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LAB REPORT: ELECTRIC FIELD AND EQUIPOTENTIALS

1 Theory

From Vector Fields to Electric Potential

In physics, vector fields are mathematical tools that allow us to visualize how a physical quantity with both magnitude and direction behaves throughout space. Rather than describing a single interaction, a field describes how one object influences every possible point in its surroundings. This perspective allows a single equation to encode the structure of a physical interaction in space.

A natural starting point for understanding electric fields is gravitation. Two objects with mass attract each other with a force proportional to the product of their masses and inversely proportional to the square of the distance between their centers of mass:

$$F_G = G\frac{Mm}{R^2}$$

where G is the universal gravitational constant.

The inverse–square dependence is not accidental. As distance increases, the interaction spreads over larger spherical surfaces, producing the $1/R^2$ behavior. This geometric interpretation will later help us understand electric fields.

From Force to Gravitational Field

Instead of focusing only on the interaction between two masses, we divide the force by one of the masses. This removes dependence on the test mass and isolates a quantity that depends only on the source mass and position:

$$g = \frac{F_G}{m} = G\frac{M}{R^2}$$

The quantity g is the gravitational field — a vector field describing acceleration due to gravity at every point in space.

At the Earth's surface:

$$g_0 = G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2}$$

At altitude H, where $R = R_{\text{Earth}} + H$:

$$g = G\frac{M_{\text{Earth}}}{(R_{\text{Earth}} + H)^2}$$

Once the field is known, the force follows immediately:

$$F_G = mg$$

Gravitational Potential Energy

Because gravity acts at a distance, an object placed in the field possesses gravitational potential energy.

With respect to the Earth's surface:

$$GPE = mgH$$

With respect to the Earth's center:

$$GPE = mg(R_{\text{Earth}} + H) = mgR$$

Since $F_G = mg$, we may also write:

$$GPE = F_G R$$

We now see the mathematical progression $F_G \to g \to GPE$: force leads to field, and field leads to potential energy.

The Electrical Analogy

Charge particles also interact through a force at distance described by Coulomb's Law:

$$F_E = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

or equivalently,

$$F_E = k\frac{Qq}{R^2}$$

where $k \approx 9 \times 10^9\,\mathrm{N{\cdot}m^2/C^2}$ in vacuum.

Structurally, this mirrors gravitation: $(Q, q) \leftrightarrow (M, m)$ and $k \leftrightarrow G$. Both forces obey an inverse–square law, although electric forces may be attractive or repulsive depending on the signs of the charges.

Vector Form and the Electric Field

To fully describe the interaction, we introduce direction using the unit vector $\hat{r}$:

$$\vec{F}_E = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

An important conceptual shift occurs when we divide the force by one of the charges. In doing so, we separate the mutual interaction from the influence of the source charge. Dividing by the test charge q:

$$\vec{E} = \frac{\vec{F}_E}{q} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$

This defines the electric field $\vec{E}$.

Here:

- $\vec{E}$ has units of N/C,
- Q is the source charge,
- r is the distance from the source,
- $\hat{r}$ determines direction.

This reflects the earlier gravitational reasoning: just as g describes the influence of a mass independently of the test mass, $\vec{E}$ describes the influence of a charge independently of any test charge.

The direction of $\vec{E}$ is defined as the direction a positive test charge would move. Electric field lines diverge from positive charges and converge toward negative charges. Unlike gravity, the electric interaction can be repulsive as well as attractive.

Electric Potential Energy and Electric Potential

Continuing the analogy, electric potential energy is defined as

$$EPE = F_E R$$

Substituting Coulomb's law:

$$EPE = k\frac{Qq}{R}$$

Since $E = kQ/R^2$, we may also write:

$$EPE = qER$$

Another fundamental electrostatic relation states

$$EPE = qV$$

where V is the electric potential (voltage), measured in volts.

This leads to another expression for the electric field:

$$E = -\frac{\Delta V}{\Delta x}$$

This equation shows that the electric field is the negative gradient of the electric potential. Therefore, the electric field has another equivalent unit:

1 N/C = 1 V/m.

Conceptually, the electric field describes how rapidly electric potential changes in space. It points in the direction of steepest decrease of potential, analogous to the slope of a hill.

Connection to the Experiment

In the laboratory, extended charge distributions are modeled using electrodes. Regions under the same electric potential are called equipotential regions. If $\Delta V \neq 0$, then $E \neq 0$.

Rather than measuring force directly, we measure potential differences between nearby points using a multimeter. From these measurements, we calculate the electric field using

$$E = -\frac{\Delta V}{\Delta x}$$

Thus the experiment completes the conceptual path:

$F_E \to \vec{E} \to EPE \to V$.

We began with inverse–square forces and arrived at a measurable scalar quantity whose spatial variation generates a vector field.

2 Conclusion

Activity 1

Our results in Activity 1 demonstrated that the data follows a curved behavior, increasing significantly near the electrodes and remaining close to zero near the center. This matches our theoretical expectation for two opposite charges, where the electric field is strongest close to the charges and weaker near the midpoint due to symmetry. The non-linear shape of the graph is consistent with the dipole-like configuration predicted by theory.

Activity 2

Our results in Activity 2 demonstrated that the data follows the prediction of the magnitude being approximately constant regardless of the position at which the electric field is measured. We measured a slope of -0.0036, which compared to the expected slope of 0, gives an absolute error of 0.0036. The R^2 value of 0.0004 further supports that there is no significant linear dependence on position. This agrees with the theoretical prediction of a nearly uniform electric field between parallel electrodes.

Activity 3

In Activity 3 we observed a strong agreement between our measurements and the theoretical cylindrical relationship $E \propto \frac{1}{r}$. From our power fit, we measured an exponent of -1.01. Compared to the expected value of -1, this corresponds to an absolute error of 0.01, or approximately 1% error. The R^2 value of 0.8841 indicates a strong correlation with the predicted inverse radial dependence. This confirms the expected cylindrical symmetry of the electric field in this configuration.

Small differences between theoretical and experimental values are likely due to systematic errors such as probe positioning, finite electrode size, non-ideal conductivity of the carbon paper, and slight asymmetries in the setup.

3 Questions

Activity 1

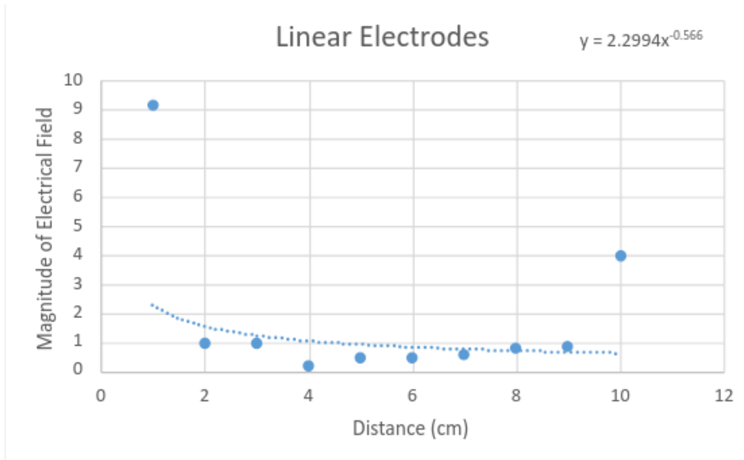

Figure 1: Table 1.

Question 1.a: How does the graph look?

The graph of ΔV versus position is nonlinear. The magnitude of ΔV is greater near each electrode and smaller near the midpoint. This occurs because the electric field is stronger close to the electrodes and weaker between them due to the superposition of two opposite charges.

Question 1.b: What would be the appropriate trendline?

The appropriate trendline is nonlinear and consistent with an inverse-square dependence. Since the electric field from each electrode follows $E \propto 1/r^2$, the combined field reflects dipole-like behavior rather than a linear relationship.

Question 1.c: Because $\Delta x = 1\,\text{cm}$, how does ΔV relate to the electric field?

Since $E = -\frac{dV}{dx}$, and Δx is constant, the electric field is approximately $E \approx -\frac{\Delta V}{\Delta x}$. Therefore, ΔV is directly proportional to the magnitude of the electric field.

Question 1.d: What does the slope of the graph represent?

The slope of the potential versus position graph represents the negative of the electric field. A steeper slope corresponds to a stronger electric field.

Activity 2

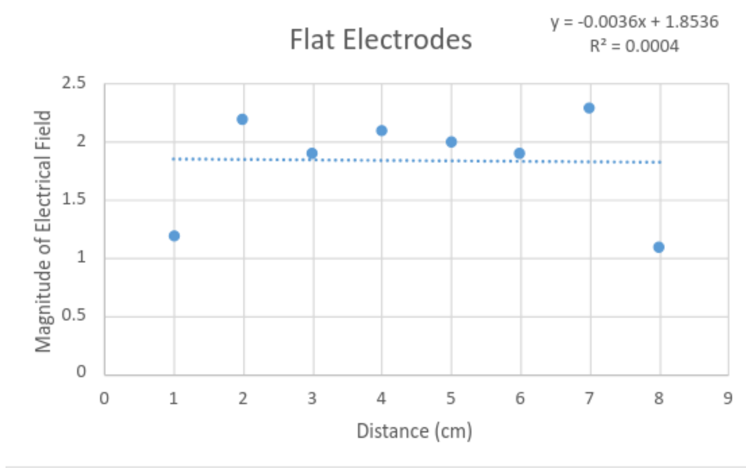

Figure 2: Table 2.

Question 2.a: What is the difference from the previous results?

Unlike the nonlinear behavior observed with point-like electrodes, the potential between parallel plate electrodes varies linearly with position. The electric field between the plates is approximately uniform.

Question 2.b: What would be the appropriate trendline?

A linear trendline is appropriate. Since the electric field between parallel plates is constant, the potential varies proportionally with distance.

Activity 3

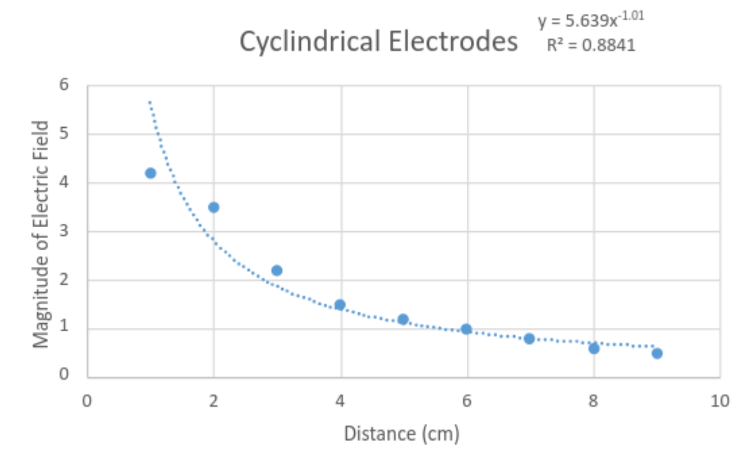

Figure 3: Table 3.

Question 3.a: Compare your results to the two previous ones.

> In the cylindrical configuration, the electric field decreases with radial distance approximately as $E \propto 1/r$. The potential varies logarithmically with distance. This behavior differs from the inverse-square dependence of point charges and the uniform field of parallel plates. The geometry of the electrodes determines how the electric field varies in space.

4 Above and Beyond

Although this lab focuses on calculating electric fields from simple charge distributions, the underlying mathematics is a powerful tool that extends far beyond electrostatics. Using integrals to sum contributions from continuous charges not only allows us to find the field of an infinite plane or a ring, but also provides a foundation for more advanced topics such as electromagnetic waves and gravitational fields. For example, the same principles of vector addition and $1/r^2$ forces appear in Newton's law of gravitation, while in wave theory, superposition and field integration are essential for understanding interference and diffraction patterns. This shows how a relatively simple concept—summing the influence of many sources—can be applied across many areas of physics.