

exercise2 (Score: 13.0 / 13.0)

1. [Test cell](#) (Score: 2.0 / 2.0)
2. [Test cell](#) (Score: 2.0 / 2.0)
3. [Coding free-response](#) (Score: 2.0 / 2.0)
4. [Written response](#) (Score: 2.0 / 2.0)
5. [Test cell](#) (Score: 1.0 / 1.0)
6. [Coding free-response](#) (Score: 2.0 / 2.0)
7. [Written response](#) (Score: 2.0 / 2.0)

Lab 3

1. 提交作業之前，建議可以先點選上方工具列的**Kernel**，再選擇**Restart & Run All**，檢查一下是否程式跑起來都沒有問題，最後記得儲存。
2. 請先填上下方的姓名(name)及學號(student_id)再開始作答，例如：

```
name = "我的名字"
student_id= "B06201000"
```

3. 演算法的實作可以參考[lab-3 \(https://yuanyuyuan.github.io/itcm/lab-3.html\)](https://yuanyuyuan.github.io/itcm/lab-3.html)，有任何問題歡迎找助教詢問。
4. **Deadline: 10/30(Wed.)**

In [1]:

```
name = "馬宗儀"
student_id = "b06201006"
```

Exercise 2

It is known that when interpolating a function $f(x)$ with a polynomial p_{m+1} of degree m that using x_j for $j = 0, 1, \dots, m$ as interpolation points the error has the form

$$|f(x) - p_{m+1}(x)| = \frac{|f^{(m+1)}(\xi_x)|}{(m+1)!} \left| \prod_{k=0}^m (x - x_k) \right|,$$

where $\xi_x \in [x_0, x_m]$.

Therefore, the polynomial $\omega_m(t) := \prod_{k=0}^m (t - x_k)$ influences the size of the interpolation error.

1. Put $m + 1$ *distinct equidistant points* in the interval $[-1, 1]$, and plot $\omega_m(t)$ for $m = 5, 10, 15, 20$.

Part 0. Import libraries.

In [2]:

```
import matplotlib.pyplot as plt
import numpy as np
```

Part 1. Define $\omega_m(t)$ function.

In [3]:

(Top)

```
def omega_m(t, x):
    # ===== 請實做程式 =====
    n=len(x)
    g_x=1
    for i in range(n):
        g_x*=(t-x[i])
    return g_x
# =====
```

In [4]:

omega

(Top)

```
# Test
print('w_5(0.5) =', omega_m(0.5, np.linspace(-1, 1, 6)))

### BEGIN HIDDEN TESTS
from random import random

rd_number = random()
x = np.linspace(-1, 1, 11)

m = len(x)
product = 1

for i in range(m):
    product *= (rd_number - x[i])

assert omega_m(rd_number, np.linspace(-1, 1, 11)) == product, 'omega_m is wrong!'
### END HIDDEN TESTS
```

w_5(0.5) = 0.017325000000000007

Part 2. Define the equidistant points function.

For example, if $m = 4$, then $m + 1$ distinct equidistant points in the interval $[-1, 1]$ should be $[-1, -0.5, 0, 0.5, 1]$.

So the results of `equidistant_points(4)` will be `[-1. -0.5 0. 0.5 1.]`.

In [5]:

(Top)

```
def equidistant_points(m):
    # ===== 請實做程式 =====
    return np.linspace(-1, 1, m+1)
# =====
```

In [6]:

points

(Top)

```
# Test
m = 4
print("Equidistant points:", equidistant_points(m))

### BEGIN HIDDEN TESTS
m = 10
assert np.mean(np.array(equidistant_points(m)) - np.linspace(-1, 1, m+1)) < 1e-7, 'equidistant_points is wrong!'
### END HIDDEN TESTS
```

Equidistant points: [-1. -0.5 0. 0.5 1.]

Part 3. plot $\omega_m(t)$ for $m = 5, 10, 15, 20$.

Please refer parts of plotting in " *lagrange.ipynb* ".

In [7]:

(Top)

```
x_range = np.arange(-1, 1, 0.01)
fig, ax = plt.subplots(figsize=(16, 9))

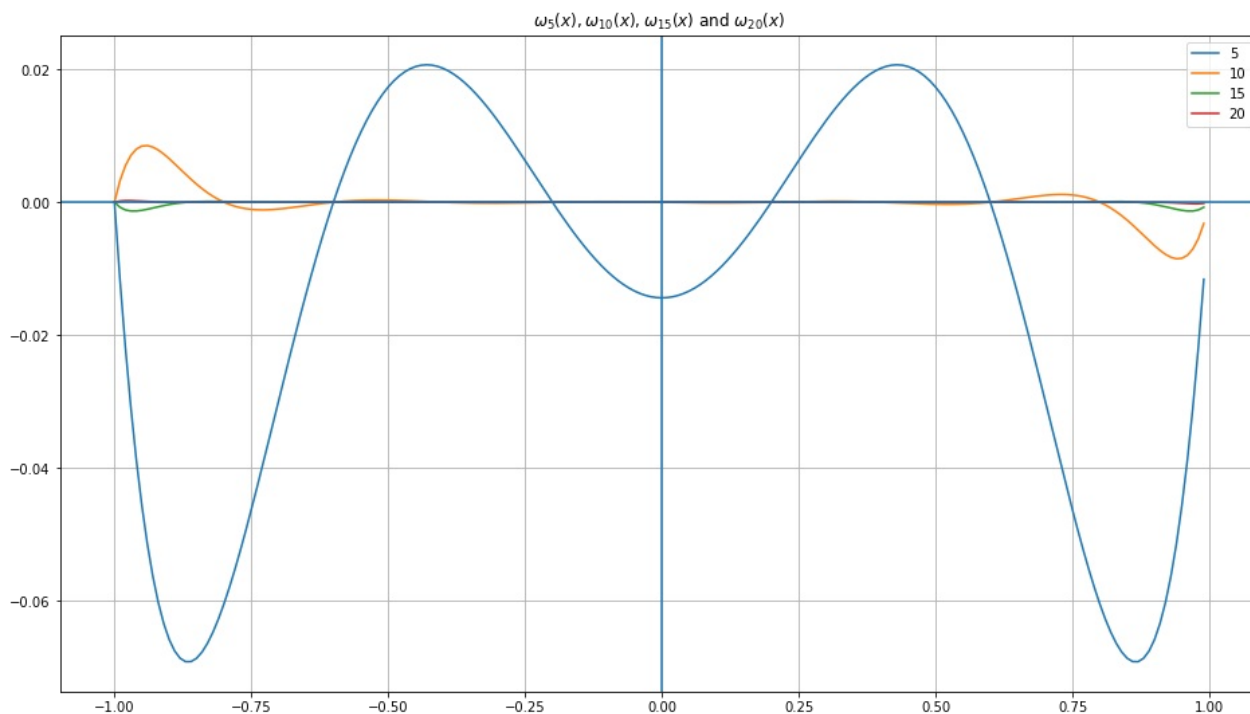
# Plot the function w_5(x), w_10(x), w_15(x) and w_20(x)
#
# Hint: ax.plot( x_points, y_points, color='?', label='?')
# ===== 請實做程式 =====
x_n=equidistant_points(5)
x=x_range
y=omega_m(x, x_n)
ax.plot(x,y,label='5')

x_n=equidistant_points(10)
x=x_range
y=omega_m(x, x_n)
ax.plot(x,y,label='10')

x_n=equidistant_points(15)
x=x_range
y=omega_m(x, x_n)
ax.plot(x,y,label='15')

x_n=equidistant_points(20)
x=x_range
y=omega_m(x, x_n)
ax.plot(x,y,label='20')
# =====

# Add other text and items
ax.set_title(r'$\omega_{5}(x)$, $\omega_{10}(x)$, $\omega_{15}(x)$ and $\omega_{20}(x)$')
plt.legend(loc='upper right')
ax.grid(True)
ax.axhline(y=0)
ax.axvline(x=0)
plt.show()
```



Part 4. What's your observation of the above figure?

(Top)

when m increase, error decrease!

2. Redo " Problem 1. " using **zeros of the Chebyshev polynomial (Chebyshev nodes) as the interpolation points.**

Part 1. Define Chebyshev nodes.

Please refer the part of Chebyshev nodes in " *lagrange.ipynb* ".

In [8]:

(Top)

```
def chebv_nodes(m):
    # ===== 請實做程式 =====
    return np.cos(np.linspace(-np.pi,0,m+1))
    # =====
```

In [9]:

(Top)

```
chebv_nodes

# Test
m = 5
print("Chebyshev nodes:", chebv_nodes(m))

### BEGIN HIDDEN TESTS
m = 10
assert np.mean(np.array(chebv_nodes(m)) - np.cos(np.linspace(0, np.pi, m+1))) < 1e-7, 'chebv_nodes is wrong!'
### END HIDDEN TESTS
```

Chebyshev nodes: [-1. -0.80901699 -0.30901699 0.30901699 0.80901699 1.]

Part 2. plot $\omega(t)$ for $m = 5, 10, 15, 20$.

Please refer parts of plotting in " *lagrange.ipynb* ".

In [10]:

(Top)

```
x_range = np.arange(-1, 1, 0.01)
fig, ax = plt.subplots(figsize=(16, 9))

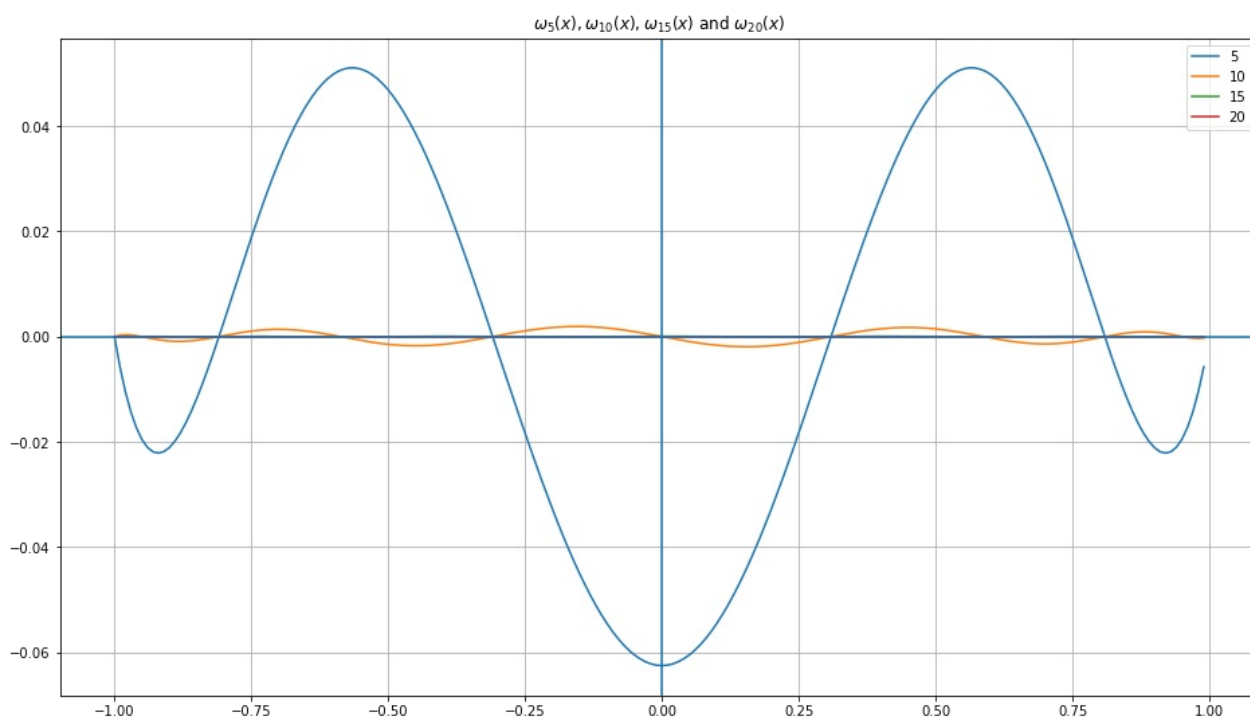
# Plot the function  $w_5(x)$ ,  $w_{10}(x)$ ,  $w_{15}(x)$  and  $w_{20}(x)$ 
#
# Hint: ax.plot( x_points, y_points, color='?', label='?')
# ===== 請實做程式 =====
x_n=chebv_nodes(5)
x=x_range
y=omega_m(x, x_n)
ax.plot(x,y,label='5')

x_n=chebv_nodes(10)
x=x_range
y=omega_m(x, x_n)
ax.plot(x,y,label='10')

x_n=chebv_nodes(15)
x=x_range
y=omega_m(x, x_n)
ax.plot(x,y,label='15')

x_n=chebv_nodes(20)
x=x_range
y=omega_m(x, x_n)
ax.plot(x,y,label='20')
# =====

# Add other text and items
ax.set_title(r'$\omega_{\{5\}}(x)$, $\omega_{\{10\}}(x)$, $\omega_{\{15\}}(x)$ and $\omega_{\{20\}}(x)$')
plt.legend(loc='upper right')
ax.grid(True)
ax.axhline(y=0)
ax.axvline(x=0)
plt.show()
```



Part 3. What's your observation of the above figure?

when m increase, error decrease !

In []:

In []:

In []: