CS 148 Written Assignment

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1. (a) True. Lambertian surfaces appear equally matte from any viewing angle, so their BRDF is constant.

   (b) True. YUV color spaces can be converted to RGB color spaces by multiplying the R, G, and B values by a 3x3 invertable matrix of fractional values.

   (c) False. The diffuse component of the Phong illumination model is equivalent to a constant BRDF.

   (d) True. This is because the human eye works with three signals: each of the three types of cones sends a different signal.

   (e) True. Assuming the flashlight has a red filter on it (which is how the red light is produced), the filter will absorb all light that is not red light.

   (f) True. Black (K) is added to account for the lack of precision in real printers. (Printers aren't really "ideal.")

   (g) False. It is possible for both to diverge.

   (h) True. This is why subsurface scattering makes liquids look more translucent than opaque.

   (i) True. The magnifying glass changes the angle of the incoming light, focusing the same amount of light on a smaller surface area.

   (j) False. Radiosity assumes that all surfaces are diffuse, and mirrored surfaces are entirely specular.

   (k) True. Damaged cones make it difficult or impossible to perceive certain colors of light.

   (l) False. Gamma correction is applied to the gamma encoded images to convert them back to the original scene luminance.

   (m) False. The intermittent light stimulus must be at or above the flicker fusion rate in order to appear steady.

2. D. Rather, the Phong illumination model approximates the global illumination model.

3. A. Radiant intensity is the total light power per unit solid angle.

4. C. An HDR image combines images taken at different exposures so that both brightly lit and shadowed areas can appear in detail.

5. B and D. Because of the conservation of energy, all light energy is reflected, absorbed, or transmitted, but none is lost.

6. B. Irradiance is the total light power per unit area, so as the surface area gets larger, irradiance decreases.

7. A and D. OpenGL shading can model the first two terms of the power series, which describe the emission of light directly from light sources and direct illumination on surfaces.

8. A and C. The reason gamma encoding allocates more bits to darker tones is because it is mimicking human perception: human eyes are more sensitive to changes in darker tones.

9. C. The HSV model is cylindrical, where hue changes around the perimeter of the cylinder, saturation changes as you go from the edge of the cylinder to the center, and value changes from the top of the cylinder to the bottom.

10. C. BRDF only accurately accounts for the reflection of light.

11. C. Global illumination can also be used to model specular highlights.

12. The YUV model defines a color space as one grayscale component (Y) that describes the intensity of the light values and two chrominance components (U and V) that describe the color values of the image. The HSV model defines a color space in terms of the hue (H), saturation (S), and value (V) of each color. The HSV model tends to be used in user interfaces as a "color picker," while the YUV model was useful for television: if a picture was being shown on a black-and-white TV, just the Y value could be used. In order to convert between the two, you could convert one to RGB and then use that to convert to the other.

13. Because you're trying to represent a 3D map in 2D, the edges of the map become especially distorted. This distortion can be reduced by using several sphere maps taken from different viewing locations, or by using a different kind of environment map, like a cube map.

14. First, you would use radiosity to compute the global ambient and diffuse illumination for the entire scene (this is view-independent). Then, you would use the Phong shading model to calculate the specular highlights "on the fly" (these are view-dependent).

15. Nusselt's analog involves placing a hemispherical projection body on a surface. A second surface is spherically projected onto the projection body, and then cylindrically projected onto the base of the hemisphere. The form factor can be calculated by taking the area projected on the base of the hemisphere and dividing it by the area of the base.

16. Cones are most densely packed in the center of the retina, near the fovea. The rods, on the other hand, have virtually no density near the fovea. Instead, they are located to each side of it.

17. Printing ink on paper is a subtractive process: light is absorbed by the ink. But the RGB model is an additive model. When you try to use it in a subtractive way, the colors are too dark and become murky. (As an example, it's easy to make green by mixing cyan and yellow. But there is no way to make yellow or cyan using R, G, and B subtractively.)

18. In order to model subsurface scattering, you would need to combine the BRDF with a BTDF, which models how light is transmitted through a material. This is how the BSSRDF works: it is a combined description for reflection and transmission.

19. Printers with more than four colors of ink can increase the range of printable colors and produce more realistic-looking photos. Some printers add an orange cartridge and a green cartridge to the original CMYK; others add lighter shades of cyan and magenta.

20. To compute the radiosity from direct light A:

    $\Phi = I_l \omega_l = \frac{I_l}{R^2} a$

    Radiance of outgoing light of diffuse surface:

    $L_o = BRDF(E_i) = k_d E_i = k_d \frac{I_l}{R^2}$

    $B_o = \int L_o cos\Theta_o d\omega_o = \pi L_o = \pi k_d \frac{I_l}{R^2}$ (Radiosity for the three corners of the cube that are located distance R away from A.)

    $= \pi k_d \frac{I_l}{2R^2}$ (Radiosity for the three corners of the cube that are distance $R\sqrt{2}$ away from A.)

    $= \pi k_d \frac{I_l}{3R^2}$ (Radiosity for the corner of the cube exactly opposite to A.)

21. Imagine that the hemicube surrounds a hemisphere. You can divide the hemicube into a series of smaller planes, then use the hemisphere to calculate the form factor of each of those planes. Then, you can project any object onto the hemicube, and based on which subdivided planes it lands in, use the form factor of those planes to calculate the form factor of the new object. This ends up being much more efficient, because the form factors of all of the planes that make up the hemicube have been pre-computed.

22. Jacobi method converges:

$$3x + y = 12$$
$$x + 3y = 15$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$$

$$x^{(k)} = (12 - y^{(k-1)})/3$$
$$y^{(k)} = (15 - x^{(k-1)})/3$$

    Initial guess: $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

x	y
0.0000	0.0000
4.0000	5.0000
2.3333	3.6667
2.7778	4.2222
2.5926	4.0741
2.6420	4.1358
2.6214	4.1193
2.6269	4.1262
2.6246	4.1244
2.6252	4.1251
2.6250	4.1249
2.6250	4.1250
2.6250	4.1250

etc.

Jacobi method diverges:

$$x + 2y = 1$$
$$2x + y = 1$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x^{(k)} = 1 - 2y^{(k-1)}$$
$$y^{(k)} = 1 - 2x^{(k-1)}$$

Initial guess: $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

x	y
0	0
1	1
-1	-1
3	3
-5	-5
11	11
-21	-21
43	43
-85	-85
171	171
-341	-341

etc.

Jacobi method goes in an infinite cycle:

$$x + y = 2$$
$$x - y = 1$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$x^{(k)} = 2 - y^{(k-1)}$$
$$y^{(k)} = x^{(k-1)} - 1$$

Initial guess: $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

x	y
0	0
2	-1
3	1
1	2
0	0
2	-1
3	1
1	2
0	0
2	-1
3	1
1	2
0	0
2	-1

etc.

23. We don't need to recompute the entire matrix. Instead, we can use the "progressive" radiosity algorithm to perform this calculation in parallel. Because we're dealing with a purely geometric relationship, we can calculate all form factors before the matrix (via parallelization). We can calculate the swept volume between objects in order to determine when we need to recalculate the form factors.

24. Without any participating media, the rendering equation looks like this:

$$L_o(x, \omega_o) = L_e(x, \omega_o) + \int_{hemi} L_o(x'_i - \omega_i) f(x, \omega_i, \omega_o) cos\theta_i d\omega_i$$

In the presence of a constant-density emitting participating medium, the light reaching point x would be $L_o(x, \omega_o)$ times $A(|x' - x|)$, where A is the accumulation of light that only depends on the distance between x and x'. The new rendering equation would look like this:

$$L_o(x, \omega_o) = L_e(x, \omega_o) + \int_{hemi} L_o(x'_i - \omega_i) A(|x' - x|) f(x, \omega_i, \omega_o) cos\theta_i d\omega_i$$

In the finite element method, the form factor is computed using the term $cos\theta_i d\omega_i$. With the emitting participating media, the form factor will also include $A(|x' - x|)$. The form factor can now be computed like this:

$$f_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{cos\theta_i cos\theta_j}{\pi r^2} A(|x' - x|) dA_i dA_j$$

If the density of the participating media is not constant, A will depend on x and x'. We would have to integrate from x to x' in order to find A, which would add one extra dimension to the problem.

25. Gamma correction maps from an input value i (in this case, $i \in 0, 1, ..., 1023$ to an output intensity $I(i)$. We know that:

$$I(i) = Ai^\gamma + I_{min}$$

The relative difference between two consecutive inputs can be described by:

$$d(i) = \frac{I(i+1) - I(i)}{I(i)} = \frac{A((i+1)^\gamma - i^\gamma)}{Ai^\gamma + I_{min}} \geq .02$$

(a) Substitute the given values for I_{min} and γ.

$$\frac{A((i+1)^2 - i^2)}{Ai^2 + 0} \geq .02$$
$$\frac{(i+1)^2 - i^2}{i^2 + 0} \geq .02$$
$$-0.498 \leq i \leq 100.498$$

Since $i \in 0, 1, ...1023$, $0 \leq i \leq 100$.

(b) From our equation for intensity, we can compute I_{max}.

$$I(i) = Ai^\gamma + I_{min}$$
$$I_{max} = I(1023) = A1023^2 + 0.03I_{max}$$
$$I_{max} = A\frac{1023^2}{0.97}$$

Let's substitute this value for I_{max} into our relative difference equation:

$$\frac{A((i+1)^2 - i^2)}{Ai^2 + 0.03I_{max}} \geq .02$$
$$\frac{A((i+1)^2 - i^2)}{Ai^2 + 0.03(A\frac{1023^2}{0.97})} \geq .02$$
$$\frac{(i+1)^2 - i^2}{i^2 + 0.03(\frac{1023^2}{0.97})} \geq .02$$
$$-122.626 \leq i \leq 222.626$$

Since $i \in 0, 1, ...1023$, $0 \leq i \leq 222$.

(c) Similar to part a, but with a different γ.

$$\frac{A((i+1)^1 - i^1)}{Ai^1 + 0} \geq .02$$
$$\frac{(i+1) - i}{i} \geq .02$$
$$\frac{1}{i} \geq .02$$
$$i \leq 50$$

Since $i \in 0, 1, ...1023$, $0 \leq i \leq 50$.

(d) Similar to part b, but with a different γ. From our equation for intensity, we can compute I_{max}.

$$I(i) = Ai^{\gamma} + I_{min}$$

$$I_{max} = I(1023) = A1023^{1} + 0.01I_{max}$$

$$I_{max} = A\frac{1023}{0.99}$$

Let's substitute this value for I_{max} into our relative difference equation:

$$\frac{A((i+1)^{1} - i^{1})}{Ai^{1} + 0.01I_{max}} \geq .02$$

$$\frac{A((i+1) - i)}{Ai + 0.01(A\frac{1023}{0.99})} \geq .02$$

$$\frac{(i+1) - i}{i + 0.01(\frac{1023}{0.99})} \geq .02$$

$$\frac{1}{i + 0.01(\frac{1023}{0.99})} \geq .02$$

$$-10.3333 \leq i \leq 39.6667$$

Since $i \in 0, 1, ...1023$, $0 \leq i \leq 39$.