PHYS 2003 4 IM Physics for Engineers

Bachelor of science in Electronics and Communication Engineering (Polytechnic University of the Philippines)

PHYS 20034
PHYSICS FOR ENGINEERS

Instructional Materials

ELIZABETH P. BISA
Department of Physical Sciences
College of Science
Polytechnic University of the Philippines

PHYS 20034 PHYSICS FOR ENGINEERS

Course Description:

This course is a study of the conceptual foundations of Classical Physics (Newtonian mechanics, fluids, classical thermodynamics, mechanical waves, electricity, magnetism and geometric optics). In examining Classical Physics concepts and their relevant consequences, vector methods, algebra, trigonometry, calculus as well as the basic concepts of differential equations will be used. Experiments and activities to develop basic laboratory skills will be performed. The results of the experiments should lead the students to form proper physics concepts and provide the basis for further explanations in the lecture class.

Course Objectives:

- Infer and predict relevant consequences and apply the laws and principles in solving physics problems.
- Analyze and interpret graphs and write the corresponding equation.
- Interpret and explain the solutions in a clear and organized manner.
- Analyze and explain the relationship between the experimental variables and formulate appropriate generalizations.
- Volunteer and share the knowledge in physics for the under-privileged.
- Identify and apply the physical laws in explaining observations from experiments.
- Determine the appropriate use of equipment and apply estimation, precision and accuracy of measurements.
- Apply the basic physical laws and principles of physics in the relation to its environment and everyday living.
- Practice proper handling of equipment and safety rules in the laboratory.
- Manifest intellectual honesty, patience, perseverance, self-discipline, and industry in performing each experiment
- Work harmoniously and productively with others.

Lesson No. _______: ____

ASSESSMENT

Name: ____________________________________

Course, Yr. & Sec: ____________________________________

Date of Submission: ____________________________________

Assessment is given after each lesson. Summarize your answers on the table given below.

QUESTION NO.	ANSWERS

Instruction: Use additional short bond papers for your solutions. Show your complete and neat solution. Identify your final answer. All must be hand written.

CONTENTS

Course Description	**2**
Assessment Form	**3**
Lesson 1: Force and Motion	**5**
Lesson 2: Statics	**25**
Lesson 3: Work, Energy, Power	**34**
Lesson 4: Impulse and Momentum	**47**
Lesson 5: Fluid Statics	**57**
Lesson 6: Fluid Dynamics	**67**
Lesson 7: Temperature and Thermal Expansion	**76**
Lesson 8: Heat Phenomena	**90**
Lesson 9: Electrostatics	**111**
Lesson 10: Direct Current and Circuit Theory	**135**

LESSON 1: **FORCE AND MOTION**

Introduction

We commonly described the motion of particles based on the definition of displacement, velocity and acceleration. This lesson will describe the motion of bodies using the concepts of force and mass.

Learning Outcomes

After successful completion of this lesson, you should be able to:

- To learn Newton's laws of motion
- To study the relationship between a force and the acceleration it causes
- To solve problems related to the Newton's second law of motion
- To define frictional force and solve problems related to it

Discussion

1.1 The Concept of Force

When you push or pull an object, you exert a force on it. If an object moves with uniform motion (*constant velocity*), no force is required to maintain the motion. Only a force can cause a change in velocity that causes a body to accelerate.

A *force* is an interaction that causes an acceleration of a body. The magnitudes of forces are defined in terms of the acceleration they give the standard kilogram. A force that accelerates that standard body by exactly 1 m/s^2 is defined to have a magnitude of one Newton (I N). The direction of the force is the direction of the acceleration. It is a vector quantity. Thus, the net force on a body is the vector sum of all the forces acting on it. Some particular forces are described below:

1. *Weight* — is a force that pulls the body directly toward a nearby astronomical body; in everyday circumstances, that astronomical body is the earth. The force is primarily due to an attraction, called gravitational attraction, between the astronomical body and any object nearby.

2. *Normal Force* — is the perpendicular force (perpendicular to the surface) experienced by a body that is pressed against a surface, or pressed against another body.

3. *Friction* — is a force, which opposes the relative motion of a body at rest or in motion.

4. *Tension* — is the force exerted by a string, rope or cable on an object to which it is attached. A tension force pulls in the direction of the rope and is exerted uniformly along its entire length.

1.2 Mass

Mass is a scalar quantity and is commonly known qualitatively as the amount of matter which an object is made. It is also defined as a measure of an object's inertia. The greater an object's mass, the greater its inertia and the less its motion changes when pushed or pulled by a force.

1.3 Newton's Laws of Motion

Sir Isaac Newton and his colleagues formulated three laws based on experimental observations which are called the laws of motion. The three laws of motion are as follows:

1. ***Newton's First Law of Motion: The Law of Inertia***
   A body at rest will remain at rest and a body in motion will continue to move in motion at constant velocity in a straight line unless, in either case, it is acted upon by an external unbalanced force.

   In an equation form, if
   $$\sum \vec{F} = 0$$
   then
   $$\vec{a} = 0$$

2. ***Newton's Second Law of Motion: The Law of Acceleration***
   An unbalanced force acting on an object will cause the object to accelerate in the direction of the force. The acceleration is directly proportional to the vector sum of all the forces acting on the object and inversely proportional to the object's mass.

In equation form,

$$\sum_i \vec{F}_i = m\vec{a} \quad \Rightarrow \quad \begin{Bmatrix} \sum_i F_{xi} = ma_x \\ \sum_i F_{yi} = ma_y \end{Bmatrix}$$

3. ***Newton's Third Law of Motion: The Law of Action and Reaction Forces***

   Whenever one object exerts a force on another object, the second object exerts a reaction force of equal magnitude but in opposite direction to the first force.

$$\vec{F}_{action} = - \vec{F}_{reaction}$$

   It is equivalent to stating that forces always occur in pairs, or that a single isolated force cannot exist.

1.4 Applications of Newton's Second Law

Newton's Second Law relates the forces acting on an object to its acceleration. Kinematics is often used to relate an object's acceleration to its changing velocity and position

Problem-Solving Strategy

The following procedure is recommended when dealing with problems involving the application of Newton's Second Law:

1. Draw a simple, neat diagram of the system.

2. Isolate the object of interest and draw the force diagram or free body diagram for the object; a diagram showing all external forces acting on the object. Do not include forces exerted by the object on its surrounding. For systems containing more than one object, draw separate diagrams for each object.

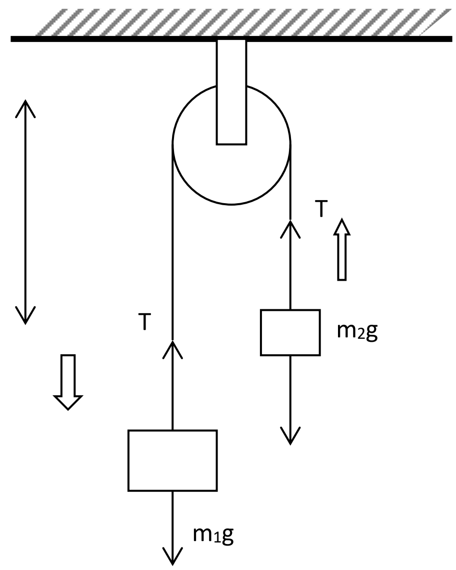
3. Establish convenient coordinate axes for each object and find the component of the forces along these axes.

4. Solve for the unknowns. You must have as many independent equations as the number of unknowns.

Sample Problems with Solutions:

1. A 7.0 kg body and 5.0 kg. body are suspended at the end of the cord that passes over a massless frictionless pulley as shown below.

   a. What is the acceleration of the system?
   b. What is the tension in the cord?



Given: $m_1 = 7.0$ kg $m_2 = 5.0$ kg

Solution:

for m_1:

$m_1g - T = m_1a$

for m_2:

$T - m_2g = m_2a$

Therefore,

$$(m_1 g - T) + (T - m_2 g) = m_1 a + m_2 a$$
$$(m_1 - m_2)g = (m_1 + m_2)a$$
$$a = \frac{(m_1 - m_2)g}{m_1 + m_2}$$
$$= \frac{(7.0\,\text{kg} - 5.0\,\text{kg})(9.8\,{}^{m}\!/_{s^2})}{7.0\,\text{kg} + 5.0\,\text{kg}}$$
$$= 1.6\,{}^{m}\!/_{s^2}$$
$$m_1 g - T = m_1 a \quad \Rightarrow \quad T = m_1 g - m_1 a \quad \Rightarrow \quad T = 57.2\,\text{N}$$

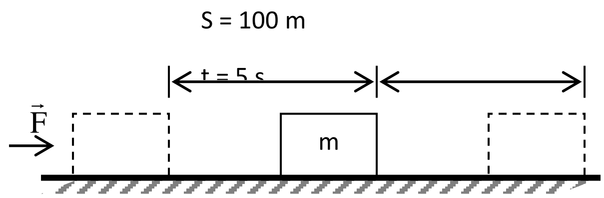
2. An unbalanced force of 50 N acts on an object weighing 100 N. What acceleration is produced?

Solution:

$$\vec{F} = m\vec{a} \;\Rightarrow\; \left|\vec{F}\right| \;\Rightarrow\; a = \frac{F}{m}, \quad F = 50\,N, \quad W = mg \;\Rightarrow\; m = \frac{W}{g}$$

$$a = \frac{Fg}{w} = \frac{(50N)(9.8\,{}^{m}\!/\!_{s^2})}{100N} = 4.9\,{}^{m}\!/\!_{s^2}$$

3. A constant horizontal force of 40 N acts on a body on a smooth horizontal surface. The body starts from rest and is observed to move 100 m in 5 s.
   a. What is the mass of the body?
   b. If the force ceases to act at the end of 5 s, how far will the body move in the next 5 s?



Solution:

a) $\vec{F} = m\vec{a} \;\Rightarrow\; \left|\vec{F}\right| = m\left|\vec{a}\right| \;\Rightarrow\; m = \frac{F}{a}$

$$S = v_o t + \tfrac{1}{2}at^2, \quad v_o = 0$$

$$a = \frac{2S}{t^2} = \frac{2 \bullet 100m}{(5\,s)^2} = 8\,{}^{m}\!/\!_{s^2}$$

$$m = \frac{F}{a} = \frac{40N}{8\,{}^{m}\!/\!_{s^2}} = 5\,kg$$

b) F = 0 $\Rightarrow$ a = 0 $\Rightarrow$ S = vt

$$v = v_o + at \qquad S = vt$$

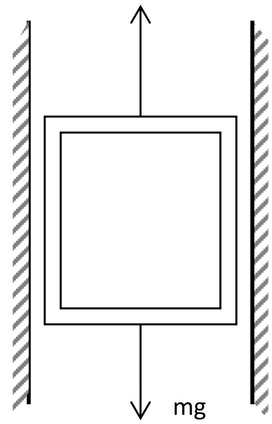
$$= 0 + 8\,{}^{m}\!/\!_{s^2} \bullet 5\,s \qquad = 40\,{}^{m}\!/\!_{s} \bullet 5\,s$$

$$= 40\,{}^{m}\!/\!_{s} \qquad = 200\,m$$

4. An elevator with a mass of 2000 kg rises with an acceleration of 1 ${}^{m}\!/\!_{s^2}$. What is the tension in the supporting cable?

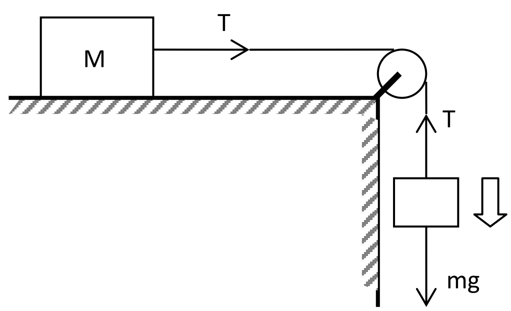
Solution:

$$\sum F_y = ma$$
$$T - mg = ma$$
$$T = m(g + a)$$
$$= (2000\,kg)(9.8\,{}^{m}\!/\!_{s^2} + 1\,{}^{m}\!/\!_{s^2})$$
$$= 21{,}600\,N$$



5. A 100 g mass lies on a frictionless table and a cord is attached to one end as shown. The cord passes over a massless, frictionless pulley at the edge of the table while a 10 g mass hangs at the other end. Find

   a) the acceleration, and
   b) the tension in the cord



Solution:

mg – T = ma where *T = Ma*

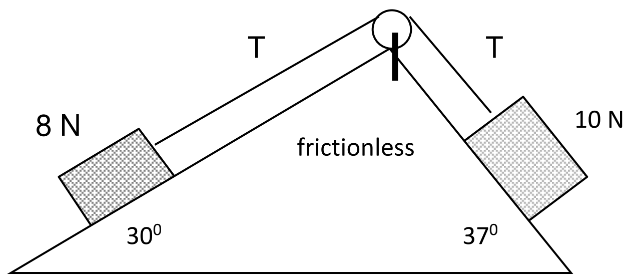
mg – Ma = ma ⇒ *mg = (m + M)a*

$$a = \frac{mg}{m + M}$$
$$= \frac{10g \bullet 980\,{}^{cm}\!/\!_{s^2}}{10g \bullet 100g}$$
$$= 49\,{}^{cm}\!/\!_{s^2}$$

$$T = Ma$$
$$= 100g \bullet 89\,{}^{cm}\!/\!_{s^2}$$
$$= 8900\,dynes$$

6. Two inclined planes are arranged as shown below. The two bodies, 8 N and 10 N are tied at the ends of a cord that passes over a massless, frictionless pulley. Find

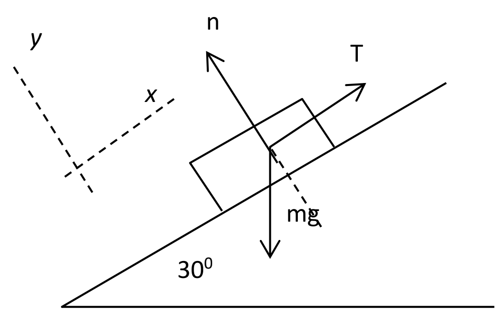
a) the acceleration of the system, and
b) the tension in the cord



Solution:
The forces acting are as shown:

For mass *m* along the x-axis $\rightarrow$ *R* – *mg* sin 30° = *ma*

For mass *M* along the x-axis $\rightarrow$ *Mg* sing 37° – T = *m*



Adding the two equations given above, we can solve for the acceleration. We find that:

$$Mg\sin 37^0 - mg\sin 30^0 = (m+M)a$$

$$a = \frac{\left(Mg\sin 37^0 - mg\sin 30^0\right)g}{mg+Mg}$$

$$= \frac{\left[(10N)(\sin 37^0) - (8N)(\sin 30^0)\right]9.8\,{}^{m}\!/\!{}_{s^2}|}{8N+10N}$$

$$= 1.1\,{}^{m}\!/\!{}_{s^2}$$

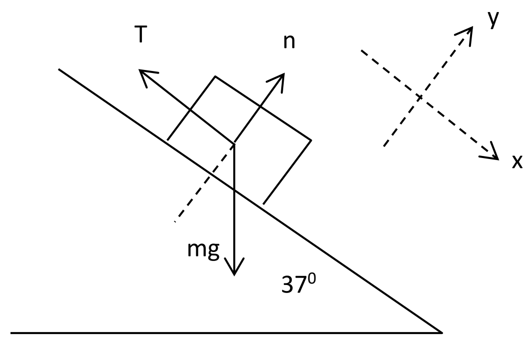
Now,

$$T - mg\sin 30^0 = ma$$

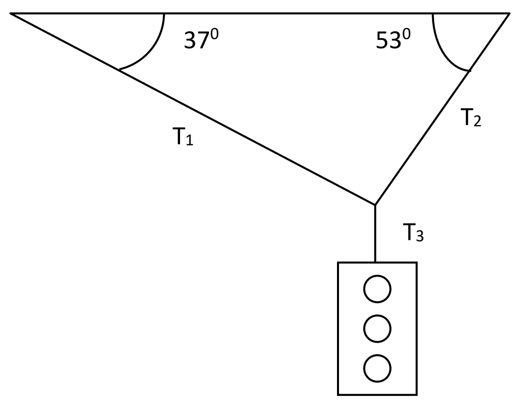
$$\mathrm{T} = \mathrm{mgsin}30^0 + ma$$

$$T = (8N)(\sin 30^0) + \left(\frac{8N}{9.8\,{}^{m}\!/\!{}_{s^2}}\right)(1.1\,{}^{m}\!/\!{}_{s^2})$$

$$= 4.9N$$

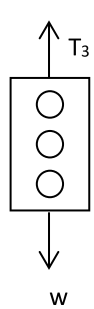


7. A traffic light, weighing 100 N, hangs from a cable tied to two other cables fastened to a support as shown in the figure below. Find the tension in the three cables.



Solution:

Free-body diagram:



$$\sum F = 0$$

$$T_3 - w = 0$$

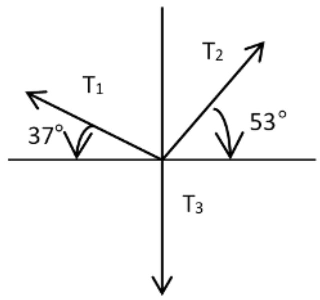
$$= w$$

$$= 100N$$

Vector Diagram:

$$\sum Fy = 0$$

$$T_1 \sin 37^0 + T_2 \sin 53^0 - T_3 = 0 \quad \rightarrow \text{eqn (1)}$$



$$\sum Fx = 0$$

$$T_2 \cos 53^0 - T_1 \cos 37^0 = 0$$

$$T_2 \cos 53^0 = T_1 \cos 37^0$$

$$T_2 = \left(\frac{\cos 37^0}{\cos 53^0}\right) T_1$$

$$= 1.33\,\mathrm{T}_1 \quad \rightarrow \quad \text{eqn (2)}$$

Substitute equation 1 to equation 2

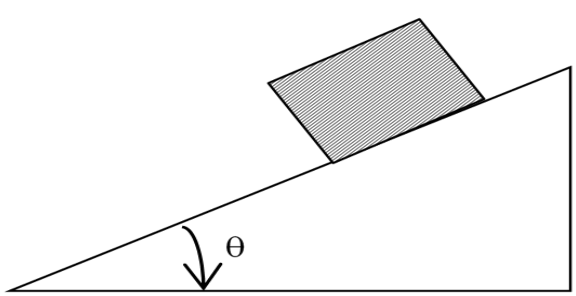
$$T_1 \sin 37^0 + (1.33T_1)\sin 53^0 - 100N = 0$$

$$T_1 = 60N$$

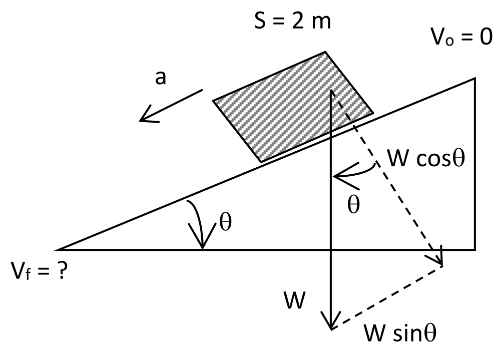
$$T_2 = 1.33T_1 = 79.8N$$

8. A block slides down a smooth plane having an inclination of θ = 15°, as shown in the figure. If the block starts from rest at the top and the length of the incline is 2 m, find

a) the acceleration of the block, and
b) its speed when it reaches the bottom of the inclined plane



Solution:



$$\sum F = ma$$

$$ma = w \sin\theta$$

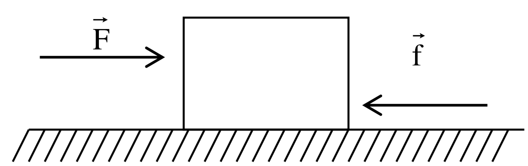
$$m\left(\frac{V_f^{\,2} - V_o^{\,2}}{2s}\right) = mg \sin 15^0$$

$$\frac{V_f^{\,2} - 0}{2(2)} = \left(9.8\,{}^{m}\!/\!_{s^2}\right)\sin 15^0$$

$$V_f = 3.185\,{}^{m}\!/\!_{s}$$

1.5 Friction

Friction opposes the motion of an object across a surface on which it rests and is directed parallel to the surface of the contact. There are two common types of friction: static friction and kinetic friction.



1. ***Static Friction*** — static frictional force exists when an object does not slide along a surface on which it rests even through a force is exerted to make it slide. If a large box is pushed but it does not slide, the static frictional force resists the applied force. The force of friction in this case is called static because the box remains stationary.

   The maximum force that static frictional force exerts depends on two factors:

   a) The relative roughness of two surfaces in contact. Roughness is measured by the coefficient of static friction μ_s. The larger the value of μ_s, the rougher the surfaces and the harder it is to move the object.

   b) The magnitude of the normal force between the object and the surface on which it rests. The larger the normal force, the harder it is to make the object move.

2. ***Kinetic Friction*** — As an object slides across a surface, kinetic frictional force opposes its motion. The word kinetic signifies that the object is moving. In the case of a moving car, for example, the road exerts kinetic frictional force on the tires of the road.

After an object, initially at rest, that is pushed or pulled, starts to move, less force is usually needed to keep the object sliding than the force required to make the object move. That is, kinetic frictional force is less than the maximum static friction.

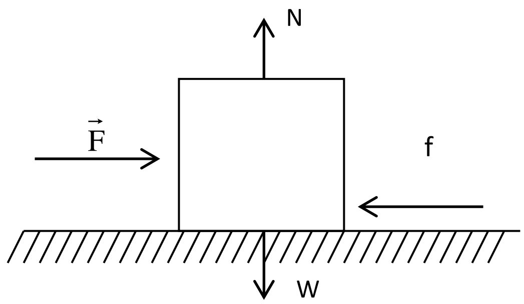
The effect of friction on the motion of an object is accounted for by defining a coefficient of kinetic friction, μ_k, a number less than the coefficient of static friction.

Properties of Frictional Force

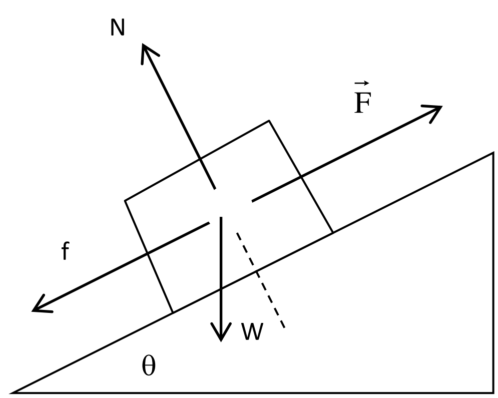
- ✓ If a body remains stationary, the static frictional force f_s, and the component of the applied force $\vec{F}$ that is parallel to the surface are equal in magnitude but opposite in direction. If the component of $\vec{F}$ parallel to the surface increases,, then f_s also increases.

- ✓ The magnitude of f_s has a maximum value that is equal to $\mu_s N$. That is, $f_s \leq \mu_s N$ where N is the magnitude of the normal force. If the component of $\vec{F}$ parallel to the surface exceeds $\mu_s N$, then the body begins to slide on the surface.

- ✓ Once a body begins to slide along a surface, the magnitude of the frictional force rapidly decreases to a smaller constant value given by $f_k = \mu_k N$, where $u_k < u_s$.



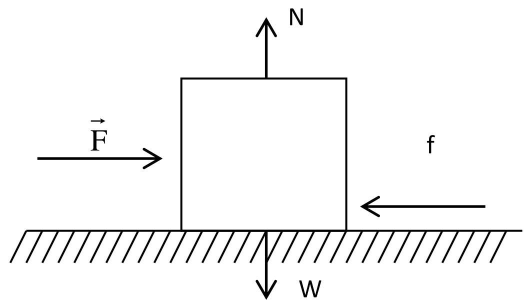
Friction on a horizontal surface



Friction on an inclined plane

Sample Problems with Solutions:

1. A box weighing 100 N starts to move across a horizontal surface when a horizontal force of 25 N is applied to it, but a force of only 20 N is needed to keep it moving in uniform motion.
   a) What are the coefficients of static and kinetic friction?
   b) If the force is not applied horizontally but in a direction 30^0 above the horizontal, find the coefficients of static and kinetic friction.



Solution:

a) $f_s \le \mu_s N, \quad f_s = F_1 = 25N$

$$F_1 = \mu_s N \quad \Rightarrow \quad \mu_s = \frac{F_1}{N}$$

$$N - W = 0 \quad \Rightarrow \quad N = W$$

$$\mu_s = \frac{F_1}{W} = \frac{25N}{100N} = 0.25$$

$$f_k = \mu_k N, \quad f_s = F_2 = 20N \quad \Rightarrow \quad F_2 = \mu_k N \quad \Rightarrow \quad \mu_k = \frac{F_2}{N}$$

$$\mu_k = \frac{F_2}{W} = \frac{20N}{100N} = 0.20$$

b) x – axis: $F\cos 30^0 - f = 0$

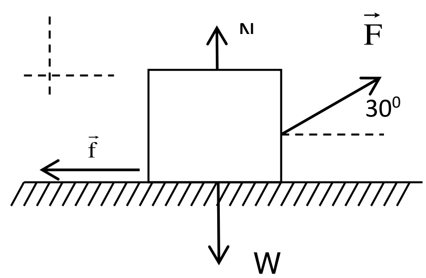
$F\cos 30^0 = f = \mu N$

y-axis: $F\sin 30^0 + N - W = 0$

$N = W - F\sin 30^0$

$\Rightarrow F\cos 30^0 = \mu N = \mu\,(W - F\sin 30^0)$

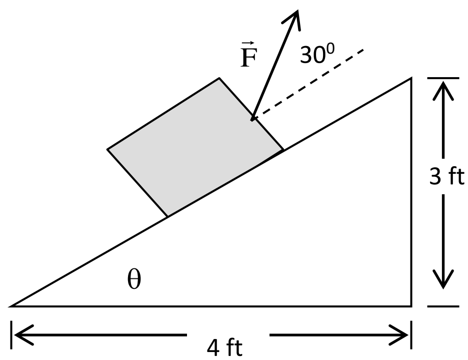
$$\Rightarrow \mu = \frac{F\cos 30^0}{W - F\sin 30^0}$$



$$\mu_s = \frac{F_2 \cos 30^0}{W - F_1 \sin 30^0} = \frac{25N\cos 30}{100N - 25N\sin 30^0} = 0.247$$

$$\mu_k = \frac{F\cos 30^0}{W - F_2 \sin 30^0} = \frac{20N\cos 30^0}{100N - 20N\sin 30^0} = 0.19$$

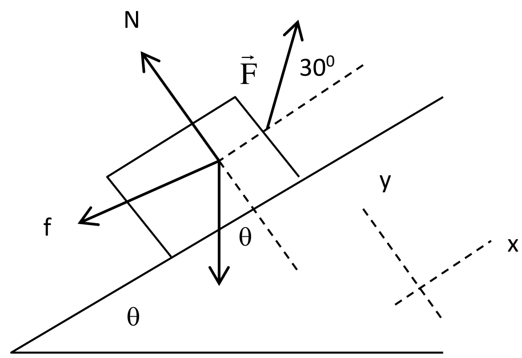
2. A 50 N body on an inclined plane, 4 ft wide at the base and 3 ft high, is pulled upward by a force 30° above the plane as shown, making the body move upward uniformly. Determine the magnitude of the applied force if the coefficient of kinetic friction between the surfaces in contact is 0.25.



Solution:

$$\theta = \arctan\frac{3}{4} = 37^0$$

The forces acting on the block are as shown:



$$\sum Fx = 0$$

x-axis: $F\cos30^0 - fW\sin37^0 = 0$

$f_k = \mu_k N$

$F\cos30^0 - \mu_k N - W\sin37^0 = 0$

y-axis: $F\sin30^0 + N - W\cos37^0 = 0$

$N = W\cos37^0 - F\sin30^0$

$$F\cos30^0 - \mu_k W\cos37^0 - F\sin30^0 - W\sin37^0 = 0$$

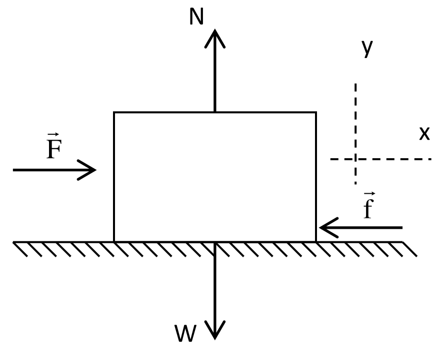
$$F\cos30^0 + \mu_k F\sin30^0 = W\sin37^0 + \mu_k W\cos37^0$$

$$F = \frac{W\left(\sin37^0 + \mu_k\cos37^0\right)}{\cos30^0 + \mu_k\sin30^0}$$

$$= \frac{50Ns\left[\sin37^0 + (0.25)(\cos37^0)\right]}{\cos30^0 + 0.25\sin30^0}$$

$$= 40.4N$$

3. A 50 kg block rests on the floor. The coefficients of static and kinetic friction are 0.70 and 0.50, respectively.



a) What is the minimum force needed to move the block?

b) If the same force continues to push the block after it starts sliding, what will be its acceleration?

Solution:

a) The forces acting are as shown in the preceding page.

x-axis: $F - f_s = 0 \quad \Rightarrow \quad F = f_s \quad , \quad f_s \leq \mu_s N$

y-axis: $N - W = 0 \quad \Rightarrow \quad N = W$

To make the block move, the maximum static friction given by $f_s = \mu_s N = \mu_s W$ must be overcome. Hence, the minimum force needed to move the block is $F = \mu_s W = \mu_s mg = \left[0.70(50kg)\left(9.8\,{}^{m}\!/\!_{s^2}\right)\right] = 34.3N$

b) As the block moves

$$F - f_k = F - \mu_k N = F - \mu_k mg = ma$$

$$\Rightarrow \quad a = \frac{F - \mu_k mg}{m}$$

$$= \frac{\mu_s \cancel{m}g - \mu_s \cancel{m}g}{\cancel{m}}$$

$$= (\mu_s - \mu_k)g$$

$$= (0.7 - 0.5)\left(9.8\,{}^{m}\!/\!_{s^2}\right)$$

$$= 1.96\,{}^{m}\!/\!_{s^2} \text{ or } 2.0\,{}^{m}\!/\!_{s^2}$$

4. A 50 kg block rests on a rough horizontal floor. The coefficient of kinetic friction between the block and the floor is 0.25. If a horizontal force of 180 N acts on the block for 3 s, find the velocity of the block at the end of that time.

Solution:
As in the preceding problem,

$$F - f_k = F - \mu_k N = F - \mu_k mg = ma$$

$$\Rightarrow \quad a = \frac{F - \mu_k mg}{m}$$

$$= \frac{F}{m} - \mu_k g$$

$$= \frac{180N}{50kg} - (0.25)\left(9.8\, {}^{m}\!/\!_{s^2}\right)$$

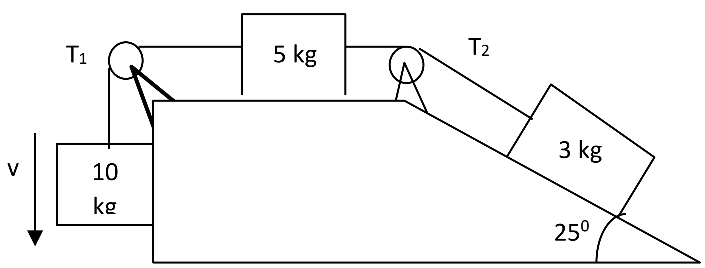
$$= 1.2\, {}^{m}\!/\!_{s^2}$$

$$v = v_o + at$$

$$= 0 + \left(1.2\, {}^{m}\!/\!_{s^2}\right)(3s)$$

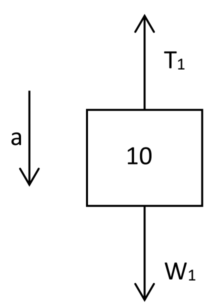
$$= 3.6\, {}^{m}\!/\!_{s}$$

5. Three blocks are connected as shown. The acceleration of the system is 2 ${}^{m}\!/\!_{s^2}$ to the left and the surfaces are rough. Find



a) the tensions in the strings, and
b) the coefficient of kinetic friction

Solution:



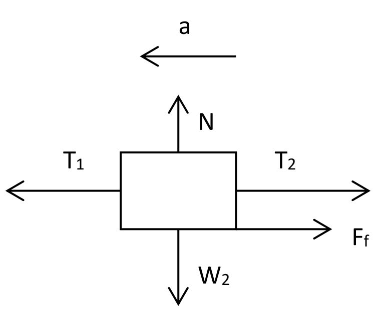
$$\sum Fy = ma$$

$$ma = W_1 - T_1$$

$$T_1 = W_1 - ma$$

$$= 10(9.8) - 10(2)$$

$$= 78N$$



$$\sum Fx = ma$$

$$m_2 a = T_1 - T_2 - F_f \quad \rightarrow \quad (1)$$

$$\sum Fy = 0$$

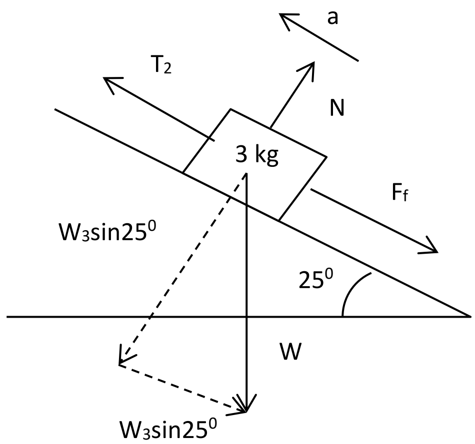
$$N - W_2 = 0$$

$$N = W_2 \quad \rightarrow \quad (2)$$

since $\mu = \dfrac{F_f}{N}$

$$F_f = \mu N = \mu W_2 \qquad \rightarrow \ (3)$$

substituting (3) into (1),



$$5kg\left(\frac{2m}{5}\right) = 78N - T_2 - \mu(5kg)(9.8\,{}^{m}/_{s^2}) \quad \rightarrow \ (4)$$

$$-68 = -T_2 - 49\mu$$

$\sum Fx = ma$ (rotated x-axis)

$$m_3 a = T_2 - F_f - W_3 \sin 25^0$$

$$= T_2 - \mu N - W_3 \sin 25^0 \qquad \rightarrow \ (5)$$

$\sum Fy = 0$ (rotated y-axis)

$$N - W_3 \cos 25^0 = 0$$

$$N = W_3 \cos 25^0 \qquad \rightarrow \ (6)$$

substituting (6) into (5)

$$(3kg)(2\,{}^{m}/_{s^2}) = T_2 - \mu(3)(9.8)\cos 25^0 - (3)(9.8)\sin 25^0$$

$$18.425 = T_2 - 26.645\mu$$

adding equations (4) and (7)

$$-68 = -T_2 - 49\mu$$

$$18.425 = T_2 - 26.645\mu$$

$$-49.575 = -75.645\mu$$

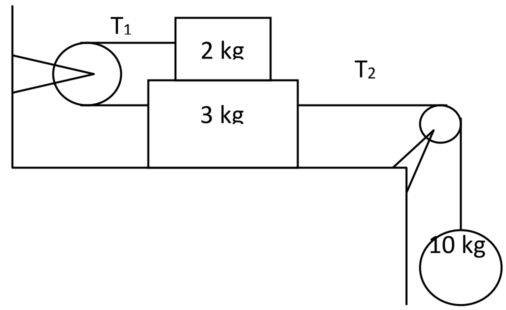
$$\mu = 0.655$$

solving for T_2 by using equation (4)

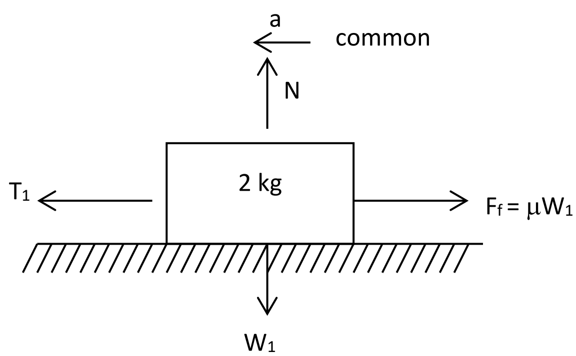
$$-68 = -T_2 - 49(0.655)$$

$$T_2 = 35.9N$$

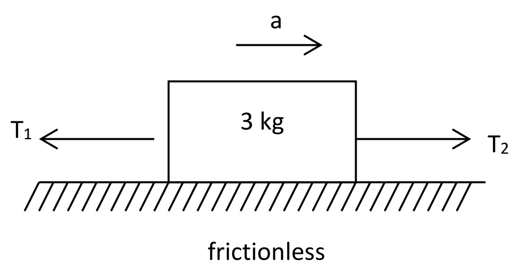
6. In the figure shown on the right, the coefficient of kinetic friction between the 2 kg and 3 kg block is 0.3. The horizontal surface and the pulleys are frictionless. The masses are released from rest. Determine the acceleration of each block and the tension in the strings.



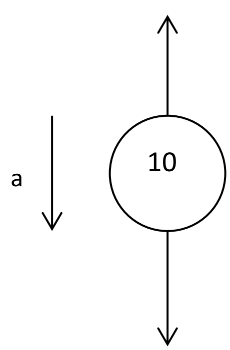
Solution:



$$\sum F = ma$$
$$m_1 a = T_1 - \mu W_1 \quad \rightarrow \quad (1)$$



$$\sum F = ma$$
$$m_2 a = T_2 - T_1 \quad \rightarrow \quad (2)$$



$$\sum F = ma$$
$$m_3 a = W_3 - T_2 \quad \rightarrow \quad (3)$$

Adding equations (1), (2) and (3)

$$
\begin{aligned}
m_1 a + m_2 a + m_3 a &= \boldsymbol{T}_1 - \mu W_1 + \boldsymbol{T}_2 - \boldsymbol{T}_1 + W_3 - \boldsymbol{T}_2 \\
a\left(m_1 + m_2 + m_3\right) &= W_3 - \mu W_1 \\
a &= \frac{W_3 - \mu W_1}{m_1 + m_2 + m_3} \\
&= \frac{10kg\left(9.8\,{}^{m}\!/\!{}_{s^2}\right) - 0.3\left(2kg\right)\left(9.8\,{}^{m}\!/\!{}_{s^2}\right)}{2kg + 3kg + 10kg} \\
&= 6.14\,{}^{m}\!/\!{}_{s}
\end{aligned}
$$

Solving for T_1 and T_2:

$T_1 = 18.16\ N$

$T_2 = 36.6\ N$

Lesson 1: Force and Motion
Assessment

Instruction; Show your complete and neat solution. Identify your final answer in the solution. See answer sheet format.

1. A force of 20 N acts upon a body whose weight is 8 N. What is the mass of the body? What is its acceleration?

2. Two masses of 8 kg and 12 kg are tied to each end of a string passing over a massless and frictionless pulley. Find the resulting acceleration of each mass.

3. A 1000–kg mass elevator has an acceleration of $1\,{}^{m}\!/_{s^2}$. What is the tension in the supporting cable if the elevator is a) ascending and b) descending?

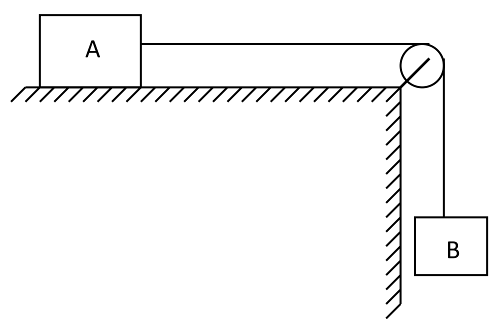
4. A 5 kg block resting on a frictionless inclined plane with an inclination angle of 20° is attached to another 5 kg block that hangs freely by a string passing over a massless, frictionless pulley. What is the resulting acceleration of the two blocks?

5. A block weighing 60 N rests on a horizontal floor. a) What is the minimum force that will make the block move if the coefficient of static friction is 0.20? b) What is the minimum force that will keep the block moving at uniform speed once it starts moving if the coefficient of kinetic friction is 0.10?

6. Blocks A and B have weigh of 44 N and 22 N, respectively (see figure below).

   a) Find the minimum weight of block C that must be placed on top of block A to keep it from sliding. Take the coefficient of static friction between A and the table top to be 0.20.

   b) Block C is suddenly lifted off A. Find the resulting acceleration of the system if the coefficient of kinetic friction between A and the table is 0.15.



7. A force of 150 N pushes a box. Starting from rest, the box achieves a velocity of 1.50 ${}^{m}\!/_{s}$ in 2.5 s. Find the coefficient of sliding friction between the box and the floor.

Introduction

In this lesson you will study a special case in mechanics wherein the net force and the net torque on an object or systems of objects are both zero. These are the two conditions for equilibrium. Statics is concerned with the calculation of forces acting on and within structures that are in equilibrium.

Learning Outcomes

After successful completion of this lesson, you should be able to:

- ☞ Study the conditions that are necessary for an extended body to remain stationary, both in terms of its position in space and its rotational ability.

- ☞ Express the conditions of equilibrium in the form of mathematical equations and apply them in solving and analyzing variety of situational problems.

Discussion

2.1 First Condition of Equilibrium

If the vector sum of all the forces acting on the body is zero, then there will not be any change in its state of motion, and the body is in a state of equilibrium under the action of these set of forces.

$$\sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... = 0$$

It satisfies then that the sum of the x components (F_1x, F_2x, F_3x . . .) of all forces acting on the object must be zero, and the sum of the y components (F_1y, F_2y, F_3y, . . .) must also be zero.

$$\sum Fx = F_1x + F_2x + F_3x + ... = 0$$
$$\sum Fy = F_1y + F_2y + F_3y + ... = 0$$

These two equations are useful in calculating one or two unknown forces acting on an object if all other forces acting on it are known. To use these equations correctly, here are some steps to follow:

1. Make a drawing of the whole structure being considered.

2. Draw a separate force diagram known as the free body diagram for one part of the structure.

3. Superimpose an x-axis and a y-axis on the force diagram.

4. Write the equations for the first condition of equilibrium.

5. Substitute the known information into the equations and algebraically solved for the unknowns.

Sample Problems with Solutions:

1. A 50-N body is supported by two ropes, one making 30° with the vertical and another making 45° with the horizontal as shown in Fig. 5.1. What is the tension in the ropes?

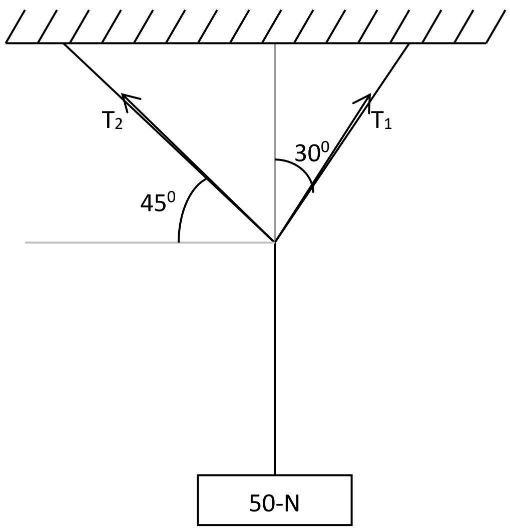


Fig. 2.1a

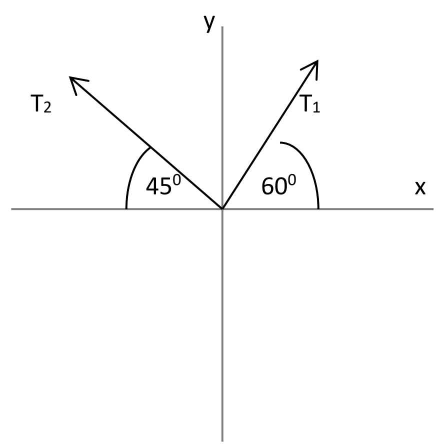


Fig. 2.1b

Solution:

$\sum Fx = 0$ $\sum Fy = 0$

from Fig. 3.2:

$T_1 \cos60° - T_2 \cos 45° = 0$ $T_1 \sin60° + T_2 \sin45° - 50\text{ N} = 0$

$0.5\, T_1 = 0.707\, T_2 \rightarrow (1)$ $0.866\, T_1 + 0.707\, T_2 = 50\text{ N} \rightarrow (2)$

Solving the two equations simultaneously,

$T_1 = 36.6\text{ N}$ $T_2 = 25.9\text{ N}$

2. Determine the weights W_1, and W_2 shown in Fig. 5.3, that cause the tension T in the horizontal cable to be 64 N.

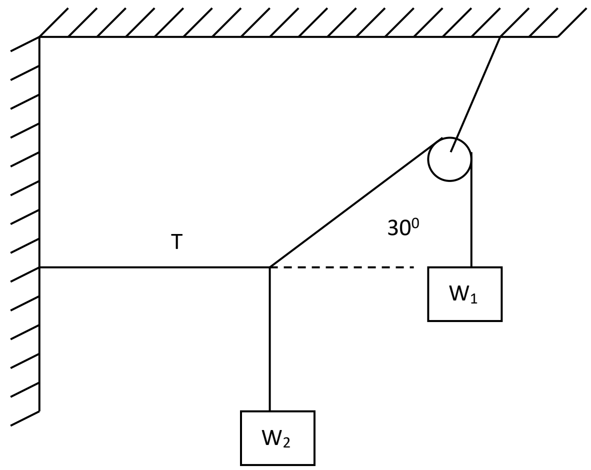
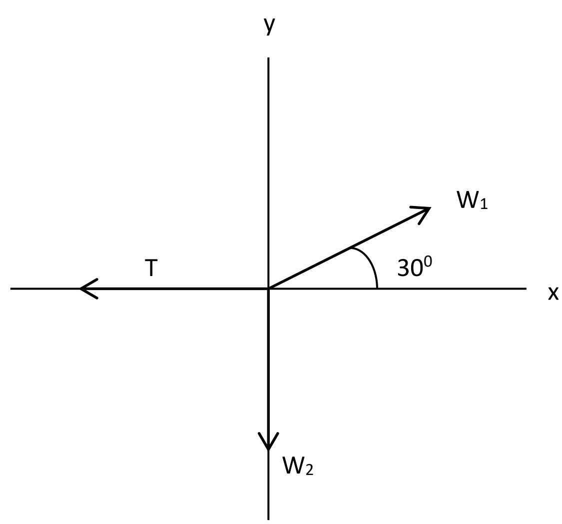


Fig. 2.2

Solution:



$$\sum Fx = 0$$
$$W_1 \cos 30^0 - T = 0$$
$$W_1 = \frac{64N}{\cos 30^0}$$
$$W_1 = 73.9N$$

$$\sum Fy = 0$$
$$W_1 \sin 30^0 - W_2 = 0$$
$$W_2 = 73.9 \sin 30^0$$
$$W_2 = 36.95N$$

2.2 Torque

Torque is a measure of the tendency of a force to cause an object to rotate. The magnitude of torque (L) is the product of the magnitude of force (F) and the moment arm (s).

The *moment arm* is the perpendicular distance from the line of action of the fore to the point about which the object rotates called *axis of rotation*.

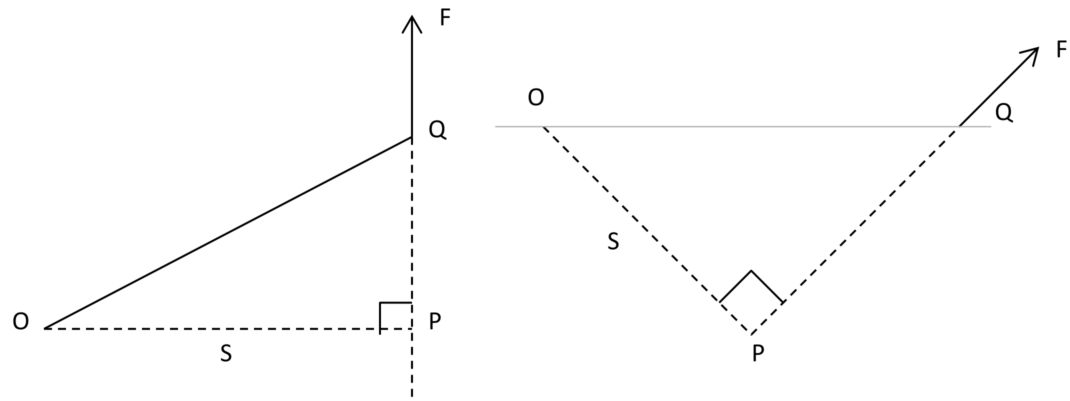


Fig. 2.3 – Measurement of Moment Arm

Point O is the axis of rotation, and S is the moment arm, the broken lines drawn from the force F is the line of action of the force.

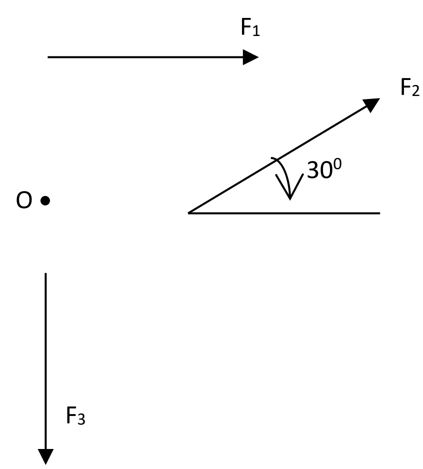
Hence,

$$L = \pm F \bullet S$$

The torque is positive if the force tends to rotate the object counterclockwise about the origin and negative if it tends to rotate the object clockwise.

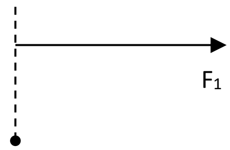
Sample Problem:

Calculate the torque about point O caused by forces F_1, F_2 and F_3. The magnitude of all the forces is 120 N and are applied a distance of 2.0 m from point O.



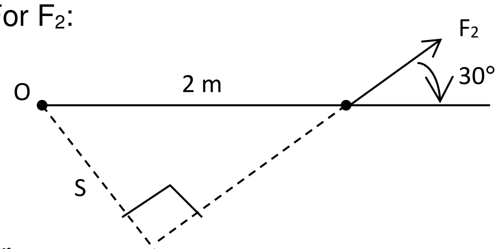
Solutions:

For F_1:



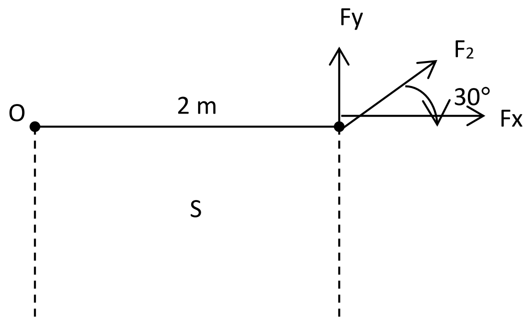
$L = F \bullet S$
$= 120\text{ N }(2\text{ m})$
$= 240\text{ N} \bullet \text{m}$

For F_2:



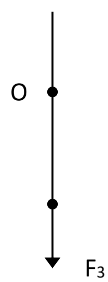
$L = F \bullet S$
$= 120\text{ N }(2\text{ m } \sin 30^0)$
$= 120\text{ N }(1\text{ m})$
$= 120\text{ N} \bullet \text{m}$

or



$L = Fy \bullet S$
$= 120\text{ N } \sin 30^0\ (2\text{ m})$
$= 60\text{ N }(2\text{ m})$
$= 120\text{ N} \bullet \text{m}$

For F_3:



$L = F \bullet S$
$= 0$ because $S = 0$

2.3 Second Condition of Equilibrium

For an object to remain in rotational equilibrium, the positive torque that tends to rotate it counterclockwise must be balanced by a torque of equal magnitude that tends to rotate it clockwise. Thus, the second condition of equilibrium is:

$$\Sigma L = L_1 + L_2 + L_3 + \ldots = 0$$

where L_1, L_2, L_3 . . . represents the torque caused by all forces F_1, F_2, F_3, . . . acting on the object.

Sample Problems with Solutions:

1. A uniform beam weighing 50 lbs and 12 ft long leans against a smooth wall with its lower end 6 ft away from the wall. What are the reactions on the ground and of the wall at the points of contact?

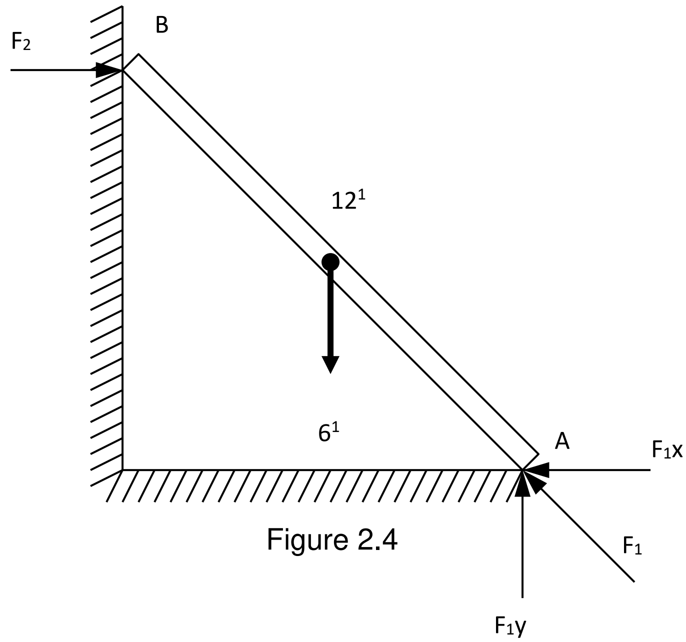


Figure 2.4

Solution:

For the first condition of equilibrium:

$$\sum Fx = 0 \qquad\qquad \sum Fy = 0$$
$$F_2 - F_1x = 0 \qquad\qquad F_1y - 50\,\text{lbs} = 0$$
$$F_2 = F_1x \qquad\qquad F_1y = 50\,\text{lbs}$$

Taking moments about A and apply the second condition of equilibrium:

$$\sum L_A = 0$$
$$F_2\left(\sqrt{108}\right) - 50\,\text{lbs}(3) = 0$$
$$F_2 = \frac{(50)(3)}{\sqrt{108}}$$
$$= 14.4\,\text{lbs}$$

thus, $F_1x = 14.4$ lbs

and $$F_1 = \sqrt{(F_1x)^2 + (F_1y)^2}$$
$$= \sqrt{(14.4)^2 + (50)^2}$$
$$F_1 = 52\,\text{lbs}$$

2. A uniform bar 3 m long is held by ropes at the ends making angle 60° and 30°, respectively, with the horizontal. A weight of 200 N is hung 0.5 m from the left end where the 60° rope is attached. Find the tension in the rope and the weight of the bar.

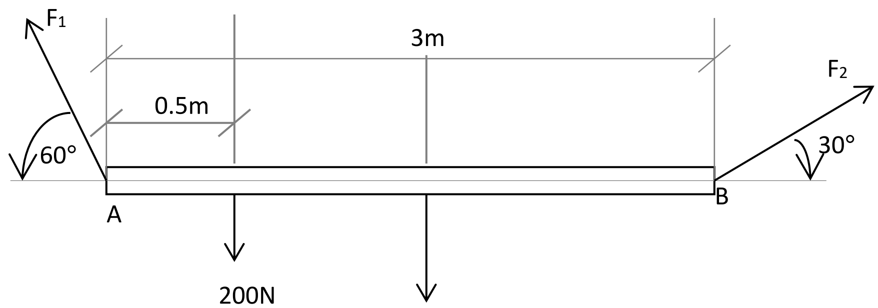


Figure 2.5

Solution:

$$\sum Fx = 0$$
$$F_2 \cos 30^0 - F_1 \cos 60^0 = 0$$
$$F_2 \cos 30^0 = F_1 \cos 60^0 \quad \rightarrow \quad (1)$$

$$\sum Fy = 0$$
$$F_1 \sin 60^0 + F_2 \sin 30^0 - 200 - W = 0$$
$$F_1 \sin 60^0 + F_2 \sin 30^0 = 200 + W \quad \rightarrow \quad (2)$$

$$\sum L_A = 0$$
$$200(0.5) + W(1.5) - F_2 \sin 30^0 (3) = 0$$

$$100 + 1.5W = 1.5F_2 \quad \rightarrow \quad (3)$$
$$W = \frac{1.5F_2 - 100}{1.5}$$

Substitute equations (1) and (3) in (2):

$$\left[\frac{F_2 \cos 30^0}{\cos 60^0}\right] \sin 60^0 + F_2 \sin 30^0 = 200 + \frac{1.5F_2 - 100}{1.5}$$
$$1.5F_2 + 0.5F_2 = 200 + F_2 - {}^{100}\!/_{15}$$
$$F_2 = 200 - {}^{100}\!/_{15}$$
$$F_2 = 133.33\text{N}$$
$$F_1 = \frac{(133.33)(.866)}{0.5}$$
$$= 230.9\text{N}$$

$$W = \frac{1.5(133.33) - 100}{1.5}$$
$$= 66.67\text{N}$$

Lesson 2: Statics
Assessment

Instruction; Show your complete and neat solution. Identify your final answer in the solution. See answer sheet format.

1. Weights of 2, 4, 6 and 8 N are fastened to the corners of a 2 m $\times$ 2 m frame. If it weighs 10 N, where would the fulcrum (the support point) be placed to balance the frame and the weights?

2. A ladder 12 m long leans against a vertical frictionless wall with its lower end 3 m from the wall. The ladder weighs 40 N. Find the magnitude and direction of the force exerted on the lower end of the ladder.

3. A uniform 25 N beam 6 m long is carried by two men A and B, one at each end of the beam.

   a) If A exerts a force of 20 N, where must a load of 45 N be placed on the beam?
   b) What force does B exert?

4. A 3 m long weightless beam is supported at each end by cables. A painter weighing 900 N stands 1 m from the left cable. Calculate the tension in each cable.

5. Where must a 500 N weight be hung on a uniform 150 N pole with a length of 5 m so that a girl at one end supports ¼ as much as a man at the other end?

6. Find the tension in each cord in Fig. 5.6 if the weight W of the suspended body is 150 N.

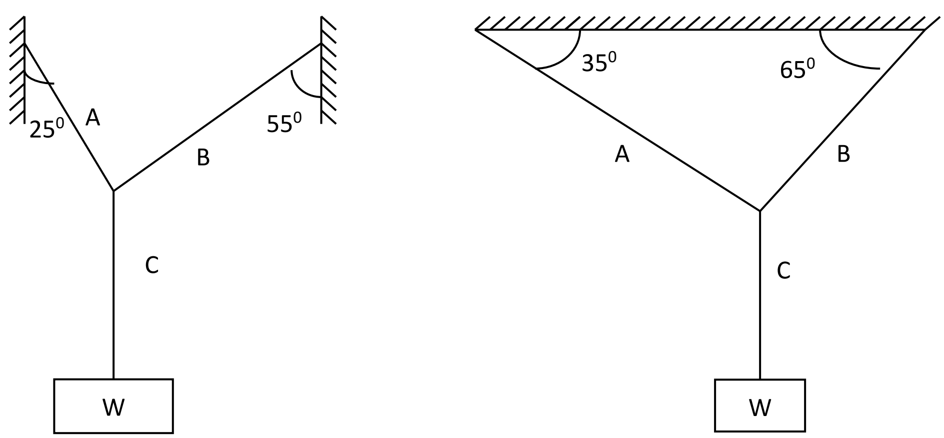


Fig. 2.6

7. Find the magnitude and direction of F_3 in Fig. 5.7 for the given system to be in equilibrium.

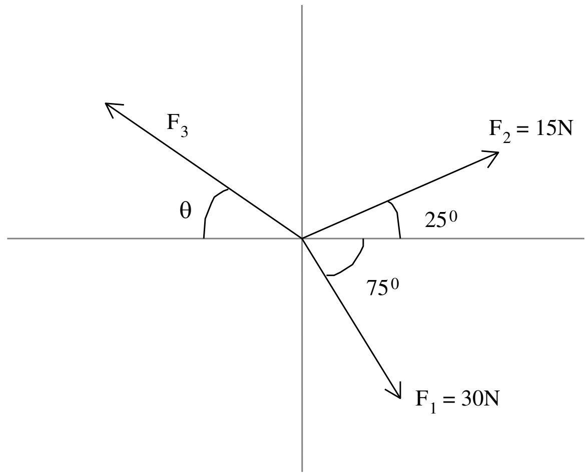


Fig. 2.7

8. A uniform pole 20 ft. long and weighing 80 lbs is held by a boy 2 ft from one end while a man carries the same pole 5 ft from the other end. At what point should a load of 100 lbs be placed so that the man will carry twice as much weight as the boy.

9. A springboard diver of weight 582 N stands at one end of a uniform 4.48 m diving board of weight 142 N. The board is attached to two pedestals, one at the other end of the board and the second pedestal 1.55 m from the first. Find the force in each of the two pedestals.

10. A ladder, whose length is 10 m and whose mass is 40 kg rests against a frictionless vertical wall. Its upper end is a distance of 7.7 m above the ground. The center of mass of the ladder is $\frac{1}{3}$ of the way up the ladder. The coefficient of static friction between the ladder and the ground is 0.53. If a carpenter climbs 85% of the way up the ladder before it starts to slip, find the mass of the carpenter.

LESSON 3: **WORK, ENERGY AND POWER**

Introduction

This lesson is devoted to the very important concepts of work and energy. These two quantities are scalars and so have no direction associated with them, which often makes them easier to work with than vectors. In this lesson, and the next we discuss an alternative analysis of the translational motion of objects in terms of the quantities energy and momentum. The significance of energy is that they are conserved. That is, in quite general circumstances they remain constant.

Learning Outcomes

After successful completion of this lesson, you should be able to:

☞ Calculate the work done by an applied force that moves an object through a certain displacement.

☞ Distinguished between kinetic and potential energy.

☞ Use the principle of conservation of energy to solve problems that involves moving objects.

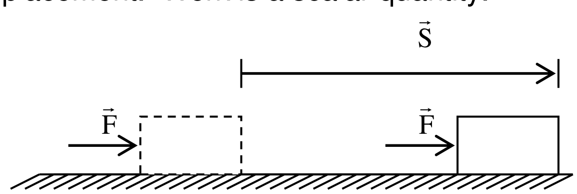
☞ Determine the power output of an energy source.

Discussion

3.1 Work

The term *work*, commonly used in connection with widely physical or mental activities, is restricted in physics, in cases wherein there is a force and a displacement along the direction of the force.

In general, *work* is defined as the product of the displacement and the component of the force along the displacement. Work is a scalar quantity.



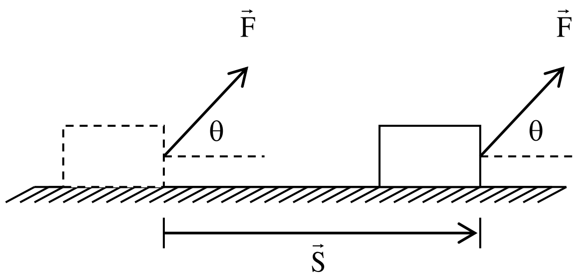
If the force and the displacement are in the same direction, as shown in the diagram above, then the work done by the force in moving the body is given by $W = \left|\vec{F}\right| = \left|\vec{S}\right| = FS$,

where: $\vec{F}$ is the applied force

$\vec{S}$ is the displacement

If the force and the displacement are not in the same direction, as shown in the diagram below, then the work done by the applied force is given by

$$W = \left|\vec{F}\right| = \left|\vec{S}\right|(\cos\theta) = FS\cos\theta$$
$$= (F\cos\theta)S$$



where θ is the angle between the two vectors $\vec{F}$ and $\vec{S}$.

There will be no work done if the displacement $\vec{S}$ and the applied force $\vec{F}$ are at right angles with each other, since $\cos 90^0 = 0$.

If the applied force does work by moving the body in the direction of the force, the work done is *positive*. If the body moves in the opposite direction of the force, the work is done by the body and is *negative*.

In general, there are three ways on how work is done.

- ✓ If the force is just to impart uniform motion on the body, the force of friction has done the same amount of work.
- ✓ In changing the position or configuration of the body system, as in the case of force applied on a body to raise the body on an inclined plane.
- ✓ In imparting acceleration to the body or system.

Units of Work

System	**Work**	**=**	**Force**	×	**Displacement**
MKS	Joule (J)	=	newton (n)	×	meter (m)
CGS	Erg	=	dyne (dyn)	×	centimeter (cm)
British	foot-pound	=	pound (lb)	×	foot (ft)

Conversion: 1 Joule = 10^7 Ergs

1 foot-pound = 1.356 Joules

Sample Problems with Solutions:

1. A wooden box is being pulled 10 m from its original position along a horizontal surface by a constant force of 25 N. Calculate the work done on the box if
   a) the force is applied horizontally, and
   b) the force makes an angle of 37^0 above the horizontal

   Solution:

   a) $W = F \bullet S$
   $= 25N \bullet 10m$
   $= 250N{\bullet}m$ (Joule)

   b) $W = F\cos 37^0 \bullet S$
   $= (25N)(\cos 37^0)(10m)$
   $= 199.66N{\bullet}m$ (Joule)

2. A 100 lb wooden box is pushed across a horizontal floor with a force of 50 lb. The coefficient of sliding friction, μ, is 0.4.
   a) Find the work done in pushing the box 60ft.
   b) How much work went into overcoming friction?
   c) How much work went into accelerating the box?

   Solutions:

   a) $W = F \bullet S$
   $= 50lbs \bullet 60\ ft$
   $= 3{,}000\ ft\text{-}lb$

   b) $W = F \bullet S$
   but, $f = \mu N = \mu W$
   $= 0.4\ (100\ lb)$
   $= 40\ lbs$

   $W = F \bullet S$
   $= 40\ lbs \bullet 60\ ft$
   $= 2{,}400\ ft\text{-}lb$

   c) $W_a = 3{,}000\ ft\text{-}lb - 2{,}400\ ft\text{-}lb$
   $= 600\ ft\text{-}lb$

3.2 Energy

The property of a body or system of bodies by virtue of which work can be performed is called *energy*. It is a scalar quantity.

Energy can exist in many forms and can be transformed from one form to another. The energy possessed by an object by virtue of its motion is called *kinetic energy*, or energy of motion. Energy of position, or configuration, is called *potential energy*.

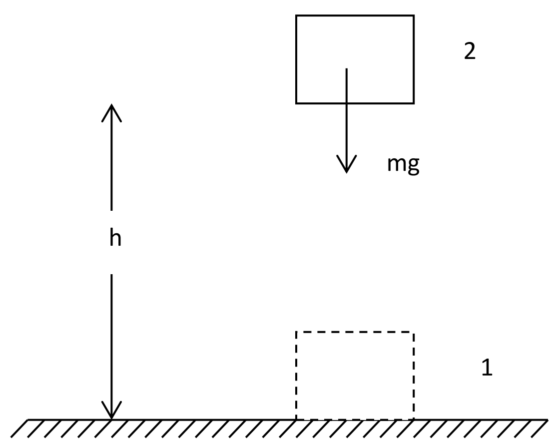
When work is done on a body in the absence of frictional force, the work done is equal to the sum of the increase in kinetic energy and the increase in potential energy. The units in which energy is expressed are the same as the units of work.

3.2.1 Potential Energy

An object may store energy because of its position. The energy that is stored is called *potential energy* (PE), because in the stored state, it has the potential to do work.

3.2.1.1 Gravitational Potential Energy

Work is required to lift objects against the earth's gravity. The potential energy due to elevated positions is called *gravitational potential energy*. The amount of gravitational potential energy possessed by an elevated object is equal to the work done against gravity on lifting it.



If a mass m is raised from position 1 to position 2, a distance h, as shown in the diagram on the right, work is done on the body against gravity with the magnitude,

$$W = -mgh$$

where mg is the force and the negative sign signifies a force against gravity. If the body is allowed to fall, the weight of the body will do the same amount of work,

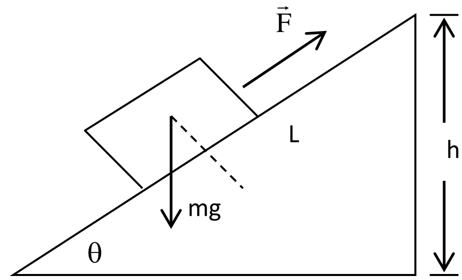
$$W = mgh$$

which in another way, is called the potential energy of the body. In other words, energy was stored in the body by virtue of its position relative to the surface. Therefore,

$$PE = mgh$$

Since weight $w = mg$, $PE = wh$

Consider now the work done in dragging a body of mass *m* along a frictionless inclined plane, as shown. Since the component of the vertical force, (the *weight* = *mg*) along the plane is ($mg\sin\theta$), the work done against this component of the weight along the plane of length *L* is,



$$W = (mg\sin\theta)\ L$$

but

$$\sin\theta = \frac{h}{L}$$

Therefore,

$$W = mgh$$

Note that the height *h* is the distance above some reference level, such as the ground or the floor of a building. The potential energy or the work done on a body raised to a height is independent of the path, or course, taken by the body. The potential energy is relative to some reference level and depends only on *mg* and the height *h*.

The potential energy of a body at high altitude with respect to the surface of the earth is given by

$$PE = GMm\left(\frac{l}{R} - \frac{l}{r}\right)$$

where the universal gravitational constant $G = 6.67 \times 10^{-11}\ {m^3}/{kg\text{-}s^2}$, *M* is the mass of the earth, *m* is the mass of the body, *R* is the radius of the earth, and *r* is the distance of the body from the center of the earth. Note that *r* is not just the altitude above the earth's surface but includes the radius of the earth as well.

3.2.2 Kinetic Energy

If an object is moving, then by virtue of that motion, it is capable of doing work. It is called energy of motion, or *kinetic energy* (KE).

The kinetic energy of the moving object can be measured by the amount of work it will do if brought to rest or by the amount of work originally need to impart velocity to it, in circumstances where the work cannot also go into potential energy.

Consider a body with initial speed V_1 on which a steady unbalanced force *F* acts as it moves a distance *s*. The body gains speed at a rate given by $a = \frac{F}{m}$ until it reaches a final speed V_2. The work done on the body by the unbalanced force that accelerated it appears as a change in its kinetic energy.

$$W = \Delta KE = Fs$$

Since $F = ma$, multiplying by s gives

$$Fs = mas$$

so that

$$\Delta KE = Fs = mas$$

Using

$$2as = V_2^{\,2} - V_1^{\,2}$$
$$as = \tfrac{1}{2}\left(V_2^{\,2} - V_1^{\,2}\right)$$

we finally obtain

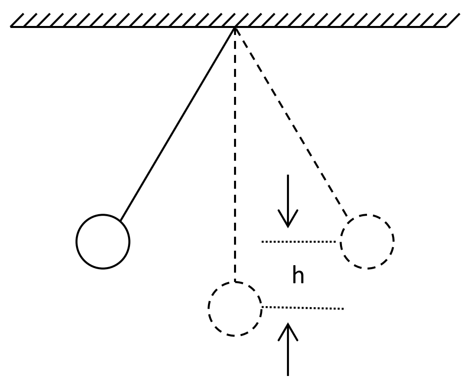
$$\Delta KE = \tfrac{1}{2}m\left(V_2^{\,2} - V_1^{\,2}\right)$$
$$= \tfrac{1}{2}mV_2^{\,2} - \tfrac{1}{2}mV_1^{\,2}$$

If the body was initially at rest at $V_1 = 0$ and the gain in kinetic energy is the final kinetic energy. Thus, the kinetic energy of a body, moving with velocity V, at any instant is

$$KE = \tfrac{1}{2}mV^2$$

3.2.3 Transformation and Conservation of Energy

Energy is given to a body or system of bodies when work is done upon it. In this process, there is merely a transfer of energy from one body to another. In such transfer no energy is created nor destroyed: it merely changes from one form to another. This statement is known as the *law of conservation of energy*.



An example of the law of the conservation of energy is the conservation of mechanical energy (potential and kinetic) in the case of a simple pendulum of mass m. If the pendulum is raised to a height h, it acquires potential energy. When it reaches the lowest point of the arc, its potential energy is minimum, but its velocity is maximum showing that the potential energy of the pendulum has been converted to kinetic energy. This conservation is 100% ; friction at the point of support and air resistance is neglected. The kinetic energy at the lowest point will carry the pendulum to the same height in the other side of the swing. The law of conservation of energy still holds even if friction and air resistance are taken into account, because in that case, when the body eventually stops swinging after some time, both its potential and kinetic energies, by then, will all have been dissipated into heat energy.

Sample Problems with Solutions:

1. Calculate the kinetic energy in joules of an 11.0 g rifle bullet travelling at 250 $^m/_s$.

Solution:

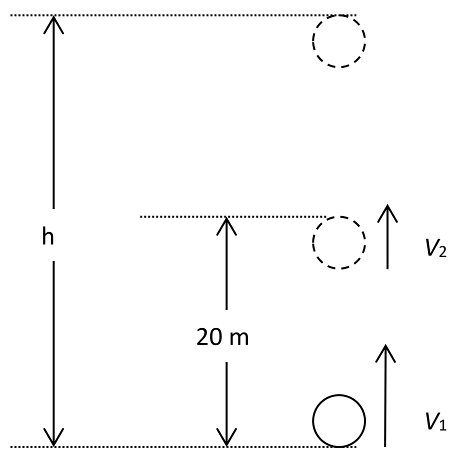
$$\begin{aligned} KE &= \tfrac{1}{2} mV^2 \\ &= \tfrac{1}{2}\left[(11.0\ g)\left(\frac{1\ kg}{1000\ g}\right)\right](250\ ^m/_s)^2 \\ &= 343.75\ joules \end{aligned}$$

2. A 40-lb stone is hoisted to the top of a building 100 ft high. By how much does its potential energy increases?

Solution:

$$\begin{aligned} PE &= mgh = wh \\ &= (40\ lb)(100\ ft) \\ &= 4000\ ft\text{-}lb \end{aligned}$$

3. A body of mass *m* is thrown vertically upward with a velocity of 25 $^m/_s$.
   a) How high will it rise?
   b) What is its velocity at a height of 20 m?



Solution:

a) KE bottom = PE at the highest point

$$\tfrac{1}{2} m V_1^{\,2} = mgh$$

$$h = \frac{V^2}{2g}$$

$$= \frac{(25)^2}{2(9.8)}$$

$$= 31.88\,m$$

b) $KE_{\leftarrow} = KE_{\uparrow} + PE_{\uparrow}$

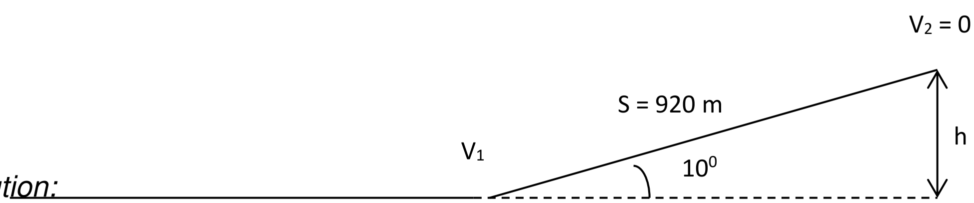
$$\tfrac{1}{2} m V_1^{\,2} = \tfrac{1}{2} m V_2^{\,2} + mgh_2$$

$$\tfrac{1}{2}\left(25\,{}^{m}\!/\!_{s}\right)^2 = \tfrac{1}{2} V_2^{\,2} + \left(9.8\,{}^{m}\!/\!_{s^2}\right)(20\,m)$$

$$V_2^{\,2} = 2\left[\tfrac{1}{2}\left(25\,{}^{m}\!/\!_{s}\right)^2 - \left(9.8\,{}^{m}\!/\!_{s^2}\right)(20\,m)\right]$$

$$V_2 = 15.26\,{}^{m}\!/\!_{s}$$

4. A volcanic ash flow is moving across horizontal ground when it encounters a 10° up slope. It is observed to travel 920 m on the upslope before coming to rest. The volcanic ash contains trapped gas, so the force of friction with the ground is very small and can be ignored. At what speed was the ash flow moving just before encountering the up slope? Consider an arbitrary mass *m* of the ash flow and see how it moves.



Solution:

$$KE_{\leftarrow} = PE_{\uparrow}$$

$$\tfrac{1}{2} m V_1^{\,2} = mgh$$

$$V_1^{\,2} = 2gh$$

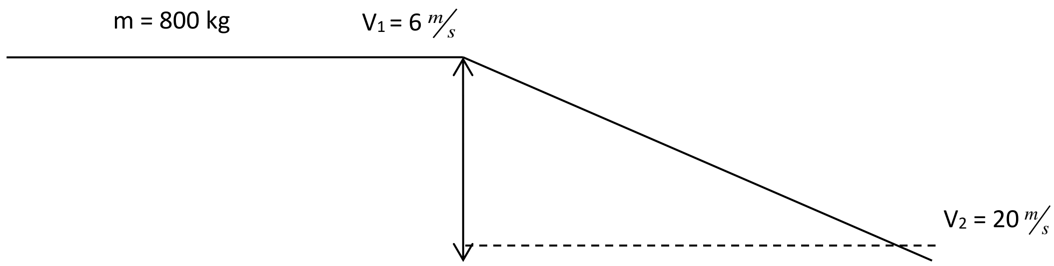
$$V_1 = \sqrt{2gh}$$

$$= \sqrt{(2)(9.8)\left(90 \sin 10^0\right)}$$

$$= 55.95\,{}^{m}\!/\!_{s}$$

5. An 800 kg car moving at 6 $^{m}/_{s}$ begins to coast down a hill 40 m high with its engine off. The driver applies the brake so that the car's speed at the bottom of the hill is 20 $^{m}/_{s}$. How much energy was lost to friction?

Solution:



$$KE_{\leftarrow} + PE_{\leftarrow} = KE_{\uparrow} + W_f$$

$$Wf = \tfrac{1}{2}(800)(6)^2 + [(800)(9.8)(40)] - \left[\tfrac{1}{2}(800)(20)^2\right]$$

$$= 14400 + 313600 - 160000$$

$$W_f = 168{,}000 \ \ joules$$

3.3 Power

The time rate of doing work is known as *power*. Power measures the amount of work done in given time:

$$\text{power} = \frac{\text{work done}}{\text{elapsed time}} = \frac{W}{t} = \frac{FS}{t} = F\,\frac{S}{t} = Fv$$

In the MKS system, the unit of power is in joule per second, also known as the watt, named after James Watt. In the CGS system, the unit of power is in erg per second. In the English system, the standard unit of power is the horsepower (hp).

$$1\,hp = 33{,}000\ ^{ft-lbs}/_{\min}$$

$$= 550\ ^{ft-lbs}/_{s}$$

$$= 746\,watt$$

$$= 0.75\,kilowatt\,(kw)$$

$$1\,kw = 1000\,watts$$

Sample Problems with Solutions:

1. How much power is expended by a man who can push a load with a force of 190 lbs to a distance of 100 ft in 4 min?

   Solution:

$$
\begin{aligned}
P &= \frac{work}{time} \\
&= (80 lbs)\left(\frac{100\,ft}{4\min}\right) \\
&= 2000\, {}^{ft-lbs}\!/\!{}_{\min} \\
&= 2\times 10^{3}\, {}^{ft-lbs}\!/\!{}_{\min}
\end{aligned}
$$

2. An engine is needed to pump 10,000 gallons of water per hour into a reservoir 100 ft above the level ground. How many horsepower is required?

   Solution:

1 gallon of water = 8.34 lbs.

100,000 gallons = 8.34×10^{5} lbs

$$
\begin{aligned}
hp &= \frac{{}^{ft-lb}\!/\!{}_{\min}}{33{,}000} \\
&= \frac{834}{19.8} \\
&= 42.1
\end{aligned}
$$

3. Water is pumped from a river with a depth of 80 m to a reservoir on the surface at a rate of $10\, {}^{m^3}\!/\!{}_{hr}$. What is the minimum power in watts required in pumping the water up?

   Solution:

$$
\begin{aligned}
P &= \frac{FS}{t} = \frac{mgS}{t} \\
&= \left(10\, {}^{m^3}\!/\!{}_{hr}\right)(9.8)(80)\left(1000\, {}^{kg}\!/\!{}_{m^3}\right) \\
&= 7{,}840{,}000\, {}^{joule}\!/\!{}_{hr} \\
&= 2{,}178\, watts
\end{aligned}
$$

Lesson 3: Work, Energy and Power
Assessment

Instruction; Show your complete and neat solution. Identify your final answer in the solution. See answer sheet format.

1. A cord is used to lower vertically a block of mass *M* a distance *d* at a constant downward acceleration of g/4. (a) Find the work done by the cord on the block. (b) Find the work done by the work of gravity.

2. A child pulls a 5.6 kg box a distance of 12 m along a horizontal surface at a constant speed. What work does the child do on the box if the coefficient of kinetic friction is 0.20 and the cord makes an angle of 45° with the horizontal?

3. To push a 25 kg crate up a 27° incline, a worker exerts a force of 120 N, parallel to the incline. As the crate slides 3.6 m, how much work is done on the crate by: (a) the worker, (b) the force of gravity, and (c) the normal force due to the incline?

4. A projectile with a mass of 2.4 kg is fired from a cliff 125 m high with an imitial velocity of 150 m/s, directed 41.0º above the horizontal. What are (a) the kinetic energy of the projectile just after firing and (b) its potential energy? (c) Use conservation of energy to find the speed of the projectile just before it strikes the ground.

5. A 220-lb man jumps out a window into a fire net 36 ft below. The net stretches 4.4 ft before bringing him to rest and tossing him back into the air. What is the potential energy of the stretched net if no energy is dissipated?

6. A body of mass *m* starts down from the top of an inclined plane 20 ft long and 10 ft high. What is its velocity at a point 12 ft from the top if coefficient of friction is 0.1?

7. Two snow-covered peaks are at elevations of 862 m and 741 m above the valley between them. If a skier starts from rest on the higher peak and just coasted down without exerting any effort, at what speed would he arrive at the lower peak?

8. Starting a race, a 68.2-kg sprinter runs the first 7.04 m in 1.60 s, starting from rest and accelerating uniformly. What average power does the sprinter generate during the 1.60-s interval?

9. An elevator with a 10-person capacity (each person with an average mass of 68 kg) raises passengers to a height of 152 m in 55.0 s at constant speed. Find the power output of the motor assuming no frictional losses?

10. A horse pulls a cart with a force of 42.0 lbs at an angle of 27º with the horizontal and moves along at a speed of 6.20 $^{mi}/_{hr}$. (a) How much work does the horse do in 12 min? (b) Find the power output of the horse in *hp*.

LESSON 4: **IMPULSE AND MOMENTUM**

Introduction

The law of conservation of energy, which discussed in the previous lesson is one of several great conservation laws in physics. Among the other quantities found to be conserved is linear momentum. I this lesson, conservation of linear momentum and energy will be used to analyze collisions. Collisions as between billiard or pool balls that illustrates vector law very effectively, The total vector momentum before collision equals the total vector momentum after collissions.

Learning Outcomes

After successful completion of this lesson, you should be able to:

- ☞ Relate Newton's second Law of Motion to impulse and momentum.
- ☞ Calculate the impulse of a force and relate it to the change in momentum.
- ☞ Distinguish between internal and external forces among interacting bodies.
- ☞ Recognize when momentum is conserved.
- ☞ Apply the impulse — momentum relation and the conservation of momentum to a variety of problems involving collisions (elastic and inelastic) and other kinds of interaction between bodies.

Discussion

4.1 Linear Momentum

Momentum is a property related to an object's motion and mass. The faster the object moves and the larger its mass, the greater is its momentum. It is the tendency of an object to continue moving at constant velocity and thereby encountering difficulty in bringing the object to rest. The linear momentum *p* is defined quantitatively as the product of its mass *m* and velocity *v*:

$$Momentum = p = mv$$

the unit of momentum is $kg\, {}^{m}\!/_{s}$.

Since velocity is a vector quantity having magnitude and direction, momentum is also a vector quantity with magnitude and direction. The direction of an object's momentum equals the direction of its velocity.

4.2 Impulse

Impulse is the product of a force acting on an object and the time that the force acts. An impulse causes the object's momentum to change:

$$\textit{Impulse } J = F \bullet t$$

An equation relating the impulse of a force and the subsequent change in momentum of the object on which the force acts is derived using Newton's Second Law of Motion $\left(\sum F = ma\right)$ and kinematics:

$$a = \frac{v - v_o}{t}$$

$$\sum F = ma$$

$$= m\left[\frac{v - v_o}{t}\right]$$

Rearranging the terms

$$\left(\sum F\right)t = m(v - v_o)$$

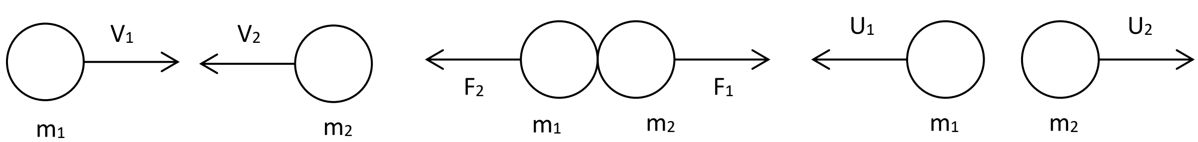
or

$$\text{impulse} = \text{change in linear momentum}$$

4.3 Conservation of Linear Momentum

As two colliding bodies interact with each other, they exert forces on each other. These are internal forces if we take the two bodies together as our system. At the same time, they may be acted upon by other external forces such as friction.

The total linear momentum of a system is conserved if there is no additional external force present before, after and during the collision. The total linear momentum of a system remains constant.



(a) before collision

(b) during collision

(c) after collision

$$(F_2 \text{ on } 1)t = m_1U_1 - m_1V_1$$
$$(F_1 \text{ on } 2)t = m_2U_2 - m_2V_2$$

Adding these two equations,

$$(F_2 \text{ on } 1 + F_1 \text{ on } 2)t = (m_1U_1 - m_1V_1) + (m_2U_2 - m_2V_2)$$

From Newton's third law,

$$F_1 \text{ on } 2 = F_2 \text{ on } 1$$

$$F_1 \text{ on } 2 + F_2 \text{ on } 1 = 0$$

Hence,

$$0 = (m_1U_1 - m_1V_1) + (m_2U_2 - m_2V_2)$$

$$m_1V_1 + m_2V_2 = m_1U_1 + m_2U_2$$

or momentum before collision = momentum after collision

4.4 Collisions

When two bodies collide, internal forces are external during the short time of impact. These forces may be conservative or dissipative. If the forces are conservative, the collision is said to be elastic. In an elastic collision, the total energy (kinetic energy) is conserved. A collision is inelastic if total energy is not conserved.

Note: All collision conserve momentum, but not all of them conserve kinetic energy.

4.4.1 Three Categories of Collisions

1. *Perfectly elastic collision* — when deformation maybe resolved in a very relatively short time interval.

2. *Partly elastic* — when deformation is resolved in a longer time interval or separation of colliding bodies is not abrupt.

3. *Inelastic collision* — deformation may be permanent that the bodies in collision may stick together and travels together with common velocity.

4.1.1 Coefficient of Restitution *(e)*

$e = 0$; perfectly inelastic collision
$e = 1$; perfectly elastic collision
$1 < e > 0$; partly elastic collision

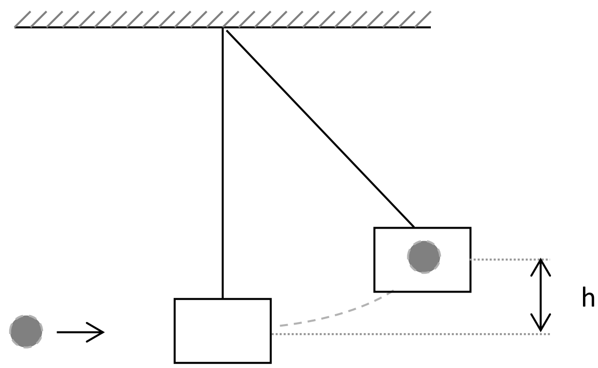
e for partly elastic collision is equal to the ratio between relative speeds.

$$e = \frac{U_2 - U_1}{V_1 - V_2} = \frac{\text{relative speed after collision}}{\text{relative speed before collision}}$$

4.5 Ballistic Pendulum

→ used to measure velocity of bullets

A bullet with mass (*m*) and unknown velocity (*v*) is fired to hit a hanging block of mass (*M*) as shown in the following figure. The block with embedded bullet swings and rises to a height (*h*).



From conservation of momentum,

$$m_1V_1 + m_2V_2 = MU_1 + MU_2$$
$$U_1 = U_2 = U = \sqrt{2gh}$$
$$m_1V_1 = MU$$
$$mV = (m + M)\sqrt{2gh}$$
$$V = \frac{m + M}{m}\left(\sqrt{2gh}\right)$$

Sample Problems with Solutions:

1. A 2200 kg car at 35 $^{m}\!/_{s}$ strikes a stationary 1500 kg car and the two locked bumpers.
   a) What is their final common velocity just after collision?
   b) What percentage of the initial energy is dissipated in the collision?

Given: $m_1 = 2200$ kg , $m_2 = 1500$ kg
$V_1 = 35\ ^{m}\!/_{s}$, $V_2 = 0$

$$m_1V_1 + m_2V_2 = m_1U_1 + m_2U_2$$
$$U_1 = U_2 = U$$

Solution:

a)

$$m_1V_1 = (m_1 + m_2)U$$

$$U = \frac{m_1V_1}{m_1 + m_2}$$

$$= \frac{(2200\,\text{kg})(35\,^{m}\!/\!_{s})}{2200\,\text{kg} + 1500\,\text{kg}}$$

$$= 20.81\,^{m}\!/\!_{s}$$

b)

$$\text{Final KE} = \tfrac{1}{2}m_1U_1^2 + \tfrac{1}{2}m_2U_2^2 \quad , \quad U_1 = U_2$$

$$= \tfrac{1}{2}(m_1 + m_2)(U^2)$$

$$= \tfrac{1}{2}(2200 + 1500)(20.81)^2$$

$$= 801153.78$$

$$\text{Initial KE} = \tfrac{1}{2}m_1V_1^2 + \cancel{\tfrac{1}{2}m_2V_2^2} \rightarrow 0$$

$$= \tfrac{1}{2}(2200)(35\,^{m}\!/\!_{s})^2$$

$$= 1347500$$

$$\frac{\text{Energy transferred}}{\text{Original energy}} = \frac{801153.78}{1347500}$$

$$= 0.595$$

hence, 59.5% of initial energy remains kinetic energy, and
40.5% of initial energy was dissipated during collision

2. Suppose that in the first example, the cars have spring loaded bumpers, so that collision is perfectly elastic.
   a) What is the velocity of each car after the collision?
   b) What are the final momenta and energies?

Solution:

from momentum conservation

a) $m_1V_1 = m_1U_1 + m_2U_2 \longrightarrow (1)$

from energy conservation:

$$\tfrac{1}{2}mV_1^2 = \tfrac{1}{2}m_1U_1^2 + \tfrac{1}{2}m_2U_2^2$$

$$e = \frac{U_1 - U_2}{V_1 - V_2}$$

$$U_2 - U_1 = V_1 - V_2$$

$$U_2 = V_1 - \cancel{V_2}(0) + U_1$$

$U_2 = V_1 + U_1 \longrightarrow$ (2)

Substitute (2) in (1)

$$m_1V_1 = m_1U_1 + m_2(V_1 + U_1)$$

$$= m_1U_1 + m_2V_1 + m_2U_1$$

$$m_1V_1 - m_2V_1 = m_1U_1 + m_2U_1$$

$$(m_1 - m_2)V_1 = (m_1 + m_2)U_1$$

$$U_1 = \frac{(m_1 - m_2)V_1}{m_1 + m_2}$$

$$= \frac{(2200 - 1500)35}{2200 + 1500}$$

$$U_1 = 6.62 \, {}^{m}\!/\!_{s}$$

$$U_2 = V_1 + U_1 = 35 + 6.62 = 41.62 \, {}^{m}\!/\!_{s}$$

b) final energies

$$KE_1 = \tfrac{1}{2} m_1 U_1^2 \qquad KE_2 = \tfrac{1}{2} m_2 U_2^2$$

$$= \tfrac{1}{2}(2200)(6.62)^2 \qquad = \tfrac{1}{2}(1500)(41.62)^2$$

$$= 48206.84 \text{ Joules} \qquad = 1299168.3 \text{ Joules}$$

final momenta

$$P_1 = m_1U_1 \qquad P_2 = m_2U_2$$

$$= 2200(6.62) \qquad = 1500(41.62)$$

$$= 14564 \text{kg} \, {}^{m}\!/\!_{s} \qquad = 62430 \text{kg} \, {}^{m}\!/\!_{s}$$

3. A 200 g block moves to the right at a speed of 100 ${}^{cm}\!/\!_{s}$ and meets a 400 g block moving to the left with a speed of 80 ${}^{cm}\!/\!_{s}$. Find the final velocity of each block if the collision is elastic.

Given: $m_1 = 200$ g $\qquad m_2 = 400$ g

$V_1 = 100 \, {}^{cm}\!/\!_{s}$ $\qquad V_2 = 80 \, {}^{cm}\!/\!_{s}$

$m_1V_1 + m_2V_2 = m_1U_1 + m_2V_2 \longrightarrow$ (1)

Solution:

$$e = \frac{U_2 - U_1}{V_1 - V_2}$$

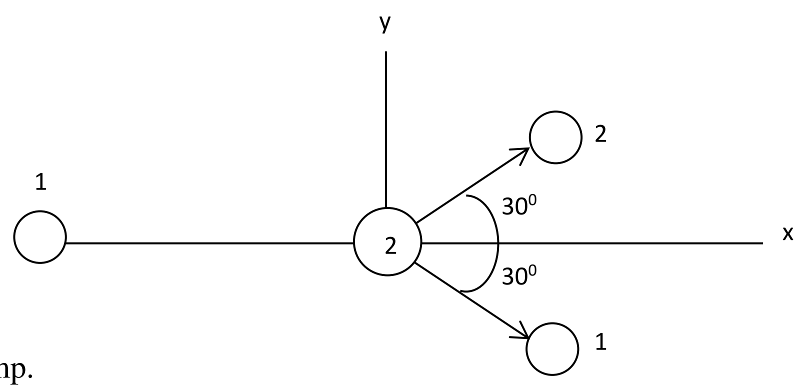
$$U_2 - U_1 = V_1 - V_2$$

$$U_2 = V_1 - V_2 + U_1 \longrightarrow \qquad (2)$$

Substitute (2) in (1)

$$\begin{aligned}
m_1V_1 + m_2V_2 &= m_1U_1 + m_2(V_1 - V_2 + U_1) \\
&= m_1U_1 + m_2V_1 - m_2V_2 + m_2U_1 \\
m_1V_1 + m_2V_2 - m_2V_1 + m_2V_2 &= (m_1 + m_2)U_1 \\
(m_1 - m_2)V_1 + 2m_2V_2 &= (m_1 + m_2)U_1 \\
U_1 &= \frac{2m_2V_2 + (m_1 - m_2)V_1}{m_1 + m_2} \\
&= \frac{2(400)(-80) + (200 - 400)(100)}{200 + 400} \\
&= -140 \, \text{cm/s} \\
U_2 &= V_1 - V_2 + U_1 \\
&= 100 - (-80) + (-140) \\
&= 40 \, \text{cm/s}
\end{aligned}$$

4. A body of mass 100 g with a velocity of 10 cm/s hits another identical body at rest, and the two recoil as shown. Find U_1 and U_2



Solution:

a) x-comp.

$$m_1V_1 + \underline{m_2V_2} \quad 0 = m_1U_1 \cos 30^0 + m_2U_2 \cos 30^0$$

$$m_1 = m_2$$

$$V_1 = U_1 \cos 30^0 + U_2 \cos 30^0$$

y-comp.

$$0 = m_2U_2 \sin 30^0 - m_1U_1 \sin 30^0$$

$$U_2 = U_1$$

hence,

$$V_1 = 2U_1\left(\cos 30^0\right)$$

$$U_1 = \frac{V_1}{2\cos 30^0}$$

$$= \frac{10 \ {}^{cm}/_{s}}{2(.866)}$$

$$= 5.8 \ {}^{cm}/_{s}$$

b)

$$\text{KE before collision} = {}^{1/2}/\ m_1V_1^{\ 2}$$

$$= \tfrac{1}{2}(100)(10)^2$$

$$= 5000\,\text{ergs}$$

$$\text{KE after collision} = \tfrac{1}{2}m_1U_1^{\ 2} + \tfrac{1}{2}m_2U_2^{\ 2}$$

$$= \tfrac{1}{2}(100+100)(5.8)^2$$

$$= 3364 \ \text{ergs}$$

Since KE before collision ≠ KE after collision, collision is inelastic.

c) KE total

KE final – KE total = 5000 – 3364

= 1636 ergs

5. A bullet weighing 0.02 lb is fired with a nozzle velocity $V_1 = 2700 \ {}^{ft}/_{s}$ into a ballistic pendulum weighing 20 lbs. Find the maximum height through which it rises.

Given: $m_1 = 0.02$ lb $\quad m_2 = 20$ lbs $\quad U_1 = U_2 = U$

$V_1 = 2700 \ {}^{ft}/_{s}$ $\quad V_2 = 0$

Solution:

$$m_1V_1 = m_1U_1 + m_2U_2$$

$$m_1V_1 = (m_1 + \ m_2)U$$

$$U = \frac{m_1V_1}{m_1 + \ m_2} = \frac{(0.02\,\text{lb})(2700\,{}^{ft}/_{s})}{0.02\,\text{lb} + 20\,\text{lbs}}$$

$$\left[U = \sqrt{2gh}\right]^2$$

$$h = \frac{U^{\ 2}}{2g}$$

$$h = 0.11 \ \text{ft}$$

6. A 1.5 kg ball moving at 2.0 $^{m}/_{s}$ to the right collides head on with a 1.0 kg ball moving at 3.0 $^{m}/_{s}$ to the left. The coefficient of restitution is 0.70. Find the speed and direction of the balls after collision.

Given: $m_1 = 1.5$ $\quad$ $m_2 = 1.0$ kg $\quad$ $e = 0.70$

$V_1 = 2.0$ $^{m}/_{s}$ $\quad$ $V_2 = 3.0$ $^{m}/_{s}$

Solution:

$$m_1V_1 + m_2V_2 = m_1U_1 + m_2U_2 \quad \rightarrow \quad (1)$$

$$\frac{U_2 - U_1}{V_1 - V_2} = 0.7 \quad \rightarrow \quad (2)$$

$$(U_2 - U_1) = (V_1 - V_2)0.7$$

$$U_2 = 0.7V_1 - 0.7V_2 + U_1$$

Substituting (2) in (1)

$$\begin{aligned}
m_1V_1 + m_2V_2 &= m_1U_1 + m_2(0.7V_1 - 0.7V_2 + U_1) \\
&= m_1U_1 + 0.7m_2V_1 - 0.7m_2V_2 + m_2U_1 \\
m_1V_1 + m_2V_2 - 0.7m_2V_1 + 0.7m_2V_2 &= m_1U_1 + m_2U_1 \\
(m_1 - 0.7m_2)V_1 + 1.7m_2V_2 &= (m_1 + m_2)U_1 \\
U_1 &= \frac{1.7m_2V_2 + (m_1 - 0.7m_2)V_1}{m_1 + m_2} \\
&= \frac{1.7(1.0)(-3.0) + [1.5 - 0.7(1.0)]2.0}{1.5 + 1.0} \\
&= -1.4 \\
U_2 &= 0.7V_1 - 0.7V_2 + U_1 \\
&= 0.7(2.0) - 0.7(-3.0) + (-1.4) \\
&= 1.4 + 2.1 - 1.4 \\
&= 2.1
\end{aligned}$$

That is,

$U_1 = -1.4$ $^{m}/_{s}$ and $U_2 = 2.1$ $^{m}/_{s}$

Lesson 4: Impulse and Momentum
Assessment

Instruction; Show your complete and neat solution. Identify your final answer in the solution. See answer sheet format.

1. A 60 gm mass with a horizontal velocity of 50 $^{cm}\!/_{s}$ collides with an 85 g mass at rest. After collision, the 60 g mass travel 30^0 below the horizontal and the 85 g mass travels 45^0 above the horizontal. Find the speed of each mass after impact.

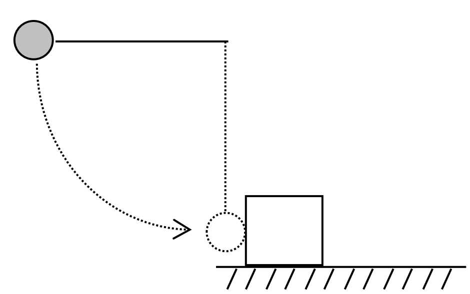
2. A 3 kg block moving to the right on a frictionless table at 4 $^{m}\!/_{s}$ makes a head on collision with another 3 kg block moving 5 $^{m}\!/_{s}$ to the left.

   a) If the collision is completely inelastic, find the final velocity of the blocks.
   b) If the collision is completely elastic, find the final velocity of the blocks.
   c) If half the initial energy is dissipated in the collision, find the final velocities of the blocks.

3. a) What is the momentum of a 12000 kg truck whose velocity is 50 $^{m}\!/_{s}$?
   b) What velocity must a 6000 kg truck attain in order to have the same momentum?
   c) What velocity must it attain to have the same kinetic energy?

4. A bullet, having a mass of 0.04 kg moving with a velocity of 450 $^{m}\!/_{s}$ penetrates a distance of 0.08 m into a wooden block firmly attached to the earth. Assuming a constant accelerating force, compute the following: a) the acceleration of the bullet, b) the accelerating force, c) the time of acceleration, and d) the impulse of the collision.

5. A steel ball of mass 0.514 kg is fastened to a cord 68.7 cm long and is released when the cord is horizontal. At the bottom of its path, the ball strikes a 2.6e-kg steel block initially at rest on a frictionless surface. The collision is elastic. Find (a) the speed of the ball and (b) the speed of the block, both just after the collision.



Introduction

In the previous lesson, we consider objects that were solid and assumed to maintain their shape except for a small amount of elastic deformation. We sometimes treated objects as point particles. This lesson will now consider materials that are deformable and can flow. These are called fluids. Fluids are substances that flows like the liquids and gases. This lesson will take first into consideration the properties and behavior as well as the principles and laws governing hydrostatics or fluid statics which is the study of fluids at rest.

Learning Outcomes

After successful completion of this lesson, you should be able to:

- Briefly describe the three states of matter and define the concepts of condensed matter fluids and fluid static
- Discuss the density and specific gravity of a substance, understand the concept of pressure at a point in a fluid, know the variation of pressure with depth, and distinguish between absolute and gauge pressure.
- Explain Pascal's law and Archimedes' principle and know the origin of buoyant force.
- Understand surface tension and the concept of capillarity.

Discussion

5.1 The Three States of Matter∗

Ordinary matter can be found in any one of the three states solid, liquid, or gas.

From one point of view, solids and liquids are classified as *condensed matter*, since they have certain properties in common, like they are both relatively incompressible so that their volumes can hardly be changed. Gases, on the other hand, are easily compressible.

From a different point of view, gases and liquids are grouped together as *fluids,* since they can flow. Gases expand to fill the whole container holding them whereas liquids settle to the bottom of any container where they are placed and take its shape. Solids do not have this property.

The atoms in a solid have relatively fixed positions in their overall structure, whereas in a fluid, the atoms can easily move relative to one another.

Once the atoms of a fluid have taken the shape of the container holding them, fluid flow stops and the fluid becomes stationary. *Fluid statics* is the study of the properties of fluids at rest.

5.2 Density and Specific Gravity

One important property of matter in general, and of fluids, in particular, is the *density* ρ (Greek letter rho), a scalar quantity defined as the ratio of the mass *m* per unit volume *V* :

$$\rho = \frac{mass}{volume} = \frac{m}{V} \tag{5.1}$$

Density may also be expressed as weight density. This is given or calculated by using the formula

$$\rho g = \frac{W}{V}$$

where ρg is the weight density

W is the weight

V is the volume

Weight density may be expressed in N/m^3 in SI or $dynes/cm^3$ in CGS system.

The density of a substance in general depends on environmental factors, like the temperature. But for liquids and solids, the variation in density is very small over wide ranges of many factors that to the first approximation, we can treat it as a constant.

The SI unit of density is kg / m^3 . The CGS unit, g / cm^3 , is also widely used. The corresponding conversion factor is $1 \ g / cm^3 = 10^3 \ kg / m^3$.

The *specific gravity* of a substance is a dimensionless quantity defined as follows:

$$specific\ gravity = \frac{weight\ of\ object\ in\ air}{weight\ of\ an\ equal\ volume\ of\ water} = \frac{density\ of\ substance}{density\ of\ water} \tag{5.2}$$

Another name for specific gravity is *relative density.* The density of water at 4.0°C is 1000 kg / m^3 .

The weight density of any substance may be determined by applying the formula

$$\rho\, g = \rho_r\, \rho_w$$

where ρ_r is the relative density $\rho\, g = \rho_r\, \rho_w$

5.3 Pressure and the Variation of Pressure with Depth

A fluid at rest exerts a force ΔF perpendicular to any surface ΔA in contact with it, such as a body immersed in the fluid or the container wall itself. The *pressure p* at a point within a fluid is defined as the ratio of the force ΔF acting perpendicular to a small element of surface area ΔA at that point:

$$p = \frac{normal\ force}{surface\ area} = \frac{\Delta F}{\Delta A} \tag{5.3}$$

It is independent of the direction of the area ΔA. Pressure is a scalar quantity.

The SI unit of pressure is the **pascal**, where 1 pascal = 1 Pa = 1 N / m^2 . The other units used are the *atmosphere* (atm), the *bar*, lb / in^2 (psi), and cm of mercury (cm Hg). The corresponding conversion factors are

1 atm = 1.013 × 10^5 Pa = 1.013 bar = 14.70 lb / in^2 = 76 cm Hg.

If the weight of the fluid is negligible, the pressure in a fluid is the same throughout its volume. But usually the fluid's weight is not negligible. Atmospheric pressure is lower at high altitude than at sea level. Deep beneath the sea, the pressure is many times larger than that on the surface.

Assuming that the acceleration *g* due to gravity is uniform throughout the volume of the fluid, the pressure in a fluid at rest increases with depth according to the following relation:

$$p = p_o + \rho g h \ , \tag{5.4}$$

where p is the pressure at depth h and p_o is the pressure on the surface. Note that the pressure is the same at any two points at the same level in the fluid, so that the shape of the container does not really matter.

5.4 Absolute and Gauge Pressure

Absolute pressure is the total pressure given in equation (1.4). The difference $p - p_a$ between absolute and atmospheric pressure is the *gauge pressure* p_g. Hence, if $p_o = p_a$ in equation (1.4), we find that

$$p_g = p - p_a = \rho g h \ . \tag{5.5}$$

If the pressure is less than atmospheric, as in partial vacuum, the gauge pressure is negative.

5.5 Pascal's Law

If the pressure p_o in equation (5.4) is increased by a certain amount, the pressure p at any depth increases by exactly the same amount. The first man to notice this fact was the French scientist Blaise Pascal (1623-1662) and a physical law, known as Pascal's law, was named after him. It states that

Pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and to the walls of the confining vessel.

Equation (153) shows that Pascal's law, maintaining that the pressure remains unchanged throughout a confined fluid, makes possible the amplification of a relatively small applied force to a much larger one if the surface area is significantly increased. This is the basis for the operation of earth-moving machineries, the brake system of cars, or even a barber's chair. Pascal's law also enables the transmission of forces over long distances to relatively inaccessible locations as in the wing flaps of an airplane.

5.6 Archimedes' Principle

The pressure at any point in a fluid at rest gives rise to a force acting perpendicular to the surface of a body submerged at that point. Since the pressure at all points on the same level in the fluid are equal, all the forces acting along the sides of the submerged body cancel out. For every sideward force on one side, we find an equivalent force on the other side at the same level as the first, which is acting in the opposite direction.

However, the forces along the top and the bottom surfaces of the submerged body do not cancel out because of the variation of pressure with depth. Along the upper surface of the sub-merged body, the pressure is less because the depth is smaller. The downward forces due to the fluid pressure at the top must also be smaller. Along the lower surface of the submerged body, the pressure is larger because the depth is greater. The upward forces due to the fluid pressure at the bottom must also be bigger. The pressure difference between the top and bottom surfaces of a submerged body therefore gives rise to a net upward force called the *buoyant force.*

According to Archimedes' principle,

A body wholly or partially immersed in a fluid is buoyed up by a force equal in magnitude to the weight of the fluid displaced by the body.

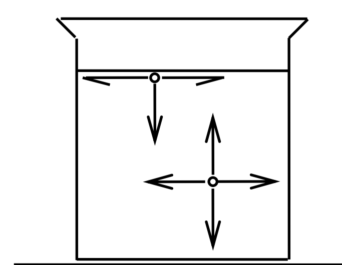
That is, if the fluid has density ρ_F and the submerged body a volume V_S, the upward buoyant force is $F_B = \rho_F V_S g$.

An object whose density ρ_O is greater than that of the fluid $(\rho_O > \rho_F)$ displaces a volume of the fluid whose weight is less than the weight of the object $(\rho_F V < \rho_O V_S g)$ so that the object sinks.

On the other hand, an object whose density ρ_O is less than that of the fluid $(\rho_O < \rho_F)$ displaces a volume of the fluid whose weight is greater than the weight of the object $(\rho_F V_S g > \rho_O V_S g)$ so that the object floats.

5,7 Surface Tension

The molecules of a liquid exert attractive forces on each other. Therefore, a molecule inside the volume of the liquid has a zero net force acting on it. But a molecule on the surface is attracted into the volume, making the surface area of the liquid as small as possible. *Surface tension* is the force on the surface of the liquid tending to minimize its area.



Surface tension can be easily demonstrated by carefully placing a needle on the surface of a body of water. It will float even if it is not partially submerged, so it is not buoyed up because of Archimedes' principle. The needle slightly depresses the surface molecules but it completely remains on the water surface. What keeps it afloat is the surface tension of the liquid.

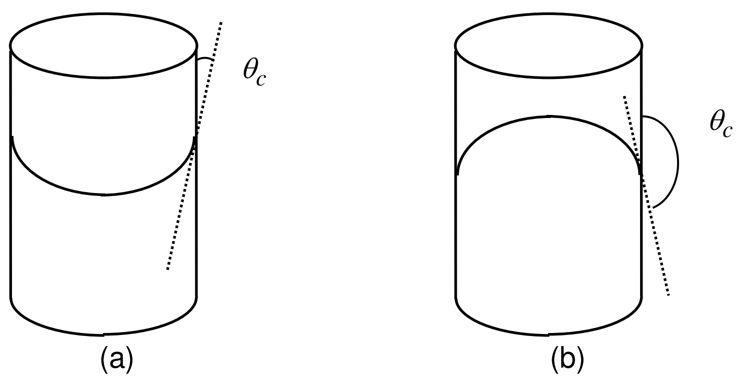
Surface tension also causes freely suspended droplets of liquid to become spherical in shape because a sphere has the smallest surface-to-volume ratio of any geometric shape.

Substances, like detergents, which reduce the surface tension when mixed with a liquid, are called *surfactants*.

5.8 Cohesion, Adhesion, and Capillarity

The attraction between similar molecules in a liquid is called *cohesion,* whereas the attraction between the molecules of a liquid and another substance, such as the wall of a thin glass tube, is called *adhesion.*

When the adhesive forces are large compared to the cohesive forces, as in the case of water and a glass surface, the surface of the liquid is concave upward, as shown in (a) in the diagram that follows, and the liquid is said to wet the surface of the other substance.



When the adhesive forces are small compared to the cohesive forces, as in the case of mercury and glass, the liquid does not wet the surface of the other substance, and the surface of the liquid is convex, as shown in (b) above.

The curved liquid surface is called a *meniscus.*

The contact angle θ_C between the wall and the tangent to the surface measures the relative magnitudes of the cohesive force and the adhesive force. For a liquid that wets the surface, the contact angle is less than 90°, as shown in (a) above. For a liquid that does not wet the surface, as in (b) above, the contact angle is greater than 90°.

When the surface of a liquid is concave upward, as shown in (a), the surface tension at the wall of the tube has a component upward. The liquid will rise in the tube until the net upward force on it due to the surface tension is balanced by the weight of the liquid. This rise is called *capillary action* or just *capillarity*.

Capillarity causes the rise of a liquid in blotting paper and of lamp oil in the wick. It is also responsible for holding water in the soil in the small spaces between soil particles. If it were not for capillarity, all the rain water would trickle downward, leaving the upper soil dry.

5.9 Sample Problems with Solutions

{1} 32 grams of a gas occupy a volume of 22 liters. What is the density of the gas in $kg \bullet m^{-3}$?

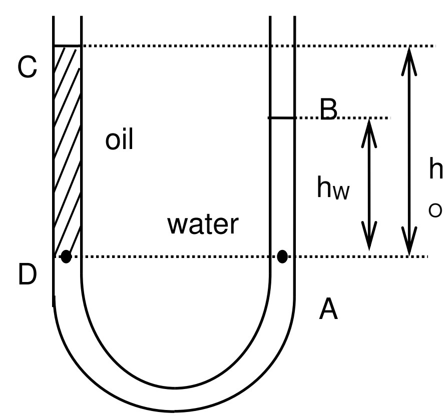
Solution:

$$1 \text{ liter} = 1000 \text{ cu. cm} = 1000\,(10^{-2}\text{ m})^3$$

$$\rho = \frac{m}{V} = \frac{32\ g}{22\ L} = \frac{32 \times 10^{-3}\ kg}{(22)(1000)\left(10^{-2}\ m\right)^3} = 1.45\ kg \bullet m^{-3}$$

(2) Water and oil are placed in the two arms of a glass U-tube as shown. If they come to rest as indicated, what is the density of the oil?

Solution:



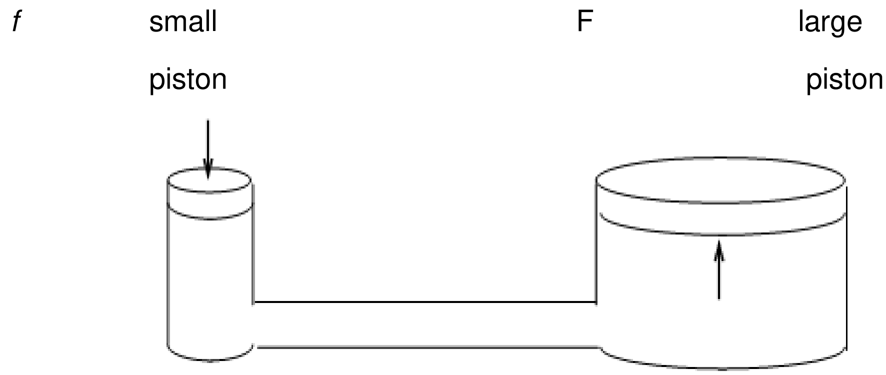
Consider the water below *D* and *A* in the tube. If the pressures at *D* and *A* were not equal, the water would flow. Since it does not, the pressure at *D* and *A* must be equal. Therefore

$$p_A = p_D$$
$$p_A = p_a + \rho_{oil}\, g\, h_o$$
$$p_D = p_a + \rho_w\, g\, h_w$$
$$\rho_{oil}\, g\, h_o = \rho_w\, g\, h_w$$
$$\rho_{oil} = \rho_w \frac{h_w}{h_o} = 916\ kg/m^3$$

where we have substituted in the above expression the following values:

$$\rho_w = 1.0 \times 10^3\ kg/m^3\ , \quad h_w = 135\ mm\ , \qquad h_w = 147.3\ mm$$

(3) The large piston in a hydraulic lift has a radius of 20 cm. What force must be applied to the small piston of radius 2 cm to raise a car of mass 1500 kg?



Hydraulic lift

Solution:

According to Pascal's law, the pressure in the small and the large piston shown in the diagram below are the same. Therefore

$$p_S = p_L \quad \rightarrow \quad \frac{f}{a} = \frac{F}{A} \quad \rightarrow \quad f = \frac{a}{A} F = \frac{\pi r^2}{\pi R^2} m g$$

The weight of the car is $mg = (1500 \ kg)(9.81 \ N/kg) = 1.47 \times 10^4 \ N$. The force that must be applied is then

$$f = \frac{(2 \ cm)^2}{(20 \ cm)^2}\left(1.47 \times 10^4 \ N\right) = 147 \ N$$

(4) A cork has a density of 200 kg/m^3. Find the fraction of the volume of the cork that is submerged when the cork floats in water.

Solution:

Let V be the volume of the cork and V_S be the volume that is submerged when the cork floats on the water. The weight of the cork is $\rho g V$, and the buoyant force is $\rho_W g V_S$. Since the cork is in equilibrium, the buoyant force equals the weight. That is $\rho_W g V_S = \rho g V$. The fraction of the cork submerged is then

$$\frac{V_S}{V} = \frac{\rho}{\rho_W} = \frac{200\ kg/m^3}{1000\ kg/m^3} = \frac{1}{5}$$

Therefore, one-fifth of the cork is submerged.

Lesson 5: Fluid Statics
Assessment

Instruction; Show your complete and neat solution. Identify your final answer in the solution. See answer sheet format.

1. A piece of copper whose density is 8.93 $g/_{cm^3}$ has a mass of 180 g in air and 162 g when submerged in a certain liquid. What is the mass density of the liquid?

2. What is the mass of one liter of methyl alcohol whose density is 190 $kg/_{m^3}$. What is its weight?

3. When placed in a pycnometer, 20 g of salt displaces 7.6 g of kerosene. If the mass density of kerosene is 0.83 $g/_{cm^3}$, find the volume and density of the salt.

4. An irregular metallic solid has a mass of 35 g. If its length is 10 cm, its width 8 cm, and its thickness 5 cm, what is the mass density of the metal?

5. A thin sheet of gold foil has an area of 3.05 cm^2 and a mass of 6.3 mg. How thick is the foil? The mass density of gold is 19,300 $kg/_{m^3}$.

6. A 60 mL flask is filled with mercury at 0°C. When the temperature rises to 80°C, 1.47 g of mercury spills out of the flask. Assuming the volume of the flask is constant, find the density of mercury at 80°C if its density at 0°C is 13,645 kg/m^3.

7. When equal masses of two substances are mixed, the resultant density is 2.5 g/cm^3. When equal volumes are mixed, the resultant density is 4.5 g/cm^3. What are the densities of the two substances?

8. A nurse applies a force of 45 N to a syringe with a fluid inside. The diameter of the syringe piston measures 1.15 cm. By how much is the pressure in the fluid increased due to the applied force?

9. Water from a spring is carried through a pipe of length 100 m down a hill inclined at 20° to the horizontal. When the pipe is full but water is not flowing, what is the pressure at the lower end of the pipe?

10. Three children each having a mass of 37.4 kg make a log raft by lashing together logs of diameter 32 cm and length 1.77 m. How many logs will be needed to keep them afloat? Take the density of the wood to be 0.75×10^3 kg/m

Introduction

Hydrodynamics or fluid dynamic is the study of fluids in motion. The discussion in this lesson is limited only for ideal fluids with the application of Bernoulli's principle.

Learning Outcomes

After successful completion of this lesson, you should be able to:

☞ Define and explain the concept of an ideal fluid.
☞ Apply the equation of continuity and Bernoulli's equation to the flow of fluids

Discussion

6.1 Quantities Used in Describing Fluid \Flow

There are three quantities frequently used in describing fluid motion. They are the density of the fluid, the pressure *p* and the velocity of flow of the fluid. These quantities, which describe the state or condition of the fluid, have definite values at each point in space and at each instant of time.

6,2 Ideal Fluid

Fluid motion is greatly simplified in the case of an ideal fluid, which has the following flow properties:

- Steady
- irrotational
- incompressible
- nonviscous.

Fluid flow is *steady* if the density, pressure, and flow velocity are constant in time. The values of these quantities generally change from one point to another, but they do not change with time at any particular point.

Fluid flow is *irrotational* if an element of the moving fluid does not rotate about an axis passing through the element itself. This can be understood by imagining a small paddle wheel immersed in the moving fluid. If the wheel moves with the fluid flow without rotating, the motion is irrotational.

Fluid flow is *incompressible* if the density of the fluid is constant, independent of position and time.

Fluid flow is *nonviscous* if no energy is dissipated through viscous forces, the analog of frictional force in the motion of solid bodies.

6.3 Streamlines

A *streamline* indicates the path of a fluid element as it flows. One can therefore imagine a streamline passing through every point in the moving fluid. In steady flow, no two stream-lines can cross one another and the pattern of streamlines does not change with time.

By drawing streamlines close to each other, one can represent regions of relatively high flow velocity. Streamlines that are farther apart indicate regions of relatively low flow velocity.

The direction of the velocity vector at any point along the streamline is always tangent to the streamline.

6.4 The Law of Conservation of Mass and the Equation of Continuity

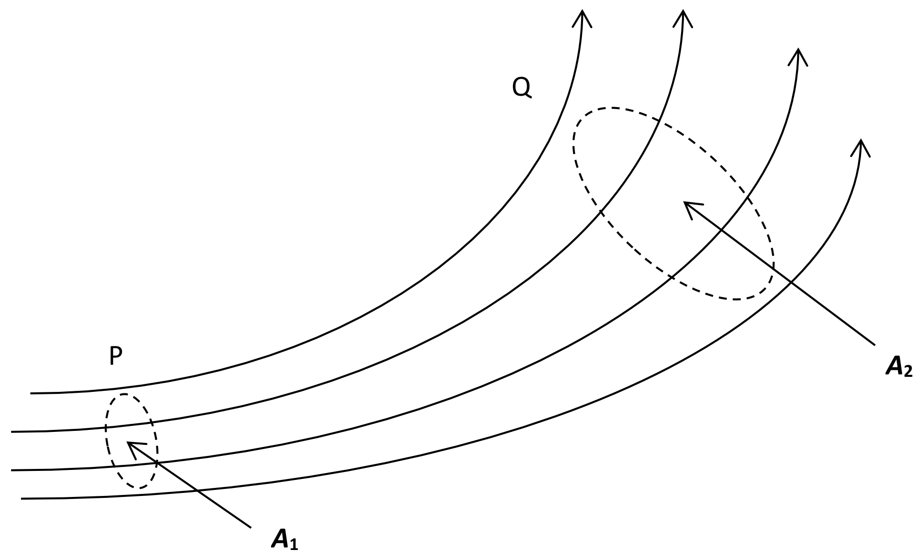
The streamlines that pass through the boundary of an area enclose a tubular region called a *tube of flow.* The boundary of such a tube is composed of streamlines to which the velocity of a fluid particle is always tangent, so that no fluid can move across it. Thus, a tube of flow behaves somewhat like a pipe of the same shape. The fluid that enters one of its end must leave through the other end. This fact expresses the *law of conservation of mass* in fluid dynamics and is expressed mathematically by the equation of continuity

$$Av = constant \tag{6.1}$$

where *A* is the cross-sectional area in one location of a tube of flow and *v* is the speed of the fluid particles passing through that cross-section.

Equation (2.1) tells us that in steady incompressible fluid flow, the speed of flow varies inversely with the cross-sectional area. The narrower the tube, the higher the flow speed.

To see why equation (2.1) is an expression of the law of mass conservation, we can apply it to two locations in a tube of flow:



Multiplying equation (2.1) by the time interval Δt, we find that

$$A_1(v_1 \Delta t) = A_2(v_2 \Delta t) \quad \to \quad A_1 L_1 = A_2 L_2 \quad \to \quad Volume_1 = Volume_2$$

Multiplying by the constant density of the fluid, we see that

mass transported across 1 = mass transported across 2

6.5 Bernoulli's Equation

Bernoulli's equation relates the pressure *p*, the elevation *y*, and the flow speed *v* in a tube of streamline flow of an ideal fluid with density ρ :

$$p + \rho g y + \tfrac{1}{2}\rho v^2 = constant \tag{6.2}$$

This does not involve a new principle but is merely an expression of the law of conservation of energy in fluid mechanics. If we multiply the preceding equation by a small volume element ΔV of the fluid, we find that

$$\begin{aligned} p\,\Delta V + (\rho\,\Delta V) g y + \tfrac{1}{2}(\rho\,\Delta V) v^2 &= (p\,\Delta A)\Delta y + (\rho\,\Delta V) g y + \tfrac{1}{2}(\rho\,\Delta V) v^2 \\ &= F\Delta x + mg y + \tfrac{1}{2} m v^2 \end{aligned}$$

The first term in the last equation on the right is just the work done by an applied force in moving a small volume element of the fluid, the second term indicates the change in the potential energy, while the third term is the change in the kinetic energy of the fluid element.

6.6 Sample Problems with Solutions

(1) Two streams join together to form a river. One stream has a width of 8.2 m, a depth of 3.4 m, and current speed of 2.3 m/s. The other stream is 6.8 m wide, 3.2 m deep, and flows at 2.6 m/s. The width of the river is 10.7 m and the current speed is 2.9 m/s. What is its depth?

Solution:

Since mass is conserved, the equation of continuity can be applied. We find that

$$A_R\, v_R = A_1\, v_1 + A_2\, v_2$$

$$(W_R\, D_R)\, v_R = (W_1\, D_1)\, v_1 + (W_2\, D_2)\, v_2$$

where W indicates the width and D the depth. The subscripts R, 1, and 2 refer to the river and the streams. Solving for D_R, we find that

$$D_R = \frac{W_1\, D_1\, v_1 + W_2\, D_2\, v_2}{W_R\, v_R}$$

$$D_R = \frac{(8.2\, m)(3.4\, m)(2.3\, m/s) + (6.8\, m)(3.2\, m)(2.6\, m/s)}{(10.7\, m)(2.9\, m/s)}$$

$$= 3.9\, m$$

(2) Water inside an enclosed tank is subjected to a pressure of two atmospheres at the top of the tank. How fast will water move out of a small hole 3 m below the top surface of the water? The density of water is 10^3 kg/m^3.

Solution:

Applying Bernoulli's principle to the upper U and lower L portions of the tank

$$p_U + \rho g y_U + \tfrac{1}{2}\rho v_U^2 = p_L + \rho g y_L + \tfrac{1}{2}\rho v_L^2$$
$$p_U - p_L + \rho g\left(y_U - y_L\right) = \tfrac{1}{2}\rho\left(v_L^2 - v_U^2\right)$$

where $p_U = 2p_a$, $p_L = p_a = 1.013 \times 10^5\ Pa = atmospheric\ pressure$, and $y_U - y_L = 3\,m$. Because the hole is very small, only a small amount of water leaks out, so that the water on the surface inside the container hardly moves, and we can set $v_U = 0$. The above equation becomes

$$2p_a - p_a + \rho g\left(y_U - y_L\right) = \tfrac{1}{2}\rho v_L^2 \ \rightarrow\ \tfrac{1}{2}\rho v_L^2 = p_a + \rho g\left(y_U - y_L\right)$$

We finally get

$$v_L^2 = \frac{2p_a}{\rho} + 2g\left(y_U - y_L\right) \ \rightarrow\ v_L = 16\ m/s$$

(3) An airplane has a wing area (each wing) of 12.5 m ^{2}. At a certain air speed, air flows over the upper wing surface at 49.8 m/s and over the lower wing surface at 38.2 m/s. Find the mass of the plane. Assume that the plane travels at constant velocity and that the lift effects associated with the planes body and tail assembly are small. The air density is 1.17 kg/m ^{3} .

Solution:

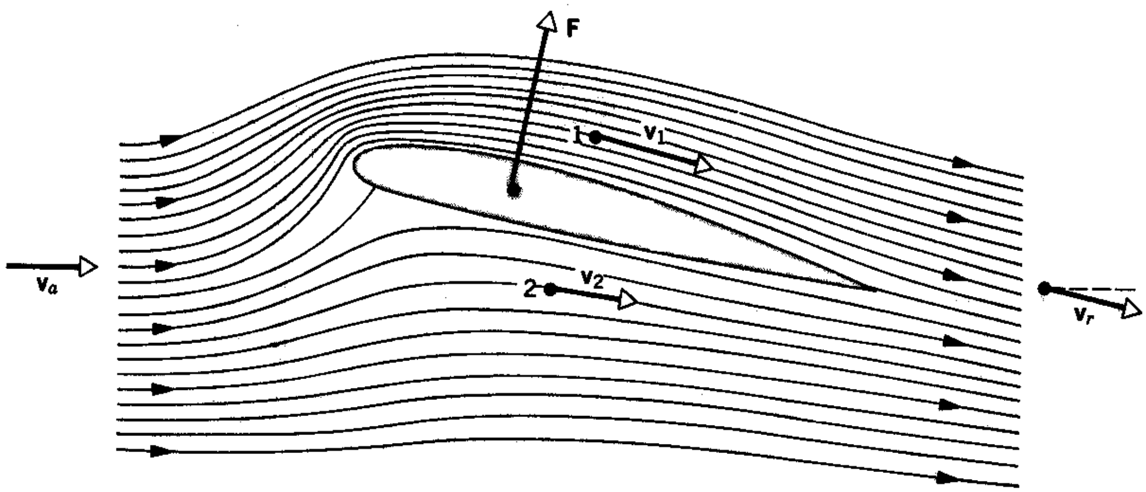
As an airplane moves, air rushes past around the wings of the plane. Wind tunnel experiments show that because of the shape of the wings, the streamlines on top are closer together than those below the wings (see illustration below).

Therefore, the air velocity on top of the wings must be greater than the air velocity at the bottom of the wings. But according to equation (2.2) above, increasing the speed *v* of flow of the fluid means a reduction in the air pressure *p*, if *y* is kept constant. A smaller pressure on top means a smaller force acting downward on the top of the wings.

The streamlines below the wings are farther apart so that the pressure at the bottom must be larger, in accordance with Bernoulli's equation. A higher pressure corresponds to a larger force acting upward at the bottom of the wings.

The unbalanced force acting on the wings gives rise to a net upward force which causes the airplane to fly.

The upward force that acts on a body by virtue of its motion through a fluid is known as dynamic lift. It is not the same as the buoyant force acting on a body, like a balloon, which arises due to Archimedes' principle.



Streamlines around an airplane wing.

Applying Bernoulli's principle to the upper *U* and lower *L* parts of the wings we get

$$p_U + \rho g y_U + \frac{1}{2}\rho v_U^2 = p_L + \rho g y_L + \frac{1}{2}\rho v_L^2$$

$$\rho g\left(y_U - y_L\right) + \frac{1}{2}\rho\left(v_U^2 - v_L^2\right) = p_L - p_U$$

The first term on the left is negligible compared to the second term because the difference $y_U - y_L$ is small. Hence, we are left with

$$\frac{1}{2}\rho\left(v_U^2 - v_L^2\right) = p_L - p_U$$

The total force F_T acting on the wings of the plane is given by

$$F_T = 2\left(F_L - F_U\right) = 2\left(p_L - p_U\right)A$$

where *A* is the wing area and the factor 2 appears because there are two wings. Hence

$$F_T = 2\frac{1}{2}\rho\left(v_U^2 - v_L^2\right)A = \rho\left(v_U^2 - v_L^2\right)A$$

This net upward force must balance the weight $m_P\, g$ of the plane so that

$$m_P\, g = \rho\left(v_U^{\,2} - v_L^{\,2}\right) A \quad \rightarrow \quad m_P = \frac{\rho A}{g}\left(v_U^{\,2} - v_L^{\,2}\right)$$

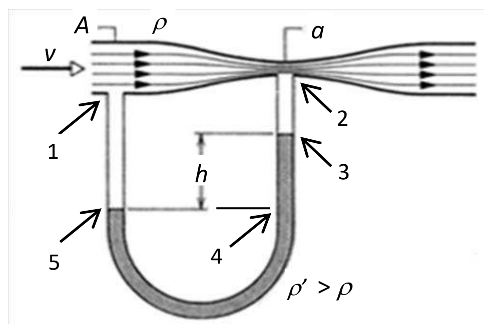
Substituting the given values, we get

$$m_P = \frac{\left(1.17\ kg/m^3\right)\left(12.5\ m^2\right)}{9.8\ m/s^2}\left\{\left(49.8\ m/s\right)^2 - \left(38.2\ m/s\right)^2\right\} = 1575.5\ kg$$

(4) A Venturi meter shown below is a device used to measure the speed of flow of a fluid in a pipe. By applying Bernoulli's equation to points 1 and 2, and the equation of continuity, show that the speed of flow at point 1 is given by

$$v = a\sqrt{\frac{2(\rho' - \rho)g\,h}{\rho\left(A^2 - a^2\right)}}$$

Solution:



With $y_1 - y_2$ negligible since the difference is very small, Bernoulli's equation gives

$$\tfrac{1}{2}\rho v_1^{\,2} + p_1 = \tfrac{1}{2}\rho v_2^{\,2} + p_2$$

By the continuity equation

$$A v_1 = a v_2 \quad \rightarrow \quad v_2 = v_1 \frac{A}{a}$$

Bernoulli's equation then becomes $\quad p_1 - p_2 = \tfrac{1}{2}\rho\left(v_2^{\,2} - v_1^{\,2}\right) = \tfrac{1}{2}\rho v_1^{\,2}\left(\dfrac{A^2 - a^2}{a^2}\right)$

Now $\quad p_5 = p_4 \;\rightarrow\; p_1 + \rho g\left(y_1 - y_5\right) = p_3 + \rho' g\left(y_3 - y_4\right) = p_3 + \rho' g\, h$

where

$$p_3 = p_2 + \rho g\left(y_2 - y_3\right) \;\rightarrow\; p_1 + \rho g\left(y_1 - y_5\right) = p_2 + \rho g\left(y_2 - y_3\right) + \rho' g\, h$$

In addition, $\quad y_1 - y_5 = \left(y_2 - y_3\right) + h$

gives

$$p_1 + \rho g\left(y_1 - y_5\right) = p_1 + \rho g\left[\left(y_2 - y_3\right) + h\right] = p_2 + \rho g\left(y_2 - y_3\right) + \rho' g\, h$$

$$p_1 - p_2 = (\rho' - \rho)g\,h = \tfrac{1}{2}\rho\,v_1^{\,2}\left(\frac{A^2 - a^2}{a^2}\right)$$

Solving for $v_1^{\,2}$, we find that

$$v_1^{\,2} = \frac{2a^2(\rho' - \rho)\rho\,g}{A^2 - a^2} \qquad \rightarrow \qquad v_1 = a\sqrt{\frac{2(\rho' - \rho)\rho\,g}{A^2 - a^2}}$$

Lesson 6: Fluid Dynamics
Assessment

Instruction; Show your complete and neat solution. Identify your final answer in the solution. See answer sheet format.

1. A garden hose having an internal diameter of 1.9 cm is connected to a lawn sprinkler that consists of an enclosure with 25 holes, each 0.13 cm in diameter. If the water in the hose has a speed of 1.1 m/s, at what speed does it leave the sprinkler holes?

2. Water is flowing smoothly through a closed pipe system. At one point, the speed of the water is 3.0 *m/s*, while at another point 1.0 *m* higher, the speed is 4.0 *m/s*. If the pressure is 2.0×10^{4} *Pa* at the lower point, what is the pressure at the upper point?

3. A tank is filled with water to a height *H*. A hole is punched in one of the walls at a depth *h* below the water surface. Show that the distance *x* from the foot of the wall at which the stream strikes the floor is given by $x = 2\sqrt{h(H-h)}$. Assume that the water leaves the hole horizontally.

4. A square plate with edge length 9.10 *cm* and mass 488 *g* is hinged along one side. If air is blown over the upper surface only, what speed must the air have to hold the plate horizontal? The air has density 1.21 kg/m^3. (Hint: Apply the principle of the dynamic li

5. Consider the Venturi meter, discussed in the sample problems, containing water without the manometer tube underneath. Let $A = 4.75a$. Suppose that the pressure at point 1 is 2.12 atmosphere.

   (a) Compute the values of *v* at point 1 and at point 2 that would make the pressure at point 2 equal to zero.

   (b) How much water in m^3 flows into the tube per minute (the volume flow rate in minutes) if the diameter at point 1 is 5.20 cm. The phenomenon at point 2 when the pressure falls to nearly zero is known as cavitation. The water vaporizes into small bubbles.

Introduction

In this lesson we begin the study of the branch of physics called thermodynamics. *Thermodynamics* is the macroscopic or large scale approach to the study of thermal properties of matter involving the transfer of heat energy. In the macroscopic approach, we focus our attention on the overall behavior of a piece matter as a whole and ignore the behavior of the individual atomic particles comprising the object. To describe the overall behavior of matter, we use only observable physical quantities characterizing the object that are measurable in the laboratory. We call these measurable, observable physical quantities the macroscopic variables. The experimentally observed relationships among some of the different macroscopic variables form what we call the physical laws of nature that are used in making predictions about the future behavior of matter.

In contrast to the macroscopic approach, we also have the microscopic approach wherein the main focus of attention now is the behavior of the individual atomic constituents of matter. This behavior is described by what we call microscopic variables that are no longer directly associated with our sense perception but exist only as some kind of mathematical formulations. The branch of physics using the microscopic approach in studying thermal properties of matter is called statistical mechanics.

Thermodynamics and statistical mechanics differ only in their approaches but they give the same results. In particular, the results of thermodynamics can be formulated in the language of statistical mechanics.

Learning Outcomes

After successful completion of this lesson, you should be able to:

- ☞ Learn to make conversions from one temperature scale to another.
- ☞ Calculate the thermal expansion of common materials.
- ☞ Know and understand the ideal gas law.

Discussion

7.1 Temperature and the Zeroth Law of Thermodynamics

There are four laws of thermodynamics and the first one is called the Zeroth Law which comes from the word zero, the only number that precedes the first non-zero number, namely one. The Zeroth Law clarifies the concept of *thermal equilibrium* and it states that

> *If two bodies A and B are separately in thermal equilibrium with a third body C, then A and B are also in thermal equilibrium with one another.*

In practice, the third body C referred to in the Zeroth Law is none other than the thermometer, a devise used to measure the *temperature*, or the hotness or coldness, of an object. Thermal equilibrium therfore means the equality of temperature, just like static equilibrium means an equality of two forces (the sum of all forces acting except one is equal to zero).

7.1.1 Thermometer

As mentioned above, a thermometer is a device capable of assigning a numerical value to the temperature of a body that it is in contact with it. There are many possible kinds of thermometers, since there are many properties of material objects that change as the temperature changes. Some physical properties that change with temperature are the length of a rod, the column of liquid inside a thin tube, and the electrical resistance of a wire. All these changes can be utilized in the construction of various types of thermometers.

7.1.2 Temperature Scales

Various temperature scales are in use: the Fahrenheit scale, the Celsius scale, the Rankine scale, and the Kelvin scale. Thus, a particular temperature could have four distinct numerical values on the four different scales. Fig. 3.1 below gives the relation of the four temperature scales just mentioned previously.

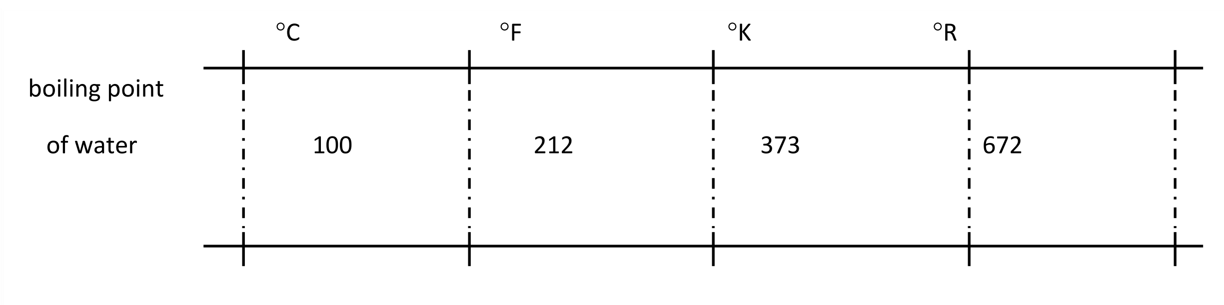


Fig. 7.1 Relationship between the different temperature scales.

In the Celsius scale, the temperature of the melting point of ice is assigned the value 0°C and the temperature of the boiling point of water is assigned the value 100°C. In the Fahrenheit scale, these same two fixed points are assigned the values of 32°F and 212°F, respectively. Thus, both the size of the degree and the zero of the scale are different in the two scales. From this information, we can derive the necessary conversion factor by ratio and proportion:

$$°C \rightarrow °F \quad \text{or} \quad °F \rightarrow °C$$

$$\frac{^\circ C - 0}{100 - 0} = \frac{^\circ F - 32}{212 - 32} \rightarrow \frac{^\circ C}{100} = \frac{^\circ F - 32}{180} \rightarrow {}^\circ C = \frac{^\circ F - 32}{1.8}$$

Therefore

$$^\circ F = 1.8\,^\circ C + 32 \tag{7.1}$$

The size of the graduation in the Kelvin scale is the same as that of the Celsius scale, while that of the Rankine scale is similar to the Fahrenheit scale.

To convert to the other temperature scales, we have the following relations:

$$^\circ K = {}^\circ C + 273.2 \tag{7.2}$$

$$^\circ R = {}^\circ F + 460 \tag{7.3}$$

7.2The Perfect Gas

7.2.1 Analytic Description of a Perfect Gas

To illustrate what was mentioned earlier in section 3.1 that the observed relationship among some of the macroscpic variables used in thermodynamics can be utilized to make predictions about the future behavior of a portion of matter, let us consider real gases which behave according to the following two physical laws:

1. Boyle's Law (1662)

   When the volume of a given mass of gas is decreased or increased while keeping its temperature constant, then the product of its pressure and volume also remains constant:

   $$pV = \text{ constant when } T = \text{constant}$$

   (3.4)

2. Charle's Law (1787)

   When a given mass of gas is heated at constant pressure, its volume increases linearly with the temperature T_C on the Celsius scale according to the equation $V = V_0(1 + aT_C)$ where V_0 is the volume at 0°C and a is a constant. Using the temperature in the absolute scale, we get the equation $V/T_K =$ constant or simply

   $$V/T = \text{constant when } p = \text{constant} \tag{7.5}$$

provided we keep in mind to use the temperature in the absolute scale.

Combining Boyle's and Charle's laws we get the equation

$$\frac{pV}{T} = \text{constant} \tag{7.6}$$

where the last constant is found to be proportional to the amount of gas in the sample. Equivalently, we have

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \tag{7.7}$$

where 1 and 2 refer to two different thermodynamic states of the gas. Knowing the values of five quantities in the last equation, we can predict the value of the sixth quantity. Just always keep in mind to use the absolute pressure and the temperature in the Kelvin scale.

In modern usage, equations (3.6) and (3.7) are two of the several forms of the *equation of state of a perfect gas,* or simply the *perfect gas equation,* or the *perfect gas law*. The perfect gas equation is an idealization because real gases do not exactly obey it and exhibit some departure instead..But as the gas pressures and densities are made smaller and smaller, the behavior of real gases becomes closer and closer to the perfect gas behavior given by the perfect gas law. The term *ideal gas* refers to a real gas that shows small enough departure from the perfect gas law. That is, an ideal gas is the same as a perfect gas provided the pressures and densities of a real gas are low or small enough. But in general, an ideal gas is not exactly the same as a perfect gas.

Using the definition of the density $\rho = M/V$ where *M* is the mass of the substance, we find that $V = M/\rho$. Applying this equation to the two states 1 and 2 of a perfect gas, we get the following result:

$$\frac{p_1\left(\frac{M_1}{\rho_1}\right)}{T_1} = \frac{p_2\left(\frac{M_2}{\rho_2}\right)}{T_2}$$

But $M_1 = M_2$ so that

$$\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2}$$

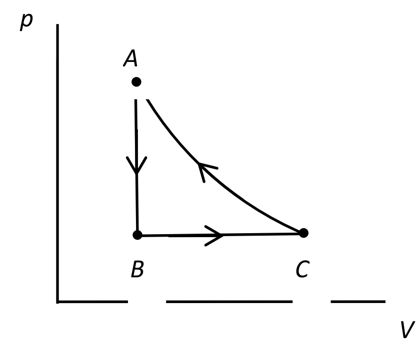
7.2.2 Graphical Description of a Perfect Gas

An alternative way of describing a perfect gas in the macroscopic point of view is by means of a curve in the three dimensional *pVT*-space which shows how the system goes from one thermodynamic state to another as it is subjected to different thermodynamic processes that cause it to change its state. Simpler than the three dimensional case is, of course, the two dimensioanal version, one of which includes the use of a *pV* diagram as shown below:

process *AB* (V = constant) $\rightarrow$ isochoric

process *BC* (p = constant) $\rightarrow$ isobaric

process *CA* (T = constant) $\rightarrow$ isothermal



7.2.3 Sample Problems with Solutions

(1) A 2500 m^3 of a confined gas is at a temperature of 35°*C* and a pressure of 75.8 *cm* of mercury (*cm Hg*). If the pressure is increased to 76.3 *cm Hg* with a final temperature of 78°*C*, what is the final volume?

Solution:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad \rightarrow \quad V_2 = \frac{p_1 V_1}{T_1}\frac{T_2}{p_2}$$

$$V_2 = \frac{(75.8\,cm\,Hg)(2500\,m^3)}{(35 + 273)K}\frac{(78 + 273)K}{(76.3\,cm\,Hg)} = 2830.36\ m^3$$

(2) A 4500 cm^3 container holds 6.8 *g* of a gas when the pressure is 75.33 *cm Hg* and the temperature is 48°*C.* What will be the pressure if 5 *g* of this gas is confined in 2800 cm^3 container at 0°*C*?

Solution:

$$\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2} \quad \rightarrow \quad p_2 = \frac{p_1}{\rho_1 T_1}\left(\rho_2 T_2\right) = \frac{p_1}{\left(\frac{M_1}{V_1}\right) T_1}\left(\frac{M_2}{V_2}\right) T_2$$

$$p_2 = \frac{\left(75.33\, cm\, Hg\right)}{\left(\frac{6.8\, g}{4500\, cm^3}\right)\left(48 + 273\right) K}\left(\frac{5\, g}{2800\, cm^3}\right)\left(0 + 273\right) K = 75.7\, cm\, Hg$$

7.2.4 The Perfect Gas from the Microscopic Point of View

The perfect gas law given by equations (3.6) and (3.7) gives an analytic description of a perfect gas from the macroscopic point of view. The following characteristics of a perfect gas come from the microscopic point of view:

1. A perfect gas consists of point particles moving with random speed in all directions following Newton's laws of motion.
2. The total number of particles comprising the gas is very large.
3. The total volume actually occupied by the gas particles themselves is a negligibly small fraction of the overall volume occupied by the gas as a whole.
4. No forces act between the partciles of the gas as they move around except the forces of collision when they hit one another.
5. Collisions are of negligible duration and perfectly elastic so no energy is lost.

Due to the large number of particles involved, each particle having its own equation of motion obtained from Newton's second law, it becomes very difficult, and impractical, to predict the future behavior of the system by solving all the equations simultaneously. Instead, we use the methods of statistics and probability to tell what will happen. Hence, the name statistical mechanics.

From the above characteristics of a perfect gas, it becomes clear why we have to carefully distinguish it from an ideal gas. Since the particles of a perfect gas are point particles, the particles of the gas are like mathematical points that have no extension in space so that a perfect gas particle can never have any internal rotational motion whatsoever. An ideal gas, on the other hand is an actual gas whose molecules have a finite small radius and can therefore posses some kind of internal rotaional motion. In fact, to properly account for some observed thermal properties of real gases, certain internal rotational motion must be taken into consideration.

Knowing the characteristics of a perfect gas, we can also understand why some macroscopic quantities used in the large scale description exist. For example, a gas occupies a certain volume because the particles comprising the gas need space to always move around. A gas also exerts pressure because the particles of the gas never cease to move and always collide with the walls of the container. Pressure arises from the force of collision of the partcles with the walls of the container. Finally, any sample of gas is always at a certain temperature because the gas partcles have some kind of average kinetic energy due to their eyernal motion. Temperature, according to the microscopic view, is simply a measure of the average kinetic energy of the gas particles.

7.3 Thermal Expansion

One effect of change in temperature is to change the linear dimensions of an object. This change in linear dimension depends on the kind of substance involved, it's initial dimensions, and the change in temperature.

7.3.1 Linear Expansion of Solids

If the temperature changes from T_1 to T_2 the change in length ΔL of a solid object is given by

$$\Delta L = \alpha L_0 \Delta T \tag{7.8}$$

where α is the coefficient of linear expansion of the object, a property of all materials, L_0 is the initial length of the object, its length at temperature T_1 , and $\Delta T = T_2 - T_1$ is the change in temperature. The coefficient of linear expansion is tabulated in reference books and is extremely small ($\approx 10^{-6}$ / C°).

Table 7.1: Thermal Expansion Coefficients

(per C° at 20°C)

Substance	$\alpha \times 10^{-6}$	$\beta \times 10^{-6}$
diamond	1.2	3.5
glass (heat resistant)	≈ 3	≈ 9
glass (soft)	≈ 9	≈ 27
concrete	≈ 10	≈ 30
iron	12	36
brass	19	57
aluminum	25	75
mercury		182
rubber	≈ 80	≈ 240
glycerin		500
		≈ 950

7.3.2 Area and Volume Expansion

Suppose that at temperature T_1 a plate made of solid material has length L_0 and width W_0. Changing the temperature to T_2 causes a change in both length and width of the object given as follows:

$$L = L_0 + \Delta L = L_0 + \alpha L_0 \Delta T = L_0 \left(1 + \alpha \Delta T\right)$$

$$W = W_0 + \Delta W = W_0 + \alpha W_0 \Delta T = W_0 \left(1 + \alpha \Delta T\right)$$

The area at temperature T_2 is therefore

$$\begin{aligned} A &= LW = L_0\left(1 + \alpha \Delta T\right) \bullet W_0 \left(1 + \alpha \Delta T\right) = L_0 W_0 \left(1 + \alpha \Delta T\right)^2 \\ &= L_0 W_0 \left[1 + 2\alpha \Delta T + \alpha^2 \left(\Delta T\right)^2\right] \end{aligned}$$

Since α is of the order 10^{-6} (see Table 3.1), α^2 is very small and decreases faster than $\left(\Delta T\right)^2$ increases so that the term involving this factor is negligible. In the first approxima-tion, we therefore have

$$A = L_0 W_0 \left[1 + 2\alpha \Delta T\right] = A_0 \left[1 + 2\alpha \Delta T\right] = A_0 + 2\alpha A_0 \Delta T$$
$$A_0 = L_0 W_0$$

Now $\Delta A = A - A_0$. The change in area is therefore given by

$$\Delta A = 2\alpha A_0 \Delta T \qquad (7.9)$$

Using the above procedure once more, but this time with thickness as the additional third dimension, we find that the change in volume of a solid is

$$\Delta V = 3\alpha V_0 \Delta T \qquad (7.10)$$

where V_0 is the initial volume at temperature T_1 .

In the case of fluids, the corresponding change in volume is given by

$$\Delta V = \beta V_0 \Delta T \qquad (7.11)$$

where β is the coefficient of volume expansion of the liquid or gas. For solids, we therefore have

$$\beta = 3\alpha$$

(3.12)

Now, the fact that change in area or volume brought about by the change in temperature ΔT is given by equation (3.9) and (3.10) means that any general problem in material expansion of solids always reduces to the problem of linear expansion in one dimension as the following considerations show.

According to equation (3.10), the final volume of a solid at temperature $T + \Delta T$ is given by $V = V_0 + \Delta V = V_0 + 3\alpha V_0 \Delta T = V_0\left(1 + 3\alpha \Delta T\right)$

In the case of a solid sphere, $V = \frac{4}{3}\pi r^3$, so that $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_0^3\left(1 + 3\alpha \Delta T\right)$. For the radius at the final temperature, we therefore have $r = r_0\left(1 + 3\alpha \Delta T\right)^{1/3}$. Using the series expansion $\left(1 + x\right)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$, which converges and therefore has a fixed finite value if $n > 0$ and $x^2 < 1$, to our particular case with $x = 3\alpha \Delta T$ and n

=1/3 > 0, we find that the stipulated condition for convergence applies since α is of the order 10^{-6}. Moreover, as has already been mentioned earlier that α^2 decreases faster than $(\Delta T)^2$ increases, we can neglect the higher order terms in x and retain only the first two terms in the expression leaving us with $(1+x)^n = 1 + nx$. That is,

$$r = r_0\{1 + \tfrac{1}{3}(3\alpha\,\Delta T)\} = r_0\{1 + \alpha\,\Delta T\}$$

which gives the final radius of the solid sphere as if it were the one dimensional case.

7.3.3 Sample Problems with Solutions

(1) If the temperature of a body is 55°C, what is it in the Fahrenheit scale?

Solution:

$$^\circ F = 1.8\,^\circ C + 32 = 1.8(55) + 32 = 131$$

(2) Calculate the temperature in which the Celsius reading is the same as in the Fahrenheit scale..

Solution:

$^\circ F = 1.8\,^\circ C + 32$ where $^\circ F = {}^\circ C$ so that

$$^\circ C = 1.8\,^\circ C + 32 \quad \rightarrow \quad ^\circ C - 1.8\,^\circ C = 32$$

$$^\circ C = {}^\circ F = -40$$

(3) A copper wire is 50 m long at a temperature of 20°C. If the temperature is raised to 60°C, what is the resulting change in length of the wire? The coefficient of linear expansion of copper is 17×10^{-6} / C°.

Solution:

$$\Delta L = \alpha L_0 \Delta T = (17 \times 10^{-6} / C^\circ)\ (50\ m)\ (60^\circ C - 20^\circ C)$$

$$\Delta L = 0.034\ m$$

(4) Three different rods *A, B*, and *C* are 25 cm long at 0°C. When the temperature increases to 100°C, the lengths of rods *A, B*, and *C* increases by 0.075 cm, 0.045 cm, and 0.065 cm, respectively. Calculate the coefficient of linear expansion of each rod.

Solution:

$$\Delta L = \alpha L_0 \Delta T \quad \rightarrow \quad \alpha = \frac{\Delta L}{L_0 \Delta T}$$

For rod *A*

$$\alpha_A = \frac{\Delta L_A}{L_{0A} \Delta T_A} = \frac{0.075\ cm}{(25\ cm)(100\ °C - 0\ °C)} = 3.0 \times 10^{-5}/C°$$

For rod *B*

$$\alpha_B = \frac{\Delta L_B}{L_{0B} \Delta T_B} = \frac{0.045\ cm}{(25\ cm)(100\ °C - 0\ °C)} = 1.8 \times 10^{-5}/C°$$

For rod C

$$\alpha_C = \frac{\Delta L_C}{L_{0C} \Delta T_C} = \frac{0.065\ cm}{(25\ cm)(100\ °C - 0\ °C)} = 2.6 \times 10^{-5}/C°$$

(5) A thin brass plate at 10°C has the same surface area as a thin steel plate at 20°C. At what common temperature, if any, will the two plates have the same area again? The coefficients of linear expansion of brass and steel are 19×10^{-6} / C° and 11×10^{-6} / C°, respectively.

Solution:

Assuming that a common temperature T_F exists where the brass and steel plates have the same area, then the following relations hold:

$$A_{0B} + 2\alpha_B A_{0B}(T_F - 10) = A_{0S} + 2\alpha_S A_{0S}(T_F - 20)$$

where A_{0B} at 10°C = A_{0S} at 20°C . Therefore

$$\alpha_B(T_F - 10) = \alpha_S(T_F - 20) \quad \rightarrow \quad \frac{\alpha_B}{\alpha_S}(T_F - 10) = T_F - 20$$

Solving for T_F, we find that

$$\left\{\left(\alpha_B/\alpha_B\right) - 1\right\}T_F = \left\{10\left(\alpha_B/\alpha_B\right) - 20\right\} \rightarrow T_F = \frac{10\left(\alpha_B/\alpha_B\right) - 20}{\left(\alpha_B/\alpha_B\right) - 1} \rightarrow T_F = -3.75\,^\circ C$$

(6) A 1.0 L glass container is filled with mercury when the temperature is 0°C. When both the glass and the mercury are heated to 100°C, 13.2 mL of mercury overflows from the glass container. Calculate the coefficient of linear expansion of the glass. The coefficient of volume expansion of mercury is 18×10^{-5} / C°.

Solution:

The mercury that overflows is equal to the excess volume V_E of mercury over the volume of the glass container given by

$$V_E = V_{Hg} - V_G = \left(V_{0Hg} + \beta_{Hg} V_{0Hg}\,\Delta T\right) - \left(V_{0G} + 3\,\alpha_G V_{0G}\,\Delta T\right)$$

where $V_{0Hg} = V_{0G} = V_0$. Therefore

$$V_E = \left(\beta_{Hg} - 3\,\alpha_G\right)V_0\,\Delta T \rightarrow \frac{V_E}{V_0\,\Delta T} = \beta_{Hg} - 3\,\alpha_G \rightarrow 3\,\alpha_G = \beta_{Hg} - \frac{V_E}{V_0\,\Delta T}$$

$$\alpha_G = \tfrac{1}{3}\beta_{Hg} - \frac{V_E}{3V_0\,\Delta T} = \frac{18 \times 10^{-5}/C^\circ}{3} - \frac{13.2 \times 10^{-3}\ L}{3\left(1.0\ L\right)\left(100\,^\circ C - 0\,^\circ C\right)}$$

$$\alpha_G = 1.6 \times 10^{-5}/C^\circ$$

Lesson 7: Temperature and Thermal Expansion
Assessment

Instruction; Show your complete and neat solution. Identify your final answer in the solution. See answer sheet format.

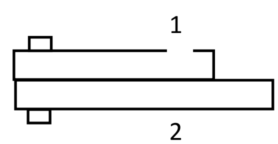
1. You work part time in a materials testing laboratory and your boss tells you to increase the temperature of a sample by 60.0°. The only thermometer you can find at your workbench reads in °F. If the initial temperature of the sample is 78.3°F, what is its temperature in °F when the desired temperature increase has been achieved?

2. Two beakers of water, A and B, initially are at the same temperature. The temperature of the water in beaker A is increased 15°F and the temperature of the water in beaker B is increased 15 K. After these temperature changes, which beaker of water has the higher temperature and by how much in °C?

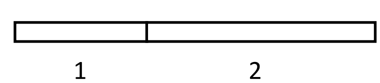
3. An automobile tire has a volume 988 in^3 and contains air at a gauge pressure of 24.2 lb/in^2 (psi) when the temperature is -2.6°C. Find the temperature of air in the tire when its volume increases to 1020 in^3 and its gauge pressure becomes 26.9 lb/in^2

4. An air bubble of 19.4 cm^3 volume is at the bottom of a lake 41.5 m deep. The bubble rises to the surface, which is at a temperature of 22.6°C, where its volume becomes 104 cm^3, before finally bursting out. Assuming the temperature of the bubble to be the same as that of the surrounding water, find the temperature at the bottom of the lake.

5. If the lengths of two bars of different solids are inversely proportional to their respective coefficients of linear expansion at the same initial temperature, the difference in length between them will be constant at all temperatures. That means one way to construct a device with two points whose separation remains the same in spite of temperature changes is to bolt together one end of two bars having different coefficients of linear expansion as shown below. What should be the lengths of a steel ($\alpha = 11 \times 10^{-6}$ / C°) and a brass ($\alpha = 19 \times 10^{-6}$ / C°) bar at 0°C so that at all temperatures their difference in length is 0.30 m?



6. A composite rod is made by joining two rods of different materials but with the same cross-sectional area as shown below. Design such a composite rod using steel ($\alpha = 11 \times 10^{-6}$ / C°) and brass ($\alpha = 19 \times 10^{-6}$ / C°) whose total length is 52.4 cm and whose effective coefficients of linear expansion is 13×10^{-6} / C°. Compute the lengths of the steel and brass rods that must be used. Neglect changes in cross-sectional area of the rods.



7. Aluminum rivets used in airplane construction are made slightly larger than the rivet holes and cooled by "dry ice" (solid carbon dioxide) before being driven. If the diameter of a hole is 4.500 mm, what should be the diameter of a rivet at 23.0°C, if its diameter is to equal that of the hole when the rivet is cooled to -78.0°C, the temperature of dry ice?

8. An aluminum cup of 110 cm^3 capacity is filled with glycerin at 22°C. How much glycerin, if any, will spill out of the cup if the temperature of the cup and glycerin is raised to 32°C?

9. Consider a mercury-in-glass thermometer. The capillary tube has a diameter of 0.004 cm, and the bulb has a diameter of 0.25 cm. Suppose that the mercury just fills the bulb at 0.00°C. Neglecting the expansion of the capillary tube so that its cross sectional area is constant, find the change in height of the mercury column if the temperature drops from 23°C to 9°C.

10. Show that the change in density $\Delta\rho$ with change in temperature ΔT is given by $\Delta\rho = -\beta\,\rho\,\Delta T$ where is the coefficient of volume expansion. Explain the minus sign. Hint: Use the series expansion method used in section 3.4.2.

Introduction

A kettle of cold water placed in a hot stove, the temperature of the water increases or brings the water to boil. We say that heat flows from the hot stove to the cold water. When two objects at different temperatures are put in contact, heat spontaneously flows from the hotter body to the colder body. The flow of heat is in the direction tending to equalize the temperature, and if contact are kept for a longer time, the temperature becomes equal. Heat and temperature are often confused. This lesson will clearly distinguish their differences and focus on heat and heat measurements.

Learning Outcomes

After successful completion of this lesson, you should be able to:

☞ Differentiate between heat and internal energy.

☞ Relate the temperature change of an object to the quantity of heat given to or taken from the object.

☞ Determine the quantity of heat required to convert a pure substance from one phase to another.

8.1 The First Law of Thermodynamics

The location of a point in the *pV* diagram representing the thermodynamic state of a perfect depends on the energy of the system. This energy, in turn, depends on the values of the macroscopic variables *p*, *V*, and *T* that characterize the system. A change in value of any one of the macroscopic variables changes the energy of the system and its cooresponding position in the *pV* diagram.

There are two general ways of changing the energy of a system of perfect gas. One has something to do with differences in temperature between the system and the environment with which the gas interacts. The other has nothing to do at all with differences in temperature but has something to do with the force acting on the system instead.

1. Thermal Interaction. Whenever a difference in tempereture exists between the system and the environment, heat energy ΔQ always goes from the body at a higher temperature to the body at a lower temperature.

   Heat absorbed by the system $\rightarrow$ $\Delta Q > 0$

   Heat given off by the system $\rightarrow$ $\Delta Q < 0$

2. Mechanical Interaction. If there is a force *F* acting on the system, the energy of the system also changes by the amount equal to the work ΔW done by the force. This change either increases or decreases the energy of the system. For clarity, let us therefore agree to use the following convention:

$\Delta W > 0 \quad \rightarrow \quad$ work done [by] the system

$\Delta W < 0 \quad \rightarrow \quad$ work done [on] the system

Now, suppose that we have a system of perfect gas that goes from the initial thermodynamic state *i* to the final thermodynamic state *f* as shown in the *pV* diagram below. Let *i1f* and *i2f* represent two possible paths the system can take

in going from states *i* to *f*. If we look separately at ΔQ and ΔW along each path, then we find that, in gereral,

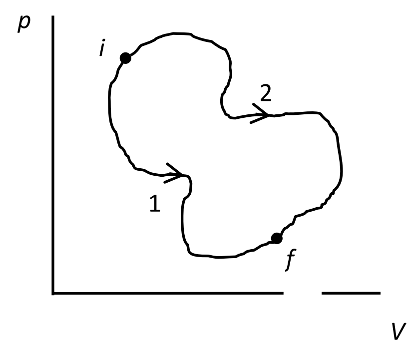


Fig. 8.1

$$\Delta Q_{i1f} \neq \Delta Q_{i2f} \quad \text{and} \quad \Delta W_{i1f} \neq \Delta W_{i2f}$$

That is, ΔQ and ΔW depend not only on the initial and final states but also on the path taken in going from *i* to *f*.

But if look instead at the combined quantity $\Delta Q - \Delta W$ along each path, then we find that

$$(\Delta Q - \Delta W)_{i1f} = (\Delta Q - \Delta W)_{i2f}$$

That is, $\Delta Q - \Delta W$ depends only on the initial and final states but not on the path taken in going from *i* to *f*. For some purposes it is therefore more convenient to look at the combined quantity $Q - W$, rather than at the two of them separately, because the change in that single quantity is path independent in the *pV* diagram.

Thermodynamic quantities that depend only on the initial and final states but not on the path taken in the *pV* diagram are called *state variables*. In our present discussion, the relevant state variable is the *internal energy* $E = Q - W$. The corresponding change in internal energy is

$$\Delta E = \Delta Q - \Delta W \tag{8.1}$$

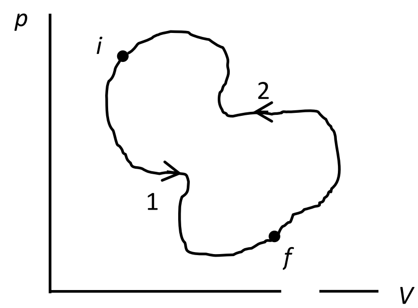
where

$\Delta Q > 0$ for heat absorbed and $\Delta Q < 0$ for heat given off

$\Delta W > 0$ for work done [by] the system and $\Delta W < 0$ for work done [on] the system.

Going back to the perfect gas whose *pV* diagram is given by Fig. 8.1, let us suppose that

$$(\Delta E)_{i1f} = E_0 > 0 \qquad \rightarrow \qquad (\Delta E)_{i1f} = +|E_0|$$



Then

$$(\Delta E)_{i1f} = (\Delta E)_{i2f} = +|E_0|$$

so that

$$(\Delta E)_{f2i} = -|E_0|$$

It follows that along the path *i*1*f*2*i* (the curve *i*1*f* then along 2 back to *i*), we find that the total cfhange in internal energy is

Fig. 8.2

$$(\Delta E)_{i1f2i} = (\Delta E)_{i1f} + (\Delta E)_{f2i} = +|E_0| + (-|E_0|) = 0$$

which is simply the law of conservation of energy for a perfect gas.

The First Law of thermodynamics is just the Law of Conservation of Energy already discussed in Physics 1 and whose one formulation can be stated as follows:

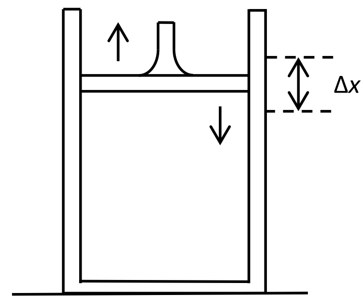
Energy cannot be created nor destroyed but it can only be transformed from one form to another.

We have shown in this section that heat is just a form of energy and that energy cannot be created nor destroyed in the case of a perfect gas using the *pV* diagram.

Mechanical equivalent of heat: one calorie of heat energy = 4.186 J

8.2 Mechanical and Thermal Interactions

To understand how the First Law of thermodynamics applies to a system of perfect gas, we must know how to compute *ΔQ* and *ΔW.* For this purpose, let us consider a gas confined in a vertical cylindrical container closed at one end by a piston light enough so that it can move up or down a distance Δ*x* in spite of its weight. In addition, let us assume that the gas is in thermal contact with the laboratory bench at the bottom of the container through which it can exchange heat energy with the environment.



For the mechanical interaction of the system with the environment through the movable piston, we find that

$$\Delta W = F\,\Delta x = (pA)\Delta x = p\,(A\Delta x) = p\,\Delta V \qquad (8.2)$$

In an isochoric process where the process takes place at constant volume, $\Delta V = 0$ and there is no work done ($\Delta W = 0$) .

To get *ΔQ* arising from the thermal interaction of the system with the environment through the bottom of the cylinder, we have to take into consideration the thermal properties of matter, like specific heat and latent heat. However, in the absence of any thermal interaction with the environment so that there is no heat exchange taking place, then $\Delta Q = 0$. Such is the case in what is called an *adiabatic process*. But in general $\Delta Q \neq 0$.

8.3 Specific Heat, Thermal Capacity, and Heat Capacity

Specific heat c (lower case letter) is the amount of heat needed to raise the temperature of a unit mass of a substance by one degree.

Units:	MKS	CGS	FPS
c	$\dfrac{kcal}{kg \bullet C^\circ}$	$\dfrac{cal}{g \bullet C^\circ}$	$\dfrac{Btu}{lb \bullet F^\circ}$ *

Examples:

* *Btu* = British thermal unit (1 *Btu* = 1055 J = 252 cal))

$$c_{ice} = 0.5 \frac{cal}{g \bullet C^\circ}, \qquad c_{water} = 1.0 \frac{cal}{g \bullet C^\circ} = 1.0 \frac{kcal}{kg \bullet C^\circ}, \qquad c_{steam} = 0.48 \frac{cal}{g \bullet C^\circ}$$

Table 4.1: Specific Heat Capacities ($cal/g \cdot C^\circ$)

aluminum	0.22	lead	0.031
copper	0.093	marble	0.21
ethanol	0.55	mercury	0.033
glass	0.2	paraffin	0.51
gold	0.031	steam	0.46
ice (-5°C)	0.50	Water (15°C)	1.0
iron	0.11	wood	0.41

Thermal capacity C (upper case letter) is the amount of heat needed to raise the temperature of the total mass of a substance. That is

$$C = mc$$

Units:	MKS	CGS	FPS
C	$\frac{kcal}{C^\circ}$	$\frac{cal}{C^\circ}$	$\frac{BTU}{F^\circ}$

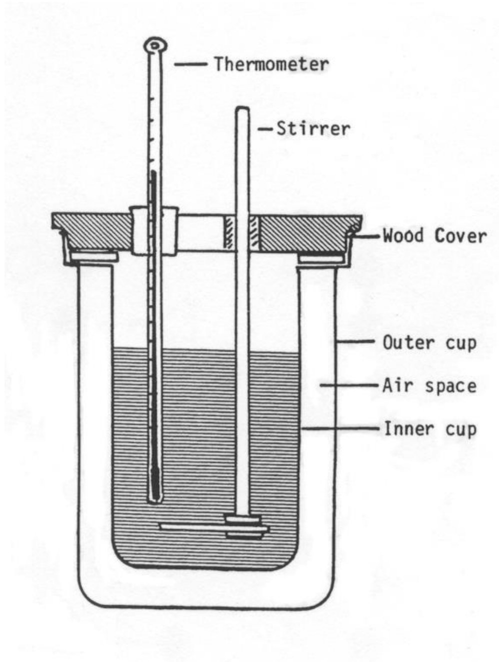
Heat quantity Q is the total amount of heat needed to change the temperature of the total mass of a substance when the change in temperature is not equal to 1.0°. That is

$$Q = mc\,\Delta T \qquad (8.3)$$

Units: joules, calories, Btu

8.4Calorimetry and the Method of Mixture

The measurement of quantities of heat is called *calorimetry.* This is done with the help of an instrument called a calorimeter. A *calorimeter* is a thermally insulated container shown that is shown below. The inner vessel is supported in the outer vessel by an insulated cover, on which a thermo-meter and a stirrer can be inserted. The inner vessel is the one that contains the substances whose heat quantities are to be determined, while the outer vessel minimizes the transfer of heat from the inside to the surrounding or vice versa.



The instrument works on the principle of the method of mixture, wherein a substance at a lower temperature is mixed with another substance at a higher temperature so that the heat gained by the cold substance is equal to the heat lost by the hot substance:

Heat gained Q_g = *Heat lost* Q_l

Heat gained is characterized by an increase in the temperature of the substance while heat lost is characterized by a decrease in temperature.

Let:

m_c = mass of the calorimeter, m_w = mass of water inside the calorimeter,

m_m = mass of the metal , c_c = specific heat of the calorimeter,

c_m = specific heat of the metal, T_w = initial temperature of water and the calorimeter

T_{mix} = temperature of the mixture, T_m = temperature of the hot metal

Then, it follows that

$$Q_g = Q_l \quad \rightarrow \quad (m_c c_c + m_w)(T_{mix} - T_w) = m_m c_m (T_m - T_{mix})$$

8.5 Change of Phase and Latent Heat

8.5.1 Change of Phase

1. Liquefaction
   a.) *melting* – the process of changing the phase of a substance from the solid state to the liquid state.
   b.) *condensation* – the process of changing the phase of a substance from the gas state to the liquid state.

2. Solidification
   a.) *freezing* – the reverse process of melting; it is the change in phase of a substance from the liquid state to the solid state.

3. Vaporization
   a.) *evaporation* – the change in phase of a substance from the liquid state to the gas state.
   b.) *sublimation* – the change in phase of a substance from the solid state to the gas state without passing through the liquid state.

8.5.2 Latent Heat

Latent heat is the amount of heat required to effect a complete change of a unit mass of a substance without a change in temperature. The two most common latent heats are latent heat of fusion and latent heat of vaporization.

1. *Latent heat of fusion* L_F is the amount of heat needed to change a unit mass of a solid to liquid or vice versa without any change in temperature ($\Delta T = 0$).

$$L_F = \frac{Q}{m} \rightarrow Q = m L_F \tag{8.4}$$

For example,

$$L_F \; of \; ice = 80 cal/g = 80 k\,cal/kg = 144 BTU/lb$$

2. *Latent heat of vaporization* L_V is the amount of heat needed to change a unit mass of a liquid to gas or vice versa without any change in temperature (ΔT = 0).

$$L_V = \frac{Q}{m} \rightarrow Q = m L_V$$

For example,

$$L_V \text{ of water} = 540 cal/g = 540 k\,cal/kg = 972 BTU/lb$$

Table 8.2: Latent Heat of Some Substances

Substance	boiling temp. (C°)	vaporization (cal/g)	melting temp. (C°)	fusion (cal/g)
nitrogen	−196	48	−210	6.1
oxygen	−183	51	−219	3.3
ethanol	+78	204	−114	25
water	100	539	0	80
mercury	357	71	−39	2.7
lead	1750	205	327	5.9

Summary:

1. When $\Delta T = 0$ is not equal to 0 and there is no change in phase,

$$Q = m c \Delta T$$

2. When $\Delta T = 0$ is equal to 0 and there is a change in phase,

$$Q = m L_F \quad \text{or} \quad Q = m L_V$$

3. When $\Delta T = 0$ and there is no change in phase,

$$Q = 0$$

8.6 Sample Problems with Solutions

(1) Calculate the total amount of heat needed to change the temperature of a 500 *g* piece of copper from 20°*C* to 120°*C*. (the specific heat of copper is 0.093 *cal /g-C°*).

Solution:

$$Q = m\,c\,\Delta T$$

$$= 500 \text{ g } (0.093\ cal/g\text{-}C^\circ)\ (120 - 20)\ C^\circ$$

$$= 4{,}650\ cal$$

(2) An aluminum (*Al*) container has a mass of 50 *g* and holds 200 *g* of water at 20°*C*. Find the final temperature of the resulting mixture when a mass of 500 *g* of lead (*Pb*) at 95°*C* is placed in the water (c_{Al} = 0.22 *cal/g-C°* ; c_{Pb} = 0.031 *cal/g-C°*).

Solution:

Heat lost by lead = Heat gained by the aluminum and water

$$Q_{Pb} = Q_{Al} + Q_{water}$$

$$(mc\Delta T)_{Pb} = (mc\Delta T)_{Al} + (mc\Delta T)_{water}$$

Substituting the given values and solving for the final temperature T_{mix} of the mixture, we get

$$500\ (0.031)\ (95 - T_{mix}) = [\,50\ (0.22) + 200\ (1.0)]\ [\,T_{mix} - 20\,]$$

$$T_{mix} = 25.13^\circ C$$

(3) How much heat is needed to change 10 *g* of ice at -20°*C* to steam at 140°*C* , assuming no heat losses due to the surrounding ?

Solution:

$$Q_T = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$$

$Q_1 = mc\Delta T$ $\leftarrow$ heat needed to raise the temperature of ice from –20°*C* to 0°*C*

$$= 10\ g\ (0.5\ cal/g\text{-}C^\circ)\ (0 - -20)\ ^\circ C$$

= 100 cal.

$Q_2 = mL_f$ ← heat needed to melt the ice at 0°C to ice water still at 0°C

= 10 g (80 cal/g)

= 800 cal.

$Q_3 = mc\Delta T$ ← heat needed to raise the temperature of water from 0°C to 100°C

= 10 g (1.0 cal/g-$C°$) (100 − 0) °C

= 1000 cal.

$Q_4 = mL_v$ ← heat needed to change water at 100°C to steam at 100°C

= 10 g (540 cal/g) = 5,400 cal.

$Q_5 = mc\Delta T$ ← heat needed to raise the temperature of steam at 100°C to 140°C

= 10 g (0.48 cal/g-$C°$) (140 − 100) °C

= 192 cal

Finally, we get

$$Q_T = 100 + 800 + 1{,}000 + 5{,}400 + 192$$
$$= 7{,}492 \text{ calories}$$

(4) Four ice cubes each of mass 15 g and at 0°C are placed in a 50 g glass which contains 200 g of water. If the glass and the water are initially at 25°C, find the temperature of the mixture and the mass of the ice remaining if there is any.

Solution:

mass of ice = 4 (15 g) = 60 g

Assuming that the final temperature of the mixture is 0°C, we find the following:

Heat needed to melt the ice = mL_f = 60 g (80 cal/g) = 4800 cal

Heat available to melt the ice $= (mc\Delta T)_{glass} + (mc\Delta T)_{water}$

$= 50\ g\ (0.16\ cal/g\ C°)\ (25 - 0)°C + 200\ g\ (1.0\ cal/g\ C°)\ (25 - 0)°C$

$= 5200$ cal

Therefore

Heat available > Heat needed → All the ice will be melted.

Solving for the final temperature T_{mix} of the mixture

Heat lost by glass and water = Heat gained by ice

$$(mc\Delta T)_{glass} + (mc\Delta T)_{water} = (mc\Delta T)_{ice} + mL_f$$

$$50\ (0.16)\ (25 - T_{mix}) + 200\ (1.0)\ (25 - T_{mix})$$

$$= 60\ (0.5)\ (T_{mix} - 0) + 60\ (80)$$

$$T_{mix} = 1.49°C$$

(5) A 0.300 *kg* lead ball was taken from a bath of hot oil and immediately placed in a 0.150 *kg* copper vessel containing 0.200 *kg* water at a temperature of 20°*C*. After stirring the mixture attained a final equilibrium temperature of 25°*C*. What was the temperature of the hot oil from which the lead ball was taken? The specific heat of lead, copper, and water are 0.031 *cal / g C°*, 0.093 *cal / g C°*, and 1.0 *cal / g C°*, respectively.

Solution:

Heat lost by the lead ball = Heat gained by the water and the copper container

$$Q_{Pb} = Q_{water} + Q_{Cu}$$

$$(mc\Delta T)_{Pb} = (mc\Delta T)_{water} + (mc\Delta T)_{Cu}$$

Substituting: the given values and solving for the final temperature T_{mix} of the mixture, we get

$$300\ (0.031)\ (T_{init} - 25) = 200\ (1.0)\ (25 - 20) + 150\ (0.093)\ (25 - 20)$$

$$T_{init} = 140°C$$

8.7 Mechanisms of Heat Transfer

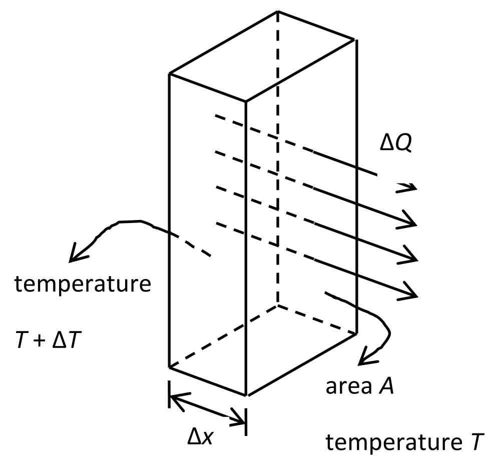
There are three mechanisms by which heat energy is exchanged between the system and the environment, namely, conduction, convection, and radiation. We discuss each in turn.

4.7.1 Conduction

Heat or *thermal conduction* is the transmission of heat energy through a medium without the transfer of mass. In all states of matter, it is caused by collision between the fast moving molecules in the region of higher temperature with the slow moving molecules in the neighboring region of lower temperature.

Consider a slab of homogeneous material

of thickness Δx as shown on the right. Experimentally, it has been found that the rate $H = \Delta Q / \Delta t$ in which a small quantity of heat ΔQ is transmitted through the slab in time Δt is given by the equation



$$H = \frac{\Delta Q}{\Delta t} = -k\,A\frac{\Delta T}{\Delta x} \qquad (8.5)$$

where *k* is the thermal property of the material called the thermal conductivity. A negative sign is included in eq.(4.5) to make *H* positive since heat flows from higher to lower temperature which makes $\Delta T/\Delta x$ negative. A substance with a large value of k is a good heat conductor; one with a small value of k is a poor heat conductor. Table 4.3 below shows some representative values of *k.*

If the temperature varies in a uniform way along the thickness of the slab so that the temperature gradient $\Delta T/\Delta x$ = constant, then we have what we call a *steady state* where *H* is the same through any cross section along its thickness.

Table 8.3: Thermal Conductivities of Some Materials

Substance	Thermal Conductivity W/(m C°)
Aluminum	238
Copper	397
Gold	314
Iron	79.5
Lead	34.7
Silver	427
Stainless steel	14
Metals (at 25°C)	

Substance	Thermal Conductivity W/(m C°)
Air	0.0234
Helium	0.138
Hydrogen	0.172
Nitrogen	0.0234
Oxygen	0.0238
Gases (at 20°C)	

Substance	Thermal Conductivity W/(m C°)
Concrete	0.8
Glass	0.8
Ice	2
Rubber	0.2
Water	0.6
Wood	0.08

In choosing building materials, one often finds them rated in terms of the *thermal resistance* or *R*-value defined by

$$R = \frac{L}{k} \quad \rightarrow \quad k = \frac{L}{R} \tag{8.6}$$

where *L* is the thickness of the material. Thus, the lower the conductivity *k*, the higher the *R*-value: good heat insulators have high *R*-values. Numerically, the *R*-value is evaluated according to eq. (4.6) expressed in the British units of $ft^2 \cdot F° \cdot h/Btu$. The *R*-value is determined for a certain thickness of material. Table 4.4 on the right shows the *R*-values of some materials.

Table 8.4: *R*-values for Some Common Building Matertials

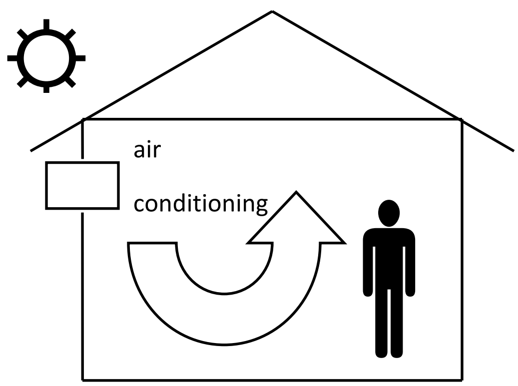
Material	*R*-value ($ft^2 \cdot F° \cdot h/Btu$)
Hardwood siding (1 in. thick)	0.91
Brick (4 in. thick)	4.00
Concrete block (filled cores)	1.93
Fiber glass batting (3.5 in. thick)	10.90
Cellulose fiber (1 in. thick)	3.70
Flat glass (0.125 in. thick)	0.89
Insulating glass (0.25-in. space)	1.54
Vertical air space (3.5 in. thick)	1.01
Air film	0.17
Dry wall (0.5 in. thick)	0.45

8.7.2 Convection

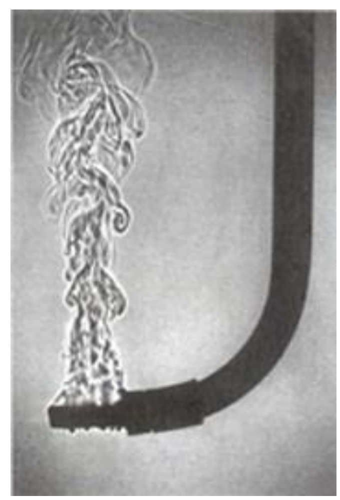
Heat or *thermal convection* is the process of transfer of heat in a fluid by the movement of the fluid itself. It occurs when a fluid comes in contact with a body whose temperature is higher than that of its surroundings. As a result of the thermal interaction of the fluid with the body at a higher temperature, the temperature of the fluid increases, and (in most cases) it also undergoes volume expansion. The density of the heated fluid therefore decreases, and being less dense and lighter than the surrounding cooler fluid, it rises because of the buoyant force. The surrounding cooler fluid falls because of its greater weight to take the place of the rising warmer fluid initially at the bottom, so that a convective circulation is set up.

In the case of a solid surface losing heat by convection in a fluid in contact with it, there is always a thin layer of fluid adjacent to the solid surface through which heat is being transferred by thermal conduction into the bulk of the convecting fluid.

For some fluids, thermal convection, rather than heat conduction, is a better basis for the conducting value of the substance, like air (see Table 4.3), which can transfer more heat by convection than by conduction, thus, allowing us to understand the significant role of the atmosphere as a powerful heat engine that creates the global climate patterns.



Cool, dense air sinks to the bottom; warm, light air rises, creating a convection current.



A heating element at the tip of a submerged tube warms the surround-ing water, producing a pattern of convection.

8.7.3 Radiation

Thermal radiation is the transfer of heat by means of electromagnetic waves, such as the radiant energy coming from the sun. In fact, all objects, even those at ordinary temperatures, emit energy in the form of electromagnetic radiation because of their temperature. The higher the temperature and the greater the surface area of the radiating body, the more the energy radiated.

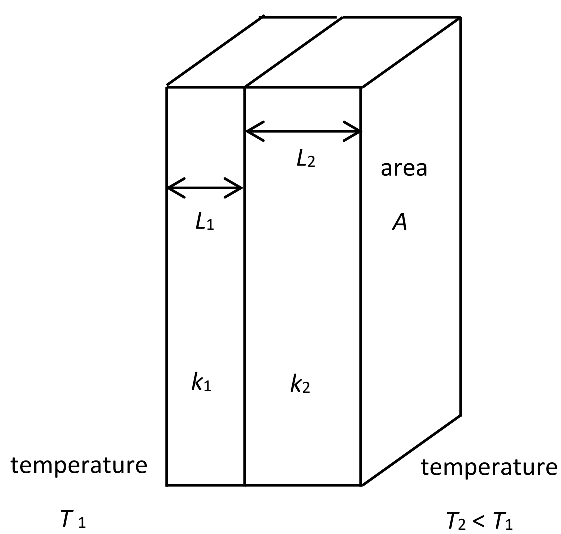
Infrared photograph of a man drinking a cold bottle of drink. Dark areas are regions of strong emission of heat energy; light areas, regions of weak ones. Note the absence of emission from the cold bottle of drink.

(1) Calculate the rate at which heat would be lost on a cold January day through a 6.2 m × 3.8 m concrete wall 25 cm thick in Baguio City. The inside temperature is 26°C and the outside temperature is 9°C; assume that the thermal conductivity of concrete is 0.8 W/(m · C°).

Solution:

$$H = \frac{\Delta Q}{\Delta t} = -k\,A\frac{\Delta T}{\Delta x} = -\left(0.8\,\frac{\mathrm{W}}{\mathrm{m}\cdot\mathrm{C}^\circ}\right)(6.2\ \mathrm{m})(3.8\ \mathrm{m})\frac{\overbrace{9^\circ\mathrm{C} - 26^\circ\mathrm{C}}^{-17\,\mathrm{C}^\circ}}{25\times10^{-2}\,\mathrm{m}} = 1280\ \mathrm{W}$$

(2) A wall of area A consists of two slabs: one of thickness L_1 and thermal conductivity k_1, the other of thickness L_2 and thermal conductivity k_2, as shown. The temperatures are T_1 and T_2, respectively, on the left-facing side and the right-facing side of the wall. Calculate the effective thermal conductivity of the wall in terms of the parameters given. (*Hint*: Calculate the temperature at the interface of the two slabs at steady state condition.)



Solution:

Consider the heat flow along each slab. Let T_x be the temperature between the two slabs. Then, for slab 1, we have using $A_1 = A$

$$H_1 = \frac{\Delta Q_1}{\Delta t} = -k_1 A_1 \frac{(\Delta T)_1}{(\Delta x)_1}$$

$$= -k_1 A_1 \frac{T_x - T_1}{L_1}$$

$$= -k_1 A_1 \frac{T_x - T_1}{L_1} = -\frac{L_1}{R_1} A_1 \frac{T_x - T_1}{L_1} = -\frac{A}{R_1}(T_x - T_1)$$

For slab 2

$$H_2 = \frac{\Delta Q_2}{\Delta t} = -k_2 A_2 \frac{(\Delta T)_2}{(\Delta x)_2} = -k_2 A_2 \frac{T_2 - T_x}{L_2} = -\frac{A}{R_2}(T_2 - T_x)$$

Now, at steady state $H_1 = H_2 = H$ (the same for all sections of the slab), we get the following:

$$-\frac{A}{R_1}(T_x - T_1) = -\frac{A}{R_2}(T_2 - T_x) \quad \rightarrow \quad T_x - T_1 = \frac{R_1}{R_2}(T_2 - T_x)$$

$$T_x\left(1 + \frac{R_1}{R_2}\right) = T_x \frac{R_1 + R_2}{R_2} = \frac{T_1 R_2 + T_2 R_1}{R_2} \quad \rightarrow \quad T_x = \frac{T_1 R_2 + T_2 R_1}{R_1 + R_2}$$

It follows that

$$T_x - T_1 = \frac{T_1 R_2 + T_2 R_1}{R_1 + R_2} - T_1 = \frac{R_1 (T_2 - T_1)}{R_1 + R_2}$$

$$T_2 - T_x = T_2 - \frac{T_1 R_2 + T_2 R_1}{R_1 + R_2} = \frac{R_2 (T_2 - T_1)}{R_1 + R_2}$$

Finally

$$H = H_1 = -\frac{A}{R_1} \frac{R_1 (T_2 - T_1)}{R_1 + R_2} = -\frac{A (T_2 - T_1)}{R_1 + R_2} = -\frac{A}{R_{eff}} \Delta T$$

where

$$R_{eff} = R_1 + R_2$$

We get the same result if we use the other slab.

8.9The Second Law of Thermodynamics

All phenomena that occur in nature conform to the known physical laws. For example, the whole process of heat flow from a body at a higher temperature to another body at a lower temperature that is in thermal contact with the first occurs in accordance with the First Law of Thermodynamics; the amount of heat absorbed by the body at a lower temperasture is exactly equal to the amount of heat given off by the body at a higher temperasture. But the reverse pro\cess, wherein heat energy flows from the body at a lower temperature to the other body at a higher temperature never occurs by itself, although it is permitted by the First Law of Thermodynamics. That means another law of nature, aside from the First Law, must be at work that prevents the reverse process just mentioned from taking place spontaneously.

The relevant physical law at work in our present case, in addition to the First Law, is the Second Law of Thermodynamics, wherein the concept of what we call entropy is the main idea involved. In the macroscopic point of view, the *entropy S* is a property of a thermodynamic system that changes, when the system undergoes a reversible process, by an amount equal to the heat ΔQ exchanged by the system with the environment divided by the temperature T at which this energy transfer takes place:

$$\Delta S = \frac{\Delta Q}{T} \qquad (8.7)$$

Now, what we call a reversible process is a process undergone by a perfect gas which has a well defined curve in the pV diagram. In a reversible process, the system can be made to retrace its original path because all the intermediate states undergone by the system are well defined. In an irreversible process, by contrast, the system cannot be made to go back through its previous path because the intermediate states that brought the system from the initial to the final states are not well defined, so that what its actual curve in the pV diagram should be is not clear. Like the internal energy E in the First Law, the entropy S is a state variable whose change depends only on the initial and final states but not on the actual path taken.

In terms of entropy, the Second Law of Thermodynamics can be stated as follows:

In a closed system, the total entropy either

i) *increases* $\left(S_i < S_f\right)$ *, as in the case of irreversible processes, or*

ii) *remains unchanged* $\left(S_i = S_f\right)$ *, as in the case of a reversible process,*

but it never decreases $\left(\text{never } S_i > S_f\right)$*.*

With the above formulation of the Second Law, we can now understand the direction of heat flow – why heat energy always goes from higher to lower temperature only and never in the opposite direction.

Suppose heat flows spontaneously from the first body at a lower temperature T_1 to the second body at a higher temperature T_2. That is, the first body losses heat energy (**-** ΔQ) while the second body gains exactly the same amount of energy (**+** ΔQ) . Because our system consists of the only two bodies which are in thermal interaction the total entropy is

$$\Delta S_{\mathrm{T}} = \Delta S_1 + \Delta S_2 = \frac{-\Delta Q}{T_1} + \frac{\Delta Q}{T_2} < 0$$

(8.8)

Since $T_1 < T_2$, the first term is a “larger” negative number than the “smaller” positive number added to it so their sum is negative.

For heat flowing in the usual manner, from T_2 to T_1 (< T_2) , we find that the total change in entropy is

$$\Delta S_{\mathrm{T}} = \Delta S_1 + \Delta S_2 = \frac{\Delta Q}{T_1} + \frac{(-\Delta Q)}{T_2} > 0 \tag{4.9}$$

The result is positive since $T_1 < T_2$ so that the second temr is a “smaller” positive number subtracted from the first. Eq. (4.9) is in accordance with the Second Law of Thermodynamics while eq.(4.8) clearly violates it.

An important application of the second law concerns the performance of actual engines, which all undergo irreversible processes, and in which only a small fraction of the input energy is converted into useful work, while the unconverted input energy is inevitably wasted away due to the inevitable increase in entropy of the system.

Lesson 8: Heat Phenomena
Assessment

Instruction; Show your complete and neat solution. Identify your final answer in the solution. See answer sheet format.

1. A copper vessel whose mass is 150 *g* contains 60 *g* of water at 15.5°*C*. How many calories of heat are required to bring the water to a final temperature of 85°*C*? The specific heat of water is 1.0 *cal*/*g* C°.

2. An aluminum container whose mass is 205 *g* contains 300 *g* of water at 20°*C*. In this container is then placed 250 *g* of iron at 150°*C* and 20 *g* of ice at −10°*C*. Find the final temperature of the mixture. The specific heat of aluminum and iron is 0.21 *cal*/*g C*° and 0.11 *cal*/*g C*°, respectively. The specific heat of ice is 0.50 *cal*/*g C*° while its heat of fusion is 80 *ca*l/g

3. A copper calorimeter whose mass is 250 *g* contains 700 *g* of water at 25°*C*. When 25 *g* of a certain metal at 98°*C* is immersed in the water, the resulting temperature is 30°*C*. What is the specific heat of the metal? The specific heat of copper is 0.093 *cal*/*g C*°.

4. If 8000 *cal* of heat is needed to change the temperature of a certain amount of ice at

   -25°*C* to steam at 135°*C*, determine the mass of the ice. The specific heat of ice is 0.50 *cal*/*g C*° while its heat of fusion is 80 *cal/g*. The specific heat of steam is 0.46 *cal*/*g C*° while its heat of vaporization is 540 *cal*/*g.*

5. An object of mass 6.50 kg falls through a height of 50.0 m and, by means of a mechanical linkage, rotates a paddle wheel that stirs 520 g of water. The water is initially at 15°C. What is the maximum possible temperature rise?

6. A small electric immersion heater is used to boil 136 g of water for a cup of instant coffee. The heater is labeled 220 watts. Calculate the time required to bring this water from 23.5°C to the boiling point, ignoring any heat losses.

7. A box with a total surface area of 1.2 m^2 and a wall thickness of 4 cm is made of an insulating material. A 10 watt electric heater inside the box maintains the inside temperature at a steady 15°C above the outside temperature. Find the thermal conductivity of the insulating material.

8. A room has a pine ceiling [k = 0.12 W/(m·C°)] that measures 3.0 m × 4.0 m × 2.0 cm. On a cold day, the temperature inside the room is 20°C, and the temperature in the attic above is 8°C. If 6.0 cm of glass wool insulation [k = 0.042 W/(m·C°)] were put in above the ceiling, how much energy would be saved in one hour?

9. A 2.0-kg block moving with an initial speed of 5 m/s slides on a rough table and is stopped by the force of friction. Assuming that the table and air remain at 24°C, calculate the entropy change of the universe.

Introduction

Electricity is the name given to a wide range of phenomena that, in one form or another, underlies just about anything around us – from lightning in the sky, sparks beneath our feet when we scuff across a rug, to what holds atoms together to form molecules. There are two branches of electricity: (1) *Electrostatics* or *static electricity* is the study of the interactions that occur when electric charges are at rest. and (2) *Electrodynamics* or *current electricity* is the study of the relations between electric, magnetic and mechanical phenomena when charges are in motion.

Electrostatics is that branch of electricity concerned with electrical charges and systems of charges that are at rest. Rubbing two bodies usually non-conductors produces such kind of electricity.

In about 600 B.C., Thales of Miletus, discovered that when amber was rubbed with fur, it attracted light objects like feather or bits of straw. This phenomenon was called "electrics", the Greek word for amber.

Sir William Gilbert found that many substances behaved like amber after being rubbed.

In the 17th century, Otto Von Guericke (1602-1686) of Magdeburg built a large sulphur sphere which when rotated about an axis and rubbed with his hand produced with his produced electric sparks.

One of the modern applications of the phenomenon of electrostatics is the Van de Graaf electrostatic generator, invented by Van de Graaf in 1931, one of the earliest devices used in accelerating charged particles

Learning Outcomes

After successful completion of this lesson, you should be able to:

- ☞ Learn the physical processes that make charged bodies.
- ☞ Determine the force between points using Coulomb's Law.
- ☞ Calculate the electric field caused by a distribution of point charges by vector summation of the electric field of the individual point charges.
- ☞ Determine the electric potential and electric potential energy caused by a distribution of point charges.

Discussion

9.1 Electric Charges

The terms positive and negative charges refer to electric charges – the fundamental quantity that underlies all electrical phenomena. Protons are the smallest positively charged particles while electrons are the smallest negatively charged.

Some important facts about atoms:

1. Every atom has a positively charged central part, called the nucleus, surrounded by the negatively charged electrons.
2. All electrons are identical, that is, they all have the same mass and the same amount of negative charges.
3. The nucleus is composed of protons and neutrons, except hydrogen with one proton but no neutron. All protons are identical. Similarly, all neutrons are identical. A proton has nearly 2000 times the mass of an electron but its positive charge is equal in magnitude to the negative charge of the electron. A neutron has slightly greater mass than a proton and has no charge.
4. Atoms normally have as many electrons as protons, so that ordinary atoms have zero net charge.

A body with more negative charges than positive charges is said to have a net negative charge. Similarly, a positively charged object has more number of positive charges than negative charges.

Ions are electrically charged atoms. A positive ion has a net positive charge with more number of protons than electrons. A negative ion has a net negative charge with the number of electrons in excess of the number of protons.

First Law of Electrostatics

Like charges repel, unlike charges attract

The attractive force between protons and electrons hold atoms together. Between neighboring atoms, the negative electrons of one atom may at times be closer to the positive protons of another atom, so the attractive force between these charges is greater than the repulsive force, and the atoms combine to form a molecule. In fact, all the chemical bonding forces that hold atoms together to form molecules are electrical forces acting in small regions where the balance of attractive and repelling forces is not perfect.

Electrification is the process of separating electrical charges through rubbing two materials together and then separating them to produce the electrical effect.

When we charge something, no electrons are created nor destroyed. Electrons are simply transferred from one material to another. The total charge is always conserved.

Conservation of Charge

In any process, the total number of positive and negative charges does not change.

9.2 Methods of Making a Charged Body

1. **By friction** – causes the transfer of electron by rubbing one matrial against another.
   Examples: a) stroking a cat's fur

   b) combing our hair in dark room – so we can see and hear the sparks

   c) scuff our shoes across a rug

2. **By contact** – caused by touching
   If the object is a good conductor, electrons will spread to all parts of its surface because the transferred electrons repel one another.
   If the object is a poor conductor, it may be necessary to touch the rod to get a more or less uniform distribution of charges.

3. **By induction**

   Electrostatic induction may be used to obtain an almost unlimited number of charges from a inducing charge without any loss of the inducing charge.

   Example: Thunderstorm

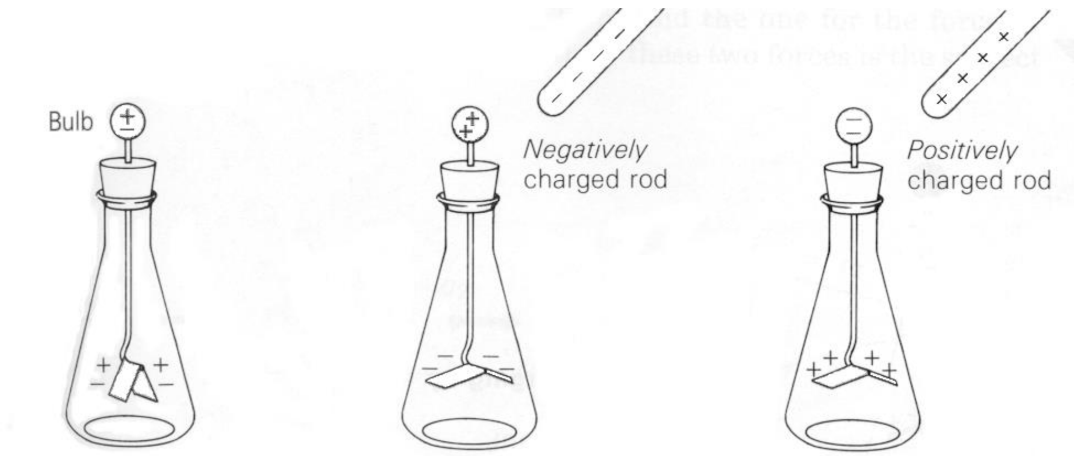
4. **By grounding** – the process where a net charge is produced by touching a metal surface

   with a finger provides a path for charge to flow to or from a very large reservoir for electric charge --- the ground.

9.3 Instruments Used in Determining and Generating Electric Charge

1. **Electroscope** – a simple instrument used to detect electrical charge or its relative amount of charge on a body. It is a sensitive instrument that consists of two pieces of very thin

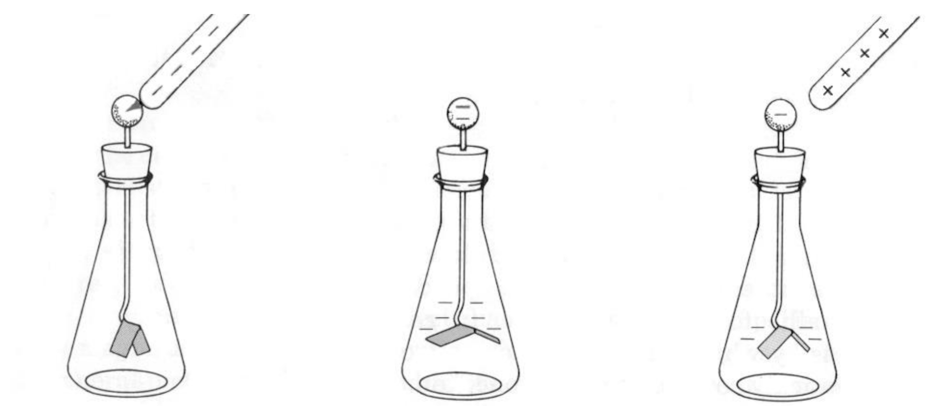
metal sheet attached at the end of a metal knob. When the metal knob is charged (either positive or negative) part of the charge goes to the gold foils, which repel each other and leaves will diverge.



Neutral electroscope has charges evenly distributed; leaves are close together.

Electrostatic forces cause leaves to diverge.

Conduction – the process on where an electroscope can be charged by touching the knob with a positive glass rod or a negative rubber rod, the electroscope acquires a charge similar to that of the charging body (see illustration on the next page).

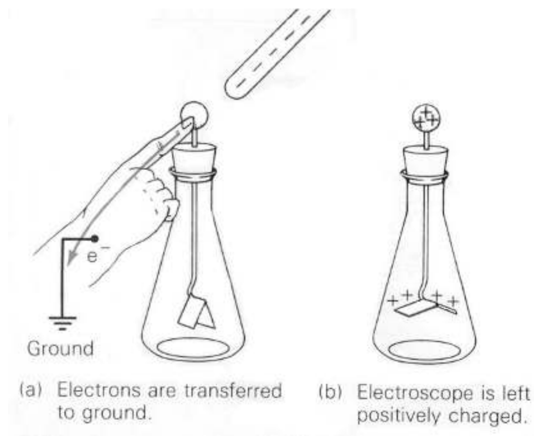


Electroscope is touched with negatively charged rod; charges transferred to bulb.

Electroscope is negatively charged (net negative charge).

Positively charged rod attracts electrons; leaves collapse.

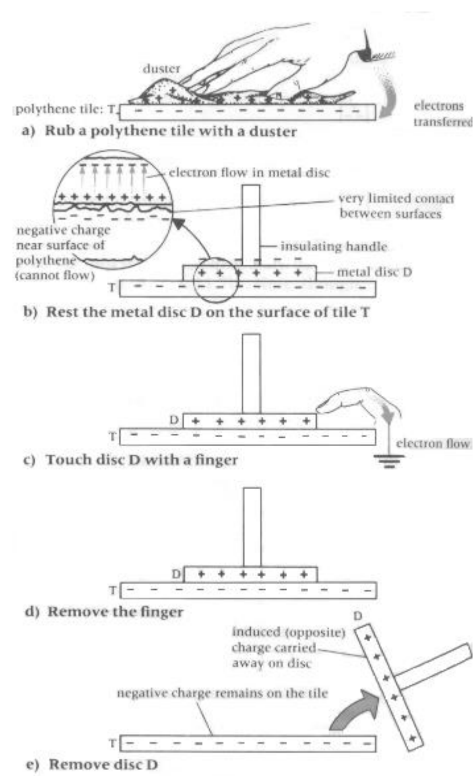
Induction is charging without physical contact. When the electroscope bulb is touched with a finger, the electroscope becomes grounded. That is, the electrons escape from the bulb and flows into the ground. Bringing a negatively charged rod close to the bulb repels electrons from the bulb. When the finger is removed, only the positive charges remain in the bulb and the electroscope becomes positively charged (see illustration on the right).



2. **Electrophorus** – a charge generator which works using the principle of induction and supplies almost unlimited charge in small amounts.

It consists of two parts, a flat plate T made of non-conducting material and a metal disk D with an insulating handle.

The non-conducting plate T is first charged by rubbing it with a piece of cloth. Depending upon the kind of material used it can become negative or positively charged. Materials like polythene, for example, gain extra electrons from the cloth by rubbing and therefore become negatively charged. Glass, on the other hand, loose electrons from their surfaces when rubbed and become positively charged.



Assuming that plate T becomes negatively charged, resting the disk D on the surface of plate T induces positive charge on the lower surface of D and negative charge on its upper

surface. No transfer of charge occurs in the

process. Since plate T is an insulator, its nega-

tive charges cannot flow to the metal disk D.

Then, touching the metal disk D with a finger allows the electrons repelled to the top of the disk to escape to the earth, leaving a deficiency of electrons on plate D, making it positively charged. Removing the finger stops the flow of electrons form D to the earth. Since plate D is insulated, electrons cannot return to it and the disk remains positively charged. The induced positive charges on D are carried away with it if the metal disk D is removed from the non-conducting plate T.

9.4 Classification of Materials Accordimg to the Flow of Charges

1. **Conductors** – usually metals wherein in them one or more of the electrons in their outer shells are not anchored to the nuclei of particular atoms but are free to wander in the material.
   Metals – good conductors of electric current as well as good heat conductors

   Examples:

   Silver - is the best conductor

   Iron

   Copper

   Aluminum

2. **Insulators** – materials wherein the atoms in them are tightly bound and belong to particular atoms. They are not free to wander about among other atoms in the material. These are poor conductors of electric current as well as poor heat conductors.
   Examples:

   Mica glass silk porcelain

   Rubber amber paraffin

3. **Semiconductors** – materials considered neither good conductors nor good insulators. They are fair insulators in their pure crystalline form but increase tremendously in conductivity when even one atom in 10 million is replaced with an impurity that adds or remove an electron from the crystal structure.
   Examples:

   Semiconductors – germanium and silicon

   Impurities – arsenic and gallium

Transistors – thin layers of semiconducting materials sandwiched together and used for the following purposes:

a) control the flow of currents in circuits

b) detect and amplify radio signals

c) produce oscillations in transmitters

d) act as digital switches

4. **Superconductors** – materials considered as perfect conductors of electricity. When these materials are cooled to a very low temperature – absolute zero Kelvin, the resistance to the flow of charge disappears.
Meike Kamerlingh Ohnes discovered these materials in 1911.

9.5 Coulomb's Law of Electrostatics

Charles Augustine de Coulomb (1736-1806), a French physicist, discovered "Coulomb's Law of Electrostatics", which states that the magnitude of the force of attraction (or repulsion) between two point charges is directly proportional to the product of the magnitude of their charges Q_1 and Q_2 , and inversely to the square of the distance r between them.

$$F = k\frac{|Q_1||Q_2|}{r^2}$$

Force of attraction – the force experienced when charges Q_1 and Q_2 have different signs (one positive and the other negative) and pulls the two charges toward each other, giving the direction of force.

Force of repulsion – the force experienced when charges Q_1 and Q_2 have the same sign (both positive or both negative) and pushes the two charges away from each other, giving the direction of force.

Equivalents:

k (proportionality constant) = 9×10^9 N-m^2/C^2

1 coulomb (C) = 6.25×10^{18} electrons

= 3×10^9 statcoulomb (stat C)

= 10^6 microcoulomb (μ C)

$= 10^{9}$ nanocoulomb (nC)

$= 10^{12}$ picocoulomb (pC)

1 electron (e) $= 1.6 \times 10^{-19}$ coulomb (C)

1 proton (p) $= 1.6 \times 10^{-19}$ coulomb (C)

1 newton (N) $= 10^{5}$ dynes

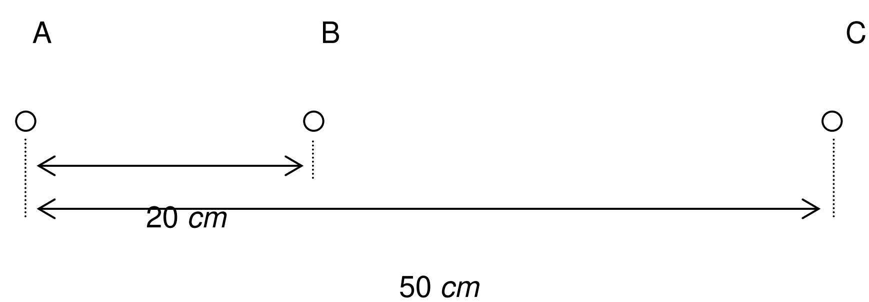
Units :

Quantity	MKS	CGS
Force (F)	newton (N)	dyne
Charge(Q)	coulomb (C)	statcoulomb (statC)
Distance (S)	meter (m)	centimeter (cm)
Constant (k)	9×10^{9} N- m²/C ²	1 dyne - cm² / statC ²

9.6 Sample Problems with Solutions

(1) Three + 100 statC charges are arranged in a straight line, the second charge B is 20 *cm* to the right of the first charge A while the third charge C is 50 *cm* to the right of A. (a) What force is exerted by charges A and B on C? (b) What force is exerted by A and C on B?

Solution:



a) Force on C due to A

$$F_1 = k\frac{|Q_1||Q_2|}{r^2} = 1\frac{\text{dyne cm}^2}{(\text{statC})^2}\frac{(100\ \text{statC})^2}{(50\ \text{cm})^2} = 4\ \text{dynes to the right}$$

Force on C due to B

$$F_2 = 1\frac{\text{dyne cm}^2}{(\text{statC})^2}\frac{(100\ \text{statC})^2}{(50\ \text{cm} - 20\ \text{cm})^2} = 11.1\ \text{dynes to the right}$$

Total force on C due to A and B

$$F_C = F_1 + F_2$$

$$= 4\ \text{dynes} + 11.1\ \text{dynes} = 15.1\ \text{dynes to the right}$$

b) Force on B due to A

$$F_3 = 1\frac{\text{dyne cm}^2}{(\text{statC})^2}\frac{(100\ \text{statC})^2}{(20\ \text{cm})^2} = 25\ \text{dynes to the right}$$

Force on B due to C

$$F_4 = -F_2 = -11.1\ \text{dynes} = 11.1\ \text{dynes to the left}$$

Net force on B due to A and C

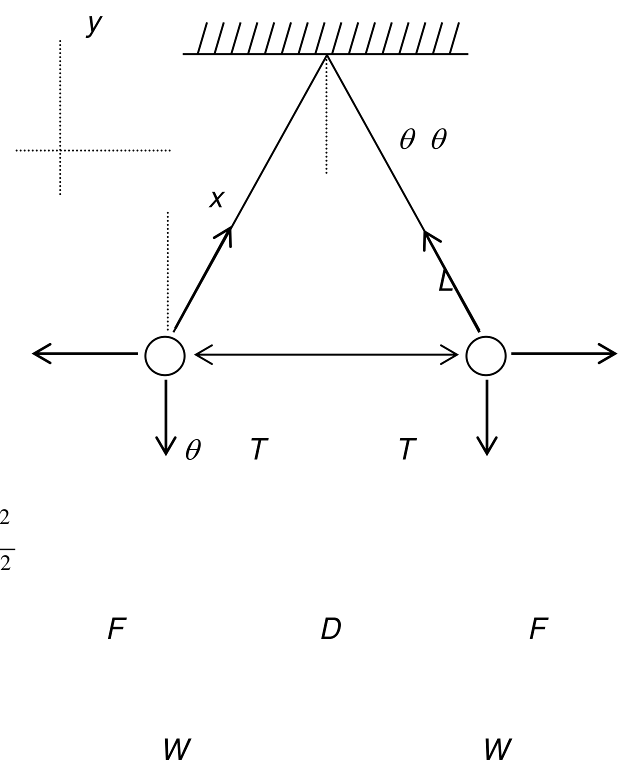
$$F_B = 25\ \textit{dynes} - 11.1\ \textit{dynes} = 13.9\ \textit{dynes}\ \text{to the right}$$

(2) Two gilded pith balls, spherical in shape and each having a mass of 1 g, are supported from a common point by a thread 1 m long. Determine the magnitude of a

positive charge that must be given to each ball to cause them to be separated by 10 *cm*.

Solution:

The forces acting on the pith balls are as shown. Using the coordinate system indicated, we find the following:



x-axis: $T\cos\theta = W = mg$

y-axis: $T\sin\theta = F = k\dfrac{|Q_1||Q_2|}{s^2} = k\dfrac{Q^2}{D^2}$

$$\frac{T\sin\theta}{T\cos\theta} = \tan\theta = \frac{k\dfrac{Q^2}{D^2}}{mg} = \frac{kQ^2}{mgD^2}$$

$$Q^2 = \frac{mgD^2\tan\theta}{k} = \frac{mgD^2}{k}\frac{D/2}{\sqrt{L^2 - (D/2)^2}}$$

$$= \frac{(1\,\text{g})\left(980\,\text{cm}/\text{s}^2\right)(10\,\text{cm})^2}{1\,\text{dyne cm}^2/\text{statC}^2}\frac{10\,\text{cm}/2}{\sqrt{(100\,\text{cm})^2 - (10\,\text{cm}/2)^2}} = 4906\ \text{statC}^2$$

$$Q = 70\ \text{statC}$$

9.7 Electric Field

An *electric field* is a region of space in which an electric charge will experience a force when placed in it. An electric field may be produced by one or more charges, and it maybe uniform or it may vary in magnitude, direction or both, from place to place. An electric field is a vector quantity whose direction is that of the force on a positive charge.

An *electric field intensity* is the Coulomb force per unit charge the electric field exerts on a positive charge placed in the region of the field. If a charge *Q*, called a test charge, experiences the Coulomb force *F*, then, the electric field intensity at the point where the test charge is located is given by

$$\vec{\mathrm{E}} = \frac{\vec{\mathrm{F}}}{Q} \text{ (vector form)}, \qquad \rightarrow \qquad \vec{\mathrm{F}} = Q\vec{\mathrm{E}}$$

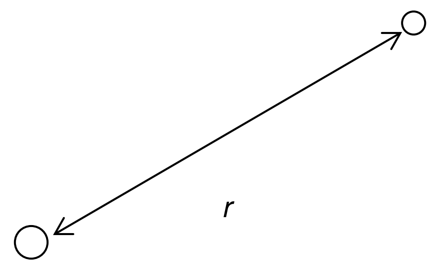
9.7.1 Electric Field Intensity of an Isolated Point Charge

Q_S = isolated point charge (in vacuum)

Q_T

Q_T = test charge

r = distance of Q_S from Q_T

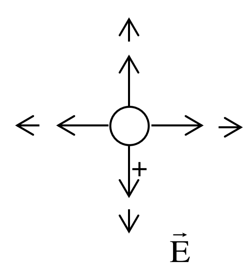


Magnitude of Coulomb force:

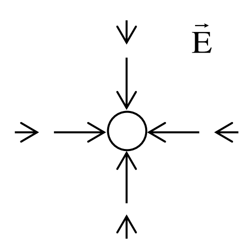
Q_S

$$F = k\frac{|Q_S||Q_T|}{r^2}$$

Magnitude of electric field *E* due to Q_S where the test charge Q_T is located:



$$E = \frac{F}{|Q_T|} = \frac{1}{|Q_T|}k\frac{|Q_S||Q_T|}{r^2} = k\frac{|Q_S|}{r^2}$$



Direction of $\vec{\mathrm{E}}$:

$+Q_S \rightarrow$ $\vec{E}$ is away from the charge in all directions –

$-Q_S \rightarrow$ $\vec{E}$ is towards the charge from all directions –

Units:

Quantity	MKS	CGS
electric field (E)	newton/coulomb (N/C)	dyne/statC
force (F)	newton (N)	dyne
charge (Q)	coulomb (C)	statC
distance (r)	meter	centimeter
constant (k)	9×10^{9} N-m^2/C^2	1 dyne-cm^2/statC^2

9.7.2 The Electric Field Intensity of a Group of Point Charges

To find $\vec{E}$ for a group of N point charges, do the following:

- Calculate $\vec{E}_i$ due to each charge i as if the other charges were not present.
- Add these separately calculated fields using vector addition to find the resultant field $\vec{E}$ at the point in question.

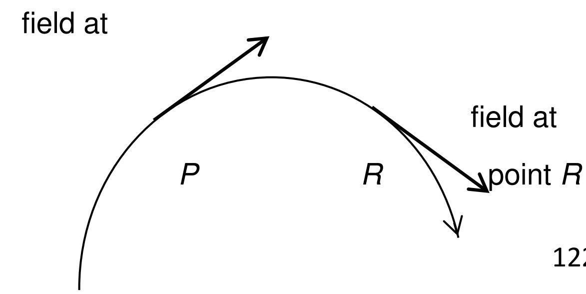
In equation form, we have

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots = \sum_{i=1}^{N} \vec{E}_i$$

The sum is a vector sum taken over all the charges present.

9.8 Lines of Force

It was Michael Faraday (1791-1867) who conceived the idea of lines of force in

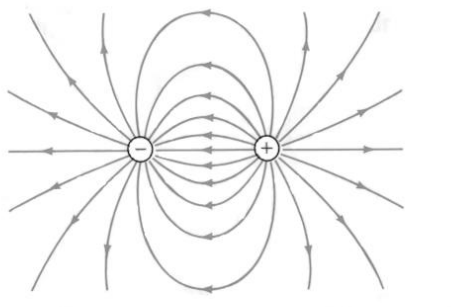


an electric and magnetic field. A line of force in an electric field is an imaginary line so drawn that a tangent to it at any point shows the direction of the electric field at that point.

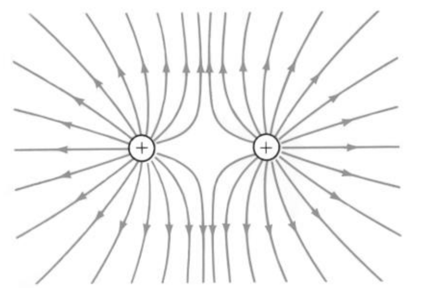
line of force

Some examples of point charges with lines of force are shown below.

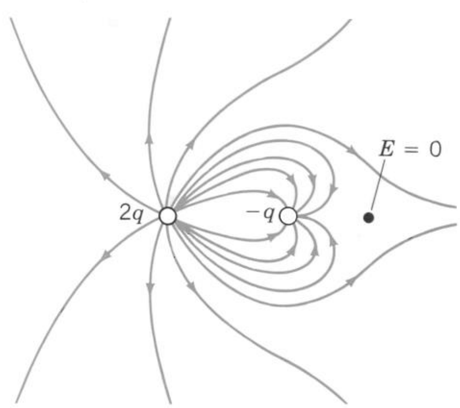
Note that no two lines of force intersect or cross each other.



Lines of force surrounding two charges equal in magnitude but opposite in sign (dipole).



Lines of force surrounding two equal positive charges.



Lines of force due to two charges 2*q* and – *q*

9.9 Sample Problems with Solutions

(1) (a) What is the electric field intensity of a charge of -1.5×10^{-9} C at a point whose distance is 20 cm from it? (b) What is the direction of the field intensity?

Solution:

(a) $$E = k\frac{|Q|}{r^2} = \left(9\times10^9\ \frac{\mathrm{N\,m^2}}{\mathrm{C^2}}\right)\frac{1.5\times10^{-9}\,\mathrm{C}}{(0.2\,\mathrm{m})^2} = 337.5\ \frac{\mathrm{N}}{\mathrm{C}}$$

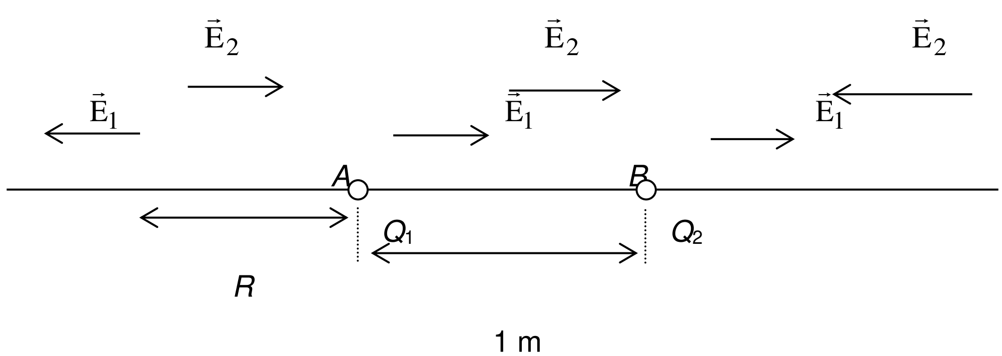
(b) Wherever the point may be, the direction of $\vec{E}$ points toward the charge as shown in section 5.7.1

(2) Two point charges +2 μC and –3 μC are fixed at points *A* and *B*, respectively, which are 1 m apart. Calculate the position on the straight line passing through points *A* and *B* where the electric field intensity is zero.

Solution:

Let $Q_1 = +2\ \mu\mathrm{C}$ $\quad\vec{E}_1$ = electric field due to Q_1

$Q_2 = -3\ \mu\mathrm{C}$ $\quad\vec{E}_2$ = electric field due to Q_2



From the diagram shown above, we find the following along the line *AB*:

- Between A and B, $\vec{E}$ cannot be zero because $\vec{E}_1$ and $\vec{E}_2$ have the same direction and they will not cancel out.
- To the right of point B, although $\vec{E}_1$ and $\vec{E}_2$ point in opposite directions, the magnitude of $\vec{E}_2$ is larger than $\vec{E}_1$ and they will not cancel out, so $\vec{E}$ cannot be zero there.
- To the left of point A, $\vec{E}_1$ and $\vec{E}_2$ point in opposite directions and they will cancell out if the distance is correct because the point is closer to the weaker charge Q_1 and farther from the stronger charge Q_2. $\vec{E}$ can therefore be zero there.

Now, let R be the distance of the point to the left of Q_1 as shown above. Then, we get the following for points to the left of A along the line AB:

$\vec{E}_1$ is to the left. $\rightarrow$ $-E_1$

$\vec{E}_2$ is to the right. $\rightarrow$ $+E_2$

so that

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = -E_1 + E_2 = 0$$

$$E_1 = E_2$$

$$k\frac{|Q_1|}{R^2} = k\frac{|Q_2|}{(1+R)^2}$$

$$|Q_1|(1+R)^2 = |Q_2|R^2$$

$$2(1+R)^2 = 3R^2$$

$$3R^2 = 2(1+2R+R^2) = 2+4R+2R^2$$

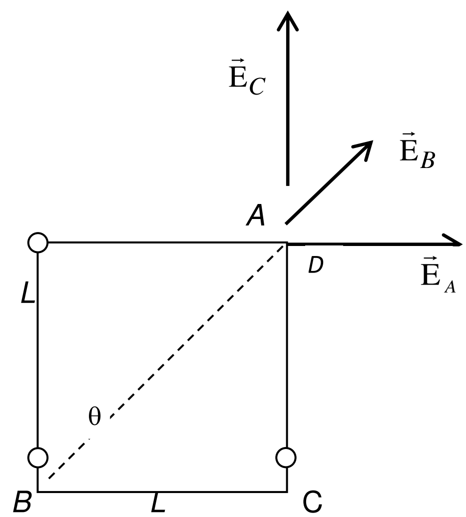
$$R^2 - 4R - 2 = 0$$

$$R = \frac{4 \pm \sqrt{16 - 4(1)(-2)}}{2} = 2 \pm \sqrt{4+2} = 2 \pm \sqrt{6}$$

$$R = 4.45 \text{ m to the left of point } A$$

(3) Three similar charges of 0.008 μC are placed at the three corners of a square 50 cm apart on each side as shown below. Find the electric field intensity at the 4th corner *D* of the square.

Solution:



$$E_A = k\frac{|Q_A|}{\overline{AD}^2} = k\frac{Q}{L^2}$$

$$E_B = k\frac{|Q_B|}{\overline{BD}^2} = k\frac{Q}{2L^2}$$

$$E_C = k\frac{|Q_C|}{\overline{DC}^2} = k\frac{Q}{L^2}$$

$$E_A = E_C = E$$

$$E_B = \tfrac{1}{2}E$$

Along the horizontal

$$E_H = E_A + E_B\cos 45^\circ = E + \tfrac{1}{2}E\frac{\sqrt{2}}{2} = \left(1 + \frac{\sqrt{2}}{4}\right)E = 1.35\,E$$

Along the vertical

$$E_V = E_C + E_B\sin 45^\circ = E + \tfrac{1}{2}E\frac{\sqrt{2}}{2} = \left(1 + \frac{\sqrt{2}}{4}\right)E = 1.35\,E$$

so that $E_H = E_V$

and

$$\theta = \arctan\frac{E_V}{E_H} = 45^\circ$$

That is, $\vec{E}_D$ is 45° above the horizontal.

It follows that if

$$E = k\frac{Q}{L^2} = \left(9\times10^9\ \frac{\mathrm{N\,m^2}}{\mathrm{C^2}}\right)\frac{8\times10^{-9}\,\mathrm{C}}{(0.5\,\mathrm{m})^2} = 288\ \frac{\mathrm{N}}{\mathrm{C}}$$

then

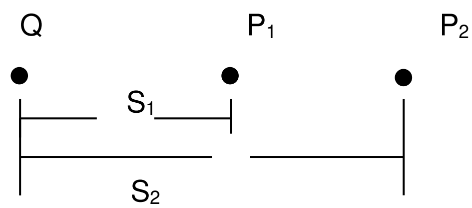
$$E_D^2 = E_H^2 + E_V^2 = 2(1.35)^2E^2$$

$$E_D = 1.35\,E\sqrt{2} = 549.8\ \frac{\mathrm{N}}{\mathrm{C}}$$

9.10 The Electric Potential

The *potential difference* (V) between two points in an electric field is the amount of work needed to take a change of 1 *coulomb* from one of the points to the other.

$$V = \frac{W}{Q}$$



Unit:

$$volt = \frac{joule}{coulomb}$$

$$W = k\,Q\,q\left(\frac{1}{S_2} - \frac{1}{S_1}\right)$$

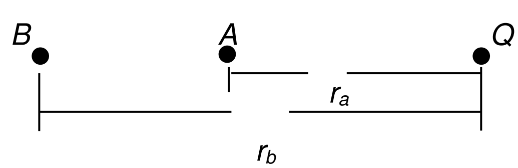
$$V = V_2 - V_1 = k\,q\left(\frac{1}{S_2} - \frac{1}{S_1}\right)$$

The *electric potential* at a point is the work per unit charge required to bring a positive test charge from a point of zero potential (like the ground) to the point in question.

Let

Q = unit test charge

r_a, r_b = distances from Q of points A and B very close to each other ($r_a \approx r_b$)



Then

$W = F\,(\,r_b - r_a\,)$

For a unit positive charge *q*, $F = k\dfrac{qQ}{r^2} = k\dfrac{Q}{r^2} = k\dfrac{Q}{r_a\,r_b}$ because $r_a \approx r_b$. Therefore

$$W = \frac{k\,Q}{r_a\,r_b}\left(r_b - r_a\right) = k\,Q\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

$$W = k\frac{Q}{r_a} - k\frac{Q}{r_b}$$

Since $V_a = k\dfrac{Q}{r_a}$, $V_b = k\dfrac{Q}{r_b}$, we get the general result

$$V = k\frac{Q}{r}$$

The electric potential *V* is a scalar quantity. The potential at a point due to several point charges may be expressed as

$$V = \sum_{i=1}^{N} k\frac{Q_i}{r_i}$$

9.11 Electric Potential Energy

If *e* designates an electron and *V* the electric potential, then the product of *e* and *V* is energy called an *electron volt*. (*eV*) , the basic unit for electric potential energy.

One electron volt is the amount of energy acquired by an electron as it moves through a potential difference of 1 volt.

For a single point charge: $PE = W = qV$

For a system of many point charges $W = \frac{1}{2} k \sum_i \sum_j \frac{Q_i\, Q_j}{r_{i\,j}}$

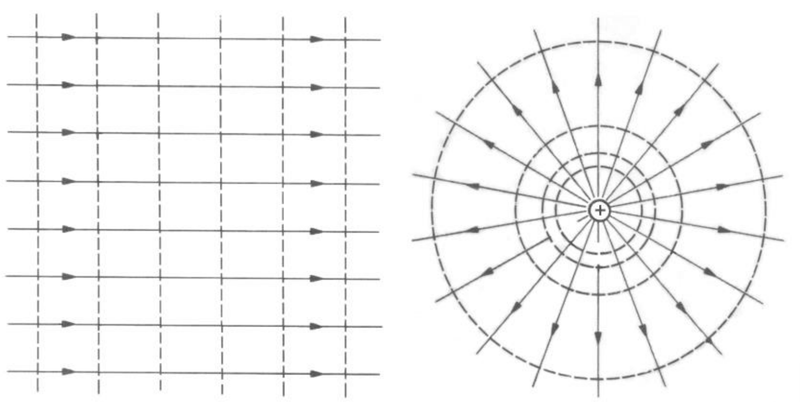
where $i \neq j$ and $r_{i\,j}$ is the distance between the two point charges Q_i and Q_j. The factor 1/2 is included to offset the effect of taking the summation twice.

Units:

Quantity	**MKS**	**CGS**
Charge (Q)	Coulomb	StatCoulomb (StatC)
Work (W)	Joule (j)	erg
Potential Energy (PE)	Joule (j)	erg
Electrical Potential (V)	Volt (V)	StatVolt (StatV)

9.12 Equipotential Surface

An *equipotential* surface is a surface all points of which are at the same potential, so that no work is done in moving a charge along that surface. The surface of a sphere with charge Q at the center is one example of an equipotential surface.

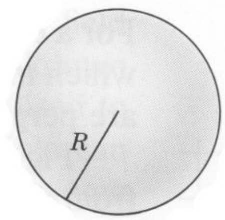


(a) (b)

Lines of force (solid lines) and cross sections of equipotential surfaces (dashed lines)

for (a) a uniform field and (b) a positive point charge.

Consider a charged spherical conductor as shown. Let

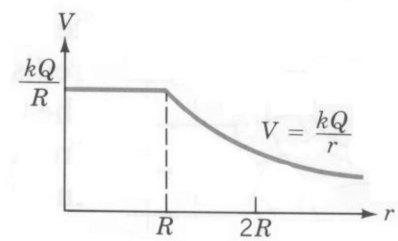


Q = charge of spherical conductor uniformly distributed on its surface

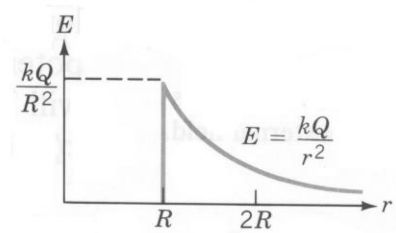
R = radius of sphere

r = distance of P from the center

P = point outside the sphere



$$E = k\frac{Q}{r^2} \quad \rightarrow \quad \text{electric field intensity at poi}$$



$$V = k\frac{Q}{r} \quad \rightarrow \quad \text{electric potential at point P}$$

On the surface of the sphere

$$E = k\frac{Q}{R^2} \qquad \text{and} \qquad V = k\frac{Q}{R}$$

Inside the sphere

$E = 0$ $\leftarrow$ There are no lines of force inside the sphere.

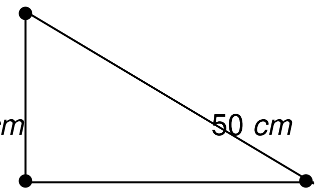
$V = V$ on the surface of sphere

9.13 Sample Problems with Solutions

(1) Two point charges +24 and –36 *statC* are 50 *cm* apart in air as shown below. What is the electric potential at a point *P which is* 30 *cm* from the first charge and 40 *cm* from the second?

Solution:

Potential due to the +24 *statC* charge



$$V = k\frac{Q}{r} = 1\frac{dyne\,cm^2}{statC^2}\,\frac{24\,statC}{30\,cm} = 0.8\,statV$$

Potential due to the –36 *statC* charge

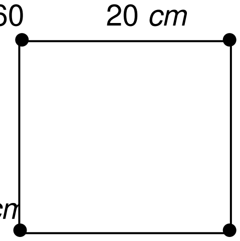
$$V = k\frac{Q}{r} = 1\frac{dyne\,cm^2}{statC^2}\,\frac{(-36\,statC)}{40\,cm} = -0.9\,statV$$

$$V_{Total} = V_1 + V_2 = (0.8 - 0.9)\,statV = -0.10\,statV$$

(2) At each center of a square 20 *cm* on a side is a small charged body. Going around these charges are +60, –30, +60 and –30 *statC*. Find (a) the field intensity (b) the electrostatic potential at the center of the square.

Solution:

a) At the center of the square



$E = 0$

Why?

b) Electrostatic potential at the center of the square

$$V = \sum_{i=1}^{4} k \frac{Q_i}{r_i} = \left(1 \frac{dyne\ cm^2}{statC^2} \right) \frac{\left[2\,(60) + 2\,(-30) \right] statC}{\left(\frac{1}{2} \right) \sqrt{20^2 + 20^2}\ cm} = 4.24\ statV$$

(3) A sphere of radius 20 *cm* is charged with +40 $\times 10^{-9}$ *C*. a) What are the field intensity and electric potential on the surface? b) What are the field intensity and electric potential at a point 10 *cm* from the center of the sphere?

Solution:

(a) $$E = k \frac{Q}{r^2} = \left(9 \times 10^9 \frac{N\,m^2}{C^2} \right) \frac{40 \times 10^{-9}\ C}{(0.20\ m)^2} = 9 \times 10^3 \frac{N}{C} = 9 \times 10^3 \frac{V}{m}$$

$$V = k \frac{Q}{r} = \left(9 \times 10^9 \frac{N\,m^2}{C^2} \right) \frac{40 \times 10^{-9}\ C}{0.20\ m} = 1.8 \times 10^3\ V$$

(b) 10 *cm* from the center of the sphere, $E = 0$ inside and $V = 1.8 \times 10^3\ V$. Why?

Lesson 9: Electrostatics
Assessment

Instruction; Show your complete and neat solution. Identify your final answer in the solution. See answer sheet format.

1. Two small similar pith balls are 3 *cm*. apart in air and carry charges of 3×10^{-9} and 12×10^{-9} *statC*, respectively. (a) Compute the force of attraction between them. (b) If the balls are touched and then separated by a distance of 3 *cm*., what will be the force between them?

2. Charges of +2 , +3 and –8 μC are placed in air at the vertices of an equilateral triangle of side 10 *cm*. Calculate the magnitude of the force acting on the –8 μC due to the other 2 charges.

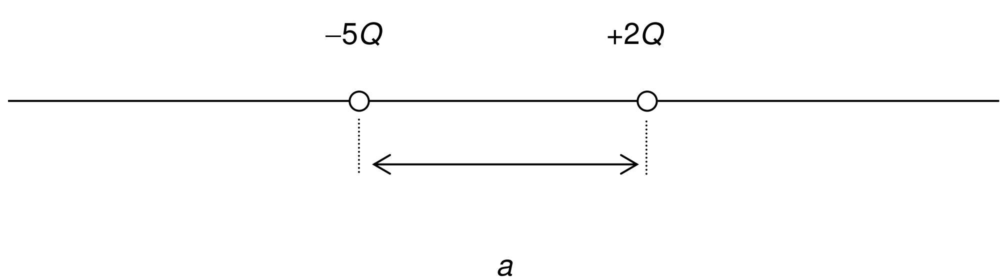
3. Two charges of $+10^{-6}$ *C* are located 1 *cm* apart. (a) What is the force on a charge $+10^{-8}$ *C* halfway between them? (b) What is the force on a charge of -10^{-8} *C* at the same place?

4. At the corners of a rectangle ABCD of sides 8 *cm*. and 6 *cm* are charges of +50, –30, +20 and –40, all in μC, respectively. AB is the longer side and BC is the shorter side. Find the force on a +15 μC charge placed at the center of the rectangle?

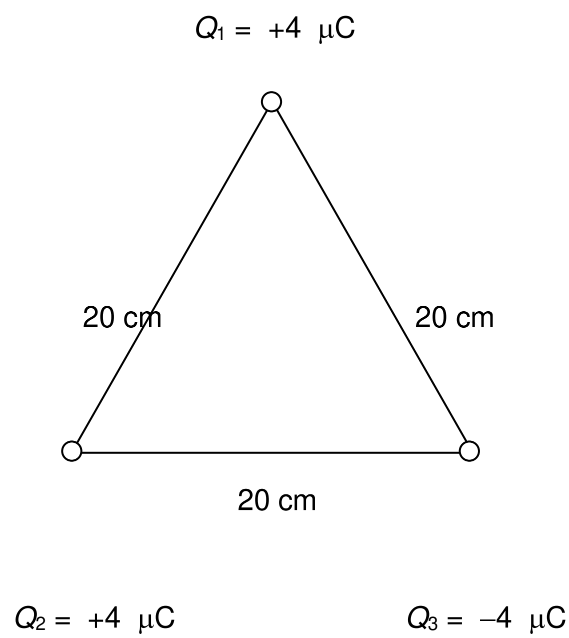
5. A small charged pith ball is suspended 2 *cm* above a second charge pith ball resting on an insulated surface. If the charge of the lower ball is +20 statC and it has a mass of 0.25 *g*, what charge must be on the upper ball to lift the lower ball?

6. A point charge Q_1 is at $x = 0$ and Q_2 is at $x = d$. What is the relation between these charges if the resultant electric field intensity is zero at the following points: (a) $x = d/2$, (b) $x = 2d$; and (c) $x = -d/2$?

7. In the diagram below, locate the point (or points) at which the electric field is zero.

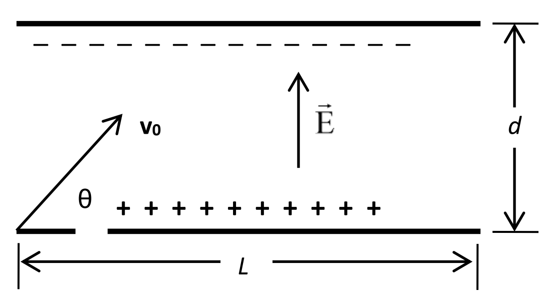


8. Three charges are located at the corners of an equilateral triangle as shown below. What is the electric field at the center of the triangle?



9. Two charges of 10 *C* and –4 *C* are 2 *m* apart. (a) What is the electric potential at a point midway these charges? (b) What is the potential at a point between the charges and 0.667 *m* away from the –4 *C* charge? (c) How much work would be required to move a 0.05 *C* charge from the midpoint to the 0.50 *m* mark?

10. An electron, inside an electric field region between two parallel plates as shown below, is projected upward at a speed of $v_0 = 5.83 \times 10^6$ m/s and at an angle of θ = 39.0°; E = 1870 N/C (directed upward), *d* = 1.97 cm, and *L* = 6.20 cm. Will the electron strike one of the two plates? If yes, which plate will it stike and at what distance from the left edge?



LESSON 10: DIRECT CURRENT AND CIRCUIT THEORY

Introduction

This lesson will discuss the basic principles of electric current and apply these principles to analyze dc circuits involving combinations of batteries, resistors and capacitors. The discussion also include the operations of some useful instruments,

Learning Outcomes

After successful completion of this lesson, you should be able to:

- ☞ Know the typical kinds of materials based on their electrical properties.
- ☞ Know what causes the flow of an electric current.
- ☞ Understand the meaning of the term electromotive force.
- ☞ Know the relationship between field theory and circuit theory.
- ☞ Learn the basic elements of direct current circuit
- ☞ Know the characteristics of series and parallel connections.
- ☞ Learn about Kirchhoff's rules and their application to simple network analysis.

Discussion

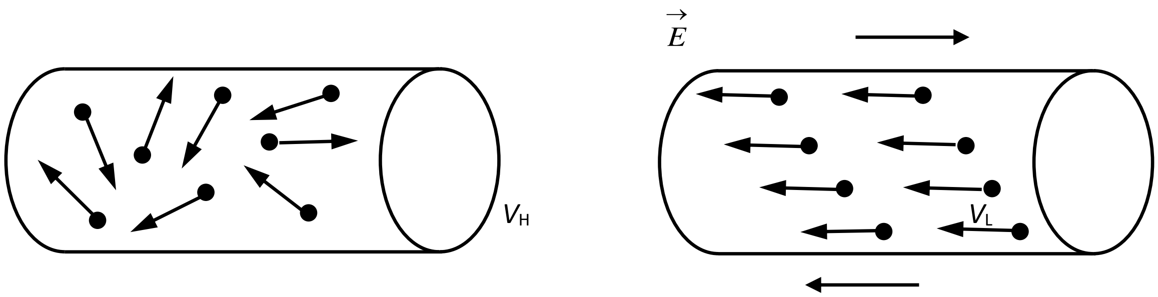
10.1 Conductors and Insulators

As already mentioned in the last chapter, we ordinarily consider three different kinds of materials. **Conductors** are those that have many free electrons that can easily move from one place to another within the material. Metals are typical electric conductor. **Insulators**, on the other hand are materials that have no free electrons because they are held fixed to the atoms. Another name for insulators are dielectric materials, or simply **dielectrics.** Glass and plastics are common examples of insulators.

Intermediate between conductors and insulators are the semiconductors, materials that are normally insulators at low temperature but become conductors if the temperature becomes high enough. One of the properties of semiconductors that make them useful is that the density of conduction electrons can be changed drastically by the addition of small quantities of impurities.

10.2 Electric Current and Electric Current Density

Consider an ordinary piece of metallic conductor that is cylindrical in shape. It has the same number of positive and negative charges so it has zero net charge. Suppose further that the conductor is isolated. Then, the free electrons of the material are in random motion without any preferred direction.



If the ends of the conductor are now placed at different electric potentials, the left end being at higher potential than the one on the right, then, according to the general rule, an electric field pointing to the right, in the direction of decreasing electric potential, is produced within the conductor. The electric field created then begins to exert a force $\left(\vec{F} = Q\vec{E}\right)$ on the free electrons of the conducting material. Electrons, being negatively charged, are pushed opposite to the direction of the electric field. As a result, the free electrons move to the left, as shown in the figure above, causing negative charges to be transported to the end at higher electric potential.

The amount of charge ΔQ transported in time Δt is defined as the electric current I

$$I = \frac{\Delta Q}{\Delta t} \tag{10-1}$$

Electric current is a scalar quantity measured in coulomb/second, or ampere (A) for short, in the SI system. Note that the charge transported can be positive or negative, so that electric current can be a flow of positive as well as negative charges. The standard reference direction for the flow of current is the direction of flow of positive charges, opposite to the flow of the free electrons.

Electric current density J is defined as the ratio of the amount of current passing perpendicular to the cross sectional area A of a conductor.

$$J = \frac{I}{A} \tag{10-2}$$

It is clear from the preceding formula that current density has the dimension of ampere/m^2. Being a vector quantity, $\vec{J}$ points in the direction of flow of charges.

For so-called linear and isotropic conducting medium, the current density is directly proportional to the applied electric field. That is

$$\vec{J} = \sigma \vec{E} \tag{10-3}$$

where σ is the **electric conductivity**, or simply conductivity, a property of the medium. A medium is *linear* if the conductivity is independent of the magnitude of the electric current. On the other hand, isotropic means that all directions are equivalent, so there is no other preferred orientation regarding the flow of current except that it is always opposite to the direction of the electric field in the case of metallic conductors. Equation (6-3) is known as **Ohm's Law** of **field theory**, the theory that looks at electricity from the microscopic point of view that we discussed in the last chapter.

Now, consider the effect of the Coulomb force acting on the free charges of the conductor. Because this force is constant, since the electric field is constant, the charges must move with uniform acceleration and gradually increase its speed. Therefore, as time elapses, the electrons in the conductor must move faster and faster, so that for a given fixed time interval, more and more charges are transported. According to equation (6-1), the electric current must continually increase.

Actually, the preceding does not occur and there appears a steady flow of electric current instead. This happens because the accelerated electrons continually collide with the positively charged particles of the atom fixed in their respective positions within the crystal structure of the material. The colliding electrons loose energy during the impact and reduce their speeds. They consequently acquire a constant drift velocity as a result of the continuous collisions, thereby causing a constant amount of current to be transported that then gives rise to a steady flow of electric current.

From the preceding paragraph, it becomes clear that there is an internal resistance to the actual flow of free electrons in a metallic conductor. As a matter of fact, there is always such a resistance to the flow of any charge carrier in any conducting medium under ordinary conditions.

10.3 Electromotive Force and Current Source

In the preceding section, it was pointed out that negative charges move away from the low potential end of a cylindrical metallic conductor toward the high potential end. In the process, the low potential end gradually acquires an increasing accumulation of positive charges that increases its potential. At the high potential end, on the other hand, negative charges gradually accumulate, hereby decreasing its potential. The potential difference between the two ends of the conductor gradually becomes zero.

With no direction with decreasing electric potential, there can be no electric field. The Coulomb force acting on the free electrons of the conductor therefore disappears, causing the transport of negative charges to stop, halting the flow of electric current. To maintain the flow of current, the potential difference between the two ends of the conductor must be maintained. Any device that accomplishes this task is called a source of **electromotive force**, abbreviated simply as EMF and denoted by the symbol $\boldsymbol{\xi}$.

A typical source of EMF is the ordinary battery, a device with two **terminals** (points at which charge can enter or leave the battery) that keeps one of them at a higher potential than the other (called the negative terminal). The terminal at a higher potential is then said to have a positive polarity while that at a lower potential, a negative polarity. The potential difference between the positive and negative terminals is called the **output voltage** of the battery. Since a battery maintains the same polarity of its terminals, the polarity of its output voltage is fixed so that the current flows in just one direction. The current supplied by a battery is therefore said to be unidirectional

A unidirectional current and fixed polarity of applied voltage are the characteristics of the so-called **direct current (dc)** source. In an **alternating current (ac)** source, the output voltage periodically reverses or alternates in polarity. The resulting electric current therefore periodically reverses or alternates in direction. But in this course, we will consider only direct current sources.

A battery, like any source of EMF, must be able to do work on charge carriers that enter it. It acts to move positive chargers from a point of low potential (the negative terminal) through its interior to a point of high potential (the positive terminal), along the direction of the EMF. The energy needed to do this is derived from the chemical energy stored in the substance making up the battery. The charges then move through the conducting medium external to the battery, dissipating energy in the process as they suffer collisions along the way, and return to the negative terminal, from which the EMF raises them to the positive terminal again, and the cycle continues. This is the usually accepted convention in the analysis of current flow. The actual motion of the free electrons is, of course, in the opposite direction.

An **ideal EMF device** is one that lacks any internal resistance to the internal movement of charge from terminal to terminal. The potential difference between the terminals of an ideal EMF device is equal to the EMF of the device. A **real EMF device,** on the other hand, such as an ordinary battery, has internal resistance to the internal movement of charge. When current flows through such a device, the potential difference between its terminals differ from its EMF. In this course, we confine ourselves to the use of ideal batteries only.

10.4 Ohm's Law and Energy Dissipation According to Circuit Theory

Suppose that the cylindrical conductor discussed in section 6.2 is now connected to the terminals of an ideal battery, so that the potential difference between its two ends is maintained, causing electric current to flow continuously through the material. From equations (6-2) and (6-3), we find that the magnitude of the electric field produced inside the conducting material, according to the field theory point of view, is given by the following expression:

$$E = \frac{J}{\sigma} = \frac{I}{\sigma A} \tag{10-4}$$

But the electric field is directly proportional to the rate of change of the electric potential *V* set up within the conductor by the battery; the larger the change in electric potential ΔV in space, the larger the electric field produced. If we now take the x-axis to be along the direction of decreasing potential parallel to the axis of the cylinder, we get the relation

$$E = \frac{\Delta V}{\Delta x} = \frac{V}{L}$$

without the negative sign since we are concerned with the magnitude of the vector only. Here, *V* is the potential difference between the two ends of the whole conductor having length *L*. Equation (6-4) therefore becomes

$$\frac{V}{L} = \frac{I}{\sigma A}$$

Rewriting the last equation, we finally get Ohm's law in the formulation of circuit theory, the theory that deals with electromagnetic phenomena from the large scale or macroscopic point of view:

$$V = IR \tag{10-5}$$

where *R*, the **resistance** characterizing the whole conducting material, is given by

$$R = \rho \frac{L}{A} \tag{10-6}$$

while ρ is the resistivity, which is simply the reciprocal of the conductivity σ, that is,

$$\rho = \frac{1}{\sigma} \tag{10-7}$$

Equation (6-6) is a quantitative measure of the resistance to the flow of accelerated electrons discussed in section 6.2. According to the above formula, the longer the conductor, the greater the resistance to the current flow. This is clear because the longer the material, the more collisions will be suffered by the moving free electrons and the more energy will be lost during collisions. The above formula also says that the resistance is inversely proportional to the cross sectional area of the cylindrical conductor. That is, the smaller the cross section, the greater the resistance. This is again clear because the thinner the material, the closer and more crowded are the electrons and more of them will therefore collide with the fixed atoms of the conductor. The greater the cross sectional area, the farther apart are the moving electrons because there is more space available and less of them will suffer hit the fixed atoms. Resistance is measured in ohm (Ω) in the SI system. Referring to equation (6-6), we find that resistivity ρ has the dimension ohm meter.

Finally, using Ohm's law as given by equation (6-5), we cam compute the energy dissipated by positive charges mentioned in the previous section, as they move through the conducting medium external to the battery, in the following way. Due to the potential drop experienced by a single charge Q_i in moving through the conductor, its energy decreases by

$$\Delta E_i = Q_i \, \Delta V = Q_i \, V$$

where we have simply denoted the total change in potential by *V.* Summing over all the charges, the total change in energy of all the moving charges is then

$$\Delta E = \sum_i \Delta E_i = \sum_i Q_i \, V = V \sum_i Q_i = V \, \Delta Q$$

Dividing by the total elapsed time Δt, we find the following time rate of change of dissipated energy or dissipated power *P*:

$$P = \frac{\Delta E}{\Delta t} = \frac{V \, \Delta Q}{\Delta t} = V \frac{\Delta Q}{\Delta t} = V \, I = (IR) I = I^2 R \tag{10-8}$$

10.5 Comparison Between Field Theory and Circuit Theory

To understand electrical phenomena, field theory requires the specification of various field vectors at every point in space, of which the electric field $\vec{\mathrm{E}}$ and current density $\vec{\mathrm{J}}$ previously discussed are just two of them. It then considers the effects of the field vectors on the different charged particles comprising a body in order to know how the material as a whole will subsequently behave. Field theory, therefore, takes the **microscopic approach**.

Circuit theory, on the other hand, takes the **macroscopic approach**. It considers a given material as an element of an **electric circuit**. Such a circuit is simply a path through which charge can flow. A **circuit element** is then accordingly viewed as a macroscopic configuration of matter with two or more points designated as terminals through which charge can enter or leave the given substance. Circuit theory, consequently, requires only the determination of the voltage drop V across the whole material and the total current I passing through it in order to understand what will happen.

While the field equations are applicable in general, relations among circuit variables are usually more convenient to use, due to the less detailed nature of work required by circuit theory, whenever the voltage\ V and current I have a simple, well-defined relationship.

In any given circuit with various circuit elements, the magnitude and the direction of the current generally change as time elapses. Therefore, as a first step in understanding the application of circuit theory, we consider only the case of dc circuit containing simply batteries and so-called resistors as circuit elements wherein the magnitude and direction of electric current do not vary with time.

10.6 Basic DC Circuit Elements and Symbols

A practical electrical device has actually, in the least, four component parts, namely

- a source of EMF
- a set of good conductors
- a load, and
- a means of control

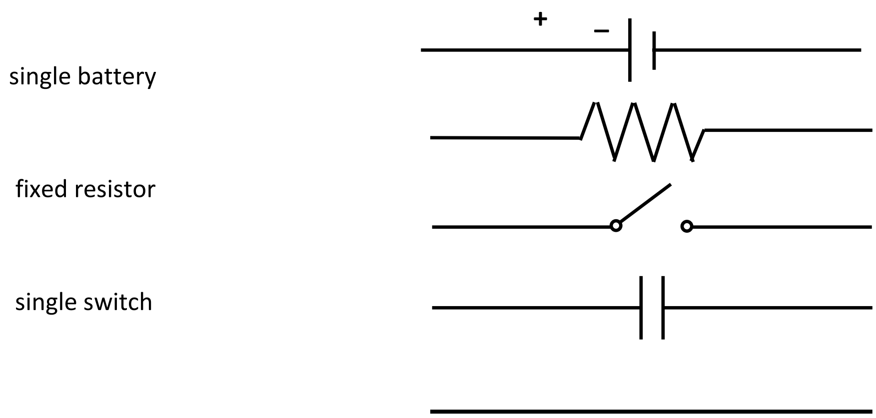
A source of EMF in a dc circuit, as we have already mentioned in section 6-3, is a battery that we consider to have no internal resistance whatsoever.

By a set of good conductors is meant here materials that offer negligible resistance to current, such as copper wires that connect the different circuit elements. Poor conductors are materials with very low conductivity and therefore provide strong resistance to the flow of current.

By load of an electric circuit, we mean here a **resistor**, a circuit element in which the stored energy is small compared to the rate of energy dissipation. Typical examples of such devices are a lamp, a buzzer, and an electric motor.

By using a switch, the simplest means of controlling any given circuit can be achieved. Turning a switch on, and thereby closing the circuit, or turning it off, and thereby opening the circuit, starts or stops the flow of current, whatever the case may be. But if only closed circuits are considered, as we will do here, switches can altogether be neglected.

Because energy is continually dissipated in the circuit, it is important to have some other source of energy aside from the battery. We therefore include a fifth essential electric device called a capacitor, aside from the four already mentioned above.



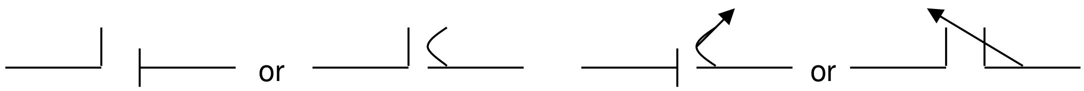
10.7Capacitance

A *capacitor* or *condenser* is a system that stores electrical energy. It consists of a pair of parallel metal plates separated by an insulator or dielectric. Dielectric may be air or any insulator like glass, mica etc.

Uses of capacitors:

1. They are used to store electrical charges or electrical energy for certain length of time.
2. They are used as low-energy on off switches in computer motherboards.
3. As a storehouse in larger amounts of energy in photoflash units that release the energy rapidly during the short duration of the flash.
4. They are also used to assist in switching a transistor on and off and to prevent excessive heat from developing.
5. They are important components in radio and television circuits.
6. They are also used in the ignition of motor cars and in much other electrical equipments.

Symbols for capacitors



capacitance C – defined as the ratio of the charge to the potential difference

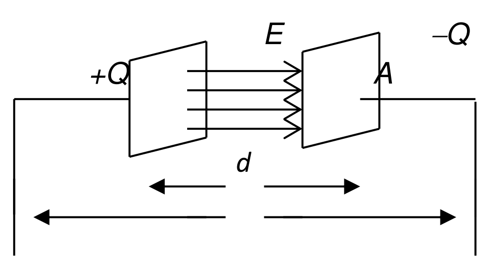
$$C = \frac{Q}{V}$$

Units:

Capacitance	Charge	Potential Difference
farad (*F*)	*Coulomb* (*C*)	*Volt* (*V*)
microfarad (μF)	*microcoulomb* (μC)	*Volt* (*V*)

1 *farad* = 10^6 *microfarad* (μF)
= 10^9 *nanofarad* (*nF*)
= 10^{12} *picofarad* (*pF*)

For a parallel plate capacitor



1) $E = \dfrac{Q}{\varepsilon_0 A}$, A = area

ε_0 = permittivity of vacuum
= 8.85×10^{-12} C² / N m²

2) $V = E d$

In a capacitor, the potential difference is proportional to the charge

3) $V = \dfrac{Q}{C}$

When a capacitor is being charged, the charge builds up from an initial value zero to final value Q. Also, potential difference builds up from zero to a final value V. The average value during charging process is ½ V making

$W = QV = ½ (Q)\ V = ½\ QV$

Also $Q = CV$

$W = ½\ CV^2$

$W = ½\ Q^2/C$

If the field is uniform (as between 2 parallel points)

$W = QV = Fr$, where F is the force on the change of Q and r is the distance between points

$V/r = F/Q = E$; $V/r = E$

V/r is the potential gradient

Energy is released when the capacitor is discharged. If the capacitor is allowed to discharge through a wire, the energy is converted to heat in the wire.

Factors affecting the capacitance of a parallel plate capacitor

1. Area of plates
   An increase in the effective area of the plates decreases potential different between plates and increases in capacitance.
2. Distance apart of the plates
   The closer the plates the lower the potential and the greater the capacitance.
3. Dielectrics between the plates
   The relative permittivity ε_r , or dielectric constant, of a medium is equal to the ratio of the capacitance C of a given capacitor with the medium as dielectric to the capacitance C_0 of the capacitor with a vacuum as the dielectric:

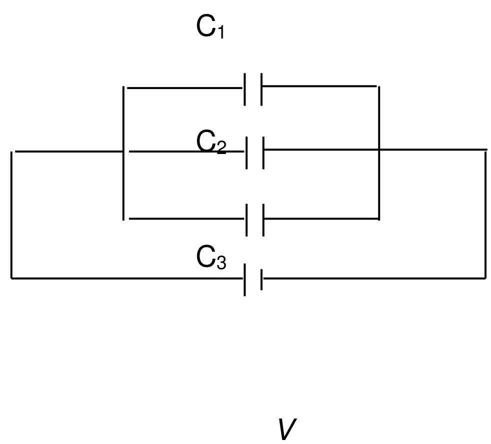
$$\varepsilon_r = \frac{C}{C_0} = \frac{\varepsilon}{\varepsilon_0} \quad \leftarrow \quad \text{dimensionless}$$

Table 10.1: Dielectric Constants of Some Dielectric Materials
(Relative permittivity ε_r)

Material	ε_r
Air	1.00026
Glass	6-10
Mica	3-6
Paper	3.5
Porcelain	6.5
Rubber	3-35
Sulfur	4
Water	80-83

10.8 Combination of Capacitors

A. Capacitors in Parallel

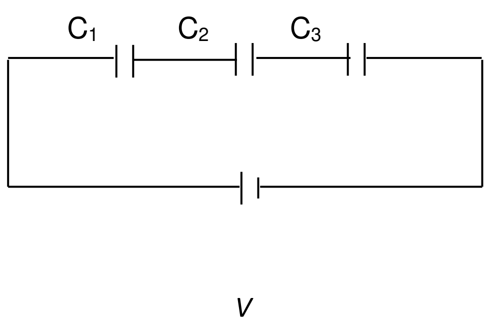


$$Q_T = Q_1 + Q_2 + Q_3$$

$$C_T = C_1 + C_2 + C_3$$

$$V_T = V_1 = V_2 = V_3$$

B. Capacitors in Series



$$Q_T = Q_1 = Q_2 = Q_3$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$V_T = V_1 = V_2 = V_3$$

10.9 Sample Problems with Solutions

(1) Calculate the capacitance of a capacitor consisting of two parallel plates separated by a layer of 0.5 *cm* thick, the area of each plate paraffin wax being 80 cm^2. The relative permittivity for the wax is 2 . (b) if the capacitor is connected to a 100 *V* source, calculate the charge on the capacitor and the energy stored in the capacitor.

Solution:

$$C = \varepsilon_r \varepsilon_0 \frac{A}{d} = 2\left(8.85 \times 10^{-12} \frac{C^2}{N m^2}\right)\left(\frac{80\, cm^2 \times 10^{-4}\, m^2}{0.005\, m}\right) = 2.8 \times 10^{-11}\, f$$

(2) A potential difference of 2.4×10^{4} *V* maintains a downward directed electric field between two horizontal parallel plates separated by 1.8 c*m*. Find the charge on an oil droplet of mass 2.2×10^{-13} *kg*, which remains stationary in the field between the plates.

Solution:

The Coulomb force $F = QE$ and $F = mg \rightarrow QE = mg \rightarrow Q = \frac{mg}{E}$

Moreover

$$E = \frac{V}{d} \rightarrow Q = \frac{mgd}{V} = \frac{\left(2.2 \times 10^{-13}\, kg\right)\left(9.8\, m/s^2\right)\left(0.018\, m\right)}{2.4 \times 10^4\, V} = 1.6 \times 10^{-18}\, C$$

(3) What distance must an electron move in a field of uniform potential gradient 20 *kV/m* in order to gain kinetic energy 3.2×10^{-18} *J*?

Solution:

$$W = Fr = (QE)r \quad \rightarrow \quad r = \frac{W}{QE}$$

potential gradient $= 20\frac{kV}{m}$ $\rightarrow$ Potential gradient has the same unit as $E = \frac{V}{d}$

$\rightarrow$ potential gradient = electric field *E*

Since work done *W* = change in energy = $\frac{1}{2}mv^2$

We finally get $r = \frac{\frac{1}{2}mv^2}{QE} = \frac{3.2 \times 10^{-18} J}{\left(1.6 \times 10^{-19} C\right)\left(2 \times 10^4 V/m\right)} = 1.0\,mm$

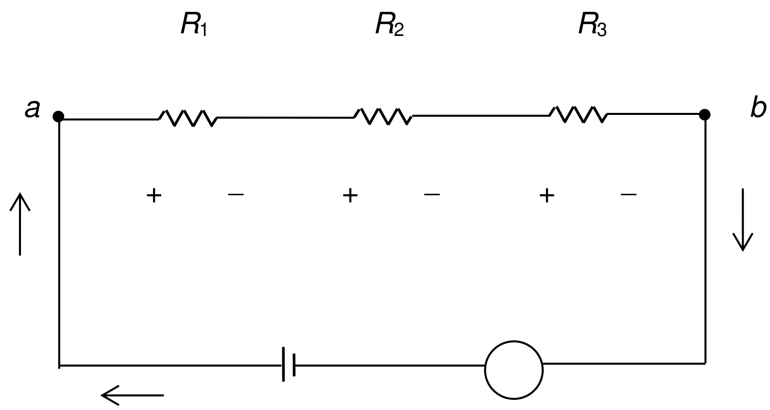
10.10 Connections of Resistances

Resistors or conductors can be connected in three different ways, namely:

a. series connection
b. parallel connection
c. series – parallel connection

10.10.1 Series Connection

In this case, resistors are joined one after the other, so that there is only one path of electric current among the conductors as shown below.



\+ −

A

I V

For series connection of resistors, current is constant in each resistor.

1. $I_{ab} = I_1 = I_2 = I_3$

Conductors have different voltages V_1 , V_2 , and V_3 so that

2. $V = V_{ab} = V_1 + V_2 + V_3$

From Ohm`s Law: $V = IR$ Therefore

$$V_{ab} = IR_{ab}$$

$$V_1 = I_1R_1 \quad , \quad V_2 = I_2R_2 \quad , \quad V_3 = I_3R_3$$

$$IR_{ab} = I_1R_1 + I_2R_2 + I_3R_3$$

since I is constant, and

3. $R_{ab} = R_1 + R_2 + R_3$

Summary:

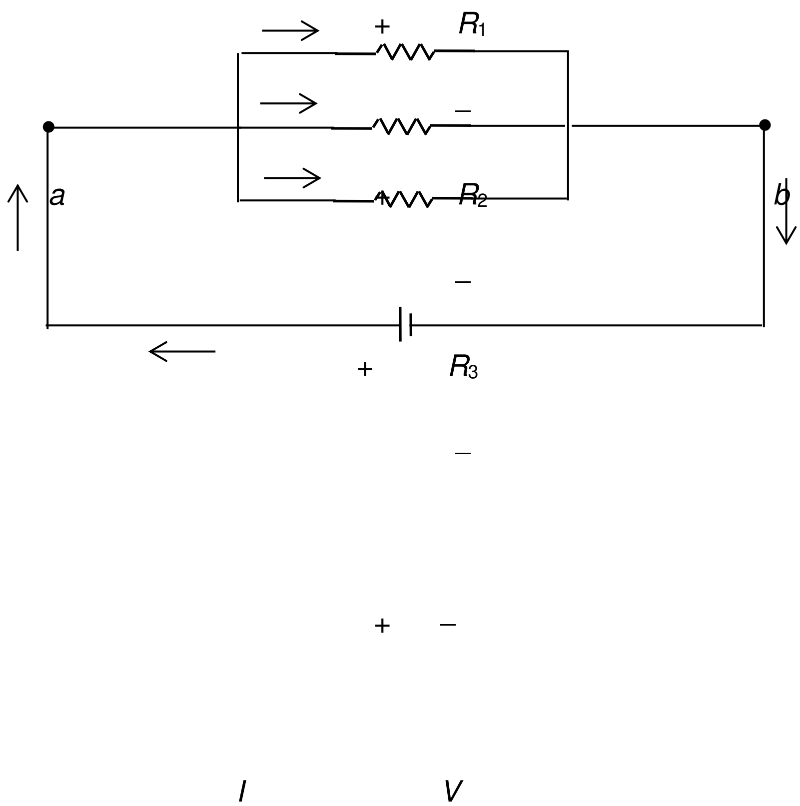
- The current in all resistors is constant.
- The voltage across the group is equal to the sum of the voltages across the individual resistors.
- The total resistance of the group is equal to the sum of the individual resistances.

10.10.2 Parallel Connection

In this particular case, resistors are connected between two common points a and b

as

shown below.



The voltage across each resistor is constant and is equal to the voltage across the whole group

1. $V_{ab} = V_1 = V_2 = V_3$

The total current I through the entire group is equal to the sum of the currents in each resistor, so that

2. $I = I_1 + I_2 + I_3$

By Ohm's Law $I = \frac{V}{R}$

Therefore

$$\frac{V_{ab}}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

Since the potential drop across each resistor and the whole circuit are equal, they all cancel out, giving us the following expression:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

R is called the *equivalent resistance* of the circuit.

Summary

- The total current is equal to the sum of the individual currents.
- The total voltage across the whole resistor is equal to the individual voltage in each resistor.
- The reciprocal of the total resistance is equal to the sum of the reciprocal of the resistances in each resistor.

10.10.3 Series–Parallel Connection

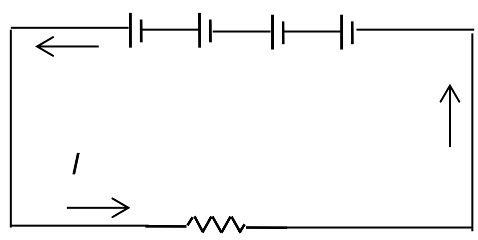
Determine the terminal where the current enters the group of resistance and the terminal where the current leaves the circuit, considering a constant current for resistances in series, and for resistances in parallel they should have two common terminals.

10.11 Connections of Dry Cells and Batteries

Batteries and group of cells may be connected in series or in parallel. Cells to be connected should be identical.

10.11.1 Cells in Series

In this case, the positive terminal of one cell is connected to the negative terminal of the next and the positive terminal of the last cell is connected to the negative terminal of the first cell and a load resistance R.



$$I = \frac{N_{\xi}}{N_r + R}$$

R

where N = total number of cells connected

ξ = emf of each cell

N_ξ = ξN = total emf

r = internal resistance of each cell

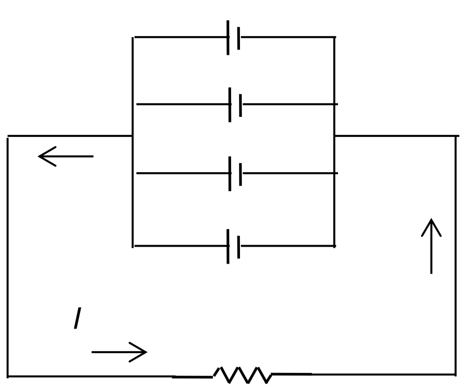
N_r = rN = total internal resistance

Moreover

$$I = I_1 = I_2 = I_3 = I_4$$

10.11.2 Cells in Parallel

In this particular case, all the positive poles of the cells are connected to a common junction and all the negative poles are connected to another common junction. The positive and negative junctions are then connected to a load resistor R.



$$I = \frac{\xi}{\frac{r}{N} + R}$$

where

N = total number of cells (identical)

ξ = emf of each cell

r = internal resistance of each cell

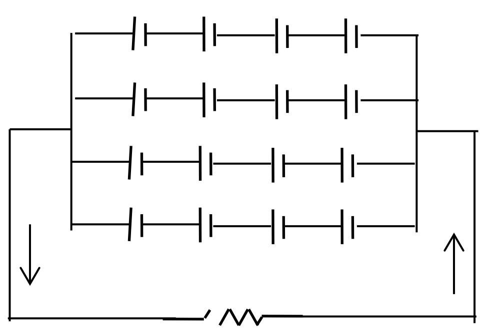
$\frac{r}{N}$ = combined internal resistance

R Moreover

I = $I_1 + I_2 + I_3 + I_4$

10.11.3 Cells in Series–Parallel Connection

$$I = \frac{\xi N}{\frac{rN}{n} + R}$$



where

N = no. of identical cells in series in one row

n = no. of rows in parallel

I

ξ = emf of each cell

\

r = internal resistance of each cell

R

10.11.4 Terminal Voltage (*TV*)

1. When delivering current (on discharge)

Terminal voltage = *emf* − voltage drop in internal resistance

$$TV = \xi - rI$$

2. When receiving current (on charging)

Terminal voltage = emf + voltage drop in internal resistance

$$TV = \xi + rI$$

3. When no current exists

Terminal voltage = emf of battery or generator

$$TV = \xi$$

10.12 Sample Problems with Solutions

(1) A 27 V battery of internal resistance 2 Ω supplies current to the resistance shown below. (a) Define the currents I_1, I_2, I_3 along the three branches *ab*, *cd*, *ef* respectively. (b) Determine the current and the potential difference across each of the resistances of branch *cd*.

Solution:

(a) From Fig. 10.12.1, we find the following:

$$\frac{1}{R_{eq1}} = \frac{1}{24} + \frac{1}{12} + \frac{1}{8} = \frac{1+2+3}{24} = \frac{6}{24} = \frac{1}{4} \qquad \rightarrow \qquad R_{eq1} = 4\,\Omega$$

$$\frac{1}{R_{eq2}} = \frac{1}{10} + \frac{1}{40} = \frac{4+1}{40} = \frac{5}{40} = \frac{1}{8} \qquad \rightarrow \qquad R_{eq1} = 8\,\Omega$$

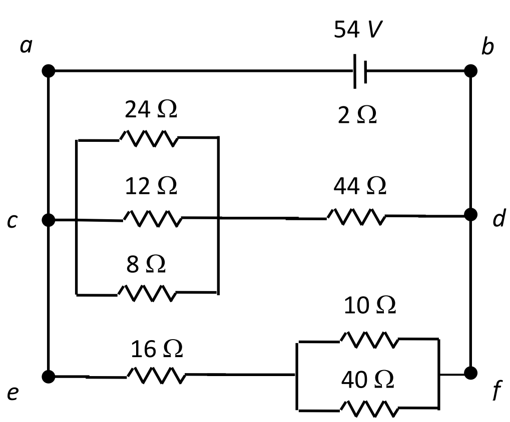


Fig. 10.12.1

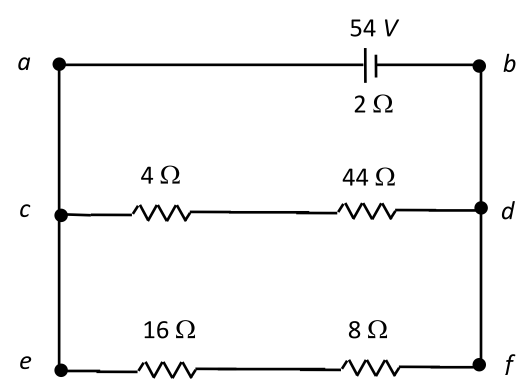


Fig. 10.12.2

From Fig. 10.12.2, we find the following:

$$R_{cd} = 4\,\Omega + 44\,\Omega = 48\,\Omega \qquad , \qquad R_{ef} = 16\,\Omega + 8\,\Omega = 24\,\Omega$$

Moreover, from Fig. 10.12.3, we find that

$$\frac{1}{R_{eq3}} = \frac{1}{R_{eq1}} + \frac{1}{R_{eq2}} = \frac{1}{48} + \frac{1}{24} = \frac{1 + 2}{48} = \frac{3}{48} = \frac{1}{16} \qquad \rightarrow \qquad R_{eq3} = 16\,\Omega$$

It follows that

$$R_T = R_{eq3} + r = 16\,\Omega + 2\,\Omega = 18\,\Omega$$

$$I_T = \frac{V_T}{R_T} = \frac{54\,V}{18\,\Omega} = 3\,A$$

$$I_{ab} = I_T = 3\,A$$

$$TV = \xi - rI_T = 54\ V - 2\,\Omega\,(3\,A) = 48\ V$$

$$I_{cd} = \frac{V_{cd}}{R_{cd}} = \frac{TV}{R_{cd}} = \frac{48\,V}{48\,\Omega} = 1\,A$$

In addition $\quad I_{ef} = I_T - I_{cd} = 3\,A - 1\,A = 2\,A$

54 *V*

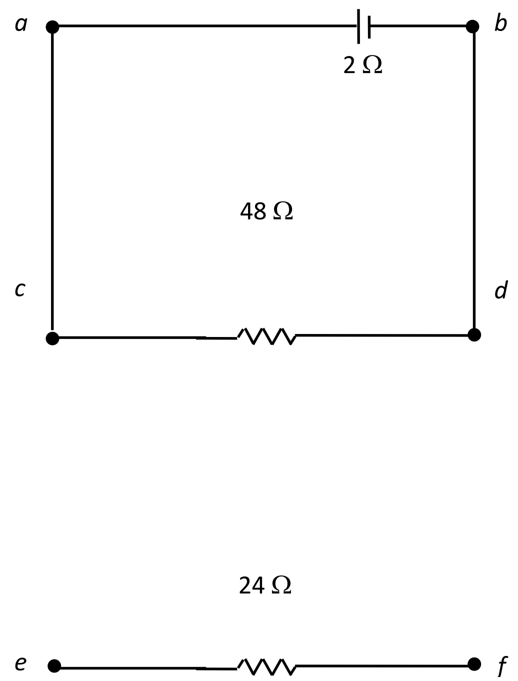


Fig. 6.12.3

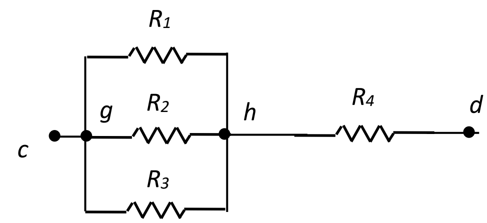


Fig. 10.12.4

b.) Along branch *cd* as shown in Fig 10.12.4, let

$$R_1 = 24\ \Omega \quad , \quad R_3 = 8\ \Omega$$

$$R_2 = 12\ \Omega \quad , \quad R_4 = 44\ \Omega$$

Therefore

$$V_4 \;=\; I_{cd}\,R_4 \;=\; (1\ A)\,(44\ \Omega) \;=\; 44\ V$$

$$V_{gh} = V_{cd} - V_4 = TV - V_4 = 48\ V - 44\ V = 4\ V$$

so that

$$V_1 = V_2 = V_3 = V_{gh} = 4\ V$$

Finally, we get

$$I_1 = \frac{V_1}{R_1} = \frac{4\,V}{24\,\Omega} = \frac{1}{6}\,A$$

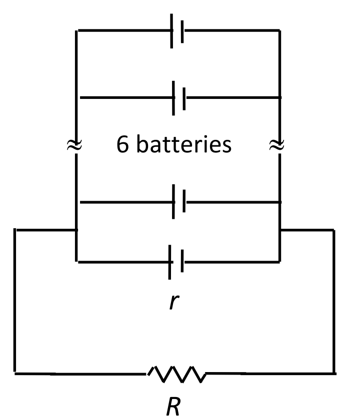
$$I_2 = \frac{V_2}{R_2} = \frac{4\,V}{12\,\Omega} = \frac{1}{3}\,A$$

$$I_3 = \frac{V_3}{R_3} = \frac{4\,V}{8\,\Omega} = \frac{1}{2}\,A$$

(2) Ten cells, each of *emf* 1.6 V and internal resistance $r = 0.08\ \Omega$, are connected in (a.) parallel (b.) series. The external resistance is 2 Ω. For each case, calculate the total resistance of the circuit, the *emf*, the terminal voltage of the battery, and the current in the external circuit.

Solution

(a) For the parallel connection (see Fig. 6.12.5)



$$R_T = \frac{r}{N} + R = \frac{0.08}{10} + 2 = 0.008 + 2 = 2.008\,\Omega$$

$$I_T = \frac{V_T}{R_T} = \frac{emf}{R_T} = \frac{1.6\,V}{2.008\,\Omega} = 0.8\,A$$

$$\frac{1}{r_T} = n\frac{1}{r} = 10\left(\frac{1}{0.08}\right) = 125 \quad \rightarrow \quad r_T = \frac{1}{125}\,\Omega = 0.008\,\Omega$$

Fig. 10.12.5 Ten cells in parallel

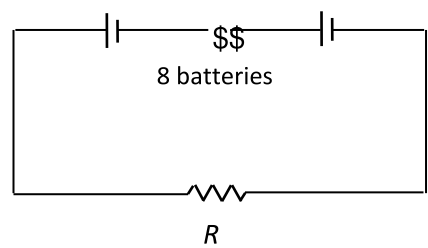
Therefore

$$TV = emf - I_T r_T = 1.6\ V - (0.8\ A)(0.008\ \Omega) = 1.6\ V - 0.0064\ V = 1.594\ V$$

(b) For the series connection (see Fig. 6.12.6), we get the following results:

$$I = \frac{N_\xi}{N_r + R} = \frac{\xi N}{rN + R} = \frac{(1.6\,V)(10)}{(0.08\,\Omega)(10) + 2\,\Omega}$$

$$= \frac{16}{2.8}A = 5.7\,A$$



$R_T = rN + R = (0.08\ \Omega)(10) + 2\ \Omega = 2.8\ \Omega$

$emf = \xi N = 10\ (1.6\ V)(10) = 16\ V$

$TV = emf - r_T I = 16\ V - (0.008\ \Omega)(5.7\ A)$

$= 15.95\ V$

Fig. 10.12.6 Ten cells in series.

10.13 Electrical Energy, Power and Heat

Work (W) in joules is the potential energy exerted by the charges flowing between two points at a potential difference ΔV.

$$W = P.E. = QV$$

$$Q = I\,t \qquad \rightarrow \qquad W = I\,t\,V$$

$$V = I\,R \qquad \rightarrow \qquad W = I^2\,t\,R$$

$$I = \frac{V}{R} \qquad \rightarrow \qquad W = \frac{V^2\,t}{R}$$

Units of Electrical Energy = Units Mechanical Energy in joule, erg, or BTU (see Chapter 4)

1 BTU = 252 calorie = 1.053 joule

1 cal = 4.18 joule

Power P is defined as the time rate of doing work.

$$P = \frac{W}{t} = \frac{E}{t}$$

Electrical power is expressed mathematically as follows:

- $P = V\,I$

- $P = I^2 R$
- $P = V^2 / R$

The unit of power is expressed in *watt* or *joule/sec* and *horsepower* (*hp*).

$$1\ kw = 1000\ watts = 1000\ joules/sec = 10^{10}\ ergs/sec$$

$$= 1.34\ hp = 4.420\ ft\text{-}lb/min = 5.67\ BTU/min$$

$$1\ hp = 0.746\ kw = 746\ watts$$

From the definition of power, we find that

$$W = Pt$$

Therefore, we also get the following units of energy:

$$1\ kilowatt\ hour\ (kwh) = 1000\ w \bullet hr = 3{,}600{,}000\ joules = 10^7\ ergs = 0.24\ calorie$$

$$1\ kwh = 1.34\ hp \bullet hr = 2.65 \times 10^6\ ft \bullet lb = 3460\ BTU$$

Finally, heat can be expressed as

$$H = W/k = VtI/k$$

where k = mechanical equivalent of heat = 4.18 *joules/cal* (1 *joule* = 0.24 *cal*).That is,

$$H = 0.24\ VtI = 0.24\ I^2 tR = 0.24\ (V^2/R)\ t$$

10.14 Sample Problems with Solutions

1. The power rating of a 10,000 Ω resistor is 4 *watts*. (a) What is the maximum allowable potential difference across the terminals of the resistor? (b) We need a 20,000 Ω resistor to be connected across the potential difference of 150 *V*. What power rating is required?

Solution:

(a.) $I = V/R \quad \rightarrow \quad P = VI = V^2/R,$

$$P = V^2/R \quad \rightarrow \quad V = \sqrt{PR} = \sqrt{(4\ W)(10{,}000\ \Omega)} = 200\ V$$

(b) $P = V^2/R = (150\ V)^2 / 20{,}000\ \Omega = 1.12\ W$

2. An electron is ejected at a potential of 20,000 *V*. What is its velocity?

Solution:

$$W = \tfrac{1}{2} m v^2 = QV$$

$$v = \sqrt{\frac{2QV}{m}} = \sqrt{\frac{2\left(1.6 \times 10^{-19}\ C\right)\left(20{,}000\ V\right)}{9.1 \times 10^{-31}\ kg}} = 8.39 \times 10^{7}\ m/s$$

10.15 Kirchhoff's Laws

In a more complicated network of resistors and sources of EMF, a systematic method of handling such networks was devised by Gustav Robert Kirchhoff, whose formulation is embodied in two laws known as Kirchhoff`s Laws.

Kirchhoff`s laws as applied to dc circuits may be stated as follows:

First Law: Conservation of Charge (Current Law)

At any point along the circuit, the sum of the currents directed toward a point is equal to the sum of the currents directed away from the same point (current directed toward a point is considered positive and current directed away from a point is negative).

$$\sum_i I_i = 0$$

Because charge cannot be created nor destroyed, charge cannot just appear from nowhere, nor can a charge already existing just simply disappear. It is therefore clear that whatever charges arrive at a particular point in a circuit are also the same charges that leave the same point.

Second Law: Conservation of Energy (Voltage Law)

It says that the algebraic sum of all the EMF`s of the batteries and the *IR* drops across the resistors that a charge encounters in moving along any closed loop of the circuit must vanish. Mathematically, this is expressed as follows:

$$\sum_i (EMF)_i + \sum_i (IR)_i = 0$$

According to equation (5-9), if we multiply the above expression with the magnitude of the charge we get a zero net change in its energy, so that the total energy of the charge remains constant as it goes around any closed loop of a given circuit.

Positive potential differences in resistors are those when going opposite to the direction of the current. Negative potential differences are those when going in the direction of the current, that is, from a point of higher potential to a lower potential. For EMF, the polarity of its voltage is positive when the path is from the negative to the positive terminal of the battery, negative when going from the positive to the negative terminal.

The procedures taken in applying Kirchhoff's laws are as follows

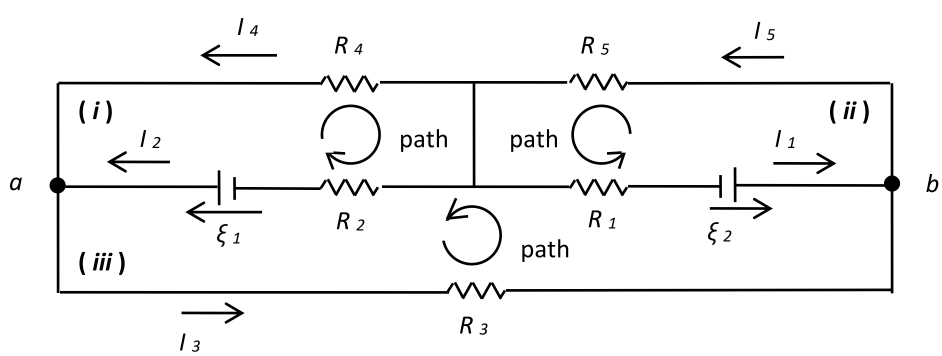
- Assume temporarily the directions of the branch currents and loop currents. A positive answer indicates that the assumed direction is correct while a negative answer indicates the opposite direction.
- Apply the first rule to a junction in a circuit where currents are directed either towards or away from that junction.
- Follow a consistent path, either clockwise or counterclockwise in going around the loop The sign of the EMF is considered positive when the loop current leaves the positive sign, and negative sign if instead, the loop current leaves the negative sign. A potential drop is considered positive if the branch current in the resistor is in the same direction as the loop current and negative if its direction is opposite to that of the loop current.
- In solving for the unknown variables, the set of equations produced can be solved by algebraic solutions.

Sample Problem

For the circuit shown on the following page,

(a) write down the equations according to Kirchoff's rules for the three subcircuits (*i*), (*ii*), and (*iii*) shown .Adopt the sign conventions as indicated by the circulating arrows indicated.

(b) Solve for the current in each resistor .



Take

$$R_1 = 1\ \Omega \qquad R_4 = 5\ \Omega \qquad \xi_1 = 6\ V$$

$$R_2 = 1\ \Omega \qquad R_5 = 4\ \Omega \qquad \xi_2 = 3\ V$$

$$R_3 = 3\ \Omega$$

Solution:

Assuming the direction of currents shown, we get the following results:

junction *a*: $I_4 + I_2 = I_3 \rightarrow I_4 = I_3 - I_2$

junction *b*: $I_3 + I_1 = I_5$

traversing loop (*i*) along the path shown:

$$R_4 I_4 - I_2 R_2 + \xi_1 = 0$$

$$R_4 (I_3 - I_2) - I_2 R_2 + \xi_1 = 0$$

$$5 (I_3 - I_2) - I_2 (1) + 6 = 0 \qquad \rightarrow \qquad 6 = 6 I_2 - 5 I_3$$

traversing loop (*ii*) along the path shown:

$$- R_5 I_5 - I_1 R_1 + \xi_2 = 0$$

$$- 4 I_5 - I_1 (1) + 3 = 0$$

$$3 = I_1 + 4 I_5$$

$$3 = I_1 + 4 (I_3 + I_1) \qquad \rightarrow \qquad 3 = 5 I_1 + 4 I_3$$

traversing loop (*iii*) along the path shown:

$$- I_3 R_3 - \xi_2 + R_1 I_1 - I_2 R_2 + \xi_1 = 0$$

$$-3 I_3 - 3 + (1) I_1 - (1) I_2 + 6 = 0$$

$$3 = 3 I_3 - I_1 + I_2$$

$$I_2 = 3 - 3 I_3 + I_1$$

Substituting the last result from the equation obtained from loop (*i*), we get

$$\begin{aligned} 6 &= 6 I_2 - 5 I_3 = 6(3 - 3 I_3 + I_1) - 5 I_3 \\ &= 18 - 18 I_3 + 6 I_1 - 5 I_3 \\ -12 &= 6 I_1 - 23 I_3 \\ 12 &= 23 I_3 - 6 I_1 \end{aligned}$$

From the last equation obtained and the equation derived from loop (*ii*), we get

$$\begin{array}{rl} & (12 = 23 I_3 - 6 I_1)(5) \\ + & (3 = 4 I_3 + 5 I_1)(6) \\ \hline & 78 = 139 I_3 \end{array}$$

$$I_3 = \frac{78}{139} A = 0.56 A$$

Substituting the above value for I_3 in the equation obtained from loop (*i*) , $I_2 = 1.46 A$

Substituting the value for I_3 in the equation obtained from loop (*ii*) , $I_1 = 0.15 A$

Substituting the values obtained for I_2 and I_3 in the equation obtained from junction *a*:

$I_4 = -0.90\ A$ (negative means the actual direction of current is opposite the one chosen)

Substituting the values obtained for I_1 and I_3 in the equation obtained from junction *b*:

$I_5 = 0.71\ A$

Lesson 10: Direct Current and Circuit Theory
Assessment

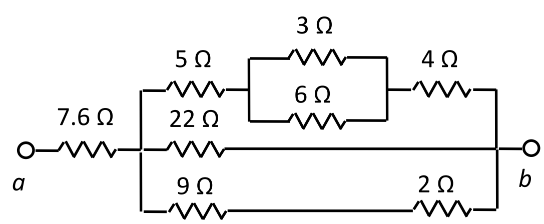
Instruction; Show your complete and neat solution. Identify your final abswer in the solution. See answer sheet format.

1. A 10 μF capacitor is charged to a voltage of 200 V and discharged thru a resistor. How much energy is given to the resistor during the discharge?

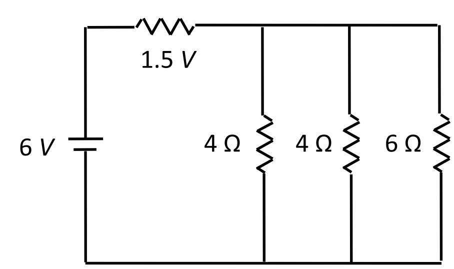
2. Three capacitors of capacitance 4 μF; 6 μF, and 12 μF, respectively, are charged to a 30 V battery. Find the energy of the stored charge when the capacitors are connected (a) in series and (b) in parallel.

3. Two capacitors of 2 μF and 3 μF are in parallel and the group is in series with another 3μF capacitor. The voltage across the system is 500 V. Find the charge and the voltage on such capacitor?

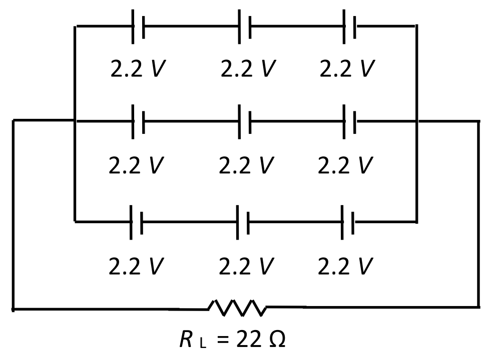
4. Find the equivalent resistance to the combination shown on the right.



5. The battery shown on the right has negligible internal resistance. Find (a) the current in each resistor and (b) the power delivered by the battery.



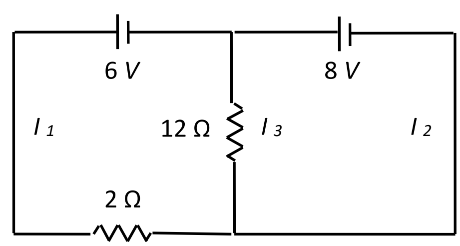
6. Find the load voltage V_L, the load current I_L, and the current supplied by each cell in the battery in the diagram shown. Assume that all cells are identical. Neglect the cells' internal resistances.



7. The capacity of a storage battery such as those used in automobile electrical system is rated in *ampere hour* (*A•h*). A 50 *A•h* battery can supply a current of 50 *A* for 1 hour. (a) What total energy is stored in a 12 *V* – 50 *A•h* battery if its internal resistance is negligible?

8. A 300 *W* driveway light is left on for 24 *hours.* How much does the energy cost if the power company's rate is 50.0 cents per kWh?

9. Find I_1, I_2, and I_3 for the circuit shown using Kirchhoff's Laws.



10. In the circuit shown, find (a) the current in each resistor, (b) the potential difference between points *a* and *b*, and (c) the power supplied by each battery.