Phys 320 homework: general requirements

Assignments are due TWO classes after assigned. No late work will be accepted.

Answers to some of the problems are given, either in the problem statement or at the end of the book. For these problems ESPECIALLY, I require that you write out all your algebra in detail. You must prove to me that you could have solved the problem if you were not given the answer.

Page and problem numbers refer to the textbook by Taylor, *Classical Mechanics*.

Problems must conform to the following standard:

- use top-down logic, line up equal signs, never put two equal signs on the same line; math and English are not written out the same way, I want you to think in terms of equations and not in terms of expressions,
- use words: set up the problem, explain your logic, interpret the answer (GRASS = given, required, analysis, substitution, solution),
- put a box around final answers,
- quote all numerical answers to at least 3 significant figures,
- include a diagram with lots of labels,
- construct your assignments as professional presentations of your solution; rough work, and untidiness, are not acceptable,
- all graphs must be computer-generated; I recommend gnuplot (www.gnuplot.info) or fooplot (www.fooplot.com).

Phys 320 homework: requirements for specific problems

- P2.22: Use Newton's method (Newton-Raphson iteration) and show me the iteration sequence to convergence to at least 5 significant figures. Use the correct value for $\pi/4$, not the author's rounded value.
- P3.8: Be careful; what is the final mass of the rocket after having burned $m\lambda_\circ$ of its fuel?
- P3.21: to find the X center of mass for the thin disk, write $X_{cm} = \frac{1}{M_{tot}} \int x \, \sigma \, dA$
  where $x = r \, \cos(\theta)$
  You can show that $X_{cm} = 0$.
- P4.2: solve parametrically; the answer to (a) is $\frac{4}{3}$.
- P4.4: the centrifugal force is doing the work, so $F = -m\omega^2 r$ and r is given in part (a); the answers to parts (b) and (c) must be identical.
- P4.9: part (b), the author's intention is that you set $U = 0$ at the new equilibrium point only AFTER you have an expression for U with the extra extension
- P4.12: $\vec{r} = \vec{x} + \vec{y} + \vec{z}$, where $\vec{x} = x \, \hat{x}$. Express your answers in parts (c) and (d) in terms of the unit vector $\hat{r}$.
- P4.21: if you read Taylor's example 4.5 you see that he chooses to work in Cartesian coordinates. I prefer that you always work with the coordinate system that takes best advantage of the geometry of the situation, thus, I prefer that you work with the spherical polar form of Laplace's operator. Whichever coordinate system you choose to work in, write out explicitly and in detail all steps in the algebra because I want you to prove to me that you can correctly perform the calculations.
- P4.23: Although the work done is path independent for a conservative force, for this problem you must explicitly specify the path in order to obtain the correct answer for the path integral. It is easiest to choose paths that are parallel to the axes.
  A different method for finding the potential, U, from that suggested by the author, is to recall that $\frac{\partial U}{\partial x} = F_x$, $\frac{\partial U}{\partial y} = F_y$, (etc); use these expressions and the info given to find the complete function, U.
- P4.34: Recall that for a point particle the moment of inertia $I = mr^2$, where r is measured perpendicularly from the axis of rotation (Taylor pg 95).
- P4.36: "Discuss the stability" means "use lots of math". Use energy methods to draw conclusions about the equilibrium. It is possible to verify your result using forces, if you consider all the forces acting upon mass m. You are not asked to perform the calculation with forces, but I suggest it as a way to double-check your answer.
- P5.23: as part of your solution, prove that $\frac{dW}{dt} = F_\circ \, v$, i.e., that the time derivative of work is equal to the product of the force and the velocity of the object.
- P5.34: write out the operator expression explicitly in terms of derivatives, and prove this relation line-by-line.

- Two driven oscillation problems:

  1. A horizontal spring has one end fixed and one end attached to a 3 kg mass, which slides without friction. The spring has a stiffness constant of 48 N/m. At time $t = 0$ the mass is at rest at the origin when a driving force of $F = 120 \cos 6t$ (F is in newtons) is applied.
     (a) Find the natural oscillation frequency, $\omega_\circ$, and show that the homogeneous solution (i.e., the solution to the motion when $F = 0$) is $x_h(t) = A \cos 4t + B \sin 4t$.
     (b) Use the method of undetermined coefficients to find the particular solution. Use an initial guess of $x_p(t) = C \cos 6t + D \sin 6t$, then find values for C and D.¡br¿ [Ans: $x_p(t) = -2 \cos 6t$]
     (c) Apply the initial conditions to show that the general solution is $x(t) = 2\left[\cos 4t - \cos 6t\right]$.
     (d) Use a trig identity to re-write the general solution in part (c) as $x(t) = 4 \sin t \sin 5t$.
     (e) The oscillator beats, with a beat frequency of $6-4 = 2$ Hz. We interpret the term $4 \sin t$ as the amplitude function, and $\sin 5t$ as the oscillation. Graph the solution from part (d) for about three beats. On the same plot, graph the envelope functions $x = \pm 4 \sin t$. COMPUTER PLOTS ARE REQUIRED.
  2. Find the full solution for a driven harmonic oscillator in the case when there is no damping, i.e., $\beta = 0$, and the driving frequency equals the natural vibration frequency, i.e., $\omega = \omega_\circ$. Use the driving function $F(t) = F_\circ \cos \omega_\circ t$.

     You will have to use the method of undetermined coefficients to find a particular solution. In this case, if you choose a form for the particular function like we did in class, $x(t) = C_1 \cos \omega_\circ t + C_2 \sin \omega_\circ t$, then the differential equation of motion cancels to zero. You must assume a form of $x(t) = t(C_1 \cos \omega_\circ t + C_2 \sin \omega_\circ t)$.

     [Ans: $x_h(t) = A \cos(\omega_\circ t - \delta)$, $x_p(t) = \frac{F_\circ}{2m\omega_\circ}\ t\ \sin \omega_\circ t$]

     When you have the solution, graph it for about three complete oscillations using the conditions $x(0) = 0$, $\dot{x}(0) = 1$, $\omega_\circ = 1$, and $F_\circ = 2m$. Interpret the result in a sentence, and include a comment on why a real system would eventually break under these conditions.

- P6.6: You must draw neat little diagrams of all of the variables in each of the coordinate systems for full points. Begin each part with a general statement for the relevant coordinate system, e.g., for the cylindrical coordinate case, you would begin with $ds^2 = dz^2 + \rho^2 d\phi^2 + d\rho^2$, then continue by setting $\rho = R$ and $d\rho = 0$ (recall that the author uses ρ to represent the perpendicular distance from the z axis).
- Both P6.3 and 6.4 are straightforward minimization problems NOT requiring the Euler-Lagrange equation. Recall that the distance between two points is given by Pythagoras $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$. Find extrema as you would normally, i.e., by setting $\frac{dt}{dx} = 0$ and $\frac{dt}{dz} = 0$.
- P6.17: In this problem, ϕ' is a function of ρ. Use an integral from the front cover of the textbook after you have written down and worked with the Euler-Lagrange equation. After you get the author's solution, I want you to PROVE TO ME that the solution is the equivalent of $y = mx + b$ (you will need to use a two-angle formula, and the transform from 2D Cartesian to polar).
- P6.19: The book suggests using y as the independent variable, in which case you need to re-write the Euler-Lagrange equation appropriately.
- P6.20: This problem extends over two pages, to include equation 6.43 of page 234. Remember the chain rule for several variables: if $F = F(g, h)$ and both g and h are functions of x, then the derivative $\frac{dF}{dx} = \frac{\partial F}{\partial g}\frac{\partial g}{\partial x} + \frac{\partial F}{\partial h}\frac{\partial h}{\partial x}$. The result that you derive in this problem is a statement of Beltrami's identity (1868).
- P7.1-4: These problems are elementary systems whose behavior we already know. Students usually find these problems difficult at first, but the more you solve the easier they become; the 3-star problems at the end of the chapter are often the fastest solved.
- P7.4: The wording in the book is unnecessarily confusing. Set up the problem as you normally would, with y being parallel to the slope and x being perpendicular to it. Set $U = 0$ wherever it is logical.
- P7.8: It will help if you find the time derivative of the location of center of mass.
- P7.20: In problems like this one, recall that coordinates (r, θ, ϕ) are orthogonal and the particle can have a component of velocity in each coordinate unless a constraint exists. Recall that $v_\phi = r\dot{\phi}$.
- P7.24: Explain in words what the mathematical expressions for force and momentum tell us.
- P7.27: You may as well solve for the acceleration of all 3 blocks. Why do the blocks move? Note that the author asks the question incorrectly: he says that the weight on either side of the upper wheel is equal, when in fact it is only the mass that is equal.
- P7.29: Follow the method of P7.35, which I solve in class. Draw an excellent diagram (use a ruler) and label it in detail.
- P7.41: Read my hint to P7.20.
- Chapter 9: A lot of the problems ask for written explanations in addition to any math you have to perform. Devote several sentences to these explanations, and make your explanations complete. Writing out your explanations will help you clarify your conceptual grasp of the material.

- Chapter 9: You solve these problems the same way that would have in Chapter 1, i.e., you have to work with Newton's 2nd law. Write down all the forces acting on a system (including normal forces), including their directions. You then apply Newton's 2nd law expression for the accelerating frame. You may find it helpful to write out vector cross products in full.
- P9.8: To solve, either draw detailed diagrams, or algebraically work out the cross products in full. The phrase "near the north pole" means "in the northern hemisphere definitely not at the pole". The phrase "across the equator" means "exactly on the equator".
- P9.10: begin from book equation 9.31, please.
- P9.14: The total potential energy is the (scalar) sum of all potentials acting on the surface, being gravitational and centrifugal. The total potential is a constant along the surface.
- P9.17 and 9.18: solve using equation 9.34.
- P10.11: make sure your answer is exactly that in the back of the
- P10.16: at the moment of collision, conserve angular momentum and show that the initial angular velocity of the cube is $\omega = \frac{3v}{4a}$.
- P10.22, 10.34, 10.35, 10.36: Do not multiply the determinants out, but rather keep them factored as long as possible; they will simplify more readily. Make hand-sketches of the mass distribution, and the directions of the principal axes, as a way to check that your answer makes sense. Recall that principal axes originate from the coordinate origin.