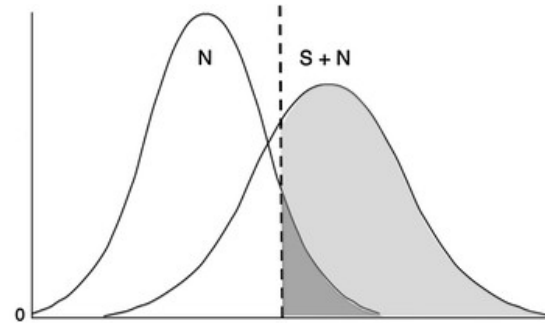
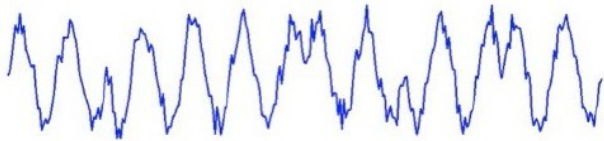


# Aplicações de sinais

## Tópicos sobre Detecção e Estimação ... 4



- Emitter - Signal -

		Does The Effect Exist?	
		Effect Exists	Effect Doesn't Exist
Receptor - Decision	Was The Effect Observed? Effect Observed	<b>Hit Rate</b> True Positive Rate Statistical Power $(1 - \beta)$	<b>False Alarm Rate</b> False Positive Rate Statistical Significance Type I Error Rate ( $\alpha$ )
	Effect Not Observed	<b>Miss Rate</b> False Negative Rate Type II Error Rate ( $\beta$ )	<b>Correct Rejection Rate</b> True Negative Rate $(1 - \alpha)$

Prof. Raul T. Rato

DEEC - 2021

Começando:

Apresentação para esta aula: (6 Mai )

**Tiago Marques de Simões Rodrigues 52856**

Que ôndula (b00, b01, b10, b11) está escondida no sinal kSs1.

Descreva o que fez e o que encontrou.

# Tópicos sobre Detecção e Estimação ... 4

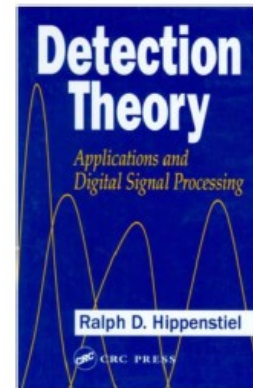
## 1.1 GENERAL PHILOSOPHY

The roots of modern detection theory can be found in the desire to protect valuable resources by detecting and destroying enemy airplanes and missiles under all types of environmental conditions

A large body of detection-related articles and books can be found in the statistics area.

Hypothesis testing relates to a topic that historically deals with statistics.

This statistics topic focuses on the detection and/or estimation of certain phenomena and usually includes the computation of the errors associated with these procedures.



### Information



#### Detection: A ~~statistical~~ problem

- The data consists of signal + noise
- Detection of a signal is always a decision based on probabilities
  - One can never be 100% sure of a detection

Goal of detection theory:

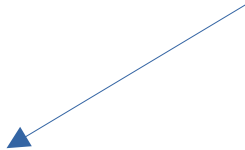
1. Assess the probability of detection
2. ~~Provide~~ confidence limits *Hard decisions must be done*
3. Often one must search within a family of signals
4. **One must decide on what is a signal before the analysis**

## Tópicos sobre Detecção e Estimação ... 4

The roots of modern detection theory can be found in the desire to protect valuable resources by detecting and destroying enemy airplanes and missiles under all types of environmental conditions

Detection must be sufficiently early  
early detection

is essential and hence, detection must usually be accomplished at very low signal-to-noise ratios (SNRs).



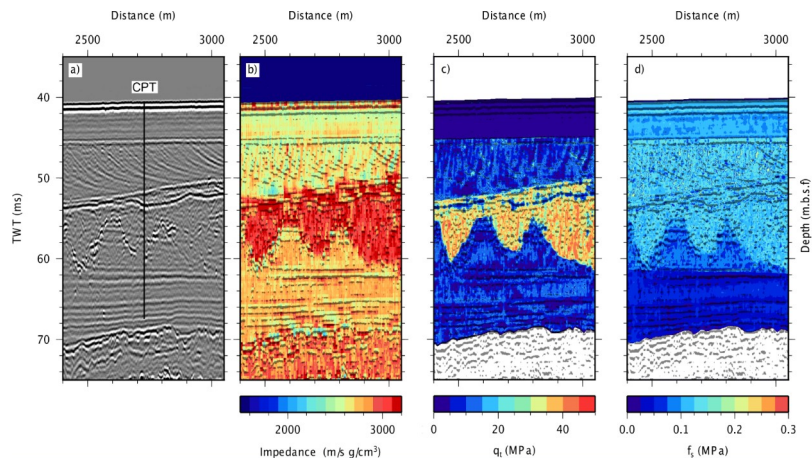
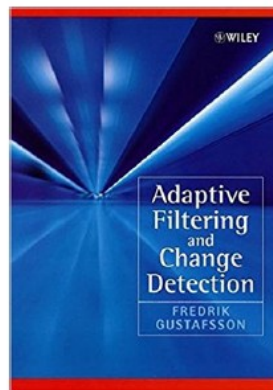
The presentation is oriented toward Hilbert spaces and projections (inner products) in these spaces. This allows a general description

## Tópicos sobre Detecção e Estimação ... 4

No entanto, há muitas mais coisas para além das aplicações na defesa.

Por exemplo, as aplicações industriais. ➔

Mas a problemática é a mesma



## 1.2 DETECTION AND ESTIMATION PHILOSOPHY

### DETECTION

declaring of

the occurrence of a particular event

with some measure of confidence.

### ESTIMATION

Estimation can be interpreted as an extension of the detection part.

how much of a particular item of interest there is

with some measure of the accuracy or confidence.



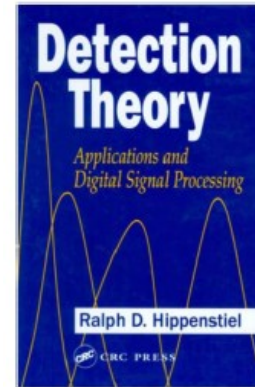
## Tópicos sobre Detecção e Estimação ... 4

typical detection examples

- (a) Radar: passive, active, or bi-static (i.e., it uses an antenna and tests for target presence)
- (b) Sonar: bi-static, active, or passive (i.e., it uses hydrophones and tests for target presence)
- (c) Digital communication: coded binary words (symbols), (i.e., one wants to detect transmitted, possible encoded, message bits consisting of binary zeros and ones)
- (d) Acoustic detection (i.e., it uses microphones): for intrusion alarm, emitter detection (i.e., gunnery, helicopter sound detection/localization, etc.)
- (e) Seismic detection (i.e., it uses geophones): to detect an earthquake, tunnel digging, nuclear testing, etc.

Nota: estes exemplos são muito “Defense oriented”.

Há muitos mais. Pensar em comunicações, controle, biomédica, ...





## Detection

In general, we obtain data (also called measurements or observations) which is thought to consist of one or several signal components embedded in additive noise. The noise may be natural (i.e., the environment, medium, channel, electronics, etc.) or man-made (i.e., jammer, power lines, shipping noise, oil exploration activities in an ocean environment, etc.). Based on the received data, we try to decide whether or not a particular event has occurred.

## Detection

three types

**I:** Known signals in additive noise.

**II:** Signals with unknown parameters in additive noise.

**III:** Random signals in additive noise.

## Tópicos sobre Detecção e Estimação ... 4

I: Known signals in additive noise.

### Example 1.1

$$s_0(t) = \cos(\omega_0 t) \quad (\text{i.e., a signal with carrier frequency } \omega_0)$$

$$s_1(t) = \cos(\omega_1 t) \quad (\text{i.e., a signal with carrier frequency } \omega_1)$$

$$r(t) = s_i(t) + n(t) \quad \text{for } 0 \leq t \leq T \quad \text{and } i = 0, 1$$

where  $r(t)$  is the received, noise corrupted signal and  $n(t)$  is an additive noise process.

## Tópicos sobre Detecção e Estimação ... 4

### I: Known signals in additive noise.

#### Example 1.2

$$s_0(t) = A_0 \text{ (i.e., a DC level of value } A_0\text{)}$$

$$s_1(t) = A_1 \text{ (i.e., a DC level of value } A_1\text{)}$$

$$r(t) = s_i(t) + n(t) ; \text{ for } 0 \leq t \leq T ; i = 0, 1$$

#### Example 1.3

$$s_0(t) = \cos(\omega_c t) \text{ (i.e., a signal with carrier frequency } \omega_c\text{)}$$

$$s_1(t) = \sin(\omega_c t) \text{ (i.e., a phase shifted version of } s_0(t)\text{)}$$

$$r(t) = s_i(t) + n(t) ; \text{ for } 0 \leq t \leq T \text{ and } i = 0, 1$$

where  $r(t)$  is the received, noise corrupted signal and  $n(t)$  is an additive noise process.

## Tópicos sobre Detecção e Estimação ... 4

### II: Signals with unknown parameters in additive noise.

Signal shape, type, etc., is known except for some signal-related parameter or parameters. Typical examples are unknown carrier frequency, amplitude, or phase.

One will decide, based on the received data  $r(t)$ , whether or not message (or signal)  $s_0$  or  $s_1$  was transmitted.

#### Example 1.4

$$r(t) = \cos(\omega_i t + \theta_i) + n(t) \quad \text{for } i = 0, 1 \text{ and for } 0 \leq t \leq T$$

where  $r(t)$  is the received, noise corrupted signal,  $\theta_0$  and  $\theta_1$  are unknown deterministic phase values, and  $\omega_0$  and  $\omega_1$  are known.

## II: Signals with unknown parameters in additive noise.

### Example 1.5

$$s(t) = \cos[\omega_c(t)] \quad \text{for } 0 \leq t \leq T \quad (\text{radar, sonar problem})$$

$$r(t) = \begin{cases} a \cos [(\omega_c + \omega_D)(t - \tau) + \theta] + n(t) \\ n(t) \end{cases} \quad \text{for } \tau \leq t \leq \tau + T$$

where  $a$  represents signal attenuation; the coefficient  $\theta$  is an unknown phase shift,  $n(t)$  is the additive noise component,  $\omega_D$  represents the unknown Doppler shift, and  $\tau$  is the unknown time delay (proportional to the round trip distance to the target).

## Tópicos sobre Detecção e Estimação ... 4

### III: Random signals in additive noise.

Signals are described statistically. That is, the unknowns are described statistically (i.e., via probability density functions, or moments, or correlation functions).

One will decide, based on the received data  $r(t)$ , whether or not the message (or signal)  $s_w(t)$  is present or not.

#### Example 1.6

Podem ser: EEG response to stimulus

$$r(t) = \begin{cases} s_w(t) + n(t) & ; \text{ if the target is present} \\ n(t) & ; \text{ if the target is not present for } 0 \leq t \leq T \end{cases}$$

where  $s_w(t)$  is a realization of a random process, and a probabilistic description of  $s_w(t)$  is available.



## Estimation

Some typical examples are

- (a) Frequency estimation (i.e., determine the actual Doppler shift)
- (b) Differential time delay estimation (i.e., determine the distance or differential distance to a target)
- (c) Amplitude estimation (i.e., target strength, size, distance)
- (d) Phase and/or bandwidth estimation (i.e., target identification)
- (e) Spectral estimation (i.e., target identification)
- (f) Analog wave form estimation (i.e., original noise-free recovery of the original wave form using optimal filtering)

## 1.2 DETECTION AND ESTIMATION PHILOSOPHY

---

### DETECTION

declaring of

the occurrence of a particular event

with some measure of confidence,

error rate

Digital communication: coded binary words (symbols), (i.e., one wants to detect transmitted, possible encoded, message bits consisting of binary zeros and ones)

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### ESTIMATION

Estimation can be interpreted as an extension of the detection part.

how much of a particular item of interest there is

with some measure of the accuracy or confidence.

# Tópicos sobre Detecção e Estimação ... 4

## Detection three types

**I:** Known signals in additive noise.

Transmitted wavelets, ...

$$s_0(t) = A_0 \text{ (i.e., a DC level of value } A_0)$$

$$s_1(t) = A_1 \text{ (i.e., a DC level of value } A_1)$$

$$r(t) = s_i(t) + n(t) ; \text{ for } 0 \leq t \leq T ; i = 0, 1$$

**II:** Signals with unknown parameters in additive noise.

Doppler echoes, ...

Signal shape, type, etc., is known except for some signal-related parameter or parameters. Typical examples are unknown carrier frequency, amplitude, or phase.

One will decide, based on the received data  $r(t)$ , whether or not message (or signal)  $s_0$  or  $s_1$  was transmitted.

**III:** Random signals in additive noise.

EEG response to stimulus, ...

Signals are described statistically. That is, the unknowns are described statistically (i.e., via probability density functions, or moments, or correlation functions).

One will decide, based on the received data  $r(t)$ , whether or not the message (or signal)  $s_w(t)$  is present or not.

$$r(t) = \begin{cases} s_w(t) + n(t) & ; \text{ if } \text{ is present} \\ n(t) & ; \text{ if } \text{ is not present} \end{cases} \quad \text{for } 0 \leq t \leq T$$

where  $s_w(t)$  is a realization of a random process, and a probabilistic description of  $s_w(t)$  is available.

# Tópicos sobre Detecção e Estimação ... 4

## SPACES INVOLVED IN THE DECISION

A detecção é uma decisão!

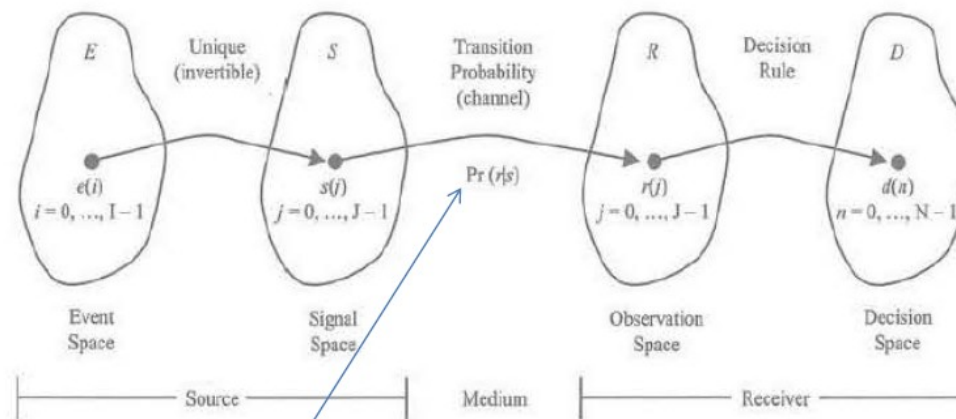


Figure 1.1: Decision model.

Ruído!

# Tópicos sobre Detecção e Estimação ... 4

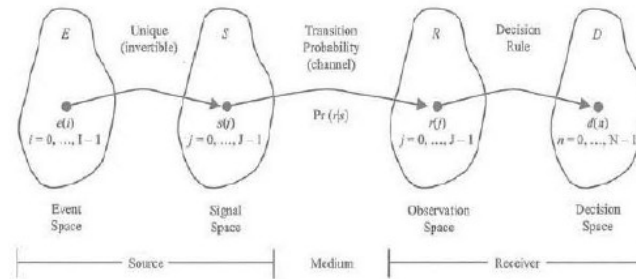


Figure 1.1: Decision model.

events are encoded as signals (i.e., quantities that propagate through the channel)

one event could be translated into or represented by several signals.

the number of signals ( $J$ ) can be different from the number of events ( $I$ ).

signals are coupled via the channel, which introduces noise, to the receiver which also is called the detector.

# Tópicos sobre Detecção e Estimação ... 4

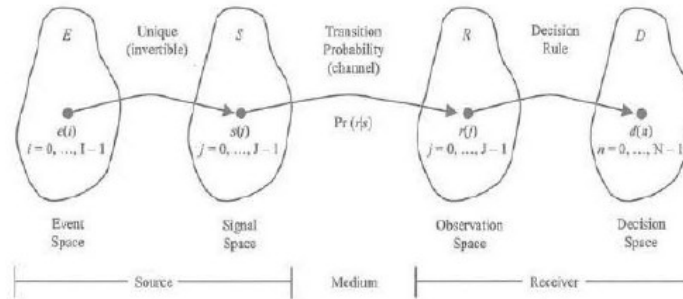


Figure 1.1: Decision model.

## E: Event Space

In the event space  $E$ , one of the  $I$  possible events can happen. We may or may not know *a priori* probabilities of the event  $e_i$  (i.e.,  $\Pr\{e_i\}$  for  $i = 0, \dots, I-1$ ). Typical events may be

- (a) Messages: alphabetical, numeric, station keeping, Morse code
- (b) Parameters: frequency, phase angle, object moving or not, hence, object speed, etc.
- (c) Targets: target present/target absent, friend/foe, etc.

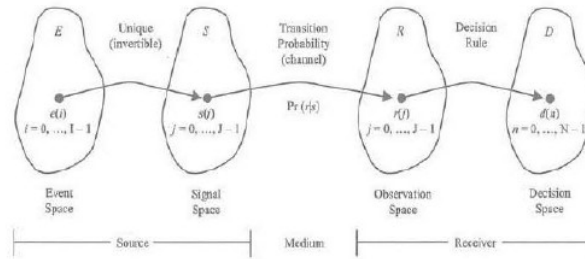


Figure 1.1: Decision model.

S: Signal Space

The events are converted into representative signals.

might use the original data (events) as the signal directly.

### Example 1.7 Phase Modulation

Events :

Signals :

$$\begin{aligned} e_0 &= 0^\circ = \theta_0 \rightarrow s_0(t) = A \cos(\omega_0 t + \theta_0) \\ e_1 &= 180^\circ = \theta_1 \rightarrow s_1(t) = A \cos(\omega_0 t + \theta_1); \quad \text{for } 0 \leq t \leq T \end{aligned}$$

### Example 1.8 Event Encoding

Events :

Signals :

EEG...	$A$	$=$	$e_0$	$\rightarrow$	$s_0(t)$	$=$	signal representing event $A$
	$B$	$=$	$e_1$	$\rightarrow$	$s_1(t)$	$=$	signal representing event $B$
	$C$	$=$	$e_2$	$\rightarrow$	$s_2(t)$	$=$	signal representing event $C$
	$\vdots$				$\vdots$		
							; for $0 \leq t \leq T$



## Tópicos sobre Detecção e Estimação ... 4

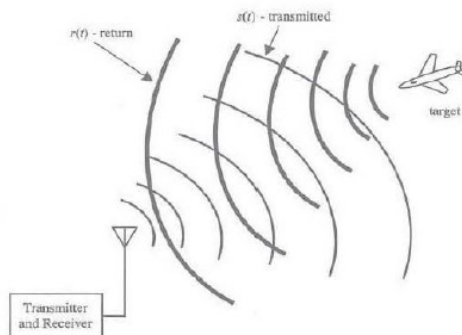


Figure 1.2: Typical target detection problem.

### Example 1.9 Radar/Sonar Problem

Figure 1.2 provides a typical target detection scenario. The signal  $s(t)$  of duration  $T$  is transmitted. A target is  $R$  units in range away. The observation  $r(t)$  is received by the receiver. The delay time is  $t_R = 2R/C$ ; where  $C$  is the propagation velocity. No noise or Doppler is considered.

Events :

Signals :

$$e_0(\text{no target}) \rightarrow \tau(t) = s_0(t) = 0 \quad ; \quad \text{for } 0 \leq t$$

$$e_1(\text{target present}) \rightarrow \tau(t) = s_1(t) = a s(t - t_R) \quad ; \quad \text{for } t_r \leq t_R \leq t_R + T$$

where  $a$  is the signal loss coefficient.

## Tópicos sobre Detecção e Estimação ... 4

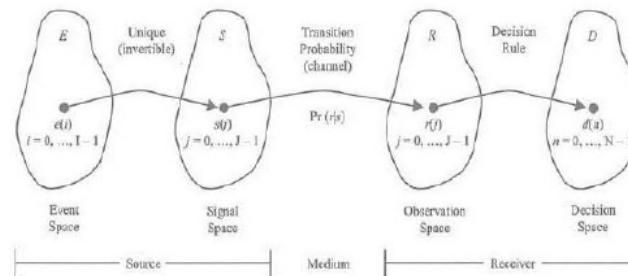


Figure 1.1: Decision model.

### R: Observation Space

The signals are observed (i.e., obtained) after having passed through some channel (i.e., the medium). Usually, this accounts for the additive noise component  $n(t)$ , which may be due to the medium, the electronics, and may also potentially be due to a jammer (i.e., man-made noise). In general, the received data is the transmitted signal plus an additive noise component:

$$\underline{r(t) = s(t) + n(t)}.$$

### R: Observation Space

#### Example 1.13 Target Detection

Events :

Received Data :

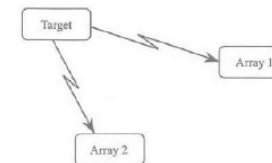
$$e_0(\text{no target}) \rightarrow r_0(t) = n(t) ; \text{ for } 0 \leq t \leq T$$

$$e_1(\text{target present}) \rightarrow r_1(t) = a s(t - t_R) + n(t) ; \text{ for } t_R \leq t \leq T + t_R$$

where  $a$  is the signal loss coefficient (i.e., no Doppler shift assumed in this particular example) and  $t_R$  is the propagation delay, which accounts for the round trip to and from the target.

## Tópicos sobre Detecção e Estimação ... 4

### R: Observation Space



#### Example 1.14 Two Array Processing

Two arrays are receiving data from a particular geographical area. The target may or may not be present and may be or not be observed at either or both locations. Details can be seen in Figure 1.3.

Denote by  $r_{ij}(t)$  the data received at array  $i$  given  $H_j$ , the  $j^{\text{th}}$  hypothesis, is true.

Events :

Received Data :

$$\begin{aligned}
 e_0(\text{no target}) &\rightarrow \begin{aligned} r_{10}(t) &= n_1(t) ; \text{ for } 0 \leq t \leq T \\ r_{20}(t) &= n_2(t) ; \text{ for } 0 \leq t \leq T \end{aligned} \\
 e_1(\text{target present}) &\rightarrow \begin{aligned} r_{11}(t) &= a_1 s(t - t_1) + n_1(t) ; \text{ for } 0 \leq t \leq T \\ r_{21}(t) &= a_2 s(t - t_2) + n_2(t) ; \text{ for } 0 \leq t \leq T \end{aligned}
 \end{aligned}$$

where  $a_i$  is the loss coefficient for the signal component at the  $i^{\text{th}}$  array, assuming a zero Doppler shift, and  $t_i$  represents the delay to reach the  $i^{\text{th}}$  array, for  $i = 1, 2$ , and  $T$  is the duration of the snapshot used in the decision.

Window

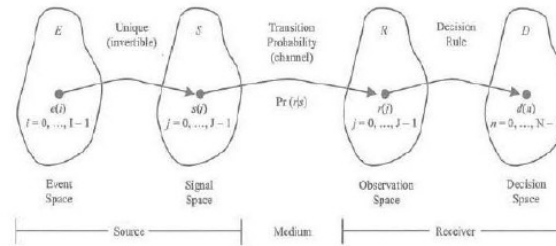


Figure 1.1: Decision model.

## D: Decision Space

The dimension of  $D$  is usually the same as the dimension of  $E$ , but not always.

If  $\dim(D) = \dim(E)$ , then we try in a best, that is in an optimal, fashion to estimate the original message. An example for  $\dim(D)$  to be less than  $\dim(E)$  is the estimation of target parameters leading to the decision yes or no, based on multiple pieces of information, such as multiple spectral lines in the passive SONAR detection problem.

# Tópicos sobre Detecção e Estimação ... 4

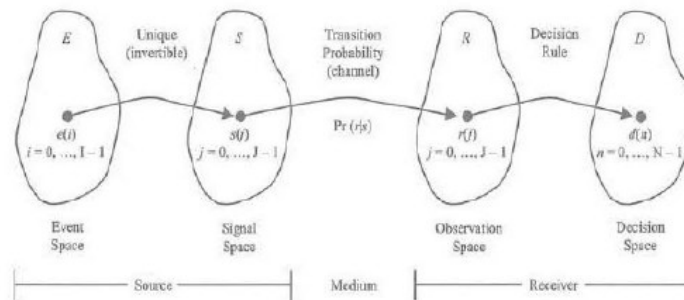


Figure 1.1: Decision model.

Muitas vezes o problema de engenharia é:

“how to obtain a mapping from  $R$  to  $D$ .”

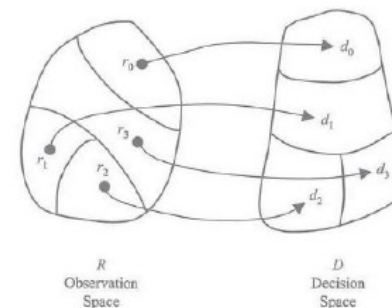
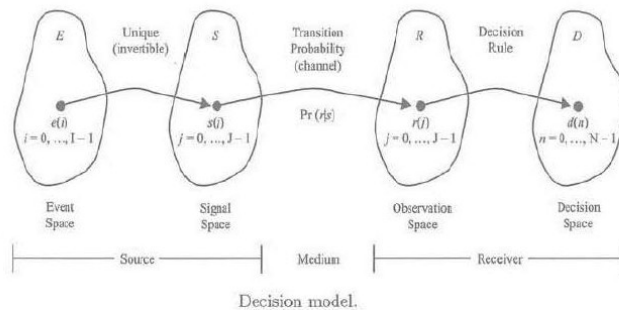


Figure 1.4: Decision regions for Example 1.16.

**Example 1.16** Multiple Decision (see Figure 1.4)

$d_i$ : choose  $H_i$ , the hypothesis that the event  $e_i$  occurred which means that the observation must lie in the space (region)  $R_i$ , for  $i = 0, 1, 2, 3$ .

## Tópicos sobre Detecção e Estimação ... 4

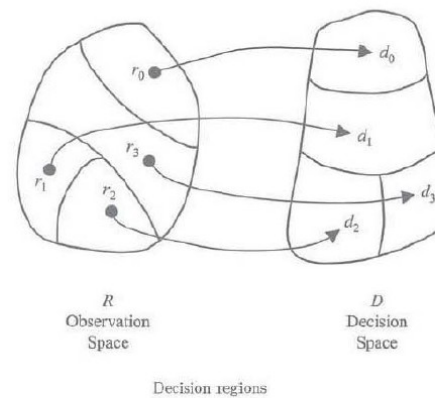


The dimension of  $D$  is usually the same as the dimension of  $E$ .

**Como decidir?**  
**Como escolher o  $d_i$ ?**

Muitas vezes o problema de engenharia é:

"how to obtain a mapping from  $R$  to  $D$ ."

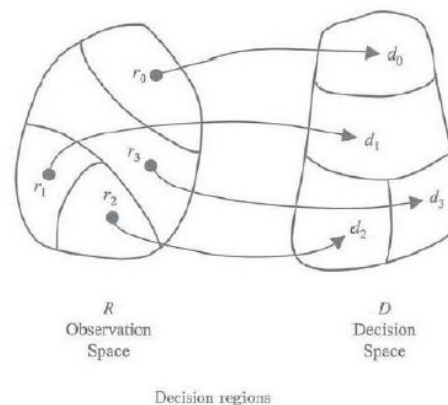


$d_i$ : choose  $H_i$ , the hypothesis that the event  $e_i$  occurred which means that the observation must lie in the space (region)  $R_i$ , for  $i = 0, 1, 2, 3$ .



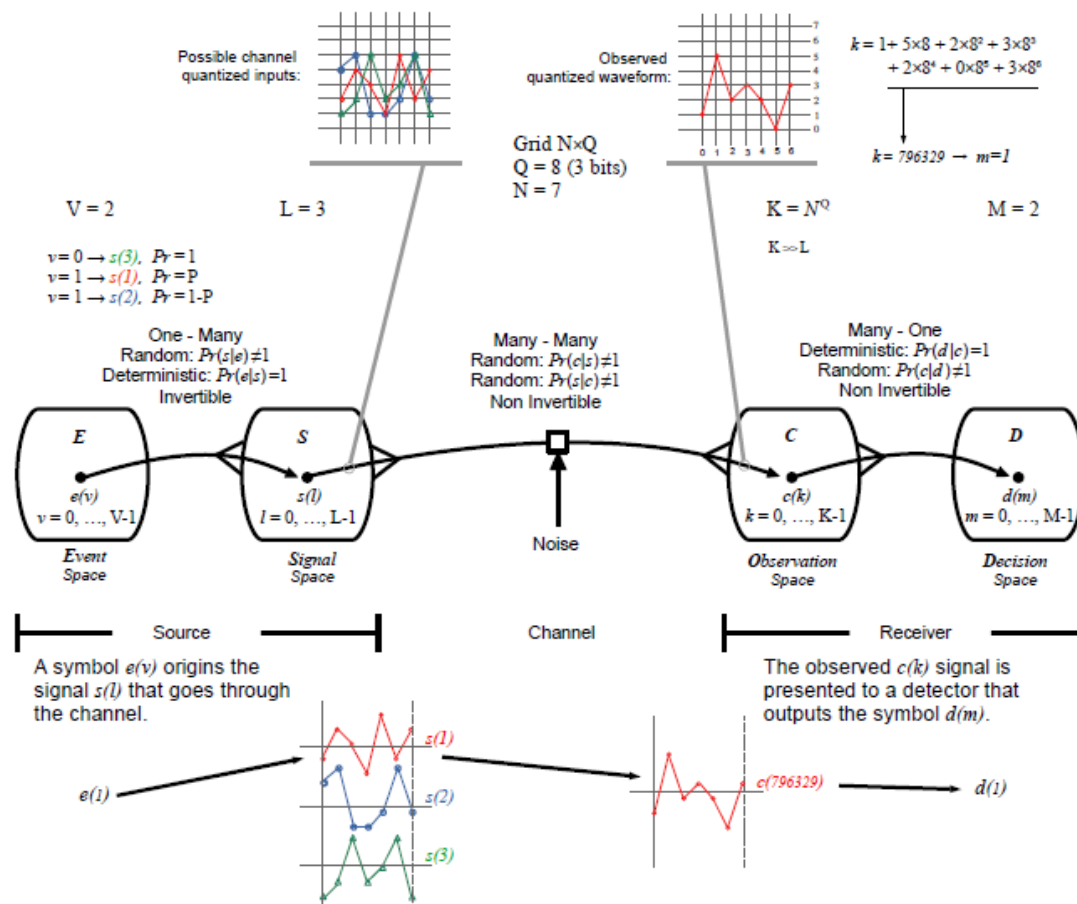
## Tópicos sobre Detecção e Estimação ... 4

### Detection



In general, we obtain data (also called measurements or observations) which is thought to consist of one or several signal components embedded in additive noise. The noise may be natural (i.e., the environment, medium, channel, electronics, etc.) or man-made (i.e., jammer, power lines, shipping noise, oil exploration activities in an ocean environment, etc.). Based on the received data, we try to decide whether or not a particular event has occurred.

# Tópicos sobre Detecção e Estimação ... 4



# Tópicos sobre Detecção e Estimação ... 4

- Emitter - Signal -

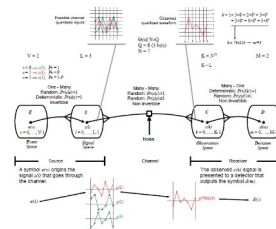
		Does The Effect Exist?	
		Effect Exists	Effect Doesn't Exist
Receptor - Decision	Effect Observed	<b><u>Hit Rate</u></b> True Positive Rate Statistical Power $(1 - \beta)$	<b><u>False Alarm Rate</u></b> False Positive Rate Statistical Significance Type I Error Rate ( $\alpha$ )
	Effect Not Observed	<b><u>Miss Rate</u></b> False Negative Rate Type II Error Rate ( $\beta$ )	<b><u>Correct Rejection Rate</u></b> True Negative Rate $(1 - \alpha)$

# Tópicos sobre Detecção e Estimação ... 4

Começando:

Apresentação para a próxima aula: (11 Mai )

Apresente e descreva a relação e os pontos de equivalência entre o que está exposto nos slides 31 e 32.



		- Emitter - Signal -	
		Does The Effect Exist?	
Receiver - Decision	Effect Observed	<b>Hit Rate</b> True Positive Rate Statistical Power $(1 - \beta)$	<b>False Alarm Rate</b> False Positive Rate Statistical Significance Type I Error Rate ( $\alpha$ )
	Effect Not Observed	<b>Miss Rate</b> False Negative Rate Type II Error Rate ( $\beta$ )	<b>Correct Rejection Rate</b> True Negative Rate $(1 - \alpha)$

## Tópicos sobre Detecção e Estimação ... 4



OBRIGADO