

## CHAPTER 6: MATHEMATICS OF FINANCE

### LEARNING OBJECTIVES

1. Distinguish between simple and compound interest.
2. Compute simple and compound interest problems.
3. Calculate interest rates and understand the time value of money.

### 6.1 INTEREST

It may be defined as the share or percentage of money or goods being invested or deposited which serves as incentive to clients.

**Examples 6.1:** Profit from an investment, pawned jewelries which are earning interest, royalty from sales of certain goods, withholding tax, and interest on money deposited in a bank, etc.

#### Kinds of Interest

1. Simple Interest
2. Compound Interest

**Simple Interest** – refers to share or percentage that does not earn another interest. It is computed by finding the product of the original capital and the rate of interest and the length of time expressed in terms of years, in symbols,

$$I = P r t$$

Where **P** – original principal, **r** – rate of interest and **t** – time in terms of years

**Examples 6.2:** Find the interest given the following:

- a. ₱ 10,000 at 4% for 2 years
- b. ₱ 275,000 at 12% in 5 years

**Solution:** Substituting the given data to the simple interest formula,

- a.  $I = P r t = (10,000)(0.04)(2) = \text{₱}800.00$
- b.  $I = P r t = (275,000)(0.12)(5) = \text{₱}165,000.00$

### 6.2 SIMPLE INTEREST FOR PERIODS OF TIME EXPRESSED IN DAYS OR MONTHS

Interest is usually computed based on **360-day/year which is equivalent to 12 months with each month having 30 days**. The interest for 30 days is computed as 30/360 or 1/12 of the interest for one year, for 60 days is 60/360 or 1/6 of the interest for one year, etc.

***Remember:** In the simple interest formula, time  $t$  is measured in the same period as the interest rate. Therefore, if the time period of a loan with an annual interest rate is given in days, it is necessary to convert the time period of the loan to a fractional part of a year.*

**Example 6.3:** Find the interest given the following conditions:

- a. Php 750 at 10% for 60 days
- b. Php 6,500 at 15% interest for 2 years and 3 months

**Solution:** We first convert 60 days and 2 years and 3 months to years before substitution

- a.  $t = 60 \text{ days} \times \frac{1 \text{ yr}}{360 \text{ days}} = \frac{1}{6}$   
 $I = P r t = (750)(0.1) \left(\frac{1}{6}\right) = \text{₱ } 12.50$
- b.  $t = 2 + \frac{3}{12} = 2 + 0.25 = 2.25 \text{ yrs}$   
 $I = P r t = (6,500)(0.15)(2.25) = \text{₱ } 2,193.00$

**Example 6.4:** Which will give more interest? An investment of Php 2,500 at 8.5% for 120 days or Php 2,500 at 9% for 60 days?

**Solution:** Solving for the interest in each of the given

Given: $P = \text{P } 2,500$	$P = \text{P } 2,500$
$r = 8.5\%$	$r = 9\%$
$t = 120 \text{ days}$	$t = 60 \text{ days}$
$I = Prt = (2,500)(0.085)\left(\frac{120}{360}\right) = \text{P } 70.83$	$I = Prt = (2,500)(0.09)\left(\frac{60}{360}\right) = \text{P } 37.50$

**Therefore, investment of Php 2,500 at 8.5% for 120 days gives more interest.**

### 6.3 Future Value or Maturity Value

The total amount due after  $t$  years can be computed as follows:

$$A = P + I$$

$$A = P + Prt$$

$$A = P (1 + rt)$$

**Where:**  $A$  – future value or maturity value,  $P$  – principal and  $I$  – total interest earned for the given number of years.

**Note 1:** This formula can be used for loans or investments. When used for a loan,  $A$  is the total amount to be repaid to the lender; this sum is called the **maturity value** of the loan.

**Note 2:** For an investment, such as a deposit in a bank savings account,  $A$  is the total amount on deposit after the interest earned has been added to the principal. This sum is called the **future value** of the investment.

Is the stated sum a maturity value or a future value?

- a. The sum of the principal and the interest on an investment?  
Answer: Future value
- b. The sum of the principal and the interest on a loan?  
Answer: Maturity value

**Example 6.5:** Find the maturity value of the loan after investing Php 15,000 at 8% interest for 3 years.

**Solution:** First, find the interest then add it to the principal to get the maturity value,

$$I = Prt = (15,000)(0.08)(3) = \text{P } 3,600.00$$

$$A = P + I = 15,000 + 3,600 = \text{P } 18,600.00$$

**Example 6.6:** At what rate must Php 4,500 be invested to accumulate to Php 4,900 in 6 months?

**Solution:** We first compute for the interest, then solve for the rate as follows,

From $A = P + I$	$I = A - P = 4,900 - 4,500 = \text{P } 400$
Then From $I = Prt$	$r = \frac{I}{Pt} = \frac{400}{4,500\left(\frac{6}{12}\right)} = \frac{400}{2,250} = 17.78\%$

**Example 6.7:** For what length of time must Php 11,200 be invested at 9% to have a sum of ₱12,300?

**Solution:** We first solve for  $I$  as follows then solve for  $t$ ;

From $S = P + I$	$I = S - P = 12,300 - 11,200 = \text{P } 1,100$
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From  $I = P r t$

$$t = \frac{I}{Pr} = \frac{1,100}{11,200(0.09)} = \frac{1,100}{1,008} = 1.09 \text{ years}$$

Note that 0.09 year may be converted into months:  $0.09 \times 12 \text{ month} = 1.08 \text{ month}$

Therefore, 1.09 years is equal to 1 year and 1 month.

**Example 6.8:** Find the future value after 1 year of ₱ 42,500 in an account earning 8.2% simple interest.

**Solution:** Substitute the given to the formula  $A = P (1 + rt)$ .

$$A = 42,500 [ 1 + 0.082 (1) ] = 42,500 (1.082) = \text{₱}45,982.00$$

The future of value of the account after 1 year is **₱45,982.00**.

**Example 6.9:** I have Php 150,000 and invested half of it at 5% interest and the other half at 7%, find the accumulated value in 5 years and 4 months?

**Solution:** Solving for the accumulated value for each investment separately.

For investment ₱ 75,000 at 5% in 5 years and 4 months or  $5 + \frac{4}{12} = 5.33 \text{ years}$

$$I = P r t = (75,000)(0.05)(5.33) = \text{₱ } 19,987.5$$

$$S = P + I = 75,000 + 19,987.50 = \text{₱ } 94,987.50$$

For investment ₱ 75,000 at 7% in 5 years and 4 months or  $5 + \frac{4}{12} = 5.33 \text{ years}$

$$I = P r t = (75,000)(0.07)(5.33) = \text{₱ } 27,982.50$$

$$S = P + I = 75,000 + 27,982.50 = \text{₱}102,982.50$$

Therefore, the Accumulated Value after 5 years and 4 months,

$$A = 94,982.50 + 102,982.50 = \text{₱ } 197,965.00$$

6.4 EXACT AND ORDINARY METHOD; EXACT AND APPROXIMATE TIME

The **exact method** is based on using  $t$  as a fraction whose numerator is the number of days in the term of the transaction and whose denominator is the exact number of days (365) in a year. If 360 days is used it is called **ordinary method**.

Exact time is the actual number of days in the term of the transaction. Approximate time arbitrarily uses 30 days for each month.

Summarizing in the table that follows:

Exact Method	Ordinary Method	Exact Time	Approximate Time
$\frac{\text{No. of days}}{\text{exact no. of days/year}}$	$\frac{\text{No. of days}}{360}$	Actual no. of days	Uses 30 days for each month

**Note:** The ordinary method is used by most businesses. Therefore, unless otherwise stated, the ordinary method will be used in this text.

*For convention, the exact number of days between two dates shall be used while a whole number of months shall be considered between two months of the same date. For instance, from Jun 8 to October 12 is 126 (22 days + 31 days + 31 days + 30 days + 12 days) while from June 8 to October 8 is 4 months (July, August, September and October).*

**Example 6.10:** Find the exact number of days between the given dates

- a. May 24, 2015 to Nov. 5, 2015
- b. Dec. 10, 2018 to Feb. 18, 2019
- c. August 15, 2020 to March 15, 2021

Solution: The table below can be used to calculate number of days required in the problem,

Month	Days	Month	Days
Jan	31	July	31
Feb	28	Aug	31
March	31	Sep	30
Apr	30	Oct	31
May	31	Nov	30
Jun	30	Dec	31
		Total	365

- a. May 24, 2015 – Nov. 5, 2015:  $7 + 30 + 31 + 31 + 30 + 31 + 5 = \mathbf{165 \text{ days}}$
- b. Dec. 10, 2014 to Feb. 18, 2015:  $21 + 31 + 18 = \mathbf{70 \text{ days}}$
- c. August 15, 2020 to March 15, 2021:  
September, October, November, December, January, February, March ) = **7 months**

**Example 6.11:** Find the different types of simple interest earned by the amount of Php 45,000.00 at 5% from May 17 to December 13, 2014.

- a. Using exact time and ordinary method.
- b. Using exact time and exact method.
- c. Using approximate time and ordinary method.
- d. Using approximate time and exact method.

**Solution:**

- a. Using exact time and ordinary method:

For exact time, Actual No. of Days:

May 17 – Dec 13:  $(14 + 30 + 31 + 31 + 30 + 31 + 30 + 13) = \mathbf{210 \text{ days}}$

Since the ordinary method is required then, use  $\frac{\text{no.of days}}{360}$  ,

$I = P r t = (45,000)(0.05) \left(\frac{210}{360}\right) = \mathbf{\text{₱ } 1,312.50}$

- b. Using exact time and exact method:

For exact time, use 210 days and for exact method use  $\frac{\text{no.of days}}{365}$  ,

$I = P r t = (45,000)(0.05) \left(\frac{210}{365}\right) = \mathbf{\text{₱ } 1,294.52}$

- c. Using approximate time and ordinary method.

For Approximate time: May 17 – Dec 13;  $(14 + 30 + 30 + 30 + 30 + 30 + 30 + 13) = \mathbf{207 \text{ days}}$

$I = P r t = (45,000)(0.05) \left(\frac{207}{360}\right) = \mathbf{\text{₱ } 1,293.75}$

- d. Using approximate time and exact method.

$I = P r t = (45,000)(0.05) \left(\frac{207}{365}\right) = \mathbf{\text{₱ } 1,276.03}$

6.5 COMPOUND INTEREST

It refers to the sum of all interest due at the end of the period be it monthly, quarterly, biannually or annually with the principal amount always combined in the computation with the interest. Thus, interest earns another interest. It is the interest paid on the original principal and on the accumulated past interest.

Understanding Compound Interest

Suppose you have ₱10,000.00 as initial deposit in a bank on January 1 with annual interest of 5%, and you would like to know how much would your money be at the end of 5 years?

If the bank would simply give 5%, your money would become ₱10,500.00 and if this amount remained in the bank for another year it would become ₱11,025.00. Notice that the remaining amount in the bank would continuously earn interest, and at the end of five years see the result in the table below, at the end of 5 years your money become ₱ 12,72.54.

Year	Beginning Balance, ₱	Interest Earned, ₱	Ending Balance, ₱
1	10,000.00	500.00	10,500.00
2	10,500.00	525.00	11,025.00
3	11,025.00	551.25	11,576.25
4	11,576.00	578.81	12,154.81
5	12,154.81	607.75	12,762.56

In the above computation, the bank pays interest on the balance at the end of the year, (based on the interest compounded annually). But bank would pay interest on your account balance at the end of every month, which means interest compounded monthly. So, what the bank would do is to divide the interest rate (5% in this problem) by 12 months and applies the rate to your balance at the end of each month.

Now let us see the following computations for the 12-month transaction.

Month	Beginning Balance, ₱	Interest Earned, ₱	Ending Balance, ₱
1	10,000.00	41.67	10,041.67
2	10,041.67	41.84	10,083.51
3	10,083.51	42.01	10,125.52
4	10,125.52	42.19	10,167.71
5	10,167.71	42.37	10,210.08
6	10,210.08	42.54	10,252.62
7	10,252.62	42.72	10,295.34
8	10,295.34	42.90	10,338.24
9	10,338.24	43.08	10,381.31
10	10,381.31	43.26	10,424.57
11	10,424.57	43.44	10,468.00
12	10,468.00	43.62	10,511.62

Notice that at the end of 1 year your money which is compounded monthly becomes ₱10,511.62 a difference of ₱11.62 from the previous computation which is compounded annually. At the end of 5 years the amount compounded monthly will be ₱ 12,833.59, even higher from the previous computation that is compounded annually.

It can be seen, that more frequent compounding occurs, the more interest is earned on a balance. A formula is given below to shorten the computation of compound interest.

**FORMULA**

$$F = P \left( 1 + \frac{j}{m} \right)^{mt}$$

**Where:**

- F – compound amount/amount of money accumulated after m years, including interest.
- P – principal/initial amount borrowed or deposited.
- j - rate of interest
- m – number of conversion periods per year/no. of times per year the interest is computed.
- t – time in years

**Examples 6.11:** A sales representative started saving for his trip to Singapore. Initially, she deposited ₱ 25,000 in a popular bank near her workplace. The bank offers an interest rate of 4.5 % twice a year, how much would her money after 2 years?

**Solution:** Substituting P = ₱ 25,000; j = 4.5%, m = 2, t = 2 years to the formula,

$$F = P \left( 1 + \frac{j}{m} \right)^{mt}$$

$$F = P \left( 1 + \frac{j}{m} \right)^{mt}$$

$$F = 25,000 \left( 1 + \frac{0.045}{2} \right)^{2(2)}$$

$$F = 25,000(1.0225)^4 = 25,000 (1.0930)$$

**Then, the Sales representatives money after 2 years: F = ₱ 27,327.08**

**Example 6.12:** A young entrepreneur start a bank account with ₱ 50,000 that compounds quarterly at an interest rate of 6%, how much money would the entrepreneur have at the end of 1 year? With the assumption that no additional deposits and withdrawals of money within the year.

**Solution:** Substitute the given P = ₱ 50,000, j = 6%, m = 4, and t = 1 to the formula,

$$F = P \left( 1 + \frac{j}{m} \right)^{mt}$$

$$F = 50,000 \left( 1 + \frac{0.06}{4} \right)^{4(1)}$$

$$F = 50,000(1.015)^4$$

$$F = ₱ 53,068.18$$

**Therefore, the entrepreneur’s bank account balance at the end of the year is ₱ 53,068.18.**

**Example 6:13:** If the balance in your savings account at the of 3 years is ₱ 73,500 and the interest rate is compounded monthly at 3.5%, what is the original amount deposited 3 years ago?

**Solution:** Substitute the given A = ₱ 73,500, j = 3.5%, m = 12 and t = 3 years to the formula,

$$73,500 = P \left( 1 + \frac{0.035}{12} \right)^{12(3)}$$

$$73,500 = P \left( 1 + \frac{0.035}{12} \right)^{36}$$

$$73,500 = 1.11 P$$

$$P = \frac{73,500}{1.11} = ₱ 66,216.22$$

**Therefore, the original amount deposited three years ago: P = ₱ 66,216.22**

**Example 6.14:** Your friend won ₱ 1,000,000 in a lottery. He seeks your advice on how to invest his money in a savings account. You accompanied him to the bank and the bank offers two plans as follows: In five years from now which plan will give the best offer?

Plan 1	Plan 2
j = 6%	j = 12%
m = monthly	m = every 2 months

**Solution:** The accumulated value for each is shown below:

Plan 1	Plan 2
$F = 1,000,000 \left(1 + \frac{0.06}{12}\right)^{12(5)}$ $F = 1,000,000(1.005)^{60}$ <b>F = ₱ 1,348,850.15</b>	$F = 1,000,000 \left(1 + \frac{0.12}{6}\right)^{6(5)}$ $F = 1,000,000(1.02)^{30}$ <b>F = ₱1,811,361.58</b>

Based on the above computation the best offer to earn more money is **Plan 2**.

**Example 6.15:** Calculate the future value of ₱ 5,000 earning 9% interest, compounded daily for 3 years.

**Solution:** Substitute the following P = ₱ 5,000, j = 9%, t = 3 years, m = 360

$$F = P \left(1 + \frac{j}{m}\right)^{mt}$$
$$F = 5000 \left(1 + \frac{0.09}{360}\right)^{360(3)}$$
$$F = 5000 (1.00025)^{1080}$$
$$F = 5000 (1.3099) = 6,549.60$$

**The future value after 3 years is ₱6,549.60.**

**Example 6.16:** Suppose your annual salary today is ₱ 1,750,000.00. You want to know what an equivalent salary will be in 20 years – that is the salary that will have the same purchasing power. Assume 6% inflation rate.

**Solution:** Using compound amount formula

$$F = P \left(1 + \frac{j}{m}\right)^{mt}$$

Substitute P = 1,750,000                      j = 6%                      t = 20 years,

The inflation rate is an annual rate, so m = 1

$$F = 1,750,000 \left(1 + \frac{0.06}{1}\right)^{1(20)}$$
$$F = 1,750,000(1.06)^{20} = 1,750,000 (3.207)$$
**F = ₱ 5,612,487.076**

**Two years from now, you need to earn an annual salary of approximately ₱5,612,487.076 in order to have the same purchasing power.**

**THE PRESENT VALUE**

The formula for finding the present value is derived from compounded interest formula,

**From**                      
$$F = P \left(1 + \frac{j}{m}\right)^{mt}$$

**The Present value formula**  $P = \frac{F}{\left(1 + \frac{j}{m}\right)^{mt}}$

Where:

- F – compound amount/amount of money accumulated after m years, including interest.
- P – principal/initial amount borrowed or deposited.
- j - rate of interest
- m – number of conversion periods per year/no. of times per year the interest is computed.
- t – time in years

**Example 6.17:** How much money should be invested in an account that earns 8% interest, compounded quarterly, in order to have ₱1, 500,000.00 in 5 years?

**Solution:** Use the present value formula,

$$F = 1,500,00 \qquad j = 8\% \qquad m = 4 \qquad t = 5$$

$$P = \frac{F}{\left(1 + \frac{j}{m}\right)^{mt}}$$

$$P = \frac{1,500,000}{\left(1 + \frac{0.08}{4}\right)^{4(5)}}$$

$$P = \frac{1,500,000}{(1.02)^{20}} = \frac{1,500,000}{1.48595} = 1,009,455.23$$

Then ₱1,009,455.23 should be invested in the account in order to have ₱1,500,000 in 5 years.



REVIEW EXERCISES 6.0

NAME \_\_\_\_\_ Score \_\_\_\_\_ Date \_\_\_\_\_ Course, Year & Section \_\_\_\_\_ Student no. \_\_\_\_\_ Professor \_\_\_\_\_

Solve the following problems.

- Find the interest using exact method and exact time and approximate time.
  - Php 11,200 at 7% from June 1 to Sept 5, 2014
  - Php 25,000 from April 20, 2015 to Aug 27, 2015.
- Find the interest using Ordinary method and exact time and approximate time.
  - Php 5,500 from Feb 5 to April 10, 2015
  - Php 375,000 from August 15 to November 15, 2014.
- At what rate of interest must be borrowed in order to double the amount of Php 75,000 in 5 years.
- How much was borrowed if the interest earned is Php 2,500 at 6% after 45 days.
- Find the compound amount and compound interest on the principal, Php 20,000 borrowed at 6% compounded annually for 3 years.
- Find the compound amount which would be obtained from an interest of Php 2,000 at 6% compounded quarterly for 5 years.
- The first credit card that you got charges 11.5 % interest rate to its customers that compounds monthly. As soon as the credit card is issued, you spent its maximum limit of ₱ 30,000. If no payments and other transactions made with the credit card, how much money would you owe to the credit card company after 6 months?
- An initial deposit of ₱ 150,000 is made into a savings account that compounds annually at an interest rate of 9%. How much is the account at the end of five years?
- After 10 years of 5% interest compounded monthly a savings account becomes ₱ 5,000. How much is the original deposit?
- A young boy started saving money from his daily allowance and when totaled ₱ 1,000, he decided to deposit in a bank. He can choose from two bank offers. The first offer is 5.3% compounded continuously for 6 years. The second is compounded quarterly for five years at 6%. Which account will yield higher amount? How much is the difference between the two offers?
- What is the compound amount on ₱ 15,000 invested at 4% with  $m = 4$  for 1 year and 8 months?
- What is the present value of ₱ 300,000 for 5 years compounded twice a year at 7.2%
- A couple plans to invest money for their child’s college education. What principal must be deposited by the parents when their child turns 10 in order to have ₱ 330,000 when the child reaches the age of 18? Assume that the money earns 8% interest, compounded quarterly.
- What principal must be deposited by parents of a newborn in order to have ₱ 248,000 in 18 years? Assume that the money earns 3%, compounded monthly.
- What principal must be deposited by parents of a newborn in order to have ₱ 150,000 in 18 years? Assume that the money earns 2.5%, compounded daily.
- Calculate the simple interest due on a 45-day loan of ₱35,000 if the annual interest rate is 8%.
- Calculate the maturity value of a simple interest, 8-month loan of \$8000 if the interest rate is 9.75%.
- A loan of ₱ 78,200 is due this coming September, what payment must be made in this coming July to repay the loan if the rate is 5.25% compounded semi-annually if the loan was made in January of the same year?
- A woman borrowed ₱ 25,000 last February 1, 2020 at 12% interest rate where  $m = 2$ . What is the total amount of loan in February 1, 2021?

20. Pretty, a grade 1 student who celebrated her 7<sup>th</sup> birthday received a total ₱ 2,500.00 gift from her Aunts and Uncles. She decided to put it in a bank that offers 3.5% interest compounded monthly. How much would her money if she decides to withdraw the money on his 15<sup>th</sup> birthday?

**References:**

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2. The Nature of Mathematics by Karl J. Smith, 12<sup>th</sup> Edition
3. Business Mathematics by Bautista, Gorospe, etc.
4. Mathematics in Society by Lippman, David. 2<sup>nd</sup> edition, 2017