3 a) $F = m\frac{dv(t)}{dt} = f_f(t) + f(t)$

$f_f = -\beta v(t)$

$\dot{v} = \frac{1}{m}\left(-\beta v(t) + f(t)\right)$ $\qquad v(t) = \dot{p}(t)$

$\ddot{p} = \frac{1}{m}\left(-\beta \overset{=v}{\dot{p}(t)} + f(t)\right)$ $\qquad \ddot{p} = \dot{v}$

b) $\begin{bmatrix} \dot{p}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{m}\beta \end{bmatrix} \begin{bmatrix} p(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f(t)$

$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} p(t) & v(t) \end{bmatrix}^T$

c) $\dot{v}(t) = -\frac{\beta}{m} v^2 + \frac{1}{m} f(t)$

Em equilíbrio $\rightarrow$ $v(t) = v_e$ $\Rightarrow f_e(t) = \beta v^2$

$v^2 = v_e^2 + 2 v_e \delta v$

$\frac{d(v_e + \delta v(t))}{dt} = -\frac{\beta}{m}\left(v_e^2 + 2 v_e \delta v\right) + \frac{\beta}{m}\left(f_e + \delta f(t)\right)$

$\frac{f_e}{m} = \frac{\beta v_e^2}{m}$

$\dot{\delta} v(t) = -\frac{f_e}{m} + 2 v_e \delta v + \frac{\beta}{m} f_e + \frac{\beta}{m} \delta f$

a) $\sum F = \dot{v} m \Rightarrow \dot{v} = \frac{1}{m}\left(f(t) - \beta v\right)$

$\dot{p} = v \Rightarrow \dot{v} = \ddot{p} = \frac{1}{m}\left(f(t) - \beta \dot{p}\right)$

$$s^2 P(s) = \frac{1}{m}\left(F(s) - s\beta P(s)\right)$$

$$H(s) = \frac{P(s)}{F(s)}$$

$$s^2 P(s) + \frac{1}{m} s\beta P(s) = \frac{1}{m} F(s)$$

$$P(s)\left(s^2 + \frac{1}{m}\beta s\right) = \frac{1}{m} F(s)$$

$$H(s) = \frac{1/m}{s^2 + \frac{\beta s}{m}} \Leftrightarrow H(s) = \frac{1/m}{s\left(s + \frac{\beta}{m}\right)}$$

b) $$\begin{bmatrix} \dot{p} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{\beta}{m} \end{bmatrix} \begin{bmatrix} p \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} f(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} p & v \end{bmatrix}^T$$

c) $$\dot{v} = \frac{1}{m}\left(f(t) - \beta v^2(t)\right)$$

$$v^2 = v_e^2 + 2 v_e \delta v$$

$$\frac{d(v_e + \delta v(t))}{dt} = \frac{1}{m}\left((f_e(t) + \delta f(t)) - \beta(v_e^2 + 2 v_e \delta v) \right)$$

$$\frac{f_e}{m} = \frac{\beta v_e^2}{m} \quad ; \quad \frac{d(v_e + \delta v(t))}{dt} = \delta \dot{v}(t)$$

$$\delta \dot{v}(t) = \cancel{\frac{1}{m} f_e(t)} + \frac{\delta f(t)}{m} - \cancel{\frac{f_e(t)}{m}} + \frac{2 v_e \delta v(t)}{m}$$

$$\delta \dot{v}(t) = -\frac{2 v_e \delta v(t) \beta}{m} + \frac{\delta f(t)}{m}$$

$$\delta x_c = \begin{bmatrix} \delta p(t) \\ \delta v(t) \end{bmatrix} \qquad \delta u_c = \delta f(t) \qquad \delta y_c(t) = \delta p(t)$$

$$\begin{bmatrix} \delta \dot{p}(t) \\ \delta \dot{v}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{2 v_e \beta}{m} \end{bmatrix} \begin{bmatrix} \delta p(t) \\ \delta v_e(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} \delta f(t)$$

$$\delta y_c(t) = [1 \quad 0] \, [\delta p(t) \quad 0]$$

d) $T = 0{,}4 \quad , \beta = 0{,}5 \quad , m = 1 \, , \quad v_e = 0{,}5$

$$x(k+1) = A_d \, x(k) + B_d \, u(k)$$

$$A_d = I + AT \implies \begin{bmatrix} 1 & T \\ 0 & 1 - \frac{2 T \beta v_e}{m} \end{bmatrix}$$

$$B_d = BT = \begin{bmatrix} 0 \\ \frac{T}{m} \end{bmatrix} \qquad C_d = C$$

$$x(k+1) = A_d \, x(k) + B_d \, u(k) \qquad y(k) = C_d \, x(k)$$

1.4) c) $\sum F(t) = f(t) - Kp(t) - \beta v(t) - \alpha v(t)^2$

$$\dot{v} = \frac{1}{m}\left(f(t) - Kp(t) - \beta v(t) - \alpha v(t)^2\right)$$

Em equilíbrio, $\dot{x} = 0 \Rightarrow \dot{p}_e = 0 \Rightarrow p_e = C^{te}$

$\dot{v}_e = 0 \Rightarrow v_e = 0$

$\dot{v}_e = 0$

$0 = f_e - Kp_e - \beta v_e - \alpha v_e^2$

$f_e = Kp_e + \beta v_e + \alpha v_e^2$

A expansão da série de Taylor de $v^2(t)$ em v_e:

$$v^2 = v_e^2 + 2 v_e \, \delta v + h.o.t$$

considerando $v(t) = v_e + \delta v(t)$ e $f(t) = f_e + \delta f(t)$:
e $p(t) = p_e + \delta p(t)$:

$$\frac{d(v_e + \delta v(t))}{dt} = \frac{1}{m}\Big(f_e + \delta f(t) - K(p_e + \delta p(t)) - \beta(v_e + \delta v(t))$$

$$- \alpha\left(v_e^2 + 2 v_e \, \delta v(t)\right)\Big)$$

$$\frac{d(\nu_e + \delta\nu(t))}{dt} = \frac{1}{m}\left(\cancel{kp_e} + \cancel{\beta\nu_e} + \alpha\cancel{\nu_e}^2 + \delta f(t) - k(\cancel{p_e} +\right.$$

$$\left. + \delta p(t)) - \beta(\cancel{\nu_e} + \delta\nu(t)) - \alpha(\cancel{\nu_e}^2 + 2\nu_e\,\delta\nu(t))\right)$$

Como $\frac{d(\nu_e + \delta\nu(t))}{dt} = \delta\dot{\nu}(t)$:

$$\delta\dot{\nu}(t) = \frac{1}{m}\left(\delta f(t) - k\,\delta p(t) - \beta\,\delta\nu(t) - 2\alpha\nu_e\alpha\nu(t)\right)$$

$$= \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & \frac{-\beta - 2\alpha\nu_e}{m} \end{bmatrix}\begin{bmatrix} \delta p \\ \delta\nu \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}\delta f(t)$$

$\delta y(t) =$

b) $$\begin{bmatrix} \delta\dot{p}(t) \\ \delta\dot{\nu}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\beta}{m} \end{bmatrix}\begin{bmatrix} \delta p \\ \delta\nu \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}\delta f(t)$$

$$\delta y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \delta p & \delta\nu \end{bmatrix}^T$$

c) $A_d \simeq I + AT$, $B_d \simeq BT$, $C_d \simeq C_e$ $\quad T = 0{,}1$

$k = 0{,}5$

$A_d = \begin{bmatrix} 1 & 0{,}1 \\ -0{,}5 & 0{,}96 \end{bmatrix}$ $\quad B_d = \begin{bmatrix} 0 \\ 0{,}1 \end{bmatrix}$ $\quad C_d = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T$ $\quad \beta = 0{,}4$

$m = 1$

$$x(k+1) = A_d\,x(k) + d_d\,u(k)$$

$y(k) = C_d\, x(k)$

d) $|\lambda I - A_d| = 0 \Leftrightarrow \lambda = 0{,}98 \pm 0{,}068\, j$

Estável

e) $M = \begin{bmatrix} 0 & 0{,}01 \\ 0{,}1 & 0{,}096 \end{bmatrix}$ $\quad C_a(M) = 2$

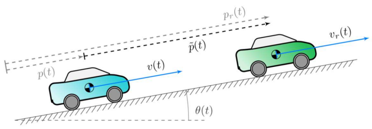
Controlável

f) $V = \begin{bmatrix} 1 & 0 \\ 1 & 0{,}1 \end{bmatrix}$ $\quad C_a(V) = 2$

Observável

Lab1

quinta-feira, 7 de outubro de 2021 09:28



Parameter	Symbol	Value
mass	m	$1500\ kg$
drag coefficient	C_d	0.3
frontal area	A	$1.8\ m^2$
air density	ρ	$1.225\ kg\,m^{-3}$
gravity accel.	g	$9.8065\ m\,s^{-2}$
typical slope	θ_e	$5\%; 0.05\ rad$
max. (engine) force	f_{max}	$2600\ N$
min. (brake) force	f_{min}	$-2000\ N$
max. force var.	Δf_{max}	$50\ N$
max. slope	θ_{max}	$10\%; 0.1\ rad$
desired distance	$\tilde{p}_r$	$10\ m$
min. distance	$\tilde{p}_{min}$	$5\ m$
typ. leader speed	v_{r_e}	$20\ m/s$
typ. wind speed	w_e	$0\ m/s$

- $\tilde{p}(t)$ – distance between the vehicles [m];
- $p(t)$ – position of the follower vehicle [m];
- $p_r(t)$ – position of the leader vehicle [m];
- $v(t)$ – speed of the follower vehicle [m/s];
- $v_r(t)$ – speed of the reference car [m/s];
- $w(t)$ – wind speed [m/s];
- $\theta(t)$ – road slope [rad];
- $f(t)$ – force generated by follower car [N].

$$x(t) = \begin{bmatrix} \tilde{p}(t) \\ v(t) \end{bmatrix}$$

$$\frac{d\tilde{p}(t)}{dt} = v_n(t) - v(t)$$

$$\frac{dv(t)}{dt} = \frac{1}{m}\sum = \frac{1}{m}\left(F_e(t) - F_w(v,w) - F_\theta(\theta) \right)$$

$$F_e(t) = f(t)$$

$$F_w(v,w) = \frac{1}{2}\rho A C_d \left(v(t) + w(t)\right)^2$$

$$F_\theta(\theta) = mg\sin(\theta(t))$$

$$\begin{bmatrix} \dot{\tilde{p}}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} v_n(t) - v(t) \\ \frac{F(t)}{m} - \frac{1}{2m}\rho A C_d \left(v(t) + w(t)\right)^2 - g\sin(\theta(t)) \end{bmatrix}$$

Em equilíbrio $\tilde{p}(t) = 10 \Rightarrow \dot{\tilde{p}}_e = 0 \Rightarrow v_{n_e} = v_e = 20$

$v_n(t) = 20$ $\qquad \dot{v} = 0$

Linearizar $\left(v(t) + w(t)\right)^2$

$$v^2(t) + 2v(t)w(t) + w^2(t)$$

Expansão Taylor $v^2(t)$ e $w^2(t)$

$$v^2 = v_e^2 + 2v_e\,\delta v$$

$$w^2 = w_e^2 + 2w_e\,\delta w$$

$$v(t) = v_e + \delta v(t)$$

$$F(t) = f_e + \delta f(t)$$

$$w(t) = w_e + \delta w(t)$$

$$\left(v_e^2 + 2v_e\,\delta v + 2vw + w_e + 2w_e\,\delta w\right)$$

$$\left(v_e^2 + 2v_e\,\delta v + 2(v_e + \delta v)(\cancel{w_e} + \delta w) + \cancel{w_e^2} +\right.$$

$$\left(v_e^2 + 2v_e\,\delta v + 2\right.$$

Expansão Taylor $\operatorname{Sen}(\theta(t))$

$$\operatorname{Sen}\theta = \operatorname{Sen}(\theta_e) + \cos(\theta_e)\,\delta\theta(t)$$

$$0 = \frac{F_e}{m} - \frac{1}{2m}\rho A C_d \left(v_e + w_e\right)^2 - g\sin(\theta_e)$$

$$F_e = \frac{1}{2}\rho A C_d \left(v_e + w_e\right)^2 + mg\sin(\theta_e)$$

Em equilíbrio:

$$F_e = \frac{20^2}{2}\rho A C_d - mg\,\mathrm{sen}(0{,}05)$$

Considera:

$$F(t) = F_e + \delta F(t) \quad ; \quad v(t) = v_e + \delta v(t)$$

$$\frac{d(v_e + \delta v(t))}{dt} = \frac{1}{m}\left[F_e + \delta F(t) - \frac{1}{2}\rho A C_d\left(v_e^2 + 2v_e\delta v(t) + w_e + 2w_e\delta w\right) - mg\left(\mathrm{Sen}(\theta_e) + \mathrm{Cos}(\right.\right.$$

$$\Leftrightarrow \delta\dot{v}(t) = \frac{1}{m}\left[\frac{v_e^2}{2}\rho A C_d - mg\,\mathrm{Sen}(\theta_e) + \delta F(t) - \frac{1}{2}\rho A C_d\left(v_e^2 + 2v_e\delta v(t)\right)\right.$$

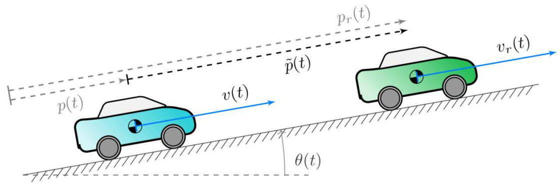
$$\delta\dot{v}(t) = \frac{1}{m}\delta F(t) - \frac{1}{\cancel{2}m}\rho A C_d\,\cancel{2}v_e\,\delta v(t)$$

$$\boxed{\delta\dot{v}(t) = -\frac{20}{m}\rho A C_d\,\delta v(t) + \frac{1}{m}\delta F(t)}$$

$$\delta\dot{\tilde{p}}(t) = \left(\cancel{v_e} + \delta v_n(t)\right) - \left(\cancel{v_e} + \delta v(t)\right) = \boxed{\delta v_n(t) - \delta v(t)}$$

$$\begin{bmatrix}\delta\dot{\tilde{p}}(t)\\ \delta\dot{v}(t)\end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix}\delta p \\ \end{bmatrix}$$

$\underline{\theta} e) \delta \theta(t)) \Big]$



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- $\tilde{p}(t)$ – distance between the vehicles [m];
- $p(t)$ – position of the follower vehicle [m];
- $p_r(t)$ – position of the leader vehicle [m];
- $v(t)$ – speed of the follower vehicle [m/s];
- $v_r(t)$ – speed of the reference car [m/s];
- $w(t)$ – wind speed [m/s];
- $\theta(t)$ – road slope [rad];
- $f(t)$ – force generated by follower car [N].

$$x(t) = \begin{bmatrix} \tilde{p}(t) \\ v(t) \end{bmatrix}$$

$$\frac{d\tilde{p}(t)}{dt} = v_r(t) - v(t)$$

$$\frac{dv(t)}{dt} = \frac{1}{m}\Sigma = \frac{1}{m}\left(F_e(t) - F_w(v,w) - F_g(\theta) \right)$$

$$F_e(t) = f(t)$$

$$F_w(v,w) = \frac{1}{2}\rho A C_d \left(v(t) + w(t)\right)^2$$

$$F_g(\theta) = mg\sin(\theta(t))$$

$$\begin{bmatrix} \dot{\tilde{p}}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} v_r(t) - v(t) \\ \frac{F(t)}{m} - \frac{1}{2m}\rho A C_d \left(v(t) + w(t)\right)^2 - g\sin(\theta(t)) \end{bmatrix}$$

modelo linear

$$\tilde{p}(t) = 1.v_r(t) - 1.v(t)$$

$$\dot{v}(t) = a_{vv}.v(t) + a_{vf}\cdot f(t) + a_{v\theta}\,\theta(t) + a_{vw}\,w(t)$$

$$a_{vv} = -\frac{1}{2m}\rho A C_d \left(v(t) + w(t)\right)^2 \Big|_{v=v_e} = \frac{-\rho A C_d v_e}{m}$$

$$a_{vv} = -\frac{1}{2m}\rho A C_d \left(v(t) + w(t)\right) \Big|_{\substack{v = v_e \\ w = w_e = 0}} = -\frac{\rho A C_d v_e}{m}$$

$$a_{vu} = \frac{f(t)}{m} \Big|_{f = f_e} = \frac{1}{m}$$

$$a_{v\theta} = -g \sin\theta \Big|_{\theta = \theta_e} = -g$$

$$a_{vw} = -\frac{1}{2m}\rho A C_d \left(v(t) + w(t)\right)^2 \Big|_{\substack{v = v_e \\ w = w_e}} = -\frac{1}{2m}\rho A C_d \, 2 \, v_e$$

$$\begin{bmatrix} \dot{\tilde{p}}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 0 & -\frac{\rho A C_d v_e}{m} \end{bmatrix} \begin{bmatrix} \tilde{p}(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} f(t) + \begin{bmatrix} 1 & 0 & 0 \\ 0 & -g & -\frac{1}{2m}\rho A C_d 2 v_e \end{bmatrix} \begin{bmatrix} v_a(t) \\ \theta(t) \\ w(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} p(t) & v(t) \end{bmatrix}^T$$

João Miguel Ferreira de Carvalho
49341

1) a) $ml^2 \cdot \dot{w} = \tau(t) - \tau_h(t) - \tau_d(t) - \cancel{l} \, mg\sin\theta(t)$

$\dot{\theta} = w$

$$\dot{w} = \frac{1}{ml^2}\left(\tau(t) - K_h w(t) - K_d w(t)^2 - lmg\sin(\theta(t))\right)$$

$$\begin{bmatrix} \dot{\theta} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} w \\ \frac{1}{ml^2}\left(\tau(t) - K_h w(t) - K_d w(t)^2 - lmg\sin(\theta(t))\right) \end{bmatrix}$$

b) Em equilíbrio:

$\theta(t) = \theta_e = \pi/6 \implies \dot{\theta}(t) = 0$

$w(t) = w_e = 0 \implies \dot{w}(t) = 0$

Considerar:

$\theta(t) = \theta_e + \delta\theta(t)$

$w(t) = w_e + \delta w(t)$

$\tau(t) = \tau_e + \delta\tau(t)$

Expansão de Taylor:

$w(t)^2 = w_e + 2w_e\,\delta w(t)$

$$\operatorname{Sen}(\theta) = \operatorname{Sen}(\theta_e) + \operatorname{Cos}(\theta_e) \cdot \delta\theta(t)$$

$$0 = \tau_e - K_h \overset{=0}{w_e} - K_d \overset{=0}{w_e}{}^2 - lmg\sin(\theta_e)$$

$$\boxed{\tau_e = lmg\sin(\pi/6)}$$

$$\frac{d(w_e + \delta w(t))}{dt} = \frac{1}{ml^2}\Big[(\tau_e + \delta\tau(t)) - K_h(\cancel{w_e} + \delta w(t)) -$$

$$- \cancel{K_d(w_e + 2 w_e \delta w(t))} - lmg\big(\operatorname{Sen}(\theta_e) + \operatorname{Cos}(\theta_e) \cdot \delta\theta(t)\big)\Big]$$

$$\frac{d(w_e + \delta w(t))}{dt} = \frac{1}{ml^2}\Big[\cancel{lmg\operatorname{Sen}(\tfrac{\pi}{6})} + \delta\tau(t)$$

$$- K_h\,\delta w(t) - lmg\big(\cancel{\operatorname{Sen}(\tfrac{\pi}{6})} + \operatorname{Cos}(\tfrac{\pi}{6})\,\delta\theta(t)\big)\Big]$$

$$\delta\dot{w} = \frac{1}{ml^2}\Big(\delta\tau(t) - K_h\,\delta w(t) - lmg\cos(\tfrac{\pi}{6})\,\delta\theta(t)\Big]$$

$$\begin{bmatrix} \delta\dot{\theta} \\ \delta\dot{w} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l}\cos(\pi/6) & -\frac{K_h}{ml^2} \end{bmatrix} \begin{bmatrix} \delta\theta \\ \delta w \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} \delta\tau(t)$$

$$\begin{bmatrix} \dot{\delta\omega} \end{bmatrix} \quad \begin{bmatrix} -\frac{g}{l}\cos(\frac{\pi}{6}) & \frac{-Kh}{ml^2} \end{bmatrix} \begin{bmatrix} \delta\omega \end{bmatrix} \quad \begin{bmatrix} \frac{1}{ml^2} \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \delta\theta & \delta\omega \end{bmatrix}^T$$

c) $A_d \approx I + AT \quad , \quad B_d \approx BT \qquad T = 0{,}1$

$$A_d = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ \frac{g}{l}\cos(\frac{\pi}{6}) & \frac{-Kh}{ml^2} \end{bmatrix} \times 0{,}1$$

$$A_d = \begin{bmatrix} 1 & 0{,}1 \\ -1{,}06 & 0{,}9063 \end{bmatrix}$$

$$B_T = \begin{bmatrix} 0 \\ 1/ml^2 \end{bmatrix} \times 0{,}1 = \begin{bmatrix} 0 \\ 0{,}1563 \end{bmatrix}$$

$$x(k+1) = A_d\, x(k) + B_d\, \tau(k)$$

$$y(k) = C_d\, x(k)$$

d) $|zI - A_d| = z \approx 1$

Marginal. Estável (Provavelmente houve um erro)

$$M_c = \begin{bmatrix} 0 & 0{,}0156 \\ 0{,}1563 & 0{,}1416 \end{bmatrix}$$

$C_a(M_c) = 2$ Controlável

$Car(M_c) = 2$ Controlável

(2) Considerando o multiplicador de Lagrange $\lambda \in \mathbb{R}$, definimos o Hamiltoniano:

$$H(x,u) = \ell(x,u) + \lambda f(x,u) \Leftrightarrow$$

$$H(x,u) = u^2 - xu - \frac{6}{5}u + \frac{x^2}{2} + \lambda(x+u-1)$$

$$H_\lambda = f(x,u) = 0 \Leftrightarrow x+u-1=0$$

$$H_x = \ell_x + \lambda f_x = 0 \Leftrightarrow x - u + \lambda = 0$$

$$H_u = \ell_u + \lambda f_u = 0 \Leftrightarrow 2u - x - \frac{6}{5} + \lambda = 0$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ u \\ \lambda \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \frac{6}{5} \end{bmatrix}$$

$$\begin{bmatrix} x^* \\ u^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} 0,36 \\ 0,64 \\ 0,28 \end{bmatrix}$$

2.2) Using the Batch approach:

$$\min_{U} J_0 = U^T \tilde{R} U + 2 U^T \tilde{S} x(0) + x(0)^T \tilde{Q} x(0)$$

wich provides at each step the optimal control sequence:

$$U^* = -\tilde{R}^{-1} \tilde{S} x(0) = [\mu^*(0)^T \quad \mu^*(1)^T \quad \mu^*(2)^T \quad \mu^*(3)^T]^T$$

$$\tilde{Q} = \text{diag}(Q, Q, Q, Q, P) = I_{10}$$

$$\tilde{R} = R \times I_4 = I_4$$

$$F = \begin{bmatrix} I \\ A \\ A^2 \\ A^3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 1 \\ 0 & 0{,}5 \\ 4 & 2{,}5 \\ 0 & 0{,}5 \\ 8 & 5{,}25 \\ 0 & 0{,}125 \\ 16 & 10{,}63 \\ 0 & 0{,}063 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ B & 0 & 0 & 0 \\ AB & B & 0 & 0 \\ A^2B & AB & B & 0 \\ A^3B & A^2B & AB & B \end{bmatrix}$$

$$\tilde{R} = G^T G + I_4 = \begin{bmatrix} 37{,}14 & 16{,}28 & 5{,}56 & 0{,}125 \\ 16{,}28 & 9{,}56 & 3{,}125 & 0{,}25 \\ 5{,}56 & 3{,}125 & 3{,}25 & 0{,}5 \\ 0{,}125 & 0{,}25 & 0{,}5 & 2 \end{bmatrix}$$

$$\tilde{Q} = F^T F = \begin{bmatrix} 341 & 224 \\ 224 & 149 \end{bmatrix}$$

$$\tilde{S} = G^T F = \begin{bmatrix} 108 & 72 \\ 48 & 32{,}14 \\ 16 & 10{,}78 \\ 0 & 0{,}06 \end{bmatrix}$$

$$K = \tilde{R}^{-1} \tilde{S} = \begin{bmatrix} 2{,}7867 & 1{,}8398 \\ 0{,}3213 & 0{,}2497 \\ -0{,}1274 & -0{,}0560 \\ -0{,}1825 & -0{,}1009 \end{bmatrix}$$

T

$$X^* = (F - GK)\, x(0) =$$

$$= [\underbrace{-4 \quad -6}_{x_0} \quad \underbrace{-14 \quad 19{,}18}_{x_1} \quad \underbrace{-8{,}81 \quad 12{,}37}_{x_2} \quad \underbrace{-5{,}25 \quad 5{,}34}_{x_3} \quad \underbrace{-5{,}16 \quad 1{,}34}_{x_4}]$$

$k=0\ ,\ x(t_0) = x_0 = \begin{bmatrix} -4 \\ -6 \end{bmatrix}\ ,\ U^* = [22{,}1853 \quad 2{,}7833 \quad -0{,}8557 \quad -1{,}3356]$

$u(t_0) = u^*_0 = 22{,}1853$

$k=1\ ,\ x(t_1) = x_0 = \begin{bmatrix} -14 \\ 19{,}18 \end{bmatrix}\ ,\ U^* = [3{,}7172 \quad -0{,}2911 \quad -0{,}7077 \quad -0{,}6186]$

$u(t_1) = u^*_0 = 3{,}7172$

$k=2\ ,\ x(t_2) = x_0 = \begin{bmatrix} -8{,}81 \\ 12{,}37 \end{bmatrix}\ ,\ U^* = [1{,}7949 \quad -0{,}2573 \quad -0{,}429 \quad -0{,}3595]$

$u(t_2) = u^*_0 = 1{,}7949$

2.3) $\min_U \ J_0 = U^T \tilde{R} U + 2U^T \tilde{S}(\bar{F}x(0) - \bar{y}) + (\bar{F}x(0) - \bar{y})^T \bar{Q} (\bar{F}x(0) - \bar{y})$

$U^* = -Kx(0) + K_Y \bar{Y}$

$Y = \bar{F}x(0) + \bar{G}U$

$$\bar{F} = \begin{bmatrix} 1 & 0 \\ 0{,}4 & 1 \\ 0{,}16 & 3{,}4 \\ 0{,}064 & 10{,}36 \end{bmatrix} \qquad \bar{G} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \\ 6{,}8 & 2 & 0 \end{bmatrix}$$

$\bar{R} = 2\mathbb{I}_3 \qquad \bar{Q} = \mathbb{I}_4$

$$\tilde{R} = \bar{G}^T \bar{G} + \bar{R} = \begin{bmatrix} 52{,}24 & 13{,}6 & 0 \\ 13{,}6 & 6 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\tilde{S} = \bar{G}^T \bar{Q} = \begin{bmatrix} 0 & 0 & 2 & 6{,}8 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K_y = \tilde{R}^{-1}\tilde{S} = \begin{bmatrix} 0 & 0 & 0{,}0934 & 0{,}1059 \\ 0 & 0 & -0{,}2117 & 0{,}0934 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K = K_y \overline{F} = \begin{bmatrix} 0{,}0217 & 1{,}4142 \\ -0{,}0279 & 0{,}2478 \\ 0 & 0 \end{bmatrix}$$

$$X^* = (F - GK)\,x(0) + G K_y \overline{X} = \begin{bmatrix} 2 \\ 2 \\ 2{,}8 \\ 0{,}654 \\ 1{,}7748 \\ 0{,}8482 \\ 1{,}5582 \\ 2{,}5487 \end{bmatrix}$$

$k=0$, $x_d(t_0) = x_{d0} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $U^* = \begin{bmatrix} -2{,}6726 & -0{,}5582 & 0 \end{bmatrix}^T$

$u(t) = u^*_0 = -2{,}6726$

$k=1$, $x_d(t_1) = x_{d0} = \begin{bmatrix} 2{,}8 \\ 0{,}6548 \end{bmatrix}$, $U^* = \begin{bmatrix} -0{,}7876 & -0{,}2025 & 0 \end{bmatrix}^T$

$u(t) = u^*_0 = -0{,}7876$

$k=2$, $x_d(t_2) = \begin{bmatrix} 1{,}7748 \\ 0{,}8482 \end{bmatrix} = U^* = \begin{bmatrix} -1{,}0388 & -0{,}279 & 0 \end{bmatrix}^T$

$u(t) = u^* = -1{,}0388$

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2.6) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

$N = 3$ $P = 1$ $Q = 1$ $R = 4$

$x(0) = [-4{,}5 \ 2]^T$ $\bar{y}(k) = 1$

$-0{,}3 \leq u(k) \leq 0{,}3$

$Y = \bar{F} x(0) + \bar{G} U$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

1.13) $u_k = -Kx$

$P_N = 12 \qquad Q = 2 \qquad R = 3 \qquad A = 0{,}5 \qquad B = 3$

$$P_k = \frac{R A^2 P_{k+1}}{B^2 P_{k+1} + R} + Q = \frac{0{,}75 P_{k+1}}{3^2 P_{k+1} + 3} + 2$$

$P_N = P_4 = 12 \qquad P_2 = 2{,}072 \qquad P_0 = 2{,}0718$

$P_3 = 2{,}081 \qquad P_1 = 2{,}072$

$$K_k = \frac{A B P_{k+1}}{B^2 P_{k+1} + R} = \frac{1{,}5 P_{k+1}}{9 P_{k+1} + 3}$$

$$K_0 = \frac{1{,}5 P_1}{9 P_1 + 3} = 0{,}1436 \qquad K_1 = 0{,}1436 \qquad K_2 = 0{,}1437$$

$K_3 = 0{,}1622$

$x_{k+1} = (A - B K_k) x_k$

$x_1 = (0{,}5 - 3 \times 0{,}1436) \times 1 = 0{,}0692$

$x_2 = 0{,}0048$

$x_3 = 0{,}00030$

$x_4 = 4{,}42 \times 10^{-6}$

$u_0 = -0{,}1436 \qquad u_2 = -6{,}88 \times 10^{-4}$

$u_3 = -5{,}35 \times 10^{-5}$

$u_0 = -0{,}1496$ ... $u_3 = -5{,}35 \times 10^{-5}$

$u_1 = 0{,}0049$

1.7) $\min\limits_{u} \; g(u) = 100\,(u_2 - u_1^2)^2 + (1-u_1)^2$

$$\nabla g = \begin{bmatrix} -400 \cdot u_1 (u_2 - u_1)^2 - 2(1-u_1) \\ 200\,(u_2 - u_1)^2 \end{bmatrix} = 0$$

Ponto crítico $= (1, 1)$

$$H(u_1, u_2) = \begin{bmatrix} -400(u_2 - 3u_1^2) + 2 & -400 u_1 \\ -400 u_1 & 200 \end{bmatrix} \Bigg|_{(1,1)} =$$

$$= \begin{bmatrix} 802 & -400 \\ -400 & 200 \end{bmatrix} > 0 \quad \text{min global}$$

1.11) $H(x, u) = g(x,u) + \lambda f(x,u)$

$$= \frac{1}{2} x^T Q x + \frac{1}{2} u^T R u + \lambda^T (Ax + Bu + c)$$

$H_\lambda = f(x,u) = 0 \iff Ax + Bu + c = 0$

$H_x = g_x + \lambda f_x = 0 \iff Qx + A^T\lambda = 0$

$H_u = g_u + \lambda f_u = 0 \iff Ru + B^T\lambda = 0$

$\cdot \quad R^{-1} B^T \lambda$

$$\begin{cases} u = -R^{-1} B^T \lambda \\ \lambda = -A^T Q x \\ x = -A^{-1}(Bu + c) = -A^{-1}(-BR^{-1}B^T \lambda + c) = A^{-1}(BR^{-1}B^T \lambda - c) \end{cases}$$

$$\Rightarrow \lambda = -A^{-T} Q A^{-1}(BR^{-1}B^T \lambda - c)$$

$$\lambda = \underbrace{A^T Q A^{-1}}_{\bar{Q}} (Bu + c)$$

$$u = -R^{-1} B^T \bar{Q} (Bu + c)$$

$$u = -R^{-1} B^T \bar{Q} B u - R^{-1} B^T \bar{Q} c$$

$$I u + R^{-1} B^T \bar{Q} B u = -R^{-1} B^T \bar{Q} c$$

$$P^{-1} \underbrace{(I + R^{-1} B^T \bar{Q} B)}_{P} u = -P^{-1} R^{-1} B^T \bar{Q} c$$

$$\Rightarrow u = -(I + R^{-1} B^T \bar{Q} B)^{-1} R^{-1} B^T \bar{Q} c$$