Math 448/548 Cpt S 430/530

Assignment 4 **due March 8**

1. Let P_2 denote the unique polynomial of degree ≤ 2 that interpolates the data

x	.8	.9	1.0
f(x)	.696707	.621610	.540302

Find the Lagrange form of P_2 and use it to calculate $P_2(.85)$, an approximation to $f(.85)$. In fact $f(x) = \cos x$, so you can check the accuracy of your approximation.

2. (a) Rework problem 1 using divided differences.

   (b) Adjoin the additional data point $x_3 = .7$, $f(x_3) = .764842$ to your table and calculate $P_3(.85)$. Compare this with $f(.85)$.

3. For $f(x) = \cos x$ we have $|f^{(n+1)}(x)| \leq 1$ for all x and all n. Use this fact to obtain crude upper bounds on $|f(x) - P_2(x)|$ and $|f(x) - P_3(x)|$, valid for all $x \in [.8, 1]$, where f, P_2, and P_3 are as in the previous problem.

4. There is a unique polynomial P_7 of degree ≤ 7 that passes through the eight points with coordinates

$$(x, 3x^2 - 6x + 5) \qquad x = -3, -2, -1, 0, 1, 2, 3, 4.$$

Find P_7.

5. Verify the identity

$$f[x_i, x_j, x_k] = \frac{f(x_i)}{(x_i - x_j)(x_i - x_k)} + \frac{f(x_j)}{(x_j - x_k)(x_j - x_i)} + \frac{f(x_k)}{(x_k - x_i)(x_k - x_j)}.$$

Conclude that if (α, β, γ) is any permutation of (i, j, k), then $f[x_i, x_j, x_k] = f[x_\alpha, x_\beta, x_\gamma]$.

6. Let P_i denote the interpolating polynomial of f of degree $\leq i$ at $x_0, \ldots, x_i$, for $i = 2, 3$. Then $P_3(x) = P_2(x) + c_3(x - x_0)(x - x_1)(x - x_2)$ for some constant c_3. Verify that $c_3 = f[x_0, x_1, x_2, x_3]$.