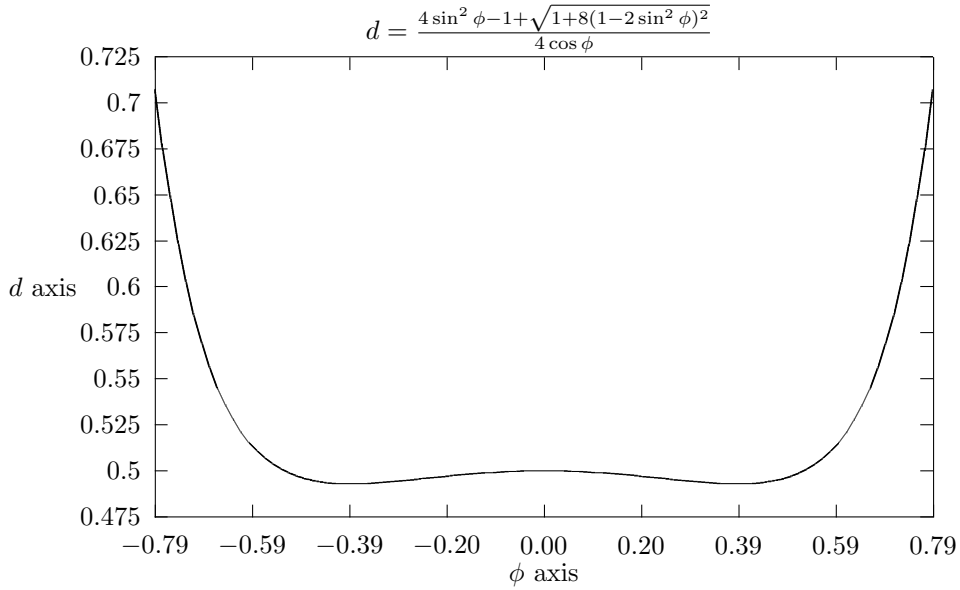
Calculations and Graphs from J. Kirk's Original 6-Strut Tensegrity

Here is the verbatim text of the gnuplot program I wrote which generates plots for the original formula for d in terms of ϕ in John Kirk's notes.

```
set terminal latex
set output "plot1.tex"
set xlabel "$\phi$ axis"
set ylabel "$d$ axis"
set nokey
set xtics -pi/4, pi/16
set ytics 0, 0.025
set size 1.0, 1.25
set nozeroaxis
set format x "$%.2f$"
set title "$d=\frac{4\sin^2\phi-1+\sqrt{1+8(1-2\sin^2\phi)^2}}{4\cos\phi}$"
d(x)=((4.*(sin(x)**2))-1.+sqrt(1.+8.*(1.-2.*(sin(x)*sin(x)))*(1.-2.*(sin(x)*sin(x)))))/(4.*cos(x))
plot [x=-pi/4:pi/4] [0.475:0.725] d(x) with lines
```

Here is the graph of d with respect to ϕ:



Here is Calc's processing of the $d(x)$ used in the gnuplot program:

```
((4.*(sin(x)**2))-1.+sqrt(1.+8.*(1.-2.*(sin(x)*sin(x)))*(1.-2.*(sin(x)*sin(x)))))/(4.*cos(x))
```

Which Calc simplifies to:

```
(4. sin(x)^2 - 1. + sqrt(1. + 8. (1. - 2. sin(x)^2)^2)) / 4. cos(x)
```

In “big” notation this becomes:

```
    ---------------------------
 2         |                       2 2
4. sin(x)  - 1. + \| 1. + 8. (1. - 2. sin(x) )
-----------------------------------------------
   4. cos(x)
```

Now to get the derivative. Here is the formula for $d(x)$ as it was when GNU Calc began it's derivation of the derivative.

```
(4. sin(x)^2 - 1. + sqrt(1. + 8. (1. - 2. sin(x)^2)^2)) / 4. cos(x)

% [calc-mode: language: nil]
% [calc-mode: symbolic: nil]
% [calc-mode: fractions: nil]
% [calc-mode: angles: rad]
```

Here is the derivative $d'(x)$:

```
(8. sin(x) cos(x)
   - 32. (1. - 2. sin(x)^2) sin(x) cos(x) / sqrt(1. + 8. (1. - 2. sin(x)^2)^2))
  / 4. cos(x)
  + 0.25 (4. sin(x)^2 - 1. + sqrt(1. + 8. (1. - 2. sin(x)^2)^2)) sin(x)
      / cos(x)^2
```

Which simplifies by collecting the $cos(x)$ terms:

```
2. sin(x) - 8. sin(x) (1. - 2. sin(x)^2) / sqrt(8. (1. - 2.  sin(x)^2)^2 + 1.)
+ 0.25 sin(x) (4. sin(x)^2 + sqrt(8. (1. - 2. sin(x)^2)^2 + 1.) - 1.) / cos(x)^2
```

In ”big” notation:

```
% [calc-mode: language: big]
 ---------------------------
      2                          2    |                   2 2
     8. sin(x) (1. - 2. sin(x) )     0.25 sin(x) (4. sin(x)  + \| 8. (1. - 2. sin(x) )  + 1.  - 1.)
2. sin(x) - ---------------------------- + ------------------------------------------------------------
      ---------------------------                                    2
     |                   2 2                                  cos(x)
    \| 8. (1. - 2. sin(x) )  + 1.
```

Here is the gnuplot program to plot this:

```
set output "plot2.tex"
set autoscale
set xlabel "$\phi$ axis"
set nokey
set ytics 0, 0.025
set size 1.25, 1.5
set nozeroaxis
set xzeroaxis
set format x "$%.2f$"
set ylabel "$d'$ axis"
set xtics -pi/4, pi/16
set ytics -1.75, 0.25
set title "$d' = \frac{2 \cos\phi \sin\phi -
\frac{8\cos\phi \sin\phi ( 1-2\sin^2\phi ) }{\sqrt{8(1-2\sin^2\phi)^2 + 1}}}{\cos\phi} +
\frac{(4\sin^2\phi + \sqrt{8(1-2\sin^2\phi)^2} -1)}{4\cos^2\phi} $\\[0.5cm]"
f(x) = 2.*sin(x) -
8.*sin(x)*(1. - 2.*(sin(x)*sin(x))) / sqrt(8.*(1. - 2.*(sin(x)*sin(x)))*(1. - 2.*(sin(x)*sin(x))) + 1.)
+ (0.25*sin(x))*(4.*(sin(x)*sin(x)) + sqrt(8.*(1. - 2.*(sin(x)*sin(x)))*(1. - 2.*(sin(x)*sin(x))) + 1.) - 1.)
/ (cos(x)*cos(x))
plot [x=-pi/4:pi/4] [-1.75:1.75] f(x) with lines
```

Here is the plot of the derivative:

$$d' = 2\sin\phi - \frac{8\sin\phi(1-2\sin^2\phi)}{\sqrt{8(1-2\sin^2\phi)^2+1}} + \frac{4\sin^2\phi-1+\sqrt{8(1-2\sin^2\phi)^2}}{4\cos^2\phi}$$