Analyzing 9-1-1 Call Data Using Bayesian Regression

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November 28, 2020

Introduction

Whitcom 9-1-1 is a regional 9-1-1 dispatch center located in Pullman, WA. The agency receives emergency (9-1-1) and non-emergency phone calls from the public. With this information, the telecommunicators dispatch first responders such as police officers and firefighters. All in all, Whitcom dispatches for law, fire, and EMS agencies spread across seven counties in Washington and Idaho. Pullman is also home to the PAC-12 school, Washington State University. Employees at Whitcom usually expect busy shifts on Saturday nights in the fall, especially when there is a home football game.

The scope of this analysis is to evaluate regression models using Bayesian and frequentist methods to predict the Whitcom call volume for the hours of noon to midnight on Saturdays in the months September[1] through December.

Data

The data set used in this analysis is one that I prepared using phone call data acquired from a Whitcom public records request, and football schedules pulled from the Washington State University Athletics' website. There are 636 observations from the years 2017 through 2019. The columns of the data are as follows:

- Count: The number of phone calls received.
- Date: The observed date.
- Hour: The observed hour of the day (12 - 23).
- Month: The observed month of the year (9 - 12).
- Game.Day: 0 = Not a game day; 1 = A game day

Poisson Regression Models

I considered four predictive models using the combination of *Hour*, *Month*, and *Game.Day*. Using R notation, they are:

- Model 1: `Count ~ Hour`
- Model 2: `Count ~ Hour + Month`

[1] Although home football games can occur in late August, Whitcom rotates to a new schedule at the beginning of September. For simplicity, this analysis excluded August to match the schedule rotation.

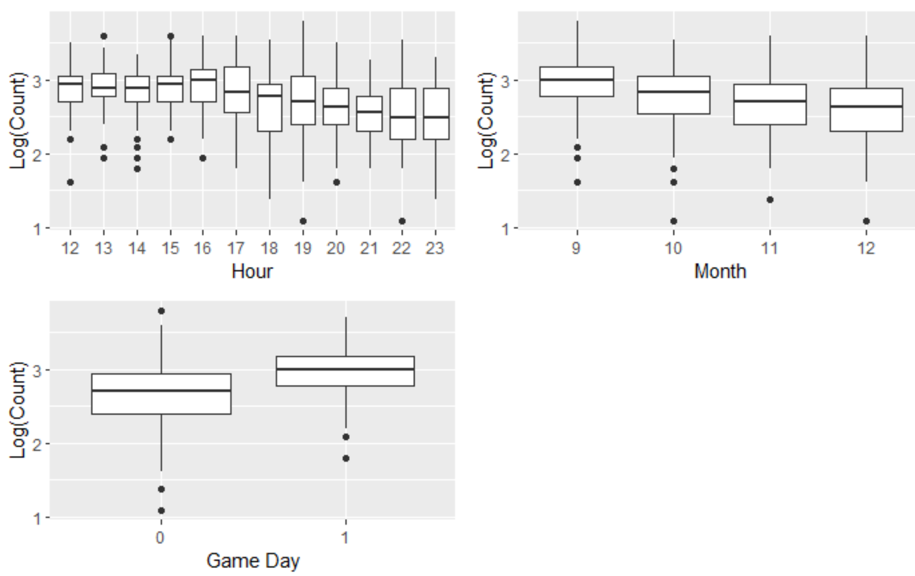
- Model 3: `Count` $\sim$ `Hour` + `Game.Day`

- Model 4: `Count` $\sim$ `Hour` + `Month` + `Game.Day`

Before creating any models, I split the data using an 80/20 random training-testing split. For the Bayesian models, I used the library, *rjags*, to fit the model and calculate the predictions. For the frequentist models, I used the standard *glm* function using the Poisson family parameter.

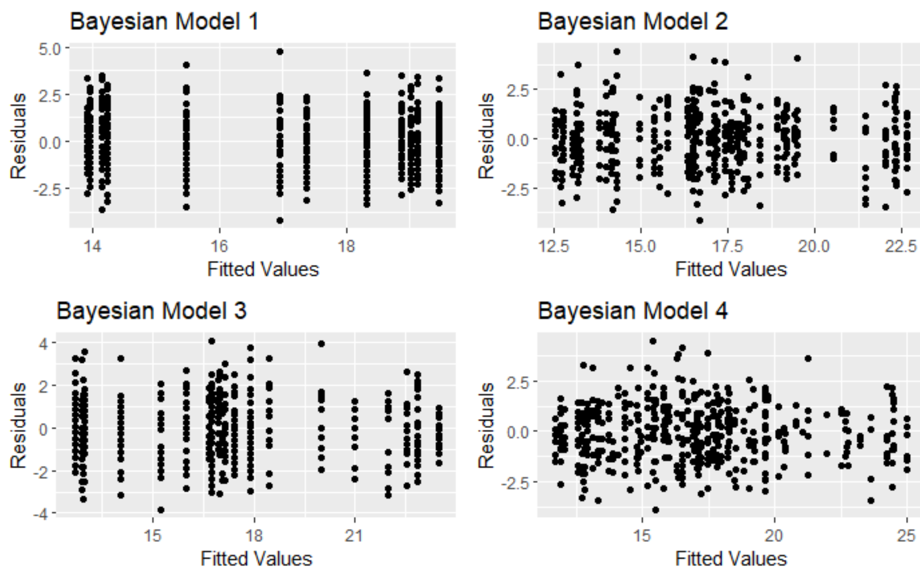
In Poisson regression, the response (Y_i) are event counts per unit of time with observations from a Poisson distribution. The assumptions of a Poisson regression model are linearity ($\log(E[Y|X] = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_p x_p$), independence ($Y_1, Y_2, ..., Y_n$ are independent random variables), and equal variance ($E[Y|x_1, ..., x_p] = V[Y|x_1, ..., x_p]$).

Treating the three predictors, *Hour*, *Month,* and *Game.Day*, as categorical variables, we can plot the box plots of $\log(Count)$ against each predictor and evaluate for any relationships.



It appears that there may be a predictive relationship between each predictor and the log of the response.

We cannot guarantee independence due to the nature the data was collected, but we can evaluate whether or not the responses may be correlated by looking at the residuals of the models.



The above graphic displays the deviance residuals against the fitted values for the four Bayesian models.[2] The tighter groupings suggest correlation might exist; however, the bulk of the residual data suggests variability remains relatively constant across the fitted values. Since the residuals are scattered about the zero line, we also show linearity.

When evaluating goodness-of-fit, I found all eight models failed the deviance goodness-of-fit test. The dispersion parameter for each model was approximately 2 suggesting each model was overdispersed. Overdispersion is when the variability of the response is greater than implied. In other words, the equal variance assumption failed in all eight models.

Negative Binomial Regression

One solution to overdispersed Poisson regression models is to use the negative binomial distribution, $NB(p,r)$, instead. The negative binomial distribution loosens the equal variance assumption of the Poisson regression model (Yufeng). The parameter p can be defined as $p = \frac{r}{E[Y|x_1,\ldots x_p]+r}$. Instead of creating a Bayesian model as

$$Y_i \sim Pois(\lambda_i)$$

$$\log(\lambda_i) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_p x_{ip}$$

[2]Plots from the frequentist models produced similar plots.

under the Poisson distribution, we can model the data under the negative binomial distribution as

$$\begin{aligned}
Y_i &\sim NB(p_i, r) \\
\log(\lambda_i) &= \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_p x_{ip} \\
p_i &= \frac{r}{r + \lambda_i}.
\end{aligned}$$

Residual analysis for all eight of the new models fared well[3]. Additionally, each model passed the deviance goodness-of-fit test. With regards to BIC, deviance, mean squared prediction error, and precision, Model 4 outperformed Models 1, 2, and 3. The Bayesian model had smaller BIC and deviance values, suggesting a better fit than the frequentist model; however, the frequentist model performed marginally better with regards to the prediction scores (less than 0.02% better).

The final coefficient estimates for Model 4 are listed below with significant coefficients (at the 95% confidence level) marked with an *.

Estimate	Bayesian	Frequentist
Intercept	2.966*	2.960*
Hour13	-0.016	-0.011
Hour14	-0.085	-0.079
Hour15	-0.007	-0.001
Hour16	0.018	0.023
Hour17	-0.038	-0.032
Hour18	-0.229*	-0.222*
Hour19	-0.161*	-0.154*
Hour20	-0.289*	-0.283*
Hour21	-0.323*	-0.316*
Hour22	-0.313*	-0.306*
Hour23	-0.309*	-0.303*
Month10	-0.075	-0.074
Month11	-0.190*	-0.189*
Month12	-0.121*	-0.121*
Game.Day	0.247*	0.247*

[3]The frequentist models were fit using the *glm.nb* function from the *MASS* library.

Conclusion

For run of the mill point prediction, the frequentist approach is fast, easy, and it gets the job done. The Bayesian approach on the other hand takes a lot more computational effort while getting you a nearly identical result. However, if you were more interested in the true distribution of calls received rather than an approximation, i.e. the posterior distribution, the Bayesian approach would be the better choice. In that event, we would be more interested in the parameters p and r for the negative binomial distribution rather than its expectation, λ.

Furthermore, the preferred model of this analysis included the *Game.Day* predictor. While game days may be a driving force for an increased call volume on Saturdays in fall, an uptick is not always solely caused by the presence of a home game. For example, on the night of August 31st, 2019 there was a home football game in Pullman and a helicopter crash 30 miles south of Pullman in another county. The helicopter crashed into the Snake River, a shared jurisdiction between Whitcom and a neighboring 9-1-1 center. As one of the telecommunicators working that night, I can tell you there was an increase in call volumes. To my knowledge, there was no connection between the crash and the football game. Increases due to unrelated factors such as this crash may be difficult to identify and model in future models.

Additionally, incoming call volumes are not a perfect representation of activity level in the 9-1-1 center. Whitcom telecommunicators are also responsible for all radio communication while they handle incoming calls, and are also responsible for making outbound calls in the course of their duties. It is possible for there to be an imbalance between incoming call volume and the rest of the employee's duties. For the sake of brevity, those factors were excluded from this analysis.

Works Cited

"2017 Football Schedule." Https://Wsucougars.com/Sports/Football/Schedule/2017.

"2018 Football Schedule." Https://Wsucougars.com/Sports/Football/Schedule/2018.

"2019 Football Schedule." Https://Wsucougars.com/Sports/Football/Schedule/2019.

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