Long Exercise No. 2

Answer Key

Total Score: 80

Note: Less 1 point for every wrong term

1.) $\int x(2x-1)^7\,dx$

Solution: Integration by Parts

let $u = x$ 1 $\quad \int dv = \int (2x-1)^7\,dx$ 1

$du = dx$ 1 $\quad v = \dfrac{\frac{1}{2}(2x-1)^8}{8}$

So, $\quad v = \frac{1}{16}(2x-1)^8$ 1

$\int u\,dv = uv - \int v\,du$

$= \dfrac{x}{16}(2x-1)^8 - \dfrac{1}{16}\int (2x-1)^8\,dx$ 1 1

2 $= \dfrac{x}{16}(2x-1)^8 - \dfrac{1}{16}\cdot\dfrac{1}{2}\,\dfrac{(2x-1)^9}{9} + C$

$= \boxed{\dfrac{x}{16}(2x-1)^8 - \dfrac{1}{288}(2x-1)^9 + C}$ 2

2.) $\int x^3\sqrt{x^2+4}\,dx$

Solution: $\int x^3(x^2+4)^{1/2}\,dx$

let $u = x^2$ 1 $\quad \int dv = \int x(x^2+4)^{1/2}\,dx$ 1

$du = 2x\,dx$ 1 $\quad v = \dfrac{\frac{1}{2}(x^2+4)^{3/2}}{3/2}$

So, $\quad v = \frac{1}{3}(x^2+4)^{3/2}$ 1

$\int u\,dv = uv - \int v\,du$

$= x^2\left[\frac{1}{3}(x^2+4)^{3/2}\right] - \int \frac{1}{3}(x^2+4)^{3/2}\,2x\,dx$

2 $= \dfrac{x^2}{3}(x^2+4)^{3/2} - \dfrac{1}{3}\int (x^2+4)^{3/2}\,2x\,dx$

2 $= \dfrac{x^2}{3}(x^2+4)^{3/2} - \dfrac{1}{3}\,\dfrac{(x^2+4)^{5/2}}{5/2} + C$

2 $= \boxed{\dfrac{x^2}{3}(x^2+4)^{3/2} - \dfrac{2}{15}(x^2+4)^{5/2} + C}$

3.) $\int x^2\cos 2x\,dx$

Solution:

let $u = x^2$ 1 $\quad \int dv = \int \cos 2x\,dx$ 1

$du = 2x\,dx$ 1 $\quad v = \frac{1}{2}\sin 2x$ 1

So, $\int u\,dv = uv - \int v\,du$

$= x^2\left(\frac{1}{2}\sin 2x\right) - \int \frac{1}{2}\sin 2x\cdot 2x\,dx$ 1

$= \dfrac{x^2}{2}\sin 2x - \int x\sin 2x\,dx$ 1 1

let $u = x$ 1 $\quad dv = \sin 2x\,dx$ 1

$du = dx$ 1 $\quad v = -\frac{1}{2}\cos 2x$

$= \dfrac{x^2}{2}\sin 2x - \left[x\left(-\frac{1}{2}\cos 2x\right) - \int -\frac{1}{2}\cos 2x\,dx\right]$

$= \dfrac{x^2}{2}\sin 2x + \dfrac{x}{2}\cos 2x - \dfrac{1}{2}\int \cos 2x\,dx$ 1

$= \boxed{\dfrac{x^2}{2}\sin 2x + \dfrac{x}{2}\cos 2x - \dfrac{1}{4}\sin 2x + C}$

or $\boxed{\left(\dfrac{2x^2-1}{4}\right)\sin 2x + \dfrac{x}{2}\cos 2x + C}$ 1

4.) $\int x^2\ln x\,dx$

let $u = \ln x$ 1 $\quad dv = x^2\,dx$ 1

$du = \dfrac{dx}{x}$ 1 $\quad v = \dfrac{x^3}{3}$ 1

So, $\int u\,dv = uv - \int v\,du$

$= \ln x\left(\dfrac{x^3}{3}\right) - \int \dfrac{x^3}{3}\cdot\dfrac{dx}{x}$ 2

$= \dfrac{x^3\ln x}{3} - \dfrac{1}{3}\int x^2\,dx$

$= \dfrac{x^3\ln x}{3} - \dfrac{1}{3}\cdot\dfrac{x^3}{3} + C$ 2

$= \boxed{\dfrac{x^3\ln x}{3} - \dfrac{x^3}{9} + C}$ 2

5.) $\int e^{-x}\cos 2x\,dx$

Solution: Integration by Parts

let $u = e^{-x}$, $du = -e^{-x}dx$ 1 $\qquad dv = \cos 2x$, $v = \frac{1}{2}\sin 2x$ 1

So, $\int u\,dv = uv - \int v\,du$

$\int e^{-x}\cos 2x\,dx = e^{-x}\cdot\frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x(-e^{-x})dx$ 1

$= \frac{1}{2}e^{-x}\sin 2x + \frac{1}{2}\int e^{-x}\sin 2x\,dx$ 1

again, use Integration by Parts; let $u = e^{-x}$, $du = -e^{-x}dx$ $\qquad dv = \sin 2x\,dx$, $v = -\frac{1}{2}\cos 2x$ 1

Hence,

$\int e^{-x}\cos 2x\,dx = \frac{1}{2}e^{-x}\sin 2x + \frac{1}{2}\left[e^{-x}\left(-\frac{1}{2}(\cos 2x)\right) - \int -\frac{1}{2}\cos 2x(-e^{-x})dx\right]$ 1

$\int e^{-x}\cos 2x\,dx = \frac{1}{2}e^{-x}\sin 2x - \frac{1}{4}e^{-x}\cos 2x - \frac{1}{4}\int e^{-x}\cos 2x\,dx$ 1

$\int e^{-x}\cos 2x\,dx + \frac{1}{4}\int e^{-x}\cos 2x\,dx = \frac{1}{2}e^{-x}\sin 2x - \frac{1}{4}e^{-x}\cos 2x$ 1

$\frac{5}{4}\int e^{-x}\cos 2x\,dx = \frac{1}{2}e^{-x}\sin 2x - \frac{1}{4}e^{-x}\cos 2x$ 1

$\frac{4}{5}\left[\frac{5}{4}\int e^{-x}\cos 2x\,dx\right] = \left[\frac{1}{2}e^{-x}\sin 2x - \frac{1}{4}e^{-x}\cos 2x\right]\frac{4}{5}$

$\therefore \int e^{-x}\cos 2x\,dx = \boxed{\frac{2}{5}e^{-x}\sin 2x - \frac{1}{5}e^{-x}\cos 2x + C}$ 1

or $\boxed{\frac{1}{5}e^{-x}(2\sin 2x - \cos 2x) + C}$

6.) $\int \frac{dx}{(x+2)^{3/4} - (x+2)^{1/2}}$

Solution: Algebraic Substitution Case 2

let $x+2 = z^4$ 1

$dx = 4z^3dz$ 1

So, $\int \frac{4z^3dz}{z^3 - z^2}$ 1

$4\int \frac{z^3dz}{z^3 - z^2} \div \frac{z^2}{z^2}$

$4\int \frac{z\,dz}{z-1}$ 1

$4\int \frac{[(z-1)+1]dz}{z-1}$ 1

$4\int \left(1 + \frac{1}{z-1}\right)dz$ 1

$4\int dz + 4\int \frac{dz}{z-1}$ 1

$4z + 4\ln|z-1| + c$ 2

but $z^4 = x+2 \Rightarrow z = \sqrt[4]{x+2}$

$\therefore \boxed{4\sqrt[4]{x+2} + 4\ln|\sqrt[4]{x+2} - 1| + c}$ 1

Alternative Solution:

5.) $\int e^{-x}\cos 2x\,dx$ Use Integration by Parts

let $u = \cos 2x$
$du = -2\sin 2x\,dx$ } 1

$\int dv = \int e^{-x}\,dx$
$v = -e^{-x}$ } 1

So, $\int u\,dv = uv - \int v\,du$

$$\int e^{-x}\cos 2x\,dx = (\cos 2x)(-e^{-x}) - \int(-e^{-x})(-2\sin 2x)\,dx \quad 1$$

$$= -e^{-x}\cos 2x - 2\int e^{-x}\sin 2x\,dx \quad 1$$

again, using Integration by parts

let $u = \sin 2x$
$du = 2\cos 2x\,dx$

$\int dv = \int e^{-x}\,dx$
$v = -e^{-x}$ } 1

Hence,

$$\int e^{-x}\cos 2x\,dx = -e^{-x}\cos 2x - 2\left[(\sin 2x)(-e^{-x}) - \int -e^{-x}(2\cos 2x\,dx)\right] \quad 1$$

$$\int e^{-x}\cos 2x\,dx = -e^{-x}\cos 2x + 2e^{-x}\sin 2x - 4\int e^{-x}\cos 2x\,dx \quad 1$$

$$\int e^{-x}\cos 2x\,dx + 4\int e^{-x}\cos 2x\,dx = -e^{-x}\cos 2x + 2e^{-x}\sin 2x \quad 1$$

$$5\int e^{-x}\cos 2x\,dx = -e^{-x}\cos 2x + 2e^{-x}\sin 2x \quad 1$$

$$\frac{\cancel{5}\int e^{-x}\cos 2x\,dx}{\cancel{5}} = \frac{-e^{-x}\cos 2x + 2e^{-x}\sin 2x}{5} + C$$

$$\therefore \int e^{-x}\cos 2x\,dx = \boxed{\frac{-e^{-x}\cos 2x + 2e^{-x}\sin 2x}{5} + C}$$

1

7.) $\int \frac{dx}{x-\sqrt[3]{x^2}}$ Case 1

Solution: Algebraic Substitution

let $z=\sqrt[3]{x} \Rightarrow z^3=x \Rightarrow x=z^3$ 2

$dx=3z^2dz$ 2

So, $\int \frac{3z^2dz}{z^3-\sqrt[3]{(z^3)^2}}$

$3\int \frac{z^2dz}{z^3-z^2}$ 2

$3\int \frac{z^2dz}{z^3-z^2} \div \frac{z^2}{z^2}$

$3\int \frac{dz}{z-1}$ 2

$3\ln|z-1|+c$ 1 but $z^3=x$

$z=\sqrt[3]{x}$

Now,

$\boxed{3\ln|\sqrt[3]{x}-1|+c}$ 1

8.) $\int \frac{(x-x^3)^{1/3}}{x^4}dx$ Case 3

Solution: Algebraic Substitution

let $x=\frac{1}{z}$ or z^{-1} 2

$dx=-z^{-2}dz$ or $-\frac{1}{z^2}dz$ 2

So, $\int \frac{\left(\frac{1}{z}-\frac{1}{z^3}\right)^{1/3}\left(-\frac{dz}{z^2}\right)}{\frac{1}{z^4}}$ 1

$-\int \left(\frac{z^2-1}{z^3}\right)^{1/3}\frac{dz}{z^2}\cdot z^4$

$-\int \frac{(z^2-1)^{1/3}}{z}\cdot z^2dz$ 1

$-\int (z^2-1)^{1/3}\,z\,dz$ Apply Power Formula

let $u=z^2-1$

$du=2z\,dz$ 1

$n.f.=1/2$

Hence, $\frac{-\frac{1}{2}(z^2-1)^{4/3}}{4/3}+c$ 1

$-\frac{3}{8}(z^2-1)^{4/3}+c$ 1 but $z=1/x$

Now, $\boxed{-\frac{3}{8}\left(\frac{1}{x^2}-1\right)^{4/3}+c}$ 1

or $\boxed{-\frac{3}{8}\left(\frac{1-x^2}{x^2}\right)^{4/3}+c}$