Three Strut Thoughts

We can imagine two equilateral triangles of identical dimensions in three-dimensional space, in planes parallel to each other, and with centers on a single axis orthogonal to both triangles. If a vertex of each triangle is associated with one of the other, so that the line connecting them is parallel to the axis between triangle centers, and lines are imagined connecting all three pairs of such vertices, we will have a shape known as a triangular prism. We can assume, for visualization concreteness, that the distance between the two triangles is on the order of two or three times the edge length of the triangles.

Now, we can imagine holding one triangle fixed and rotating the other triangle clockwise (looking toward the fixed triangle) on the common axis, while allowing the lines connecting the two triangles to move keeping their connectivity the same. When the rotation reaches one hundred twenty degrees, the two triangles will align in the way they did at the beginning, but the lines between them will trace diagonals of the shape of the original prism. If we continue rotating the triangles from one hundred twenty degrees to one hundred eighty degrees, the lines connecting them will intersect at that point. Continuing the rotation further would result in a configuration identical to a rotation in the opposite direction through one hundred eighty degrees, progressing down through one hundred twenty degrees and then further to zero, so we can think of this further rotation beyond one hundred eighty degrees of no interest.

Now it is interesting to consider only the rotation beyond one hundred twenty degrees, and less than one hundred eighty degrees, as well as thinking of the size of the moving triangle changing from equal to the fixed triangle progressively smaller down to, but never equalling zero. Also, it's interesting to consider the distance between the two triangles moving from any value greater than zero up to any finitely large value.

We now have in mind a model for a structure that can be parameterized by three parameters, one for the rotation, one for the distance between the triangles, and one for the relative size of the moving triangle. We can think of the distance between the triangles as being relative to the edge length of the fixed triangle, thinking of that as unit length.

This abstract model can be thought of as instructive as we imagine constructing something similar in the real world using real materials. In the real world, the struts and cables will have non-zero thickness that will limit how close to the ends of the ranges several parameters may come, before interfering with each other or before force equilibrium is threatened by three or more forces becoming close to co-planar and, thus, possibly reversing the direction of their effects due to stretch of component materials.

The rotation parameter will not become of interest, as long as it is kept near the middle of its allowed range. As a result, we have a two-parameter model. Examination of this two-dimensional space of possibilities, we will see a large middle ground where structures can be characterized as primarily volumetric. These will range between roughly cylindrical and roughly pyramidal, as the difference between sizes of the two triangles range from small to large. At the extremes of the ratio of distance between the cylinder-end triangles to the size of the triangles, we see either a mostly one-dimensional structure (long and thin), or a mostly two-dimensional structure (wide and flat). In this latter case, we will see two extremes of shape along with a middle ground. The middle ground will resemble a hexagon of cables, cross-trussed with a triangular shape of struts. At the extremes, we'll see a triangle of cables, with either a comparable triangle of struts or a three-pointed star shape of struts within the triangle.

— John Kirk