Republic of the Philippines
POLYTECHNIC UNIVERSITY OF THE PHILIPPINES
Office of the Vice President for Academic Affairs
Department of Mathematics and Statistics

STAT 012 – Engineering Data Analysis
LAS 2.1: The Normal Distributions
2nd Semester

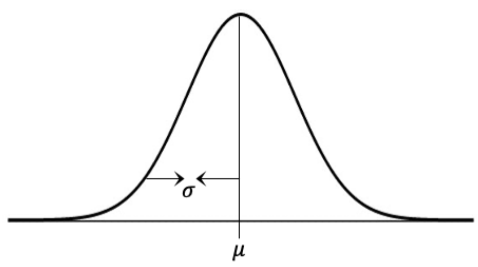
Learning Objectives:

1. Determine the types of Continuous Probability Distribution.
2. Convert raw score to z-score and vice versa.
3. Use the 68%-95%-99.7% and z-table to find an area under the normal curve.
4. Solve problems involving Normal Distribution.

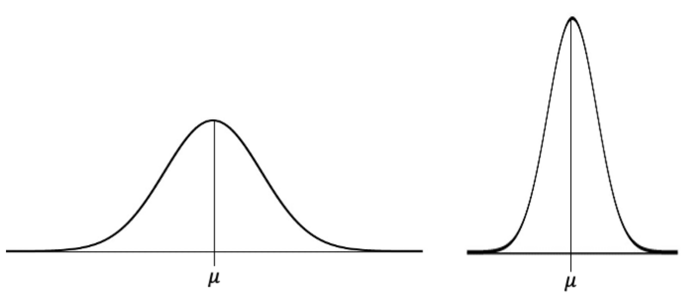
Definition. The Normal Distribution

The normal distribution is a distribution where the mean, median, and mode are equal and the distribution is clustered at the center. It's graph is symmetrical bell-shaped curve. It has two parameters, the mean and standard deviation which are denoted as μ and σ, respectively. It was developed by Carl Friedrich Gauss and sometimes called as Gaussian Curve.

$$n(x;\ \mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



The Mean and Standard Deviation as Parameters - The mean μ and standard deviation σ helps us to construct the normal curve. The mean is always the center of the data, that is, the middle part of the normal curve is always the μ. On the other hand, the standard deviation tells us the width of the normal curve. If the σ is too small, the normal curve is squeezed, compressed and tall. Otherwise, it is widely spread and flat.



High Standard Deviation   Low Standard Deviation

Let us illustrate the concept of the normal curve. Consider the following ages of a senior high students:

17	18	15	17	18	15	17	17	19	15
17	20	19	16	17	19	17	17	16	14
18	16	16	17	17	17	18	17	18	16

Note. The normal distribution tells us that if the most frequent value in each data occurred in the middle, the given data is normally distributed. The word "normal" means the middle part or the mean value is the normal value, the typical, or the usual value that always happen in an event. Examples of normal distributions are height of the people, shoe sizes, blood pressure, birth weight, and IQ scores.

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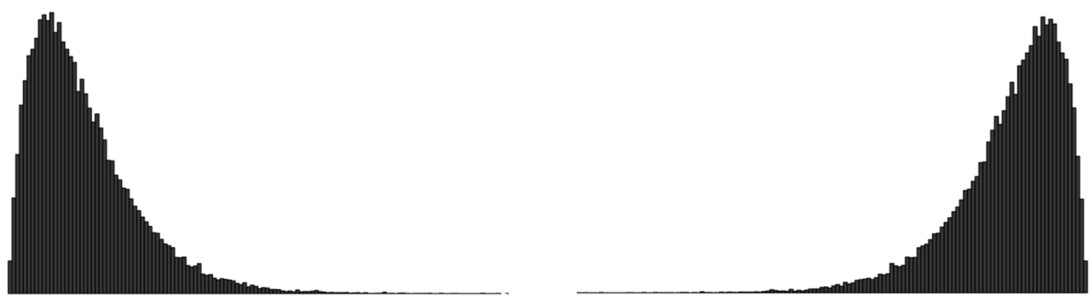
Skewness of the Data - The distributions that are not in normal are called skewed. We have two types of skew, the skewed to the left and skewed to the right.



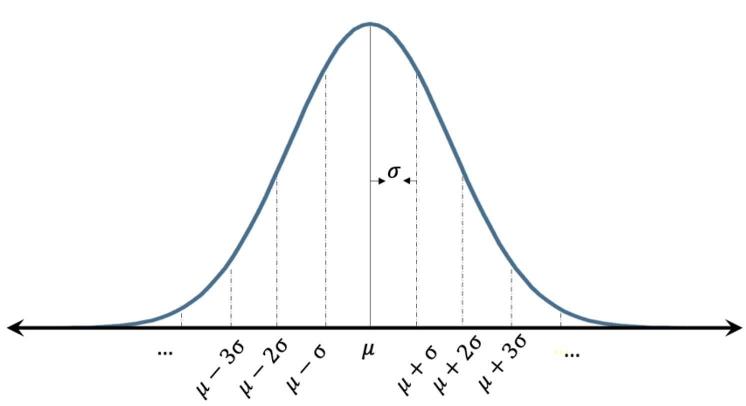
Skewed to the Right　　　　Skewed to the Left

An example of a real-life variable that has a skewed left distribution probably is a test score, Most students would have high test scores, and a smaller number of people would have low scores and most people retire in their 60s, but some might retire early, leading to a left-skewed distribution of retirement ages, On the other hand, most people earn in the low/medium range of salaries, with a few exceptions (CEOs, professional athletes etc.) that are distributed along a large range (long "tail") of higher values, most of the players in basketball scores below average, and most of the people lives below average life span are examples of skewed to the right.

Properties of a Normal Distribution - The following are the properties of a normal distribution:

1. The mean, median, and mode are equal.
2. The curve is symmetric about the mean.
3. The total area is 100% or 1.
4. The normal curve is asymptotic to the horizontal axis.
5. The normal curve is tall and clustered if the standard deviation is small. Otherwise, the normal curve is flat and spread.

Construction of a Normal Curve and The 68%-95%-99.7% Rule - The μ is the central data of a normal curve, hence, we treat this as our zero mark. The σ pertains to how well-dispersed the data is from the center. Thus, we can make it as our unit count.



Example 1. Construct the normal curve whose mean is 9 with a standard deviation of 2.

Example 2. Draw the normal curve with the following parameter values: $\mu = 1.8$ and $\sigma = 0.4$.

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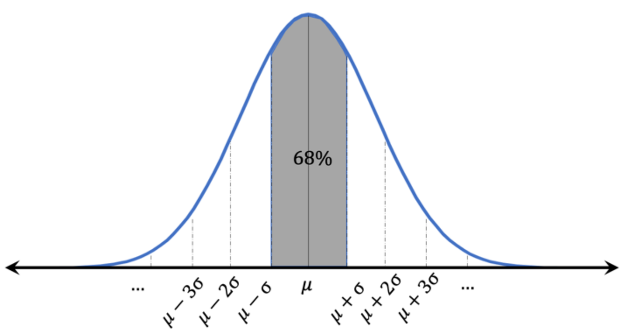
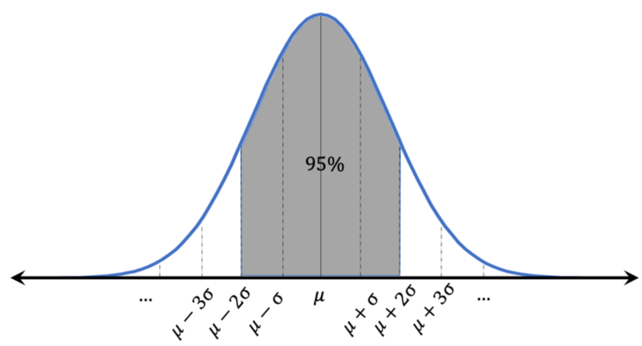
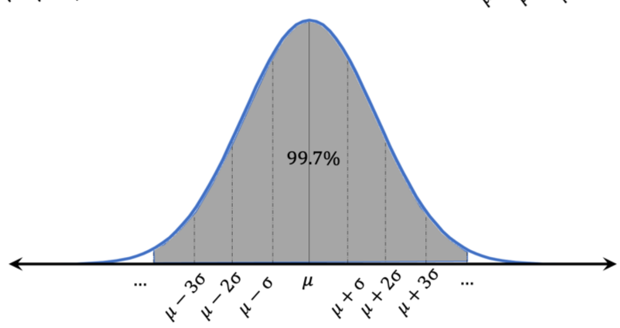
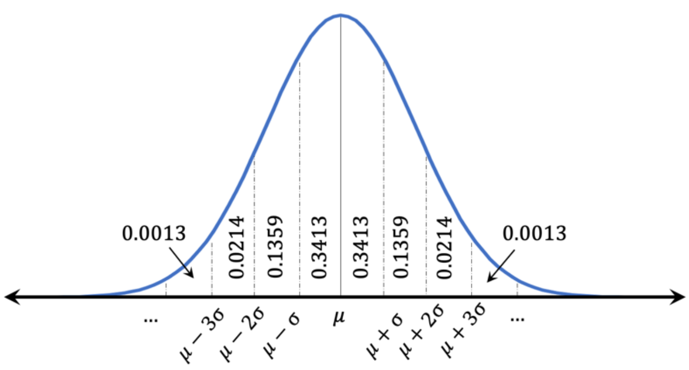
Consider the normal curves with the following percentage of areas.







With these information, we can illustrate the distributions of the areas as:



The 68%-95%-99.7% Rule – is a procedure on how to find the area under the normal curve based on an observation. Here, about 68% of values fall within one standard deviation of the mean. 95% of the values fall within two standard deviations from the mean. Lastly, almost all of the values—about 99.7%—fall within three standard deviations from the mean.

Example 3. The normal distribution has a mean of 30 and a standard deviation of 1. Approximate the area between 28 to 30.

Example 4. Draw a normal curve whose mean is 7 and a standard deviation of 0.4. Approximate the area at most 8.2.

How to find the probability of continuous random variables? The answer is to use the normal distribution, and how is this done? By simply finding an area under the normal curve. Moreover, how do we get the area under the normal curve? Use a z-table or a 68%-95%-99.7% rule!

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Definition. The Z – Score

It is a statistical measure that quantifies the distance between a data point and the mean of a dataset. It's expressed in terms of standard deviations. It indicates how many standard deviations a data point is from the mean of the distribution.

$$z = \frac{x - \mu}{\sigma}$$

Example 5. If $\mu = 5$ and $\sigma = 2.1$. What is the equivalent z score if $x = 7$?

Example 6. If $\mu = 21$ and $\sigma = 0.5$. What is the equivalent raw score if $z = 24$?

The Z-Table. The standard score table or simply the z-table is a table that contains z-scores with a specific distance away from the mean and with a corresponding values (area of the normal curve). This values can be treated as the probability of the given event.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

Example 7. Find the area of the desired region base on the following given.

a. From mean to $x = 23, \mu = 16, \sigma = 5$.
b. From mean to $z = -2.23$
c. From $z = -2.23$ and onwards.
d. At least $z = 1.71$.
d. Between $z = -1$ and $z = 1$.

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Example 8. The average number of acres burned by forest and range fires is 4,300 acres per year with a standard deviation of 750 acres. The distribution of the number of acres is normal. What is the probability that between 2,500 and 4,200 acres will be burned in any given year?

Example 9. An electrical firm manufactures light bulbs that have a life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.

Example 10. The average height of Grade 11 students in PUPSHS is 167 centimeters with a standard deviation of 5 centimeters. If there are a total of 90 students, find the following:

A. How many students and what is the probability that in random selection, the height of the student is
   A1. Above 150cm?
   A2. Between 140cm and 160cm?
   A3. Below 167cm?

Example 11. The mean weight of 545 women is 99 lbs. and std. dev. is 5. Assuming that the weights of these woman are normally distributed, how many would you expect to weigh less than 105?

Example 12. The mean reading achievement score of a group of 4th year HS students is 78 and std. dev. of 4. Granting that the scores of these students are normally distributed, how many would you expect to score less than 85? If the number of students who took the test is 200, how many would that be? How many students would score more than 85.