Long Quiz No. 1

Answer Key

Total Score: 100

Note: Less 1 point for every wrong term

1.) $\int (8x^5-5x^4+12x^2-16x+21)\,dx$

Solution:

$\int 8x^5\,dx - \int 5x^4\,dx + \int 12x^2\,dx - \int 16x\,dx + \int 21\,dx$

$8\int x^5\,dx - 5\int x^4\,dx + 12\int x^2\,dx - 16\int x\,dx + 21\int dx$

$\frac{8x^6}{6} - \frac{5x^5}{5} + \frac{12x^3}{3} - \frac{16x^2}{2} + 21x + C$

$\boxed{\frac{4}{3}x^6 - x^5 + 4x^3 - 8x^2 + 21x + C}$

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2.) $\int x^2(x^3-1)^{11}\,dx$

Solution:

let $u = x^3-1$ 2

$du = 3x^2\,dx$ 2

$n.f. = 1/3$ 2

So,

$\frac{1}{3}\cdot\frac{(x^3-1)^{12}}{12} + C$ 2

$\therefore \boxed{\frac{(x^3-1)^{12}}{36} + C}$ 2

3.) $\int \frac{dx}{x\ln^2 x}$

Solution: $\int (\ln x)^{-2}\frac{dx}{x}$ 2

let $u = \ln x$ 2

$du = \frac{1}{x}\,dx$ or $\frac{dx}{x}$ 2

So, apply power formula,

$\frac{(\ln x)^{-2+1}}{-2+1} + C$ 2

$\frac{(\ln x)^{-1}}{-1} + C$ 6

$\therefore \boxed{-\frac{1}{\ln x} + C}$ 2

4.) $\int \sqrt{1+2\sin 3x}\,\cos 3x\,dx$

Solution:

$\int (1+2\sin 3x)^{1/2}\cos 3x\,dx$

let $u = 1+2\sin 3x$ 2

$du = 6\cos 3x\,dx$ 2

$n.f. = 1/6$ 2

So, apply Power Formula,

$\frac{1}{6}\cdot\frac{(1+2\sin 3x)^{3/2}}{3/2} + C$ 2

$\therefore \boxed{\frac{(1+2\sin 3x)^{3/2}}{9} + C}$ 2

5.) $\int \frac{\cos^3 x}{1-\sin x}\,dx$

Solution:

Identity $\cos^2 x = 1-\sin^2 x$

$\int \frac{\cos^2 x\cdot\cos x}{1-\sin x}\,dx$ 2

$\int \frac{(1-\sin^2 x)\cdot\cos x}{1-\sin x}\,dx$ 2

$\int \frac{(1+\sin x)\cancel{(1-\sin x)}\cos x}{\cancel{1-\sin x}}\,dx$ 2

$\int (1+\sin x)\cos x\,dx$

$\int \cos x\,dx + \int \sin x\cos x\,dx$ 2

let $u = \sin x$ or $u = \cos x$

$du = \cos x\,dx$ $du = -\sin x\,dx$

$n.f. = -1$

Apply Power Rule

$\therefore \boxed{\sin x + \frac{\sin^2 x}{2} + C}$ or 2

$\boxed{\sin x - \frac{\cos^2 x}{2} + C}$

Alternative Solution:

5.) $\int \frac{\cos^3 x}{1-\sin x}\,dx$

$\int \frac{\cos^2 x \cdot \cos x}{1-\sin x}\,dx$ ②

$\int \frac{(1-\sin^2 x)\cos x\,dx}{1-\sin x}$ ②

$\int \frac{(1+\sin x)(1-\sin x)\cos x\,dx}{1-\sin x}$

$\int (1+\sin x)\cos x\,dx$ ②

let $u = 1+\sin x$ ①

$du = \cos x\,dx$ ①

So, apply power formula

$\boxed{\frac{(1+\sin x)^2}{2} + C}$ ②

6.) $\int e^{\sin 4x}\cos 4x\,dx$

Solution:

let $u = \sin 4x$ ②

$du = 4\cos 4x\,dx$ ②

$n.f. = 1/4$ ②

So,

$\boxed{\frac{1}{4}e^{\sin 4x} + C}$ ④

7.) $\int(\tan^2 x + \cot^2 x + 4)\,dx$

Solution: Use the identity

$\tan^2 x = \sec^2 x - 1$

$\cot^2 x = \csc^2 x - 1$

$\int \tan^2 x\,dx + \int \cot^2 x\,dx + 4\int dx$

$\int(\sec^2 x - 1)\,dx + \int(\csc^2 x - 1)\,dx + 4\int dx$ ① ① ①

$\int \sec^2 x\,dx - \int dx + \int \csc^2 x\,dx - \int dx + 4\int dx$

$\tan x - x - \cot x - x + 4x + C$ ① ① ① ① ①

$\therefore \boxed{\tan x - \cot x + 2x + C}$ ②

8.) $\int y\csc 3y^2 \cot 3y^2\,dy$

Solution:

$\int \csc 3y^2 \cot 3y^2\; y\,dy$

let $u = 3y^2$ ②

$du = 6y\,dy$ ②

$n.f. = 1/6$ ②

So,

$1/6\,(-\csc 3y^2) + C$

$\therefore \boxed{-\frac{1}{6}\csc 3y^2 + C}$ ④

or $\boxed{-\frac{\csc 3y^2}{6} + C}$

9.) $\int \frac{e^{3x}}{5+2e^{3x}}\,dx$

Solution:

let $u = 5 + 2e^{3x}$ 2

$du = 6e^{3x}\,dx$ 2

$n.f. = 1/6$ 2

So,

$\boxed{\frac{1}{6}\ln|5+2e^{3x}| + c}$

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10.) $\int \sin x \ \sin(\cos x)\,dx$

Solution:

$\int \sin(\cos x) \ \sin x\,dx$

let $u = \cos x\,dx$ 2

$du = -\sin x\,dx$ 2

$n.f. = -1$ 2

So,

$-1[-\cos(\cos x)] + c$

$\therefore \boxed{\cos(\cos x) + c}$

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