CHAPTER 2: LOGIC AND SET THEORY

LEARNING OBJECTIVES

At the end of this chapter the student should be able:

1. To define logic, propositions, and types of statements.
2. To learn how to use logical connectives to combine statements.
3. To translate the different types of statements into symbolic logic.
4. To construct a truth table and evaluate the truth value of statements.

2.1 LOGIC

- It Is a science that deals with the principles and criteria of validity of inference and demonstration.
- It is a method of reasoning that involves a series of statements, each of which must be true if the statement before it is true.
- It is the science of correct reasoning; science which describes relationships among propositions in terms of implication, contradiction, contrariety, conversion, etc.
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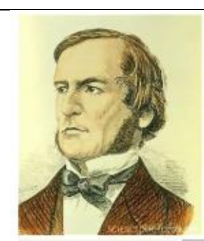
2.2 Some Mathematician who contributed to the development of Logic:

Gottfried Wilhelm Leibniz (1646–1716) was one of the first mathematicians to make a serious study of symbolic logic. He tried to advance the study of logic from a merely philosophical subject to a formal mathematical subject. He never completely achieved this goal; however, several mathematicians, such as **Augustus De Morgan** (1806–1871) and **George Boole** (1815–1864), contributed to the advancement of symbolic logic as a mathematical discipline.	
George Boole published The Mathematical Analysis of Logic in 1848. In 1854 he published the more extensive work, An Investigation of the Laws of Thought. Many of Boole's mathematical ideas, such as Boolean algebra, have applications in the areas of computer programming and the design of electronic circuits.	

2.3 PROPOSITION

- It is the basic building blocks of logic.
- It is a declarative sentence or statement that is either true or false but not both true and false.
- Opinions are avoided in Logic, Mathematics and Science, because they can be true for some people and false for others.
- Commands and questions are not considered propositions
- Can be represented by propositional variables such as P, Q, R, S, p, q, r and s.
- The Truth Value of a proposition is True (denoted as T) if it is a true statement, and False (denoted as F) if it is a false statement

2.4 CONNECTIVES

The following connectives are generally used in propositional logic:

Connective	Symbolic Form	Statement	Type of Statement
not	$\sim p$ or $\neg p$	not P	negation
and	$p \wedge q$	p and q	conjunction
or	$p \vee q$	P or q	disjunction
If…then	$p \rightarrow q$	If p then q P implies q	conditional
If and only if	$p \leftrightarrow q$	P if and only if	biconditional

2.5 TYPES OF STATEMENTS IN PROPOSITIONAL LOGIC

1. Simple Statement
2. Compound Statement

SIMPLE STATEMENT is a statement that conveys a single idea or simple facts about the world.

Example 1: P: Willy has an expensive yacht.

Q: Today is Monday.

R: Avatar was not selected as best picture at the 82nd Academy Awards ceremony.

2.6 NEGATION OF A STATEMENT is the exact opposite of a statement; it is false when the original is true and true when the original is false.

➢ The negation of statement P is denoted by **~P or ¬ P**

Example 2: The following are negation of the above statements P, Q and R:

~P: Willy does not have an expensive yacht.

~Q: Today is not Monday

~R: Avatar was selected as best picture at the 82nd Academy Awards ceremony.

Take Note:

a. The negation of a statement involving "all", "every" and "none" is a statement which will involve "There is at least one".

Example 3: M: All mammals require sleep.

~M: There is at least one mammal that does not require sleep.

b. The negation of a statement involving "some" will involve some variety of "none".

Example 4: N: Some children fear the dark.

~N: No child fears the dark.

Example 5: Identify the following sentences as statements or not statement. Give the negation of each statement in the second column of the table 2.1

Table 2.1

Statement	If statement, give the negation.
1. The moon is full tonight.	
2. All horses are mammals.	
3. Buy me a milk tea.	
4. There are 12 kinds of amino acid	
5. Fibbonacci was genius.	
6. Some pirates love other pirates.	
7. Titanic is the world's largest cruise ship	
8. Granite is not a metamorphic rock.	
9. Every day there is much work to be done.	
10. FPJ was a good actor.	

Solution: Answers are provided in the second column of Table 2.1.

Table 2.1

Statement	**If statement, give the negation.**
1. The moon is full tonight.	The moon is not full tonight.
2. All horses are mammals.	At least one horse is not a mammal.
3. Buy me a milk tea.	Not a statement, it is a Command.
4. There are 12 kinds of amino acid	There are not exactly 12 kinds of amino acid.
5. Fibbonacci was genius.	This is an opinion.
6. Some pirates love other pirates.	(Assuming love is well-defined) No Pirate loves other pirates.
7. Titanic is the world's largest cruise ship	Titanic is not the world's largest cruise ship.
8. Granite is not a metamorphic rock.	Granite is a metamorphic rock.
9. Every day there is much work to be done.	There is at least one day that there is not much work to be done.
10. FPJ was a good actor.	This is an opinion.

2.7 COMPOUND STATEMENT is a statement that conveys two or more ideas.

- It is connected by phrases "and", "or", "If…then" and "if and only if".

KINDS OF COMPOUND STATEMENT/PROPOSITION

1. Conjunction   2. Disjunction   3. Conditional   4. Bi-conditional

CONJUNCTION is a compound statement formed by using the word "and" to join two simple sentences.

- It is represented the symbol $\wedge$.
- **P and Q** is expressed in symbols $P \wedge Q$.
- **A conjunction is true if and only if both P and Q are true.**

Example 6: a. P: Joey eats hamburger.

Q: Mary drinks cola.

$P \wedge Q$: Joey eats hamburger and Mary drinks cola.

b. M: 5 is prime number

N: Water boils at 100° C

$M \wedge Q$: 5 is a prime number and water boils at 100° C.

Note:

a. A negation symbol will only apply to the statement which immediately comes after it.

b. Negation of Compound Statement, **$P \wedge Q$ is $\sim P \vee \sim Q$**

Example 7: P: Today is Friday.

Q: It is not raining.

$\sim P \wedge Q$: Today is not Friday and It is not raining.

$P \wedge \sim Q$: Today is Friday and it is raining.

$\sim (P \wedge Q)$: It is not true that both Today is Friday, and It is not raining.

Example 8: Consider the following simple statements.

A: Today is Friday.

B: It is raining.

C: I am going to a movie.

D: I am not going to the basketball game.

Write the following compound statements in symbolic form.

1. Today is Friday, and it is raining.
2. It is not raining, and I am going to a movie.
3. I am going to the basketball game and I am not going to a movie.
4. It is raining, and I am going to the basketball game.

Solutions: 1. $A \wedge B$ 2. $\sim B \wedge C$ 3. $\sim D \wedge \sim C$ 4. $B \wedge \sim C$

DISJUNCTION

- It Is a compound sentence formed by using the word "or" to join two simple sentences.
- **P or Q** is expressed in symbols **"$P \vee Q$"**
- **A disjunction is true if either statement is true or if both statements are true and the only way that it is false is if both halves are false.**
- The negation of disjunction **$P \vee Q$ is $\sim P \wedge \sim Q$**

Example 9: a. P: The clock is slow.

Q: The time is correct.

$P \vee Q$: The time is slow, or the time is correct.

b. R: The principal will attend the class on Tuesday

S: I will skip my mathematics class on Wednesday.

$R \vee S$: The principal will attend the class on Tuesday, or I will skip my mathematics class on Wednesday.

c. A: It is cold outside.

B: It is snowing.

$A \vee B$: It is either cold outside or it is snowing.

$\sim (A \vee B)$: It is not the case that it is either cold outside or snowing.

2.8 De Morgan's Laws for Negation of Conjunction and Disjunction statements:

Given the statements p and q:

1. $\neg(p \wedge q) \equiv \neg p \vee \neg q$
2. $\neg(p \vee q) \equiv \neg p \wedge \neg q$

From the previous example of conjunction,

$\sim (P \wedge Q)$: It is not true that both Today is Friday, and It is not raining.

This statement is also equivalent to:

$\sim P \vee \sim Q$: Today is not Friday or It is raining.

From the previous example of disjunction,

$\sim (A \vee B)$: It is not the case that it is either cold outside or snowing.

The above statement is equivalent to,

$\sim A \wedge \sim B$: It is not cold outside, and it is not snowing.

Example 9a: Use De Morgan's Laws for Statements

P: Kyla has a cellphone.

Q: Jerome has a laptop computer.

$P \wedge Q$: Kyla has a cellphone and Jerome has a laptop computer.

$P \vee Q$: Kyla has a cellphone or Jerome has a laptop computer.

Solution: Express the negation of original statement P and Q by De Morgan's Laws as follows:

a.) $\neg(\mathbf{P} \wedge \mathbf{Q}) \equiv \sim\mathbf{P} \vee \sim\mathbf{Q}$

$\sim P \vee \sim Q$: Kyla does not have a cellphone or Jerome does not have a laptop computer.

b.) $\boldsymbol{\sim (P \vee Q) \equiv \sim P \wedge \sim Q}$

$\sim P \wedge \sim Q$: Kyla does not have a cellphone and Jerome does not have a laptop computer.

Example 13b: Express the following P and Q in equivalent negation form.

P: I am rich

Q: I am happy.

Using De Morgan's laws for Statements:

$\sim (P \wedge Q) \equiv \sim P \vee \sim Q$

I am not rich, or I am not happy.

$\sim (P \vee Q) \equiv \sim P \wedge \sim Q$

I am not rich, and I am not happy.

2.9 TRUTH VALUE AND TRUTH TABLE

1. The truth value of a simple statement is either true (T) or false (F).

2. The truth value of a compound statement depends on the truth values of its simple statements and its connectives.

Example 10: Suppose statement A, B and F are true and statements C, D and E are false. Find the truth value (true or false) of the ff.

STATEMENTS	TRUTH VALUE	STATEMENTS	TRUTH VALUE
1. $A \wedge C$		6. $\sim B \wedge D$	
2. $(\sim B) \vee (\sim F)$		7. $\sim (B \wedge D)$	
3. $A \wedge B$		8. $(A \vee B) \wedge C$	
4. $E \vee F$		9. $A \vee (B \wedge C)$	
5. $(A \wedge B) \vee C$		10. $(E \vee F) \wedge \sim D$	

Solution: The truth value in each number are provided bellow:

STATEMENTS	TRUTH VALUE	STATEMENTS	TRUTH VALUE
1. $A \wedge C$	False	6. $\sim B \wedge D$	False
2. $(\sim B) \vee (\sim F)$	False	7. $\sim (B \wedge D)$	True
3. $A \wedge B$	True	8. $(A \vee B) \wedge C$	False
4. $E \vee F$	True	9. $A \vee (B \wedge C)$	True
5. $(A \wedge B) \vee C$	True	10. $(E \vee F) \wedge \sim D$	True

2.10 THE TRUTH TABLE

A truth table is a table that shows the truth value of a compound statement for all possible truth values of its simple statements. Table 2.2 shows a Standard Truth Table.

Table 2.2: Standard Truth Table

Given Statement	
P	**Q**
T	T
T	F
F	T
F	F

Note 1: With two statements P and Q, there are four possibilities: both could be true, both could be false, one of the statements could be true and the other is false and vice versa.

Note 2: The number of rows of the truth table can be verified by the formula 2^n where n is the number of statements. So, for the above truth table with n = 2 has $2^2 = 4$ rows.

THE TRUTH TABLE OF COMPOUND STATEMENTS

TABLE 2.3							
		Conjunction	**Disjunction**	**Negation**			
P	**Q**	**P ∧ Q**	**P ∨ Q**	**~ P**	**~ P ∧ Q**	**~ (P∧ Q)**	**~ (P ∨ Q)**
T	T	T	T	F	F	F	F
T	F	F	T	F	F	T	F
F	T	F	T	T	T	T	F
F	F	F	F	T	F	T	T

The first two columns of table 2.3, shows the four possibilities for statements P and Q, there are four possibilities, that both P and Q could be true, both could be false, P could be true and Q is false, or P could be false and Q is true.

In the third column shows a conjunction P ∧ Q, is true only if and only if both P and Q are true, the rest of the possibilities are false.

In the fourth column shows a disjunction P ∨ Q, is true if either statement is true or if both statements are true and the only way that it is false is if both halves are false.

The negation of P, ~P is false when the original is true and true when the original is false as shown in column 5.

The possibilities in columns 6 to 8 can be based on the possibilities in the previous columns and is left as an exercise.

Example 11: Construct a truth table for the following statements.

a. ~ (~P ∨ Q) ∨ Q

b. (P∧~Q) ∨ (~P ∨ Q)

Solution:

a. The Truth Table for ~ (~P ∨ Q) ∨ Q is constructed in table 2.4,

Table 2.4

P	**Q**	**~P**	**P ∨ Q**	**~P ∨ Q**	**~ (~P ∨ Q)**	**~ (~P ∨ Q) ∨ Q**
T	T	F	T	T	F	T
T	F	F	T	F	T	T
F	T	T	T	T	F	T
F	F	T	F	T	F	F

b. The Truth Table for (P∧~Q) ∨ (~P ∨ Q) is constructed in table 2.5,

Table 2.5

P	**Q**	**~P**	**~Q**	**P∧~Q**	**~P ∨ Q**	**(P∧~Q) ∨ (~P ∨ Q)**
T	T	F	F	F	T	T
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

2.11 EQUIVALENT STATEMENTS

When two statements have the same truth value under all possible situations indicated by the symbol ≡.

Example 12: Prove that A ≡ ~ ~ A.

Solution: Answer is listed in table 2.6.

Table 2.6

A	**~ A**	**~ ~ A**
T	F	T
F	T	F

Example 13: Show that ~ (P ∨ ~Q) and ~ P ∧ Q are equivalent statements.

Solution: The statement ~ (P ∨ ~Q) is shown in table 2.7 and ~ P ∧ Q in table 2.8,

Table 2.7

P	Q	~	(P	∨	~ Q)
T	T	**F**	T	T	F
T	F	**F**	T	T	T
F	T	**T**	F	F	F
F	F	**F**	F	T	T

Table 2.8

P	Q	~p	∧	Q
T	T	F	**F**	T
T	F	F	**F**	F
F	T	T	**T**	T
F	F	T	**F**	F

Therefore, ~ (P ∨ ~Q) and ~ P ∧ Q are equivalent statements.

~ (P ∨ ~Q) ≡ ~ P ∧ Q

TAUTOLOGY is a statement which is true in every case, (always true).

SELF CONTRADICTION is a statement always false.

Example 14: Use the truth table to prove that A ∨~ A is a tautology.

Solution: Table2.9 illustrates A ∨~ A.

Table 2.9

A	~A	A ∨ ~A
T	F	T
F	T	T

Example 15: Is the statement x + 2 = 5 a tautology of self-contradiction.

Answer: Neither. The statement is not true for all values of x and it is not false for all values of x.

2.12 CONDITIONAL STATEMENTS

- Are compound statement that takes the form "if…then"
- It is represented by symbols P → Q, read if P then Q or P implies Q,

  where P is called the antecedent (hypothesis) and Q is the consequent (conclusion)

Example 16: In each statement, identify the antecedent and consequent.

1. If <u>our school was this nice</u>, then <u>I would go there once a week</u>.

   Antecedent Consequent

2. If <u>I have ₱ 1,000.000.00</u>, then <u>I'll buy you a car</u>.

   Antecedent Consequent

Table 2.10

Truth Table for P → Q

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

Note: The only way the statement P → Q could be false is if P is true, but Q is false.

Example 17: For each of the following conditional statements, state the hypothesis, the conclusion and whether the conditional statement is true.

a. If 7 is an even number, then 7 can be divided evenly by 2.

Answer: The hypothesis 7 is an even number, and the conclusion is 7 can be divided evenly by 2. This statement is **true** because the hypothesis is false.

b. If 7 is a prime number, then 9 is a prime number.

Answer: This conditional statement is **not true** (false) because the hypothesis is true and the conclusion is false.

c. If water boils at 100°C, then granite is a metamorphic rock.

Answer: Logically, the statement is **true** because the two parts are both true. This statement does not say that granite is a metamorphic rock because water boils at 100°C, only that if the first part is true then the second part will also be true.

d. If I receive my check tomorrow, then I will pay the ₱1,000.00 that I owe you.

Answer: This conditional statement is **true.**

e. If 2 is a negative number, then 22 is a positive number.

Answer: This conditional statement is **true** because the hypothesis is false.

Example 18: Use the truth table to prove that conditional statement P → Q is logically equivalent to ~P ∨ Q.

Solution: Table 2.10 illustrates the answer to this problem.

Solution: The table below shows **P → Q ≡ ~P ∨ Q.**

Table 2.11

P	Q	P → Q	~ P	~P ∨ Q
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Therefore, P → Q ≡~P ∨ Q.

Example 18: Write each of the following in its **equivalent disjunctive form**, (~P ∨ Q).

a. If I could play the guitar, then I would join the band.

Answer: I cannot play the guitar, or I would join the band.

b. If Daniel cannot play, then his team will lose.

Answer: Daniel can play, or his team will lose.

c. If I am rich then I am happy.

Answer: P ∧ ~ Q: I am rich, and I am not happy.

Transforming Simple Statement into If-then form, then identify the hypothesis and conclusion.

Example 18a. Good citizens obey rules and regulation.

Answer: If they obey rules and regulations, then they are good citizens.

H: They obey rules and regulations.

C: They are good citizens.

Example 18b: The sum of the measures of complementary angles is 90°.

Answer: If the sum of the measures is 90°, then they are complementary angle.

H: The sum of the measures is 90°.

C: They are complementary angles.

Simple rule for transforming statements which start with "all",

All Xs are Ys.

If it is an X, then it is a Y.

Example 18c: All squares are rectangle.

If a figure is a square, then it is a rectangle.

Example 18 d: All paintings by Juan Luna are valuable.

If it is a painting of Juan Luna, then it is valuable.

2.13 NEGATION OF CONDITIONAL STATEMENT, P → Q

Truth Table of Negation of P → Q

$\sim(P \rightarrow Q) \equiv P \wedge \sim Q$

P	Q	~ Q	P → Q	~ (P → Q)	P∧~Q
T	T	F	T	F	F
T	F	T	F	T	T
F	T	T	T	F	F
F	F	T	T	F	F

Example 19: Write the negation of each conditional statement.

a. If they pay me the money, then I sign the contract.

Answer: They paid me the money and I did not sign the contract.

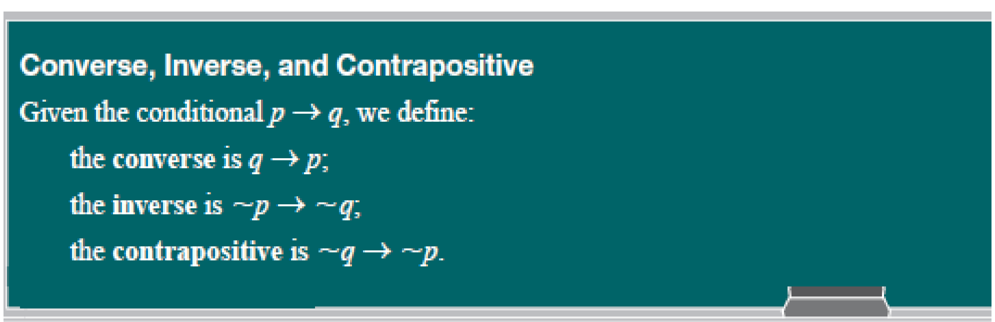
b. If the lines are parallel then they do not intersect.

Answer: The lines are parallel, and they intersect.

c. If all rich people are happy, then all poor people are sad.

Answer: All rich people are happy but there exists a poor person who is not sad.

2.14 CONVERSE, INVERSE AND CONTRAPOSITIVE



Source: Nature of Mathematics, 12th edition, Karl J. Smith

LAW OF CONTRAPOSITION

A conditional may always be replaced by its contrapositive without having its truth value affected.

Example 20: Find the converse, inverse and contrapositive of the given statement.

If you are ten years old, then you cannot legally drive a car.

Answer: **Converse**: If you cannot legally drive a car, then you are ten years old. (This statement is false because it is not true in every instance.)

Contrapositive: If you can legally drive a car, then you are not ten years old. (This statement is true no matter to whom you refer just as the original statement)

Inverse: If you are not ten years old, then you can legally drive a car.

2.15 THE BICONDITIONAL

- The statement (P → Q) ∧ (Q → P) is called a biconditional and is denoted by P ↔ Q, which is read "P if and only if Q". The symbol ↔ is a double headed arrow.

Table 2.12
TRUTH TABLE FOR BICONDITIONAL STATEMENT

P	Q	P ↔ Q
T	T	T
T	F	F
F	T	F
F	F	T

- A biconditional statement is defined to be true whenever both parts have the same truth value.

- In the truth table above, p↔q is true when p and q have the same truth values, (i.e., when either both are true, or both are false.)

Equivalent Statements:

$$P \leftrightarrow Q \equiv [(P \rightarrow Q) \wedge (Q \rightarrow P)]$$

Proof: P ↔ Q ≡ [(P → Q) ∧ (Q → P)]

p	q	P ↔ q	P → q	$q \rightarrow p$	$(p \leftrightarrow q) \wedge (q \rightarrow p)$
T	T	**T**	T	T	**T**
T	F	**F**	F	T	**F**
F	T	**F**	T	F	**F**
F	F	**F**	T	T	**T**

Example 21: Let P, Q and R represent the following statements:

P: She will go on vacation.
Q: She cannot take the train.
R: She cannot get a loan.

Write the following symbolic statements in words.

1. P ↔~ Q   2. ~R ↔ ~ P

Solution:

1. She will go on vacation iff she can take the train.
2. She can get a loan iff she will not go on vacation.

Example 22: State whether the biconditional statement is true or false.

a. x + 4 = 7 if and only if x = 3

Solution: Both equations are true when x = 3 and both are false when $x \neq 3$. Both equations have the same truth value for any given value of x, so this is a true statement.

b. $x^2 = 36$ if and only if x = 6.

Solution: If x = -6, the first equation is true and the second equation is false, thus this is a false statement.

2.16 LOGIC PUZZLES

It can be solved by using deductive reasoning and a chart that enable us to display the given information in visual manner.

Example 23: Each of the four neighbors, Sean, Marie, Sarah and Brian has a different occupation (editor, banker, chef or dentist). From the following clues, determine the occupation of each neighbor.

a. Maria gets home from work after the banker but before the dentist.
b. Sarah, who is the last to get home from work, is not the editor.
c. The dentist and Sarah leave for work at the same time.
d. The banker lives next door to Brian.

Solution:

From clue 1, Maria is not the banker or the dentist. In the chart that follows, write X1 (stands for ruled out by clue 1) in the Banker and the Dentist columns of Maria's row,

	Editor	Banker	Chef	Dentist
Sean				
Maria		X1		X1
Sarah				
Brian				

From clue 2, Sarah is not the editor, so we put X2 (ruled out by clue 2)in the Editor column of Sarah's row. We know from clue 1 that the banker is not the last to get home and from clue 2, Sarah is the last to get home, therefore Sarah is not the banker, then mark X2 is Sarah's banker column of Sarah's row.

	Editor	Banker	Chef	Dentist
Sean				
Maria		X1		X1
Sarah	X2	X2		
Brian				

From clue 3, Sarah is not the dentist. Write X3 for this condition. There are now Xs for three of the four occupations in Sarah's row; therefore, Sarah must be the chef. Place a √ in that box. Since Sarah is the chef, none of the other three people can be the chef. Write X3 for these conditions. There are now Xs for the three of the four occupations in Maria's row; therefore, Maria must be the editor. Insert a √ to indicate that Maria is the editor and write X3 to indicate that neither Sean nor Brian is the editor.

	Editor	Banker	Chef	Dentist
Sean	X3		X3	
Maria		X1	X3	X1
Sarah	X2	X2	√	X3
Brian	X3		X3	

From clue 4, Brian is not the banker. Write X4 for this condition. Since there are three Xs in the Banker column, Sean must be the banker. Place a √ in that box. Write X4 in that box. Since there are 3 Xs in the Dentist column, Brian must be the Dentist. Place a √ in that box.

	Editor	Banker	Chef	Dentist
Sean	X3	√	X3	X4
Maria	√	X1	X3	X1
Sarah	X2	X2	√	X3
Brian	X3	X4	X3	√

The final answer: Sean is the banker, Maria is the editor, Sarah is the chef and Brian is the dentist.

REVIEW EXERCISES 2.0

Name ________________________________ Score ___________ Date ________________
Course, Year & Section _____________Student No. __________ Professor________________

Answer the following.

I. Write the negation of the following statements:

1. All mathematicians are ogres.
2. All dogs have fleas.
3. No even integers are divisible by 5.
4. No triangles are rectangles.
5. Some integers are not odd.
6. Some bananas are rotten.

II. Given the following statements, translate each into symbols.

P: Prices will rise
Q: Taxes will rise.

1. Prices will rise, or taxes will not rise.
2. Prices will rise, and taxes will not rise.
3. Prices will rise, and taxes will rise.
4. Prices will not rise, and taxes will not rise.

III. Translate the following statements into *if-then* form.

1. All prime numbers greater than 2.
2. All good people go to heaven.
3. All work is noble (Thomas Carlyle)
4. All useless life is an early death (Goethe)

IV. Tell which of the statements are true.

1. Let p: 5 + 3 = 8; q: 15 – 8 = 7

a. $\sim p \vee q$ b. $\sim p \wedge \sim q$ c. $\sim (p \wedge q)$

2. Let p: 2 is prime; q: 1 is prime.

a. $\sim p \vee \sim q$ b. $\sim (\sim p)$ c. $(p \wedge q) \vee \sim q$

V. How many different truth possibilities are there with three statements A, B and C. List them.

VI. How many truth possibilities are there with a.) 4 statements, b.) 5 statements and c. n statements

VII. Use truth table to prove that $A \wedge (B \vee C) \equiv (A \wedge B)$.

VIII. Find the converse, inverse and contrapositive, given the following statements.

If the temperature drops below freezing point, then the rain will turn to snow.

IX. For each of the conditional statements, state the negation, contrapositive and converse.

a. If there is no oxygen, then the candle will go out.
b. If water freezes, then the temperature is below 32°F.
c. If x^2 is even, then x is even.
d. If you obey the law, then you will not go to jail.
e. The moon is made of green cheese only if Mickey Mouse is president.

X. State whether each biconditional is true or false.

a. x > 7 iff > 6

b. x + 5 > 7 iff x > 2

XI. Use De Morgan's laws to write the given statement in an equivalent form.

1. It is not true that, it rained, or it snowed.
2. I did not pass the test and I did not complete the course.
3. She did not visit Thailand and she did not visit Bangkok.

XII. Use the tautology $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$ to write each statement.

1. If I go, then I paid ₱150.00.
2. If cherries have turned red, then they are ready to be picked.
3. We will not visit Baguio, or we will visit Burnham Park.
4. The sun is shining, or I will not go to park.

XIII. Write each symbolic statement in words, given the statements p, q, r, s, t and u.

p: The tour goes to Italy.
q: The tour goes to Spain.
r: We go to Venice.
s: We go to Florence.
t: The hotel fees are included.
u: The meals are not included.

1. $p \wedge \sim q$ 2. $r \vee s$ 3. $r \rightarrow \sim s$ 4. $p \rightarrow r$ 5. $s \leftrightarrow \sim r$

XIV. Brianna, Ryan, Tyler and Ashley were recently elected as the new class officers (president, vice president, secretary and treasurer) of the sophomore class at Summit College. From the following clues, determine which position each holds.

a. Ashley is younger than the president but older than the treasurer.
b. Brianna and the secretary are both the same age, and they are the youngest members of the group.
c. Tyler and the secretary are next door neighbors.

XV. There are three people (Alex, Ben and Cody), one of whom is a knight, one a knave, and one a spy. The knight always tells the truth, the knave always lies, and the spy can either lie or tell the truth. Alex says: "Cody is a knave." Ben says: "Alex is a knight." Cody says: "I am the spy." Who is the knight, who the knave, and who the spy?