Derivation of Six-Strut (Tetrahedral) Tensegrity Geo

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If we now specify the geometry to be a six-strut (tetrahedral) tensegrity structure, and considered wi
coordinate system:

1. whose origin, S, is at the center of gravity of the entire spherical tensegrity, and
2. whose Z axis passes through point P of strut OPQ, where point P is distance d from point
3. whose X, Y and Z units are each equal to half the length of the strut (i.e. $\overline{PO}$), and
4. such that the axis of the strut lies at a 45 degree angle from the XZ and YZ planes, with both X and Y are positive.

Then the coordinates of each of the labelled points in the figure can be written by inspection at th
geometry is known at these points:

$\phi = 0 \Longrightarrow d = \frac{\sqrt{2}}{2}$

point	*coordinates*
$O:$	$\left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$
$P:$	$\left[0, 0, \frac{\sqrt{2}}{2}\right]$
$Q:$	$\left[-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$
$A:$	$\left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$
$B:$	$\left[-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$
$C:$	$\left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right]$
$D:$	$\left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$
$E:$	$\left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right]$
$F:$	$\left[-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$
$G:$	$\left[\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right]$
$H:$	$\left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right]$
$I:$	$\left[\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right]$
$J:$	$\left[\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right]$
$S:$	$\left[0, 0, 0\right]$

$\phi = \frac{\pi}{4} \Longrightarrow d = \frac{1}{2}$

point	*coordinates*
$O:$	$\left[0, 1, \frac{1}{2}\right]$
$P:$	$\left[0, 0, \frac{1}{2}\right]$
$Q:$	$\left[0, -1, \frac{1}{2}\right]$
$A:$	$\left[\frac{1}{2}, 0, 1\right]$
$B:$	$\left[-\frac{1}{2}, 0, 1\right]$
$C:$	$\left[-1, \frac{1}{2}, 0\right]$
$D:$	$\left[1, \frac{1}{2}, 0\right]$
$E:$	$\left[0, 1, -\frac{1}{2}\right]$
$F:$	$\left[-1, -\frac{1}{2}, 0\right]$
$G:$	$\left[\frac{1}{2}, 0, -1\right]$
$H:$	$\left[-\frac{1}{2}, 0, -1\right]$
$I:$	$\left[0, -1, -\frac{1}{2}\right]$
$J:$	$\left[1, -\frac{1}{2}, 0\right]$
$S:$	$\left[0, 0, 0\right]$

$\phi = \frac{\pi}{2} =$

point	*c*
$O:$	$[-$
$P:$	$[0$
$Q:$	$[\frac{\sqrt{2}}{2}$
$A:$	$[\frac{\sqrt{2}}{2}$
$B:$	$[-$
$C:$	$[-$
$D:$	$[\frac{\sqrt{2}}{2}$
$E:$	$[\frac{\sqrt{2}}{2}$
$F:$	$[\frac{\sqrt{2}}{2}$
$G:$	$[\frac{\sqrt{2}}{2}$
$H:$	$[-$
$I:$	$[-$
$J:$	$[-$
$S:$	$[0$

From this position, strut OPQ can be rotated about the Z axis through the angle ϕ (where ϕ is b
distance d from the origin to get coordinate formulas for the general-case position of the strut. To do t
first three rows above, but converted to *homogeneous coordinates*, by adding a fourth coordinate who
Then we post-multiply this coordinate matrix by the bilinear transformation matrix for rotation abou

$$\begin{bmatrix} \frac{\sqrt{2}}{2}, & \frac{\sqrt{2}}{2}, & \frac{\sqrt{2}}{2}, & 1 \\ 0, & 0, & \frac{\sqrt{2}}{2}, & 1 \\ -\frac{\sqrt{2}}{2}, & -\frac{\sqrt{2}}{2}, & \frac{\sqrt{2}}{2}, & 1 \end{bmatrix} \begin{bmatrix} \cos\phi, & \sin\phi, & 0, & 0 \\ -\sin\phi, & \cos\phi, & 0, & 0 \\ 0, & 0, & 1, & 0 \\ 0, & 0, & -(\frac{\sqrt{2}}{2}-d), & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2}(\cos\phi-\sin\phi), & \frac{\sqrt{2}}{2}(\mathrm{s} \\ 0 & , \\ \frac{\sqrt{2}}{2}(\sin\phi-\cos\phi), & -\frac{\sqrt{2}}{2}(\end{bmatrix}$$

Top to back (for CD) — rotate strut OPQ -90 w/o Z, then -90 w/o X:

$$\begin{bmatrix} 0, & -1, & 0, & 0 \\ 1, & 0, & 0, & 0 \\ 0, & 0, & 1, & 0 \\ 0, & 0, & 0, & 1 \end{bmatrix} \begin{bmatrix} 1, & 0, & 0, & 0 \\ 0, & 0, & -1, & 0 \\ 0, & 1, & 0, & 0 \\ 0, & 0, & 0, & 1 \end{bmatrix} = \begin{bmatrix} 0, & 0, & 1, & 0 \\ 1, & 0, & 0, & 0 \\ 0, & 1, & 0, & 0 \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

For D and C ($O \to D$ and $Q \to C$):

$$\begin{bmatrix} \frac{\sqrt{2}}{2}(\cos\phi-\sin\phi), & \frac{\sqrt{2}}{2}(\sin\phi+\cos\phi), & d, & 1 \\ \frac{\sqrt{2}}{2}(\sin\phi-\cos\phi), & -\frac{\sqrt{2}}{2}(\sin\phi+\cos\phi), & d, & 1 \end{bmatrix} \begin{bmatrix} 0, & 0, & 1, & 0 \\ 1, & 0, & 0, & 0 \\ 0, & 1, & 0, & 0 \\ 0, & 0, & 0, & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2}(\sin\phi+\cos\phi), & \\ -\frac{\sqrt{2}}{2}(\sin\phi+\cos\phi), & \end{bmatrix}$$

Top to left (for BH) — rotate strut OPQ -90 w/o Z, then -90 w/o Y:

$$\begin{bmatrix} 0, & -1, & 0, & 0 \\ 1, & 0, & 0, & 0 \\ 0, & 0, & 1, & 0 \\ 0, & 0, & 0, & 1 \end{bmatrix} \begin{bmatrix} 0, & 0, & 1, & 0 \\ 0, & 1, & 0, & 0 \\ -1, & 0, & 0, & 0 \\ 0, & 0, & 0, & 1 \end{bmatrix} = \begin{bmatrix} 0, & -1, & 0, & 0 \\ 0, & 0, & 1, & 0 \\ -1, & 0, & 0, & 0 \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

For BH ($O \to B$ and $Q \to H$):

$$\begin{bmatrix} \frac{\sqrt{2}}{2}(\cos\phi-\sin\phi), & \frac{\sqrt{2}}{2}(\sin\phi+\cos\phi), & d, & 1 \\ \frac{\sqrt{2}}{2}(\sin\phi-\cos\phi), & -\frac{\sqrt{2}}{2}(\sin\phi+\cos\phi), & d, & 1 \end{bmatrix} \begin{bmatrix} 0, & -1, & 0, & 0 \\ 0, & 0, & 1, & 0 \\ -1, & 0, & 0, & 0 \\ 0, & 0, & 0, & 1 \end{bmatrix} = \begin{bmatrix} -d, & \frac{\sqrt{2}}{2}(\sin\phi-\cos \\ -d, & \frac{\sqrt{2}}{2}(\cos\phi-\sin \end{bmatrix}$$

Top to right (for AG) — rotate strut OPQ 90 w/o Z, then 90 w/o Y:

$$\begin{bmatrix} 0 & ,1 & ,0 & ,0 \\ -1 & ,0 & ,0 & ,0 \\ 0 & ,0 & ,1 & ,0 \\ 0 & ,0 & ,0 & ,1 \end{bmatrix} \begin{bmatrix} 0 & ,0 & ,-1 & ,0 \\ 0 & ,1 & ,0 & ,0 \\ 1 & ,0 & ,0 & ,0 \\ 0 & ,0 & ,0 & ,1 \end{bmatrix} = \begin{bmatrix} 0 & ,1 & ,0 & ,0 \\ 0 & ,0 & ,1 & ,0 \\ 1 & ,0 & ,0 & ,0 \\ 0 & ,0 & ,0 & ,1 \end{bmatrix}$$

For AG ($O \to A$ and $Q \to G$):

$$\begin{bmatrix} \frac{\sqrt{2}}{2}(\cos\phi - \sin\phi) \, , & \frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) & , d & , 1 \\ \frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) \, , & -\frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) & , d & , 1 \end{bmatrix} \begin{bmatrix} 0 & ,1 & ,0 & ,0 \\ 0 & ,0 & ,1 & ,0 \\ 1 & ,0 & ,0 & ,0 \\ 0 & ,0 & ,0 & ,1 \end{bmatrix} = \begin{bmatrix} d \, , & \frac{\sqrt{2}}{2}(\cos\phi - \sin\phi) \, , \\ d \, , & \frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) \, , \end{bmatrix}$$

Top to front (for FJ) — rotate strut OPQ 90 w/o Z, then -90 w/o X:

$$\begin{bmatrix} 0 & ,1 & ,0 & ,0 \\ -1 & ,0 & ,0 & ,0 \\ 0 & ,0 & ,1 & ,0 \\ 0 & ,0 & ,0 & ,1 \end{bmatrix} \begin{bmatrix} 1 & ,0 & ,0 & ,0 \\ 0 & ,0 & ,-1 & ,0 \\ 0 & ,1 & ,0 & ,0 \\ 0 & ,0 & ,0 & ,1 \end{bmatrix} = \begin{bmatrix} 0 & ,0 & ,-1 & ,0 \\ -1 & ,0 & ,0 & ,0 \\ 0 & ,1 & ,0 & ,0 \\ 0 & ,0 & ,0 & ,1 \end{bmatrix}$$

For F and J ($O \to J$ and $Q \to F$):

$$\begin{bmatrix} \frac{\sqrt{2}}{2}(\cos\phi - \sin\phi) \, , & \frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) & , d & , 1 \\ \frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) \, , & -\frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) & , d & , 1 \end{bmatrix} \begin{bmatrix} 0 & ,0 & ,-1 & ,0 \\ -1 & ,0 & ,0 & ,0 \\ 0 & ,1 & ,0 & ,0 \\ 0 & ,0 & ,0 & ,1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) \\ \frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) \end{bmatrix}$$

Top to bottom (for EI) — rotate strut OPQ 180 w/o X:

$$\begin{bmatrix} 1 & ,0 & ,0 & ,0 \\ 0 & ,-1 & ,0 & ,0 \\ 0 & ,0 & ,-1 & ,0 \\ 0 & ,0 & ,0 & ,1 \end{bmatrix}$$

For E and I ($O \to I$ and $Q \to E$):

$$\begin{bmatrix} \frac{\sqrt{2}}{2}(\cos\phi - \sin\phi) , & \frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) & , d \, , 1 \\ \frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) , & -\frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) & , d \, , 1 \end{bmatrix} \begin{bmatrix} 1 \, , & 0 & , & 0 & , 0 \\ 0 \, , & -1 & , & 0 & , 0 \\ 0 \, , & 0 & , & -1 & , 0 \\ 0 \, , & 0 & , & 0 & , 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2}(\cos\phi - \sin\phi) \, , - \\ \frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) \, , \end{bmatrix}$$

So, in the general case:

$$\begin{array}{cl} \textit{point} & \qquad\qquad\qquad \textit{coordinates} \\ O: & \left[\frac{\sqrt{2}}{2}(\cos\phi - \sin\phi) \, , \frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) \, , \qquad d \right. \\ P: & \left[\qquad 0 \qquad , \qquad 0 \qquad , \qquad d \right. \\ Q: & \left[\frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) \, , -\frac{\sqrt{2}}{2}(\sin\phi + \cos\phi), \qquad d \right. \\ A: & \left[\qquad d \qquad , \frac{\sqrt{2}}{2}(\cos\phi - \sin\phi) \, , \frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) \right. \\ B: & \left[\qquad -d \qquad , \frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) \, , \frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) \right. \\ C: & \left[-\frac{\sqrt{2}}{2}(\sin\phi + \cos\phi), \qquad d \qquad , \frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) \right. \\ D: & \left[\frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) \, , \qquad d \qquad , \frac{\sqrt{2}}{2}(\cos\phi - \sin\phi) \right. \\ E: & \left[\frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) \, , \frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) \, , \qquad -d \right. \\ F: & \left[\frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) \, , \qquad d \qquad , \frac{\sqrt{2}}{2}(\cos\phi - \sin\phi) \right. \\ G: & \left[\qquad d \qquad , \frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) \, , -\frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) \right. \\ H: & \left[\qquad -d \qquad , \frac{\sqrt{2}}{2}(\cos\phi - \sin\phi) \, , -\frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) \right. \\ I: & \left[\frac{\sqrt{2}}{2}(\cos\phi - \sin\phi) \, , -\frac{\sqrt{2}}{2}(\sin\phi + \cos\phi), \qquad -d \right. \\ J: & \left[-\frac{\sqrt{2}}{2}(\sin\phi + \cos\phi), \qquad d \qquad , \frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) \right. \\ S: & \left[\qquad 0 \qquad , \qquad 0 \qquad , \qquad 0 \right. \end{array}$$

Next, rotate the coordinate system $-\pi/4$, then ϕ around the Z axis, by use of the following transf

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & , \frac{\sqrt{2}}{2} & , 0 \, , 0 \\ -\frac{\sqrt{2}}{2} & , \frac{\sqrt{2}}{2} & , 0 \, , 0 \\ 0 & , 0 & , 1 \, , 0 \\ 0 & , 0 & , 0 \, , 1 \end{bmatrix} \begin{bmatrix} \cos\phi \, , & -\sin\phi & , 0 \, , 0 \\ \sin\phi \, , & \cos\phi & , 0 \, , 0 \\ 0 \quad , & 0 & , 1 \, , 0 \\ 0 \quad , & 0 & , 0 \, , 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) \, , & \frac{\sqrt{2}}{2}(\cos\phi - \\ \frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) \, , & \frac{\sqrt{2}}{2}(\sin\phi + \\ 0 \qquad , & 0 \\ 0 \qquad , & 0 \end{bmatrix}$$

Applying the transformation to the points:

$$\begin{bmatrix} \frac{\sqrt{2}}{2}(\cos\phi - \sin\phi) & , & \frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) & , & d & , 1 \\ 0 & , & 0 & , & d & , 1 \\ \frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) & , & -\frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) & , & d & , 1 \\ d & , & \frac{\sqrt{2}}{2}(\cos\phi - \sin\phi) & , & \frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) & , 1 \\ -d & , & \frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) & , & \frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) & , 1 \\ -\frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) & , & d & , & \frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) & , 1 \\ \frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) & , & d & , & \frac{\sqrt{2}}{2}(\cos\phi - \sin\phi) & , 1 \\ 0 & , & 0 & , & 0 & , 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2}(\sin\phi + \cos\phi) & , \\ \frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) & , \\ 0 & , \\ 0 & , \end{bmatrix}$$

The first point (O) is:

$$\begin{aligned} O &= \Big[\ \tfrac{\sqrt{2}}{2}\tfrac{\sqrt{2}}{2}(\cos\phi + \sin\phi)(\cos\phi - \sin\phi) - \tfrac{\sqrt{2}}{2}\tfrac{\sqrt{2}}{2}(\cos\phi + \sin\phi)(\cos\phi - \sin\phi)\ ,\ \tfrac{\sqrt{2}}{2}\tfrac{\sqrt{2}}{2}(\cos\phi - \sin \\ &= \Big[\ 0\ ,\ \tfrac{1}{2}\left(\cos^2\phi - 2\cos\phi\sin\phi + \sin^2\phi\right) + \tfrac{1}{2}\left(\cos^2\phi + 2\cos\phi\sin\phi + \sin^2\phi\right),\ d\ ,\ 1\ \Big] \\ &= \Big[\ 0\ ,\ \tfrac{1}{2}\left(\cos^2\phi - 2\cos\phi\sin\phi + \sin^2\phi + \cos^2\phi + 2\cos\phi\sin\phi + \sin^2\phi\right),\ d\ ,\ 1\ \Big] \\ &= \Big[\ 0\ ,\ 1\ ,\ d\ ,\ 1\ \Big] \end{aligned}$$

The second point (A) is:

$$\begin{aligned} A &= \Big[\ \tfrac{\sqrt{2}}{2}d(\sin\phi + \cos\phi) - \tfrac{\sqrt{2}}{2}\tfrac{\sqrt{2}}{2}(\cos\phi - \sin\phi)^2\ ,\ -\tfrac{\sqrt{2}}{2}d(\sin\phi - \cos\phi) + \tfrac{\sqrt{2}}{2}\tfrac{\sqrt{2}}{2}(\cos\phi + \sin\phi)(\mathrm{co} \\ &= \Big[\ \tfrac{\sqrt{2}}{2}d(\sin\phi + \cos\phi) - \tfrac{1}{2}(\cos\phi - \sin\phi)^2\ ,\ \tfrac{1}{2}\left(\cos^2\phi - \sin^2\phi\right) - \tfrac{\sqrt{2}}{2}d(\sin\phi - \cos\phi)\ ,\ \tfrac{\sqrt{2}}{2}(\cos\phi \end{aligned}$$

The third point (B) is:

$$\begin{aligned} B &= \Big[\ -\tfrac{\sqrt{2}}{2}d(\sin\phi + \cos\phi) + \tfrac{\sqrt{2}}{2}\tfrac{\sqrt{2}}{2}(\sin\phi - \cos\phi)^2\ ,\ \tfrac{\sqrt{2}}{2}d(\sin\phi - \cos\phi) + \tfrac{\sqrt{2}}{2}\tfrac{\sqrt{2}}{2}(\sin\phi + \cos\phi)(\mathrm{si} \\ &= \Big[\ -\tfrac{\sqrt{2}}{2}d(\sin\phi + \cos\phi) + \tfrac{1}{2}(\sin\phi - \cos\phi)^2\ ,\ \tfrac{\sqrt{2}}{2}d(\sin\phi - \cos\phi) + \tfrac{1}{2}\left(\sin^2\phi - \cos^2\phi\right),\ \tfrac{\sqrt{2}}{2}(\cos \end{aligned}$$

The fourth point (C) is:

$$\begin{aligned} C &= \Big[-\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}(\sin\phi+\cos\phi)^2 + \frac{\sqrt{2}}{2}d(\sin\phi-\cos\phi) \,,\, \frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}(\sin\phi+\cos\phi)(\sin\phi-\cos\phi) + \frac{\sqrt{2}}{2}d(\sin \\ &= \Big[-\frac{1}{2}(\sin\phi+\cos\phi)^2 + \frac{\sqrt{2}}{2}d(\sin\phi-\cos\phi) \,,\, \frac{1}{2}\left(\sin^2\phi-\cos^2\phi\right) + \frac{\sqrt{2}}{2}d(\sin\phi+\cos\phi) \,,\, \frac{\sqrt{2}}{2}(\sin \end{aligned}$$

The fifth point (D) is:

$$\begin{aligned} D &= \Big[\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}(\sin\phi+\cos\phi)^2 + \frac{\sqrt{2}}{2}d(\sin\phi-\cos\phi) \,,\, \frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}(\sin\phi+\cos\phi)(\cos\phi-\sin\phi) + \frac{\sqrt{2}}{2}d(\sin \\ &= \Big[\frac{1}{2}(\sin\phi+\cos\phi)^2 + \frac{\sqrt{2}}{2}d(\sin\phi-\cos\phi) \,,\, \frac{1}{2}\left(\cos^2\phi-\sin^2\phi\right) + \frac{\sqrt{2}}{2}d(\sin\phi+\cos\phi) \,,\, \frac{\sqrt{2}}{2}(\cos \end{aligned}$$

So, gathering all the points together:

$$\begin{array}{ll} O: & \Big[\qquad 0 \qquad , \qquad 1 \\ A: & \Big[\frac{\sqrt{2}}{2}d(\sin\phi+\cos\phi) - \frac{1}{2}(\cos\phi-\sin\phi)^2 \,, -\frac{\sqrt{2}}{2}d(\sin\phi-\cos\phi) + \frac{1}{2}\left(\cos^2\phi-\sin^2 \right. \\ B: & \Big[-\frac{\sqrt{2}}{2}d(\sin\phi+\cos\phi) + \frac{1}{2}(\sin\phi-\cos\phi)^2 \,,\, \frac{\sqrt{2}}{2}d(\sin\phi-\cos\phi) + \frac{1}{2}\left(\sin^2\phi-\cos^2\phi \right. \\ C: & \Big[-\frac{1}{2}(\sin\phi+\cos\phi)^2 + \frac{\sqrt{2}}{2}d(\sin\phi-\cos\phi) \,,\, \frac{1}{2}\left(\sin^2\phi-\cos^2\phi\right) + \frac{\sqrt{2}}{2}d(\sin\phi+\cos\phi \\ D: & \Big[\frac{1}{2}(\sin\phi+\cos\phi)^2 + \frac{\sqrt{2}}{2}d(\sin\phi-\cos\phi) \,,\, \frac{1}{2}\left(\cos^2\phi-\sin^2\phi\right) + \frac{\sqrt{2}}{2}d(\sin\phi+\cos\phi \end{array}$$

We can subtract 1 from the y-coordinate and d from the z-coordinate to center our coordinate syst

$$\begin{array}{ll} O: & \Big[\qquad 0 \qquad , \qquad 0 \\ A: & \Big[\frac{\sqrt{2}}{2}d(\sin\phi+\cos\phi) - \frac{1}{2}(\cos\phi-\sin\phi)^2 \,, -\frac{\sqrt{2}}{2}d(\sin\phi-\cos\phi) + \frac{1}{2}\left(\cos^2\phi-\sin^2\phi\right) - \\ B: & \Big[-\frac{\sqrt{2}}{2}d(\sin\phi+\cos\phi) + \frac{1}{2}(\sin\phi-\cos\phi)^2 \,,\, \frac{\sqrt{2}}{2}d(\sin\phi-\cos\phi) + \frac{1}{2}\left(\sin^2\phi-\cos^2\phi\right) - \\ C: & \Big[-\frac{1}{2}(\sin\phi+\cos\phi)^2 + \frac{\sqrt{2}}{2}d(\sin\phi-\cos\phi) \,,\, \frac{1}{2}\left(\sin^2\phi-\cos^2\phi\right) + \frac{\sqrt{2}}{2}d(\sin\phi+\cos\phi) - \\ D: & \Big[\frac{1}{2}(\sin\phi+\cos\phi)^2 + \frac{\sqrt{2}}{2}d(\sin\phi-\cos\phi) \,,\, \frac{1}{2}\left(\cos^2\phi-\sin^2\phi\right) + \frac{\sqrt{2}}{2}d(\sin\phi+\cos\phi) - \end{array}$$

Recall the equilibrium equation for our class of symmetrical tensegrities:

$$(z_{OB}+z_{OC})(x_{OA}+x_{OD}) = (x_{OB}+x_{OC})(z_{OA}+z_{OD})$$

By simple sustituting of coordinates we get:

$$(z_B + z_C)(x_A + x_D) = (x_B + x_C)(z_A + z_D)$$

where z_p and x_p are the z and x components of point p in our coordinate system centered at O. Now

$$\begin{aligned}
z_B + z_C &= \frac{\sqrt{2}}{2}(\cos\phi + \sin\phi) - d + \frac{\sqrt{2}}{2}(\sin\phi - \cos\phi) - d = \sqrt{2}\sin\phi - 2d \\
x_A + x_D &= \frac{\sqrt{2}}{2}d(\sin\phi + \cos\phi) - \frac{1}{2}(\cos\phi - \sin\phi)^2 + \frac{1}{2}(\sin\phi + \cos\phi)^2 + \frac{\sqrt{2}}{2}d \\
&= 2\cos\phi\sin\phi + \sqrt{2}d\sin\phi \\
x_B + x_C &= -\frac{\sqrt{2}}{2}d(\sin\phi + \cos\phi) + \frac{1}{2}(\sin\phi - \cos\phi)^2 - \frac{1}{2}(\sin\phi + \cos\phi)^2 + \frac{\sqrt{2}}{2} \\
&= -2\cos\phi\sin\phi - \sqrt{2}d\cos\phi \\
z_A + z_D &= \frac{\sqrt{2}}{2}(\cos\phi + \sin\phi) - d + \frac{\sqrt{2}}{2}(\cos\phi - \sin\phi) - d = \sqrt{2}\cos\phi - 2d
\end{aligned}$$

Now we can set these factors equal to one another and solve for d.

$$(\sqrt{2}\sin\phi - 2d)(2\cos\phi\sin\phi + \sqrt{2}d\sin\phi) = (-2\cos\phi\sin\phi - \sqrt{2}d\cos\phi)(\sqrt{2}$$

$$\Longleftrightarrow -\left(\sqrt{2}\cos\phi + \sqrt{2}\sin\phi\right)d^2 + (1 - 4\sin\phi\cos\phi)\,d + \sqrt{2}\cos\phi\sin^2\phi + \sqrt{2}\mathrm{si}$$

Using the quadratic formula $\left[ax^2 + bx + c = 0 \Longrightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}\right]$ we get

$$\begin{aligned}
d(\phi) &= \frac{1 - 4\cos\phi\sin\phi \pm \sqrt{(1 - 4\cos\phi\sin\phi)^2 + 4(\sqrt{2}\cos^2\phi\sin\phi + \sqrt{2}\sin^2\phi\cos\phi)}}{2\sqrt{2}(\sin\phi + \cos\phi)} \\
&= \frac{1 - 4\cos\phi\sin\phi \pm \sqrt{32\cos^2\phi\sin^2\phi + 1}}{2\sqrt{2}(\sin\phi + \cos\phi)}
\end{aligned}$$

The negative root can be shown to be absurd (its values for $d(\phi)$ are negative which is not possible in

We can also calculate $|OA|$ which is the length of the "short" tendon and $|OB|$, the length of the "

$$
\begin{aligned}
|OA| &= \sqrt{(x_A - x_O)^2 + (y_A - y_O)^2 + (z_A - z_O)^2} \\
&= \sqrt{\left(\frac{\sqrt{2}}{2}d(\sin\phi + \cos\phi) - \frac{1}{2}(\cos\phi - \sin\phi)^2\right)^2 + \left(-\frac{\sqrt{2}}{2}d(\sin\phi - \cos\phi) + \frac{1}{2}\left(\cos^2\phi - \sin^2\phi\right)\right.} \\
&= \sqrt{2d^2 - 2\sqrt{2}\cos\phi d + \cos^2\phi + \frac{1}{2}\sin^4\phi + 2\sin^2\phi + \frac{1}{2}}
\end{aligned}
$$

$$
\begin{aligned}
|OB| &= \sqrt{(x_B - x_O)^2 + (y_B - y_O)^2 + (z_B - z_O)^2} \\
&= \sqrt{\left(-\frac{\sqrt{2}}{2}d(\sin\phi + \cos\phi) + \frac{1}{2}(\sin\phi - \cos\phi)^2\right)^2 + \left(-\frac{\sqrt{2}}{2}d(\sin\phi - \cos\phi) + \frac{1}{2}\left(\cos^2\phi - \sin^2\right.\right.} \\
&= \sqrt{2d^2 + d\left(\sqrt{2}\cos(2\phi)\sin\phi - \sqrt{2}\cos\phi\cos(2\phi) - 2\sqrt{2}\cos\phi\right) + \sin^2\phi + \frac{1}{2}\cos^4\phi + 2\cos^2\phi +}
\end{aligned}
$$

Since the mininima and maxima points seem interesting, let's calculate the derivative of $d(\phi)$:

$$
d' = \frac{\frac{32\cos(2\phi)\cos\phi\sin\phi}{\sqrt{32\cos\phi^2\sin\phi^2+1}} - 4\cos(2\phi)}{2\sqrt{2}(\sin\phi + \cos\phi)} - \frac{\frac{\sqrt{2}}{2}(\sqrt{32\cos\phi^2\sin\phi^2 + 1} - 4\cos\phi\sin\phi + 1)}{2(\sin\phi + \cos\phi)^2}
$$

$d(\phi)$ is at a maximum when $d'(\phi) = 0$. By numerical methods the derivative is 0 when...

Sometimes it is desired to determine an unknown strut length given on the lenght of the two string