

B’ = {1, 2, 6, 9}

C’ = {1, 2, 3, 5, 6, 8, 9}

4. (A ∩ B)’ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

A ∩ B = { }

5. B – C = {3, 5, 8}

6. C – A = {4, 7, 10}

7. A ∩ B ∩ C = { }

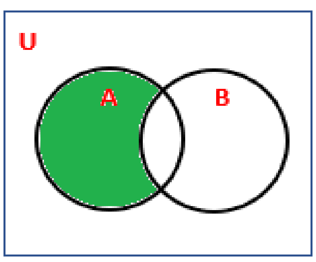
8. (A ∩ B ∩ C)’ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Example 17. A Venn diagram showing two sets is given below, find the indicated sets:

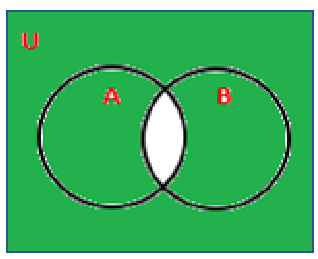
1. A ∩ B
2. (A ∩ B)'
3. A ∪ B'
4. A ∩ B'
5. B'
6. B - A

Solution:

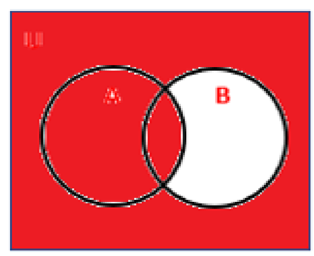
1. A ∩ B



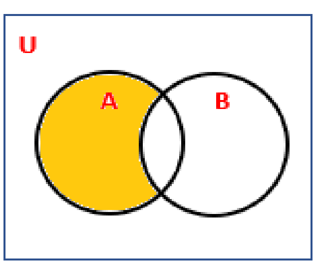
2. (A ∩ B)'



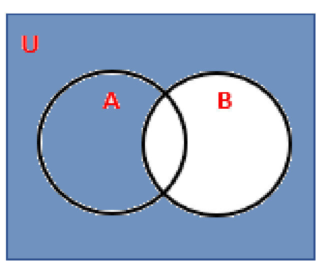
3. A ∪ B'



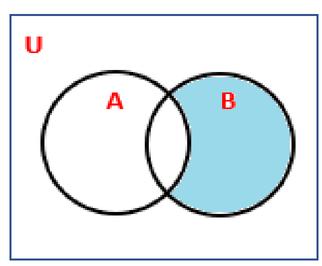
4. A ∩ B'



5. B’

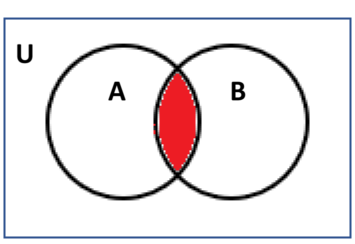


6. B - A



Example 17. Please take note of the correction in no. 1.

1. A ∩ B



REVIEW EXERCISES 2.1

Name ________________________________ **Score** __________ **Date** _______________
Course, Year & Section _____________**Student no.** ________**Professor** _______________

Answer the following problems:

I. Write T if the statement is true and F if not.

_____ 1. Equal sets are always equivalent sets.

_____ 2. The set of negative integers are finite.

_____ 3. {1, 2, 3,…10} is an infinite set.

_____ 4. {w,a,s,p} ~ {s,w,a,p}

_____ 5. If A { a,e,I,o,u} then a ∈ A

_____ 6. Given C = $\{x | x \epsilon\ I\ \wedge 5x\ \leq 15\}$ and D = $\{x | x\epsilon\ I\ \wedge x^2 < 25\}$, then C ≠ D

_____ 7. Using the given in problem 6, Q = R.

_____ 8. The set of even numbers less than 100 is infinite.

_____ 9. The cardinality of a null set is zero.

_____10. The set of dots in a plane is an infinite set.

II. Tell whether each of the following set is a well-defined set or not.

____________ 1. The set of whole numbers more than 407
____________ 2. The set of all months with 31 days
____________ 3. The set of competent mathematicians in all state colleges and universities
____________ 4. The set of all actors in the movie “Harry Potter”.

III. Using the Roster Method, list down the elements of each of the following sets described below.

1. $\mathrm{A} = \{$ all elements that belong to the Halogen family $\}$

2. $\mathrm{B} = \{$ all positive integers between 5 and 10 $\}$

3. $\mathrm{C} = \{$ all letters in the word “Philippines” $\}$

4. $\mathrm{D} = \{$ all negative odd integers greater than -13 but less than $-11\}$

IV. Using the Rule Method and the Set builder notation to describe each of the following sets whose elements are listed below.

1. $\mathrm{D} = \{\mathrm{Rudy, crocodile}, \pi\}$ ____
2. $\mathrm{A} = \{\mathrm{h, e, l, o}\}$ ____
3. $\mathrm{B} = \{0\}$ ____
4. $\mathrm{C} = \{2,4,6, \dots ,200\}$ ____

V. Tell whether finite or infinite set.

____________ 1. A = { set of even numbers}

____________ 2. B = { set of intergers less than 20}

____________ 3. M = $\{-5 \leq \mathrm{x}\ \leq 5\}$

____________ 4. R = { set of elements in the periodic table}

____________ 5. $\mathrm{S} = \{\mathrm{x} | \mathrm{x} = 2^{\mathrm{n}}, \mathrm{n} \in \mathrm{N}\ \wedge \mathrm{n}\ \leq 5\}$

VI. Write the following in the Roster Method.

1. $\left\{\frac{x}{5}\middle|x < 5\right\}$ 2. $\{x|x \text{ is an odd integer between } 0 \text{ and } 10\}$

2. $\{x^2|x^2 - 1|0 \le x \le 5, x \in Z\}$ 4. $\left\{\frac{1}{x}\middle|x \in N\right\}$

VII. Given the following: $\mathrm{U} = \{\mathrm{x}|0 \le \mathrm{x} \le 20\}$

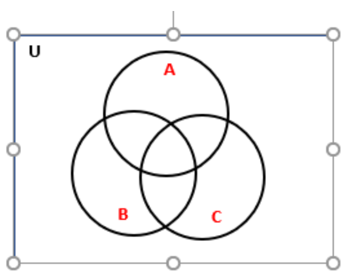
$\mathrm{A} = \{2\mathrm{x}|0 \le \mathrm{x} \le 5\}$

$\mathrm{B} = \{2\mathrm{x} - 1|1 \le \mathrm{x} \le 7\}$

Find the following:

1. A U B
2. A ∩ B
3. A' U B'
4. A' ∩ B'
5. A – B
6. B – A
7. (A ∩ B)'
8. (A U B)'
9. A'
10. B'

VIII. Given the following Venn diagram of Universal set U and subsets A, B, and C, shade the portion indicated in the following sets:



1. A U B U C
2. A ∩ B ∩ C
3. A – (B ∩ C)
4. (A U B) – C
5. A' U B' U C'
6. A' ∩ B'∩ C'
7. A – (B ∩ C)'
8. (A U B)' – C
9. (A – B)'
10. (B – C)'

IX. Solve the following problems. Use Venn diagram to answer the questions below:

In the College of Engineering, 100 students were asked about their favorite subjects, the result is as follows:

45 liked Algebra
30 liked Chemistry
38 liked English
25 liked both Algebra and Chemistry
26 liked both Algebra and English
22 liked both Chemistry and English
10 like all three subjects

a. How many students like Algebra and Chemistry but not English? ______________________
b. How many students like Algebra and English but not Chemistry? ______________________
c. How many students like English and Chemistry but not Algebra? ______________________
d. How many students like exactly 1 subject? ______________________
e. How many students did not like any subject? ______________________

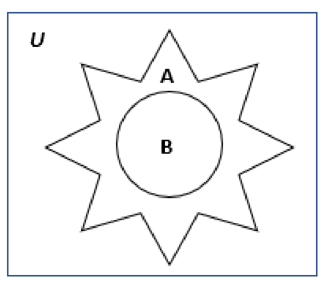
X. The Mathematics Club is preparing for year end team building and outing. There are 50 members from which the members of standing committees are taken. There are 8 members of Finance Committee, 5 are members of Program committee and 10 are members of planning committees. Also 4 are also members of Finance and Program, 8 are both members of Finance and Planning and 4 are also members of Program and Planning and 1 of whom was of whom is in Finance too. How many members are on exactly 2 committees?

XI. Suppose a survey of 60 freshmen accounting students at the Main Campus of the Polytechnic University of the Philippines produced the following results:

19 of the students read Manila Bulletin Today
18 read Philippine Daily Inquirer
50 read Philippine Star
13 read Manila Bulletin Today and Philippine Daily Inquirer
11 read Philippine Daily Inquirer and Philippine Star
13 read Manila Bulletin Today and Philippine Star
9 read all three.

1. How many students read none of the publications?
2. How many read only Philippine Star?
3. How many read Manila Bulletin Today and Philippine Daily Inquirer, but not Philippine Star?

XII. Copy the Venn diagram below and shade the region or region corresponding to the given set.



1. A’ 2. $A \cup B$ 3. $A \cap B$ 4. $A \cap B'$ 5. $A' \cup B$