

Econ 330: Urban Economics

Lecture 5

John Morehouse
January 20th, 2021

Lecture V: Rents

Schedule

Today

- 1) **Intro to Rents**
- 2) City "shape"

Upcoming

- **!! HWI due** (Sunday, Jan 28) **!!**
- **Reading** (Chapter IV *ToTC*)

Taking Stock

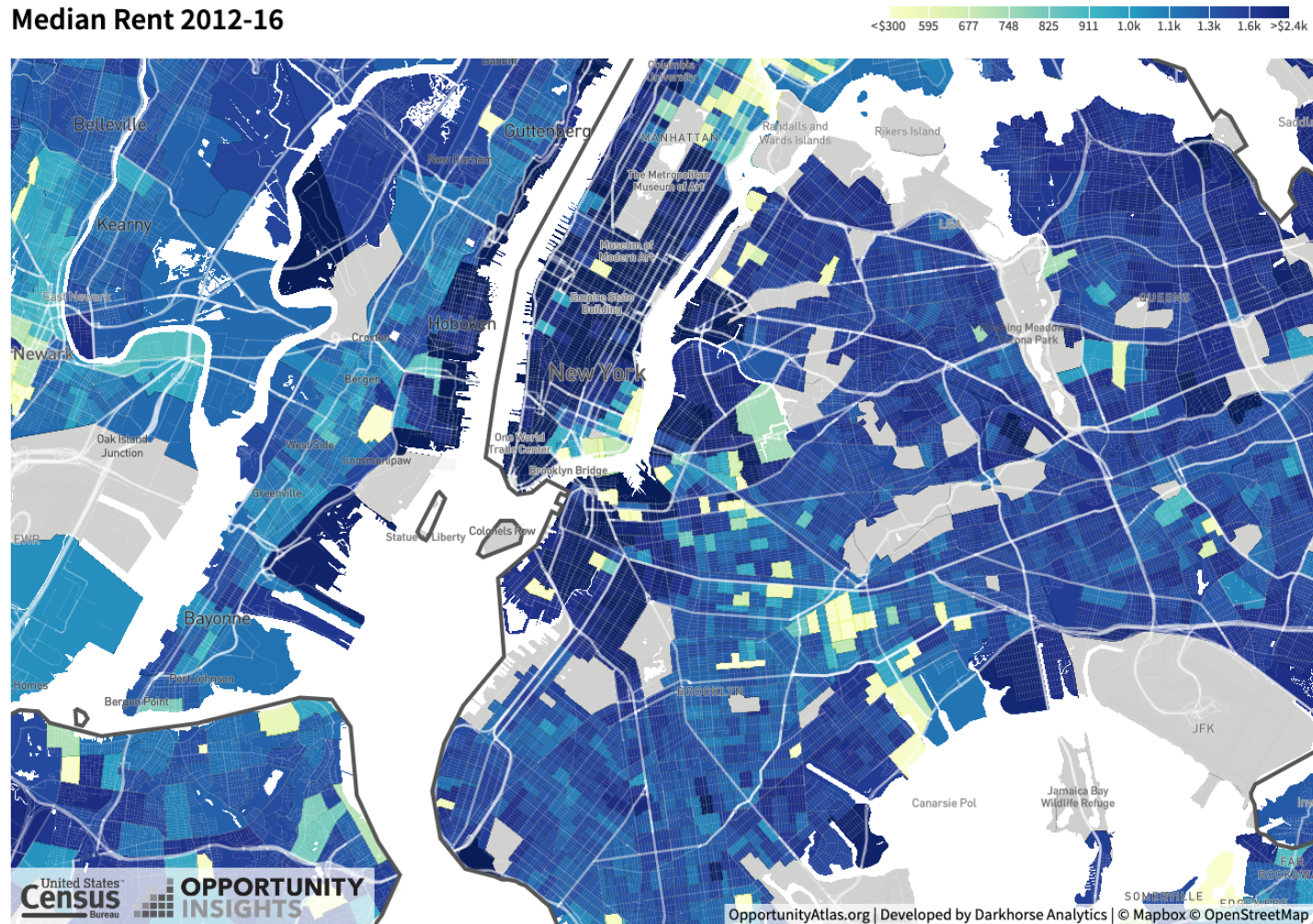
First Two Weeks: Introduction and **existence, size & growth** (philosophical-ish questions)

Now: Fundamentals of **city shape**. **Questions**

- Why do people and firms choose a particular location? What influences these choices?
 - **Today:** How do these choices impact rental prices **within** cities
 - **Later:** How do these choices impact rental prices **across** cities
 - Formalize this. Learn **basics** of **discrete choice theory**
- Can we explain the current *and* historical "shape" of cities?

Rents: NY

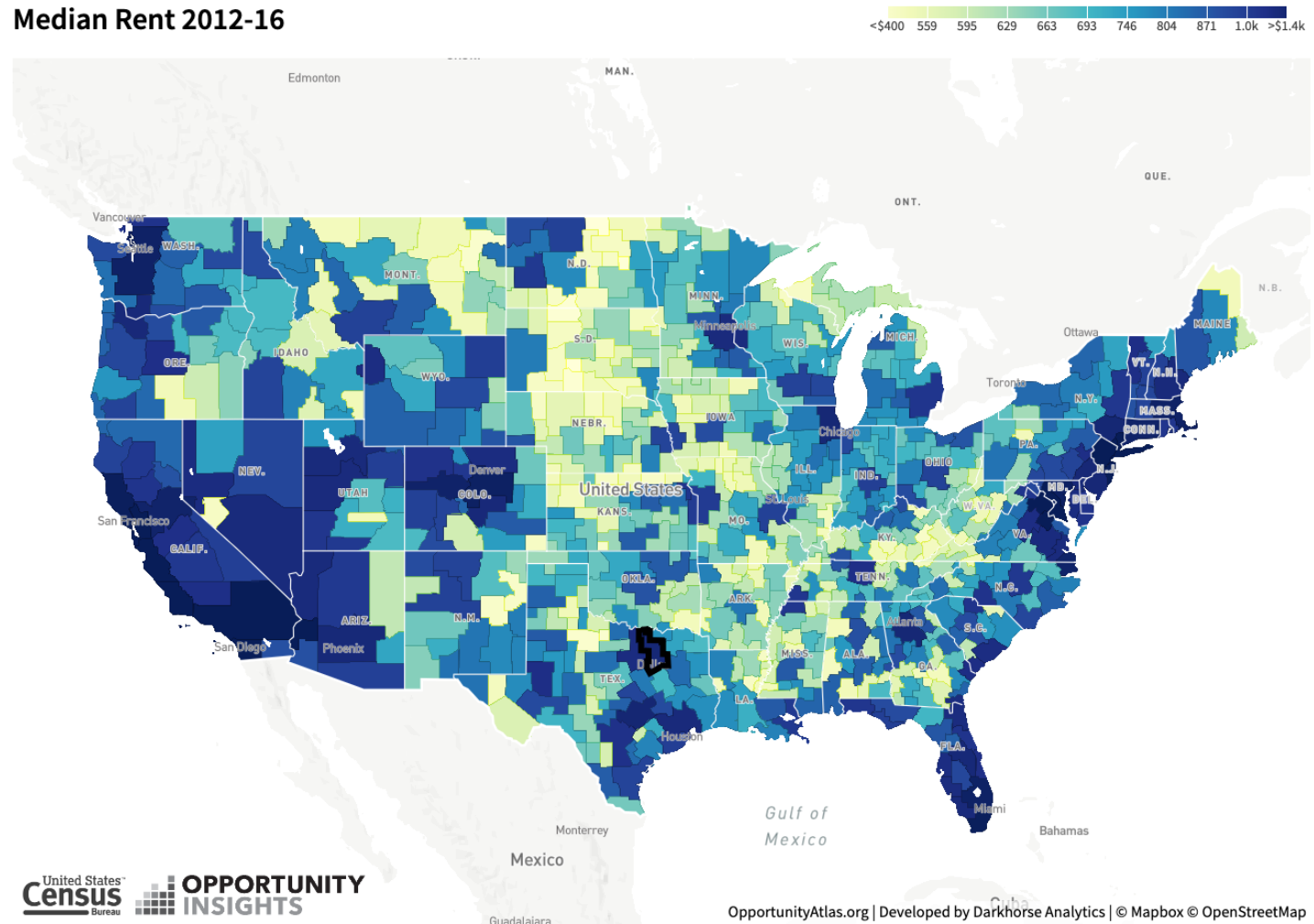
Median Rent 2012-16




source: [Opportunity Atlas](https://www.opportunityatlas.org/)

Rents: US

Median Rent 2012-16



Checklist

- 1) **Intro to Rents** 
- 1.5) **Bonus: COVID19 and Cities**
- 2) **Consumer Bid Rent**
- 3) **Manufacturing Bid Rent**
- 4) **City Shape**

Bonus: COVID19 and Cities research

Questions:

Q1) How does COVID19 impact housing/rental prices?

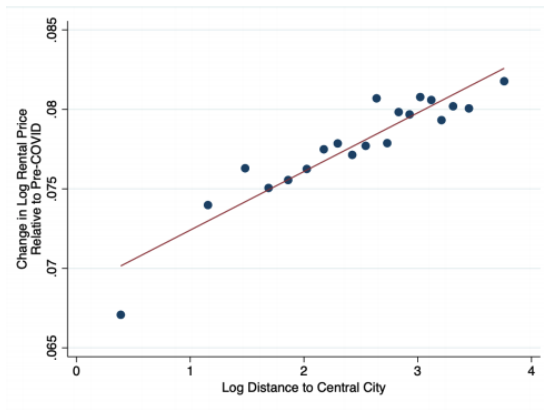
- Is the effect the same everywhere? Why or why not?

Q2) How many jobs can be done remotely? Does this vary systematically across sectors? Cities?

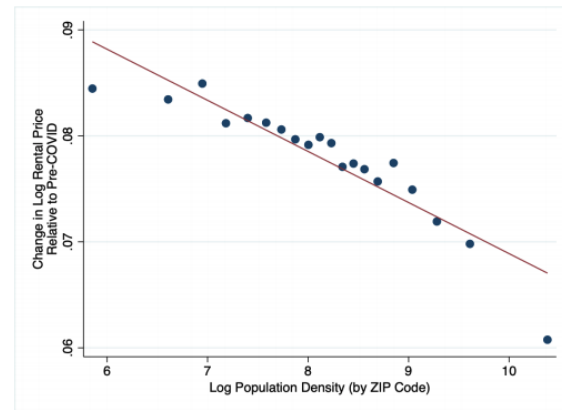
Q3) What do we think will happen to city structure as a result of increased (potentially permanent) WFH

Bonus: COVID19 and Cities Research

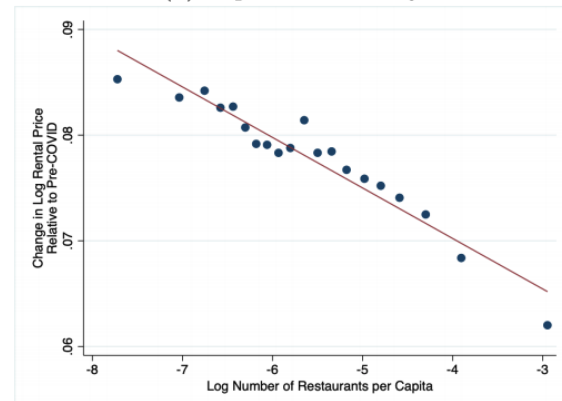
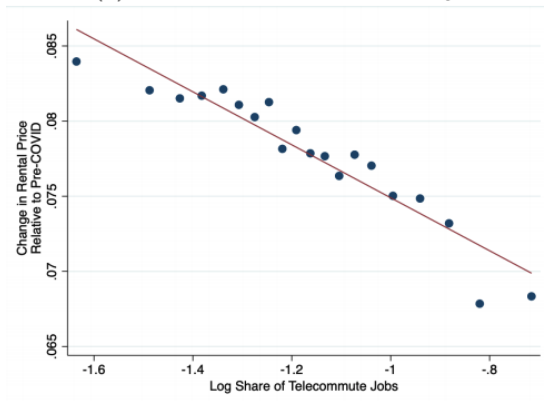
- **A1:** *The Impact of the COVID-19 Pandemic on the Demand for Density: Evidence from the U.S. Housing Market* (Liu & Su, 2020)



(a) Distance to the Central City

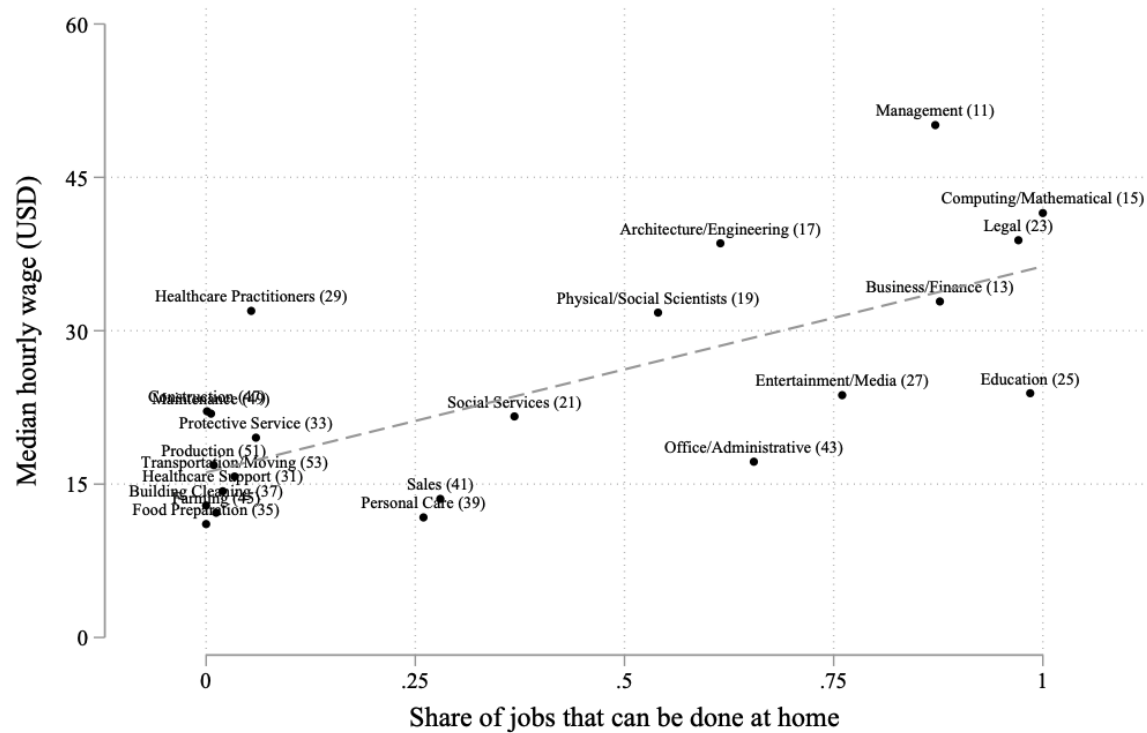


(b) Population Density



Bonus: COVID19 and Cities research

- **A2:** *How many jobs can be done at home?* (Dingel & Nieman, 2020)

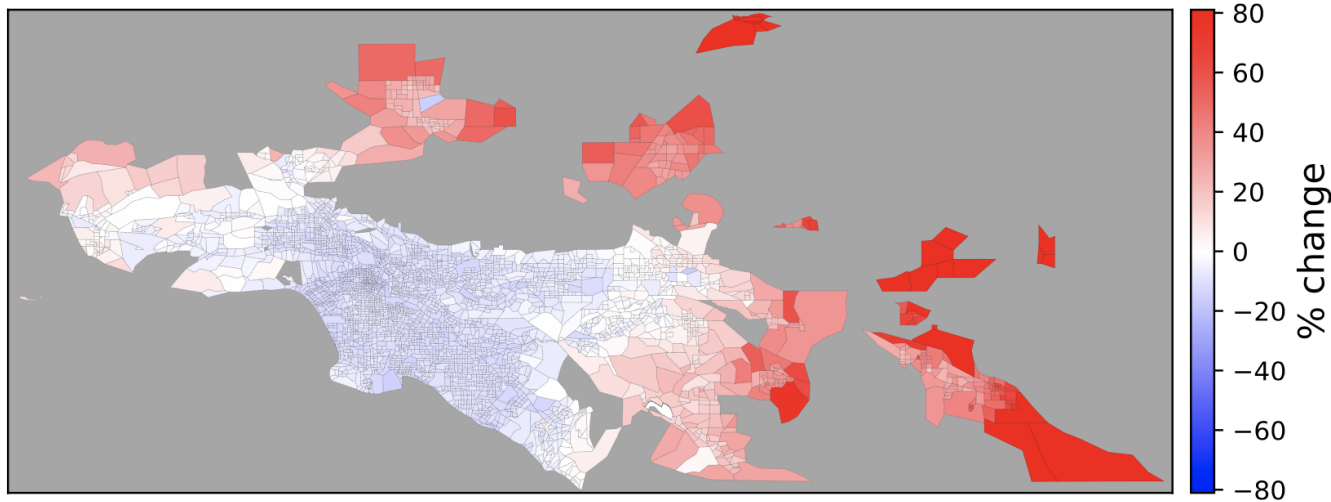


Bonus: COVID19 and Cities research

Q3) What do we think will happen to city structure as a result of increased (potentially permanent) WFH?



- **A3:** *How Do Cities Change When We Work from Home?* (Delventhal et. al, 2020)

Figure 2: House prices



Note: Percentage change relative to benchmark economy in counterfactual with $\psi = 0.33$. See main text for details.

Checklist

- 1) **Intro to Rents** 
- 1.5) **Bonus: COVID19 and Cities** 
- 2) **Consumer Bid Rent**
- 3) **Manufacturing Bid Rent**
- 4) **City Shape**

The Bid-Rent Curve

A **Bid - Rent Curve** is the *relationship between housing prices and the distance of land from the city center*[†]

These curves vary across sectors

- **Consumer Bid:** commuting costs
- **Manufacturing:** Accessibility to consumers and suppliers
- **Tech/info/offices:** Accessibility to Information

[†] It actually does not have to be the city center -- can be a point of attraction. In this class we will always use the city center though.

Housing Prices Model

We now build a simple model of rental/housing prices **within** a city

- 1) Commuting cost is **only location factor** in decision making
 - **All locations** are otherwise identical
- 2) Only **one member** of household commutes to employment area
- 3) Only considers the **monetary (not time) cost of commuting**
- 4) Noncommuting travel is **insignificant**
- 5) Public services, **taxes, amenities** are the **same everywhere** (implication from 1)

Locational Indifference

Axiom 1: *Housing prices adjusts until there is locational indifference (and prices in general)*

- IE: until an increase in rent for a closer location just offsets the lower commuting costs

In math:

$$\Delta P \cdot h + \Delta x \cdot t = 0$$

- P: **price** of housing (price per square foot)
- h: **amount** of housing (in ft^2)
- x: **distance** to employment area
- t: **commuting cost** per mile

Slope of the Housing Bid-Rent Curve

If there is locational indifference we can derive the **slope** of the **bid-rent** curve:

$$\underbrace{\Delta P \cdot h}_{\text{Marginal change in housing cost}} + \underbrace{\Delta x \cdot t}_{\text{Marginal change in commuting cost}} = 0$$

Slope of the Housing Bid-Rent Curve

If there is locational indifference we can derive the **slope** of the **bid-rent** curve:

$$\begin{aligned}\Delta P \cdot h + \Delta x \cdot t &= 0 \\ \Delta P \cdot h &= -\Delta x \cdot t\end{aligned}$$

Slope of the Housing Bid-Rent Curve

If there is locational indifference we can derive the **slope** of the **bid-rent** curve:

$$\begin{aligned}\Delta P \cdot h + \Delta x \cdot t &= 0 \\ \Delta P \cdot h &= -\Delta x \cdot t \\ \frac{\Delta P}{\Delta x} &= -\frac{t}{h}\end{aligned}$$

Notice: $\frac{\Delta P}{\Delta x}$ is the **slope** of the **bid-rent** curve

- price is on the vertical axis, distance is on the horizontal. So this is rise over run

$\Delta P \cdot h = -\Delta x \cdot t$: Another way of putting this: $MC = MB$!

Another Derivation

Suppose you have decided that the optimal amount of money to spend on housing and commuting per month is M^*

- You can allocate this as

$$P \cdot h + x \cdot t = M^*$$

- Since we graph the bid rent curve in the (x, P) space, we solve for P :

$$P \cdot h + x \cdot t = M^*$$

$$P \cdot h = M^* - x \cdot t$$

Another Derivation

Suppose you have decided that the optimal amount of money to spend on housing and commuting per month is M^*

- You can allocate this as

$$P \cdot h + x \cdot t = M^*$$

- Since we graph the bid rent curve in the (x, P) space, we solve for P :

$$P \cdot h + x \cdot t = M^*$$

$$P \cdot h = M^* - x \cdot t$$

$$P = \frac{M^*}{h} - \frac{t}{h} \cdot x$$

- Slope: $\Delta P = 0 - \frac{t}{h} \cdot \Delta x \implies \frac{\Delta P}{\Delta x} = -\frac{t}{h}$
 - Can also take derivative if p w.r.t x and get the same thing, if that is easier for you

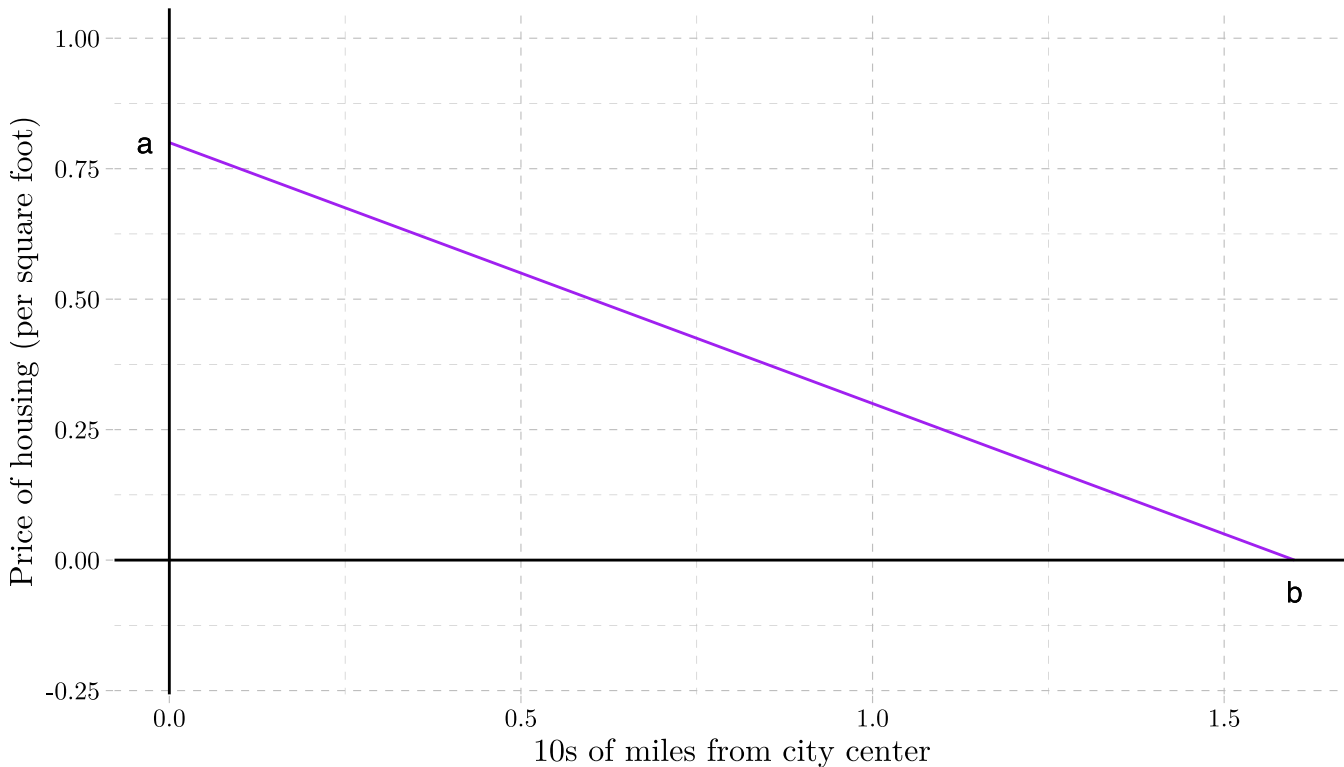
No Substitution

Example Suppose the following:

- Each household has \$800 a month to spend on housing and commuting
- All rental units are the same size, with each HH occupying a rental unit that is 1000 sq ft
- Monthly commuting cost is \$50 dollars per mile from employment center

Task: Draw the housing - price curve. Put miles from city center on **x axis** and price per square foot on **y axis**

Example: The housing price curve



a: max WTP for a square foot (at center of city) (80 c per square foot)

b: furthest away from center HH is willing to live (16 miles)

Substitution

Q1: If you really wanted to live closer to campus -- or an exciting downtown in a big city -- would you be willing to live in a smaller apartment to do so?

A1: Most peopleTM: Yes. You are willing to **substitute**

Q2: What do I mean by **substitute**? Substitute what?

A2: Substitute housing consumption for **lower commuting cost** (and whatever else being close to the center of the city gets you)

Substitution

Let's formalize the mechanism for substitution a bit:

higher prices \implies higher opportunity cost per square foot of housing (for the consumer)

- As price of rent increases, consumers are likely to substitute (atleast somewhat) towards other goods, decreasing the square footage of housing demanded
- **Housing units closer to city centers are thus likely to be smaller in size**

Adding substitution to the model

Q3: Did our model of locational indifference accomdate for substitution?
Why or Why not?

$$\Delta P \cdot h + \Delta x \cdot t = 0$$




A3: No because h (the quantity of housing consumed) is **independent of distance** from center, x

If consumers can substitute, our locational indifference condition becomes:

$$\Delta P \cdot h(x) + \Delta x \cdot t = 0$$

- Where $h(x)$ is an *increasing* function of x
 - **Ex:** $h(10) > h(5)$ (the quantity of housing demanded 10 miles from the center exceeds that of 5 miles)

Checklist

- 1) **Intro to Rents** 
- 1.5) **Bonus: COVID19 and Cities** 
- 2) **Consumer Bid Rent** 
- 3) **Manufacturing Bid Rent**
- 4) **City Shape**

Manufacturing Bid Rent

WTP for land from manufacturing firms is a function of the land's accessibility (similar to consumers)

- **Fact:** Urban manufacturing employment is largely decentralized and dispersed
 - Most firms locate close to the highway. **Why?** This has not always been the case
- Firms are balancing **freight** and **labor costs**
 - Further from labor \implies **higher wage** (to compensate for increased commuting cost)
 - Further from shipping center \implies higher **freight cost**

Manufacturing Bid Rent

Let's start with a simple modelTM. **Assumptions**

1) Input & Output **prices** & **quantities** are fixed

- Firm only decides location

2) Firms **import intermediate goods** and **export output** to other cities via a **central terminal** (train)

3) Wage paid is to compensate workers for commuting. Workforce is suburban so wage is **highest at center**

4) Firms use horse carts to transport inputs and output to the **central terminal**

- We will relax this one soon

Firm's Bid Rent

What do we use to get the firm's bid - rent equation?

Axiom 5: *Competition generates zero economic profit*

Recall the profit equation:

$$\pi = TR - TC$$

In this model:

- $TR = P * Q$ (fixed, exogenous)
- TC is a function of freight cost, labor cost, and intermediate goods cost:

$$TC(x) = \text{Freight Cost}(x) + \text{Labor Cost}(x) + \text{Land Cost}(x) + \text{Intermediate Input Cost}$$

Firm Bid Rent

From here on out, let's call Intermediate Input Cost = \bar{I}

- Invoking zero economic profit, from the last slide we can write:

$$TR - (\text{Freight Cost}(x) + \text{Labor Cost}(x) + \text{Land Cost}(x) + \bar{I}) = 0$$

- **In words:** The most a firm would be willing to pay for land then is revenue net of non land cost. Rearranging:

$$\text{Land Cost}(x) = TR - \text{Freight Cost}(x) - \text{Labor Cost}(x) - \bar{I}$$

Note: Land Cost = $P(x) * L_m$, where:

- $P(x)$ is the *price of land at x miles away from the center*
- L_m is the *amount of land the manufacturer uses in production* (fixed input at L_m)

Firm Bid Rent: Equation

We can replace land cost with $P(x) * L_m$ to get the equation for the **manufacturing bid rent** curve

$$P(x) * L_m = TR - \text{Freight Cost}(x) - \text{Labor Cost}(x) - \bar{I}$$

Firm Bid Rent: Equation

We can replace land cost with $P(x) * L_m$ to get the equation for the **manufacturing bid rent** curve

$$P(x) * L_m = TR - \text{Freight Cost}(x) - \text{Labor Cost}(x) - \bar{I}$$
$$P(x) = \frac{TR - \text{Freight Cost}(x) - \text{Labor Cost}(x) - \bar{I}}{L_m}$$

In words, this equation says:

- Higher revenues \implies higher land prices **for every distance** x
- An increase in freight costs, labor costs, or intermediate input costs will **decrease** the price for every distance x

Example

Suppose

$$P = 5, Q = 2, FC(x) = 4x, \text{Labor}(x) = 1 - 3x, L_m = 1, \bar{I} = 0$$

- 1) Derive the firm's bid rent curve. Carefully write down your steps
- 2) What is the price the firm is willing to pay for land at $x = 1$?
- 3) Is the WTP higher or lower when we move away from the center?
- 4) What distance away from the center is the WTP zero?

Example

1) Start with zero profit: $\pi(x) = 0 \implies TR - FC(x) - LC(x) - P(x) = 0$

Plugging in:

$$\begin{aligned} 5 \times 2 - 4x - (1 - 3x) - P(x) &= 0 \\ 9 - x &= P(x) \end{aligned}$$

2) $P(1) = 8$

3) Lower (if $x_2 > x_1$, $P(x_2) < P(x_1)$)

4) $P(x) = 0 \implies x = 9$

Back to Reality

How can a model like this help us understand the industrial revolution?

- What happened to freight costs? **They fell** A few innovations:

Transportation Innovations:

- Omnibus (1827)
- Cable Cars (1873)
- Electric Trolley (1886)
- Subways (1895)

In our model, what do these innovations do? **Decrease labor costs relative to freight**

More History

- The *intracity* truck (1910): twice as fast and half as costly as the horse-drawn wagon[†]
- Truck decreased the cost of moving **output** relative to the cost of moving **workers**
- Manufacturing Firms moved closer to low-wage suburbs

The *intercity* truck (1930): alternative to ships and rail^{††}

- **Highways**: orientation **shifted** from **ports** and **railroad terminals** to **roads**
- **Modern cities**: manufacturers oriented toward highways and beltways (freight costs decreased relative to labor)

[†] Intra = Within ^{††} Inter = Across

Checklist

- 1) **Intro to Rents** ✓
- 1.5) **Bonus: COVID19 and Cities** ✓
- 2) **Consumer Bid Rent** ✓
- 3) **Manufacturing Bid Rent** ✓
- 4) **City Shape**

Office Space

Last bid - rent curve we will work on: that for **offices**

Assumptions

- Office firms use high skilled labor that need *face to face* interaction for production
- Proximity to other office firms is an important input
- Opportunity cost of high skilled labor time is greater than other types of labor

Office Bid Rent

- So as office firms get further from center their "transit" cost goes up. So what must happen to WTP?

City Organization

So how do we put all of this together? And why are these called **bid** rent curves anyways?

- **Land will be allocated to highest bidder**
- This will vary by location in the city

Example

- Assume profit for office and manufacturing is given by
- $\pi_{\text{office}} = 105 - r(x_{\text{Office}}) - (5 + 4 \times x_{\text{office}})$
- $\pi_{\text{manufact}} = 75 - r(x_{\text{manufact}}) - (5 + 2 \times x_{\text{manufact}})$

For consumers, they can allocate money between housing and commuting:

- $r(x_{\text{commuter}}) = \frac{100}{4} - \frac{2}{4} \times x_{\text{commuter}}$

Example

Task:

- Derive the bid rent curve for office space, manufacturing, and commuters. Plot all of them.
- Find how land is allocated. Specifically, over what range from the center is office space, manufacturing space, and housing space?

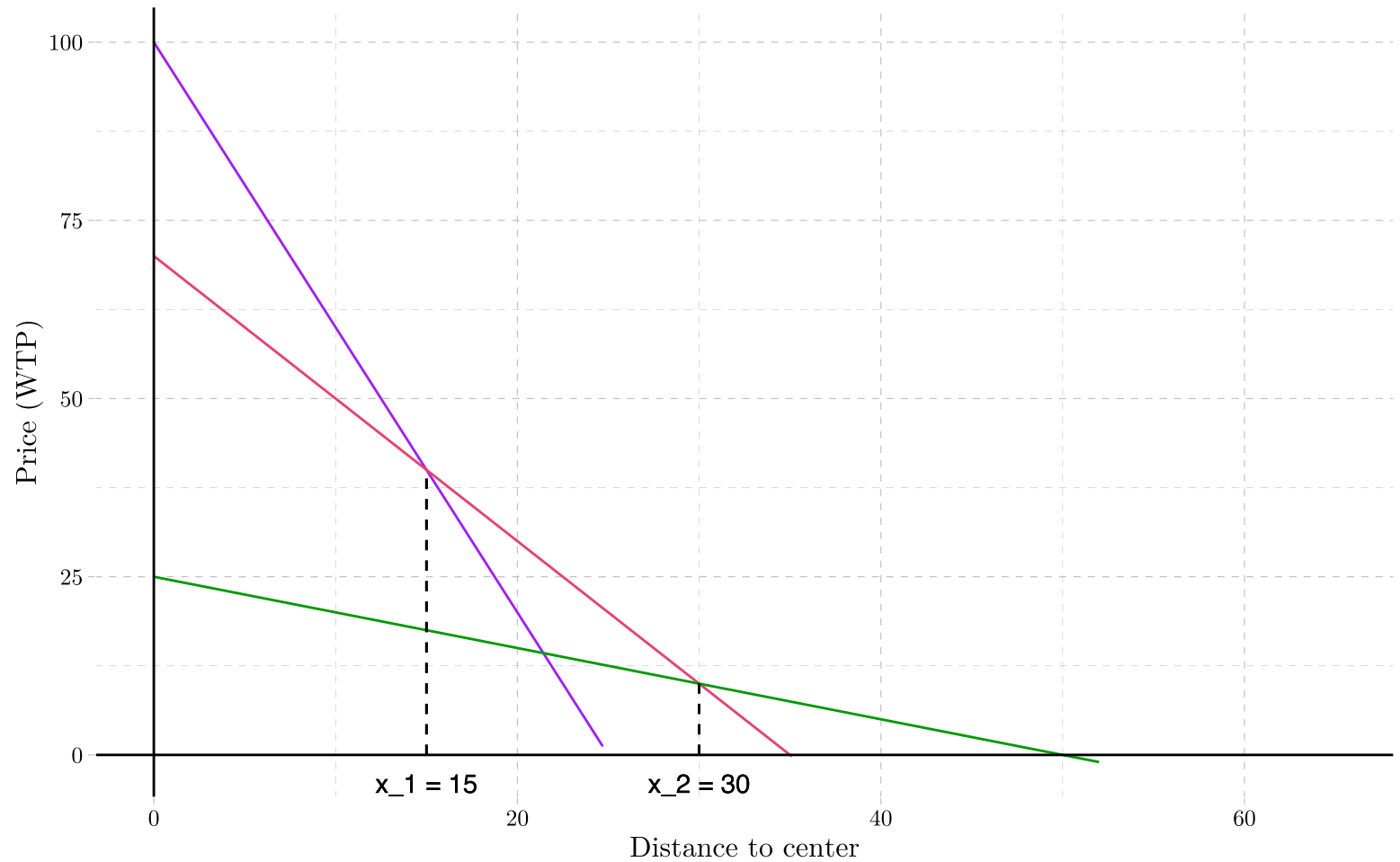
Example

Task:

Bid rent curves for office and manufacturing come from zero profit. Commuters curve was given.

- Office: $r(x_{\text{office}}) = 105 - (5 + 4 \times x_{\text{office}})$
- Manufacturing: $r(x_{\text{manufact}}) = 75 - (5 + 2 \times x_{\text{manufact}})$
- Commuters: (given) $r(x_{\text{commuter}}) = \frac{100}{4} - \frac{2}{4} \times x_{\text{commuter}}$
- Office firms locate in the range of x in $[0, 15]$
- Manufacturing firms locate in the range of x in $[15, 30]$
- Commuters locate in the range of x in $[30, 50]$

Example



Checklist

- 1) **Intro to Rents** ✓
- 1.5) **Bonus: COVID19 and Cities** ✓
- 2) **Consumer Bid Rent** ✓
- 3) **Manufacturing Bid Rent** ✓
- 4) **City Shape** ✓