Formal $Epsilon - N$ Proof

Prove: $\lim_{n\to\infty} \frac{2n}{n+1} = 2$

Compute:

$$\begin{aligned}\left|\frac{2n}{n+1} - 2\right| &= \left|\frac{2n-2(n+1)}{n+1}\right| \\ &= \left|\frac{2n-2n-2}{n+1}\right| \\ &= \left|\frac{-2}{n+1}\right| \\ &= \frac{2}{n+1}\end{aligned}$$

Given any $\varepsilon > 0$, choose N such that

$$\frac{2}{N+1} < \varepsilon$$

Solving for N gives:

$$N + 1 > \frac{2}{\varepsilon} \implies N > \frac{2}{\varepsilon} - 1$$

A convenient choice is:

$$N = \left\lceil \frac{2}{\varepsilon} \right\rceil$$

Then for every $n \geq N$, we have

$$|a_n - 2| \leq \frac{2}{n+1} \leq \frac{2}{N+1} \leq \varepsilon$$

Thus, $a_n \to 2$ as $n \to \infty$.

$$\boxed{\textbf{This completes the proof for } a_n = \frac{2n}{n+1}\textbf{.}}$$

The sequence converges to 2, as required.