Event generator normalization

Carlos Muñoz

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1 Luminosity

The number N of events collected in a phase space $\Delta\Omega$ can be written as a function of the integrated luminosity $\int L \cdot dt$ of the experiment as :

$$N = \left(\int L \cdot dt \right) \int \frac{d\sigma}{d\Omega} \Delta\Omega \,, \tag{1}$$

where $d\sigma/d\Omega$ is the differential cross section of the process.

2 Uniform generation

In the case where events are generated uniformely in Q^2, x_B, t and ϕ, in a phase space volume

$$\Delta\Omega = \Delta Q^2 \Delta x_B \Delta t \Delta\phi \,, \tag{2}$$

the number of events expected in the experiment can be calculated by weighting each of the generated events by its differentical cross section :

$$N = \left(\int L \cdot t \right) \frac{\sum_{i=1}^{N_{gen}} \left(\frac{d\sigma}{d\Omega} \right)_i \cdot \Delta\Omega_i}{N_{gen}} \,, \tag{3}$$

where N_{gen} is the total number of generated events. Notice that the phase space factor $\Delta\Omega$ defined in Eq. 2 is typically different for each event due to variations in the Δt range kinematically allowed for different values of Q^2 and x_B. It should be included under the sum running over the events i in Eq. 3, as $\Delta\Omega_i$.

Most usually, we want to plot the number of events as a function of one particular variable by binning the events on that variable. The number of events in a bin j is then given by :

$$N_j = \left(\int L \cdot t \right) \frac{\sum_{i=1}^{N_{gen}}{}^{\text{if } i \in (\text{bin } j)} \left(\frac{d\sigma}{d\Omega} \right)_i \cdot \Delta\Omega_i}{N_{gen}} \,, \tag{4}$$

3 Generation according to a process cross section

If events are generated according to the differential cross section of the process being simulated, then events do not need any *individual* weighting. However, they need to be normalized by the total cross section integrated over the phase space where events were generated.

The total number of events produced for a given integrated luminosity still follows Eq. 1. In this case, the total number of events produced in the full phase space is :

$$N_{TOT} = \left(\int L \cdot dt \right) \int \frac{d\sigma}{d\Omega} \Delta\Omega \,. \tag{5}$$

Typically, generation routines will provide a numerical integral of the total cross section, in a multi-dimensional space space which is normalized to 1. For example, in the TFoam class this value is returned by the method `TFoam::GetIntegMC`. It then needs to be multiplied by the phase space volume $\Delta\Omega = \Delta Q^2 \Delta x_B \Delta t \Delta\phi$ were events where generated.

In order to normalize the events generated in this way, each event should be multiplied by N_{TOT}/N_{gen} :

$$N_j = \frac{N_{TOT}}{N_{gen}} \sum_{\substack{i=1 \\ \text{if } i \in (\text{bin } j)}}^{N_{gen}} 1 \,. \tag{6}$$