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Toward Robotic Musicianship

We present the development of a robotic percussionist named Haile that is designed to demonstrate musicianship. We define *robotic musicianship* in this context as a combination of musical, perceptual, and interaction skills with the capacity to produce rich acoustic responses in a physical and visual manner. Haile listens to live human players, analyzes perceptual aspects of their playing in real time, and uses the product of this analysis to play along in a collaborative and improvisatory manner. It is designed to combine the benefits of computational power, perceptual modeling, and algorithmic music with the richness, visual interactivity, and expression of acoustic playing. We believe that combining machine listening, improvisational algorithms, and mechanical operations with human creativity and expression can lead to novel musical experiences and outcome. Haile can therefore serve as a test bed for novel forms of musical human–machine interaction, bringing perceptual aspects of computer music into the physical world both visually and acoustically.

This article presents our goals for the project and the approaches we took in design, mechanics, perception, and interaction to address these goals. After an overview of related work in musical robotics, machine musicianship, and music perception, we describe Haile's design, the development of two robotic arms that can strike different locations on a drum with controllable volume levels, applications developed for low- and high-level perceptual listening and improvisation, and two interactive compositions for humans and a robotic percussionist that use Haile's capabilities. We conclude with a description of a user study that was conducted in an effort to evaluate Haile's perceptual, mechanical, and interaction functionalities. The results of the study showed significant correlation between humans' and Haile's rhythmic perception as well as strong user satisfaction from Haile's perceptual and mechanical capabilities. The study also indicated areas for improvement, such as the need for better timbre and loudness control as well as more advanced and responsive interaction schemes.

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Goals and Motivation

Most computer-supported interactive music systems are hampered by their inanimate nature, which does not provide players and audiences with physical and visual cues that are essential for creating expressive musical interactions. For example, motion size often corresponds to loudness, and gesture location often relates to pitch. These cues provide visual feedback and help players anticipate and coordinate their playing. They also create a more engaging experience for the audience by providing a visual connection to the sound. Computer-supported interactive music systems are also limited by the electronic reproduction and amplification of sound through speakers, which cannot fully capture the richness of acoustic sound.

Our approach for addressing these limitations is to employ a mechanical apparatus that converts digital musical instructions into physically generated acoustic sound. We believe that musical robots can bring together the unique capabilities of computational power and the expression and richness of acoustic sounds created through physical and visual interaction. A musical robot can combine algorithmic analysis and response capabilities that are not humanly possible with rich sound and visual gestures that cannot be reproduced by loudspeakers. We hope that such novel human–machine interaction can lead to new musical experiences and new music that cannot be conceived by traditional means. The engaging power of musical robots can also be useful as an educational tool, introducing learners not only to music but also to the mathematics, physics, and technology behind it in an interactive, hands-on manner.

Current research directions in musical robotics focus mostly on sound production and rarely address perceptual aspects of musicianship, such as listening, analysis, improvisation, or group interaction. Such automated devices can be classified in one of two ways: *robotic musical instruments,* which are mechanical constructions that can be played by live musicians or triggered by prerecorded sequences (Singer, Larke, and Bianciardi 2003; Jordà 2002; Dannenberg et al. 2005); or *anthropomorphic musical robots*—hominoid robots that attempt to imitate the action of human musicians (Takanishi and Maeda 1998; Sony 2003; Toyota 2004). Only a few attempts have been made to develop perceptual robots that are controlled by neural networks or other autonomous methods (Baginsky 2004).

The work described in this article addresses our preliminary research goal in this area: the development of a robotic percussionist that can demonstrate perceptual, physical, and social aspects of musicianship. The application we developed is aimed at allowing the robot to analyze live rhythmic input in real time and to react in an expressive manner through responsive and engaging acoustic drumming. We conclude by presenting a number of possible future research directions that expand the concept of robotic musicianship to pitch-based instruments, addressing melodic and harmonic perceptual and interaction models.

Related Work

We have identified a number of research fields that relate to our attempt to achieve robotic musicianship. These fields are *musical robotics,* which focuses on the construction of automated mechanical sound generators; *machine musicianship,* which centers on computer models of music theory, perception, and interaction (Rowe 2004); and *rhythmic perceptual modeling,* which can be seen as a subset field of machine musicianship that bears particular relevance to our initial focus on percussion.

Musical Robotics

Early work on musical robotics primarily addressed mechanical keyboard instruments such as the Pianista (1863) by French inventor Jean Louis Nestor Fourneaux. (See Kapur 2005 for a comprehensive historical review of musical robots.) In recent years, the field has received significant commercial, artistic, and academic interest, expanding to anthropomorphic designs (Rosheim 1994) as well as other robotic musical instruments, including chordophones, aerophones, membranophones, and idiophones. Several approaches have been recently explored for robotic stringed instruments. GuitarBot (Singer, Larke, and Bianciardi 2003), for example, is a mechanical guitar with four strings, each equipped with a sliding bridge, picking mechanism, and damper solenoid. Both the sliding and picking mechanisms are operated with DC servomotors under closed-loop feedback control, and the entire system is controlled via MIDI signals. Jordà's Electric Guitar Robot (Jordà 2002), on the other hand, has six strings that can be plucked by twelve picks, driven by an electro-valve hammer-finger. Current approaches for mechanical guitars, however, are not designed to explore the full range of sonic variety through string techniques such as bouncing, bowing, strumming, scratching, or rubbing.

Other attempts have been made to develop expressive wind instrument robots. Takanishi's Anthropomorphic Flutist Robot (Takanishi and Maeda 1998), for example, uses a complex mechanical imitation of human organs in an effort to accurately reproduce human flute playing. The elaborate apparatus includes robotic lungs, lips, fingers, and tongue. The robot, called WF-4RII, has also been used for educational purposes, teaching beginners the basics of flute playing (Solis et al. 2004). Other examples for aerophone robotic instruments are Toyota's Robotic Trumpeter (2004) and the Autosax (Rae 2005), which are programmed to follow deterministic rules.

More varied work has been done on robotic percussionists, both for idiophone and membranophone instruments. The ModBots (Singer et al. 2004), for example, are miniature modular instruments designed to affix to virtually any structure. Each ModBot consists of only one electromechanical actuator (a rotary motor or a linear solenoid) that responds to varying degrees of supply voltage regulated by a microcontroller. A more elaborated mechanism by Singer is used in the TibetBots (Singer et al. 2004), which consist of six robotic arms that strike three Tibetan singing bowls. Here, an effort was made to capture a wider timbral variety by using two robotic arms, controlled by solenoids, for each bowl to produce a richer set of sounds.

Another approach for broadening timbre and pitch versatility is employed by the Thelxiepeia (Baginsky 2004). The instrument consists of a mechanical drumstick and a motorized mechanism to rotate the drum circumference that can lead to the production of a range of pitches. Researchers such as Pongas, Billard, and Schaal (2006) have also studied lower-level control aspects such as joint synchronization and phase locking with rhythmic robots. Other robotic instruments and anthropomorphic robots that influence our work were developed by Trimpin (2000), Crick, Munz, and Scassellati (2006), Dorsenn (2006), and Brooks et al. (2004).

Machine Musicianship

In his book *Machine Musicianship,* Robert Rowe (2004) describes systems that demonstrate musicianship as those that analyze, perform, and compose music with computers based on theoretical foundations in fields such as music theory, computer music, cognition, artificial intelligence, and human–computer interaction. Scholars from a variety of fields have explored different approaches for the design of such analysis-performance-composition musical systems. One of the earliest research directions in this area was the score follower, in which the computer tracks a live soloist and synchronizes accompaniment MIDI data (Dannenberg 1984; Vercoe 1984)—and more recently, audio data (Orio, Lemouton, and Schwarz 2003)—to musical input.

The classic score-following approach focuses on matching predetermined musical events to real-time input. A more improvisatory approach is taken by systems such as Robert Rowe's Cypher (1992) or George Lewis's Voyager (2000). Here, the software analyzes musical input and responds by controlling and manipulating a variety of parameters such as melody, harmony, rhythm, timbre, and orchestration. David Cope (1996) has taken a non-real-time approach to machine musicianship in his system for analyzing composers' styles based on MIDI renditions of their compositions. Mr. Cope's algorithm learns the style of a given composer by modeling aspects such as expectation, memory, and musical intent. It can then generate new compositions with stylistic similarities to the originals. Francois Pachet's Continuator (2002), on the other hand, takes a real-time approach in learning the improvisation style of musicians as they play polyphonic MIDI instruments. Continuator can then continue the improvisation by performing in the style of the analyzed performer.

Other research in the field of machine musicianship that informed our work addresses areas such as theoretical modeling of improvisation and musical interaction (Pressing 1994; Johnson-Laird 2002) and experimentations with real-time human-robotic interaction such as Mari Kimura's composition *GuitarBotana* for the Guitarbot (Singer, Larke, and Bianciardi 2003).

Rhythmic Perceptual Modeling

Computational modeling of rhythm perception can be seen as a subset of the field of machine musicianship that is particularity relevant to the rhythmic phase of our work. Research in this area addresses computational approaches for modeling both low- and high-level rhythmic percepts. Lower-level cognitive rhythmic modeling addresses percepts from onset detection to tempo and beat detection using audio sources (e.g., Puckette, Apel, and Zicarelli 1998; Scheirer 1998; Foote and Uchihashi 2001) as well as MIDI (e.g., Winkler 2001). Higher-level rhythmic percepts include more subjective concepts such as rhythmic stability, similarity, and tension. Similarity comparison modeling typically focuses on how well two rhythms overlap. For instance, Paulus and Klapuri (2002) correlate low-level features of the audio signal, Tanguiane (1993) counts the number of coincident onsets, and Coyle and Schmulevich (1998) correlate a sequence of note duration ratios. Desain and Honing (2002), on the other hand, use a computational model for rhythmic stability as a potential measure of similarity and predictability that is based on the relationship between pairs of adjacent interval durations. In our research, we attempted to implement some of these models in an effort to extract cognitive meaning from real-time live drumming, and to allow Haile to respond to the acoustic input based on its cognitive understanding of low- and high-level rhythmic concepts.

Challenges

As part of our effort to construct a robotic percussionist that can demonstrate musicianship, we identified a number of challenges in design, mechanics, perception, and interaction. Our main challenge in designing Haile's physical body was to create a mechanical device that encourages humans to collaborate with a machine in an expressive and intuitive manner. The robot's shape, construction materials, and the manner in which technology is embedded in the physical form had to support intuitive and engaging musical interaction. Mechanically, our main challenge was to develop a dexterous robotic apparatus that could accurately translate perceptually based performance algorithms into a rich acoustic and visual performance. Ultimately, we aimed to control actions that explore the rich sonic variety of percussive musical instruments through multiple playing techniques and sound-production methods. In perception, our challenge was to implement models for low-level percepts such as note onset and pitch as well as high-level musical percepts such as rhythmic stability and similarity, allowing the robot to obtain a meaningful representation of the music to which it listens. Based on this perceptual analysis, an additional challenge was to develop responsive improvisation algorithms, generating musical responses that intuitively relate to human input.

In interaction design, our goal was to develop performance algorithms that would enable the robot to collaborate with human players in a meaningful and inspiring manner, using transformative and generative methods both sequentially and synchronously. Later in this article, we present future educational challenges in developing constructionist musical activities for Haile that allow novices to interact with the robot while learning music, mathematics, acoustics, engineering, and programming.

Implementation

To encourage familiar and expressive interactions with human players, Haile's design is anthropomorphic, employing two percussive arms that can move to different locations along the drum's radius and strike with varying velocities. We now consider our implementation of this robot, including aspects of its physical design, mechanics, perception, and interactivity.

Physical Design

Haile is designed to play a Native American powwow drum, a unique multi-player instrument that supports the collaborative nature of the project. To match the natural aesthetics of the Native American powwow ritual, we chose to construct the robot from wood. A design made by Clint Cope was used as a basis for Haile's appearance. The wooden parts were made on a CNC wood-cutting machine and constructed from several layers of plywood that were glued together. Metal joints were designed to allow shoulder and elbow movement as well as leg adjustability for different drum heights. While attempting to create a warm and organic look for the robot, it was also important that the technology would not be completely hidden, so that players and learners would be able to see and understand the robot's operation. We therefore left the mechanical apparatuses uncovered and embedded a number of

Figure 1. Haile's anthropomorphic design.

Figure 2. The right arm slider mechanisms.

Figure 3. The right arm striking mechanism.

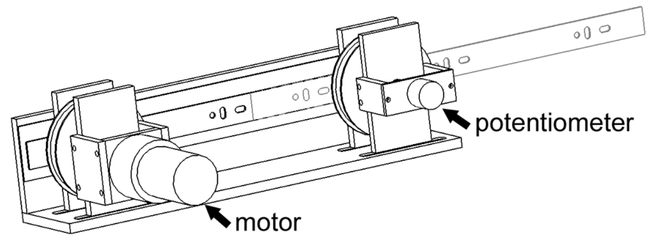


Figure 2

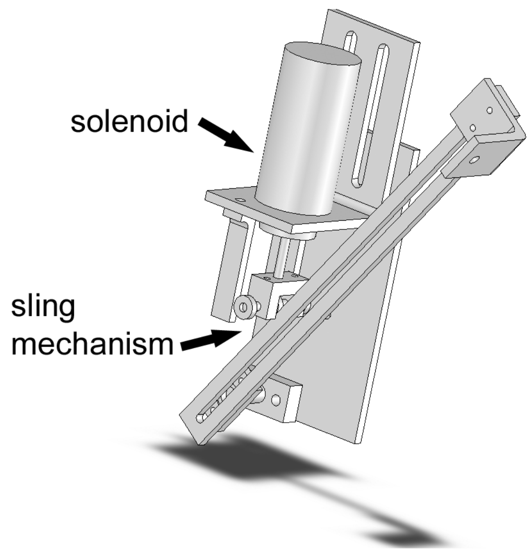


Figure 3

LEDs on Haile's body that provide an additional representation of the mechanical actions (see Figure 1).

Mechanics

Haile controls two robotic arms; the right arm is designed to play fast notes, and the left arm is designed to produce larger and more visible motions that produce louder sounds in comparison to the right arm. Both arms can adjust the sound of strikes in two manners: Different pitches are achieved by striking the drumhead in different locations along its radius, and volume is adjusted by hitting with varying velocities. To move to different positions over the drumhead, each arm employs a linear slide, a belt, a pulley system, and a potentiometer that provides feedback (see Figure 2). Unlike robotic drumming systems that allow hits at only a few discrete locations (Jordà 2002; Rae 2005), Haile's arms can strike anywhere on a line between the center and the rim of the drum, moving the 10 inches (about 25 cm) between these two points in about 250 msec.

The right arm's striking mechanism is loosely based on a piano hammer action and consists of a solenoid-driven device and a return spring (see Figure 3). The arm strikes at a maximum speed of 15 Hz, faster than the left arm's maximum speed of 11 Hz. (For comparison, the human winner of the 2004 International Fastest Drummer award played 1,180

Figure 4. Haile's right arm design.

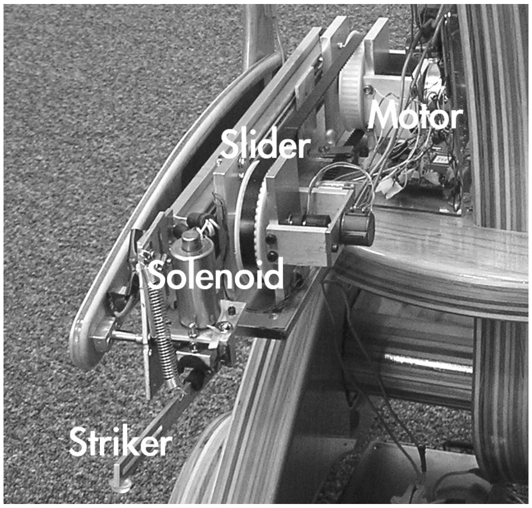
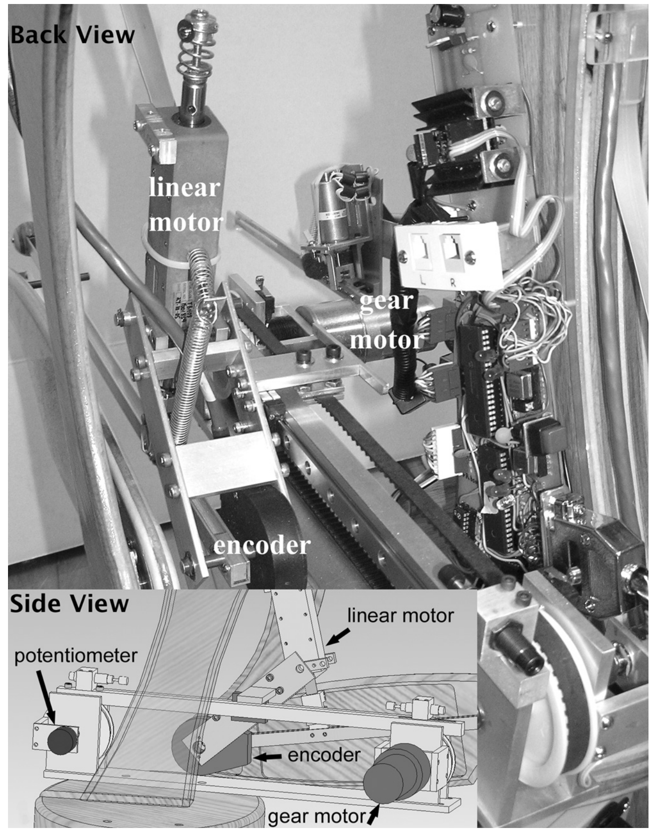


Figure 5. The linear motor and encoder provide closed-loop position and velocity over the left arm height while the gear motor and potentiometer control distance from the center of the drum.

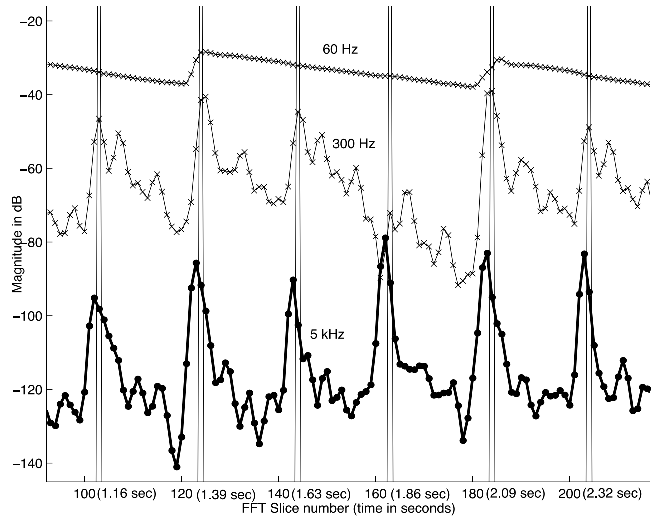


beats per minute—less than 10 Hz per arm—as reported on drummagazine.com.) However, the right arm cannot generate a wide dynamic range or provide easily noticeable visual cues, which limits Haile's expression and interaction potential. The left arm was designed to address these shortcomings using larger visual movements and a more powerful and sophisticated hitting mechanism. Whereas the striking component of the right arm is about the size of a finger and can move only 2.5 inches (about 6.4 cm) vertically, the entire left forearm takes part in the striking motion and can move up and down 8 inches (about 20 cm). A linear motor and an encoder located at the elbow are used to provide sufficient force and control for the larger mass and motions. Additional images showing the mechanical construction of the arms are provided in Figures 4 and 5.

In an effort to provide an easy-to-program environment for Haile, we decided to use Max/MSP, an intuitive graphical programming environment that can make the project accessible to composers, performers, and students. Our first one-armed prototype incorporated the USB-based Teleo System from makingthings.com as the main interface between Max/MSP and Haile's sensors and motors. Low-level control of the solenoid-based right arm's position was computed within Max/MSP, which required a continuous feed of position updates to the computer. This consumed much of the communication bandwidth as well as processor time on the main computer.

The current two-arm mechanism uses multiple onboard microprocessors for local low-level control as well as Ethernet communication with the main computer. The new system, therefore, facilitates much faster and more sophisticated control (with its 2-msec control loop) and requires only low-bandwidth communication with the operating computer. Each arm is locally controlled by an 18F452 PIC microprocessor, both of which receive RS232 communications from a Modtronix Ethernet board (SBC68EC). The Ethernet board receives 3-byte packets from the computer: a control byte and two data bytes. The protocol uses an address bit in the control byte to send the information onto the appropriate arm processor. The two data bytes typically contain position and velocity set points for each hit, but they can also be used to update the control parameters.

Figure 6. Magnitude plots from 60-Hz, 300-Hz, and 5-kHz frequency bands over several low- and high-pitched hits showing the relatively slow decay of the low-pitched hits. (A 2,048-point FFT and a 512-point hop size were used.)



Two onboard PIC microprocessors are responsible for controlling the arms' sliding and hitting mechanisms, ensuring that the impacts occur at the desired positions and velocities. To allow enough time for the arms to move to the correct locations and execute the strokes, a 300-msec delay line is implemented between signal reception and impact. It has been shown that rhythmic errors of only 5 msec are detectable by average listeners (Coren and Ward 1984); therefore, it was important to ensure that this delay remained accurate and constant regardless of different hit velocities, allowing us to easily compensate for it in the higher-level interaction application.

Both arms store incoming hit commands in a first-in-first-out queue, moving toward the location of a new note immediately after each hit. Owing to its short vertical hitting range, the solenoid-driven right arm has a fairly consistent stroke time for both soft and loud hits. We therefore implemented the 300-msec delay as a constant for this arm. The left arm, on the other hand, undergoes much larger movements, which require complex feedback control to ensure that impacts occur at the correct time regardless of hits velocity. While waiting for incoming notes, the left arm remains about one inch (about 2.5 cm) above the surface of the drum. When a new note is received, the arm is raised to a height proportional to the loudness of the hit. After a delay determined by the desired velocity and elevation, the arm starts descending toward the drumhead under velocity control. After impact, the arm returns back to its standby position above the drumhead. Extremely fast notes employ a slightly different control mode that makes use of the bounce of the arm in preparation for the next hit. The left arm, therefore, controls a wide dynamic range and provides performers and viewers with anticipatory and real-time visual cues, enhancing expression and enriching the interaction representation.

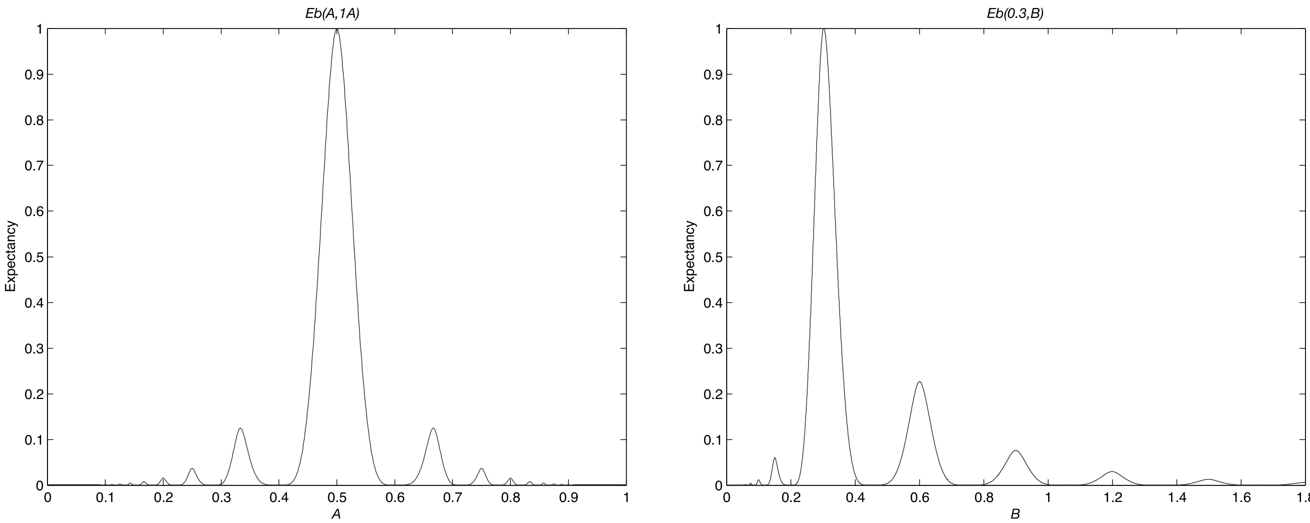
Perception

As a test bed for musical human–robot interaction, we developed a number of independent perceptual modules for Haile that can be embedded in a variety of combinations in compositions and other interaction schemes. Each module addresses one perceptual aspect—from hit onset, amplitude, and pitch detection, through beat and density analysis, to rhythmic stability and similarity perception. We base our low-level modules for hit onset and amplitude detection on the Max/MSP `bonk~` object (Puckette, Apel, and Zicarelli 1998), and we adjust its output to the unique character of the powwow drum. The `bonk~` object provides effective onset attack detection, but its frequency band output is insufficient for accurate pitch detection of the powwow's low and long-reverberating sounds. Because `bonk~` is hard-coded with a 256-point analysis window, the lowest frequency it can analyze is 172Hz—too high for the powwow drum, which has a natural fundamental frequency of about 60 Hz. Moreover, pitch detection is complicated when high-frequency hits are masked by the long decay of the previous low-pitched strikes.

To address these issues, we wrote a Max/MSP external object that uses 2,048-point analysis windows to determine the magnitude and derivative of lower-frequency bins. By taking into account the spectral changes in addition to magnitudes, we can better determine whether energy in a particular frequency band came from the current hit or from previous ones (see Figure 6).

Other relatively low-level perceptual modeling

Figure 7. Cognitive expectancy of neighboring time intervals: (left) the expectancy of intervals A and 1-A is higher at integer ratios, 1:1 being the highest; (right) bias toward expecting intervals of 600 msec (after Desain and Honing 2002).



modules provide beat detection, using Tristan Jehan's `beat~` Max/MSP object based on Scheirer (1998), and density detection, where we examine the number of note onsets per unit time to represent the density of the rhythmic structure. We also implement a number of higher-level rhythmic analysis modules for percepts such as rhythmic stability, based on Desain and Honing's computational model (2002), and similarity using the model of Tanguiane (1993). The stability model is based on the relationship between pairs of adjacent note durations that are rated according to their perceptual expectancy. This depends on three main criteria: Perfect-integer relationships are favored, ratios have inherent expectancies (i.e., 1:2 is favored to 1:3, and 3:1 is favored to 1:3), and durations of 600 msec are preferred. The expectancy function may be computed as

$$E_b(A,B) = \int_0^r \left(\text{round}(r) - r\right) \times \left|2\left(r - \text{floor}(r) - 0.5\right)\right|^p \times \text{round}(r)^d \, dr$$

where A and B are the durations of the two neighboring intervals, $r = \max(A|B, B|A)$ represents the near-integer relationship between note durations, p controls the shape of the peaks, and d is negative and affects the decay rate as the ratios increases. This function is symmetric around $r = 1$ when the total duration is fixed (see Figure 7, left). Generally, the expectancy function favors small near-integer ratios and becomes asymmetric when the total duration varies, exhibiting the bias toward the 600-msec interval (see Figure 7, right).

Our similarity rating is derived from Tanguiane's binary representation, where two rhythms are first quantized and then given a score based on the number of note-onset overlaps and near-overlaps. To support real-time interaction with human players, we developed two Max/MSP externals that analyze and generate rhythms based on these stability and similarity models. These externals have recently been embedded in a live interaction module that reads measure-length rhythmic phrases and modifies them based on desired stability and similarity parameters. Both parameters vary between 0 and 1 and are used together to select an appropriate rhythm from a database of pre-analyzed rhythms. A stability rating of 1 indicates the most stable rhythm in the database, and 0 corresponds to the least stable rhythm. The similarity parameter determines the relative contribution of similarity and stability (for detail, see Weinberg, Driscoll, and Thatcher 2006).

Interaction Design

The main challenge in designing the interaction with Haile was to implement our perceptual mod-

Figure 8. Model of sequential decentralized interaction. Musical actions are taken in succession without synchronous input from other participants and with no central system to coordinate the interaction.

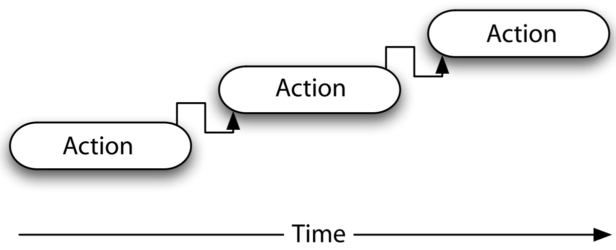
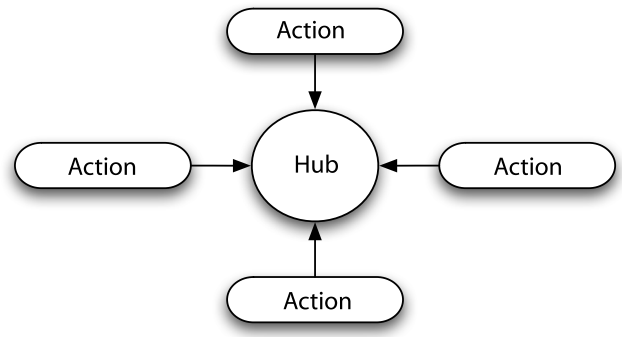


Figure 9. Model of synchronous centralized interaction. Human and machine players take musical actions simultaneously and interact through a computerized hub that interprets and analyzes the input data.



ules in a manner that would lead to an inspiring human–machine collaboration. The approach we took to address this challenge is based on our theory of interdependent group interaction in interconnected musical networks (Weinberg et al. 2005). At the core of this theory is a categorization of collaborative musical interactions in networks of artificial and live players based on sequential and synchronous operations with centralized and decentralized control schemes. For example, in sequential decentralized interactions, players create their musical materials with no influence from a central system or other players and can then interact with the algorithmic response in a sequential manner (see Figure 8). In a synchronous centralized network topology, on the other hand, players modify and manipulate their peers' music in real-time, interacting through a computerized hub that performs analysis and generative functions (see Figure 9). More sophisticated schemes of interaction can be designed by combining centralized, decentralized, synchronous, and sequential interactions in different directions, and by embedding weighted gates of influence between participants (see Figure 10).

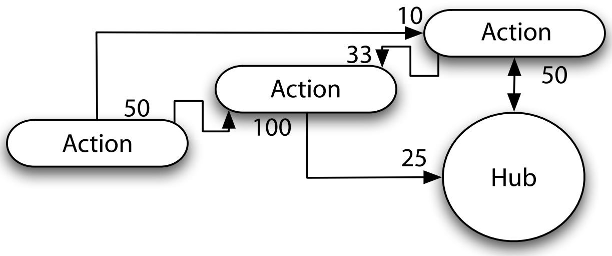
Based on these ideas, we developed six different interaction modes for Haile: *Imitation, Stochastic Transformation, Perceptual Transformation, Beat Detection, Simple Accompaniment,* and *Perceptual Accompaniment.* These interaction modes use different perceptual modules and can be embedded in different combinations in interactive compositions and educational activities. In the first mode, Imitation, Haile merely repeats what it hears based on its low-level onset, pitch, and amplitude perception modules. Players can play a rhythm, and after a couple of seconds of inactivity, Haile imitates it in a sequential call-and-response manner. Haile uses one of the arms to play lower pitches close to the drumhead center and the other arm to play higher pitches close to the rim.

In the second mode, Stochastic Transformation, Haile improvises in a call-and-response manner based on players' input. Here, the robot stochastically divides, multiplies, or skips certain beats in the input rhythm, creating variations of users' rhythmic motifs while keeping their original feel. Different transformation coefficients can be adjusted manually or automated to control the level of similarity between their motifs and Haile's responses.

In the Perceptual Transformation mode, Haile analyzes the stability level of users' rhythms and responds by choosing and playing other rhythms that have similar levels of stability to the original input. In this mode Haile automatically responds after a specified phrase length. Imitation, Stochastic Transformation, and Perceptual Transformation are all sequential interaction modes that form decentralized call-and-response routines between human players and the robot. Beat Detection and Simple Accompaniment modes, on the other hand, allow synchronous interaction where humans play simultaneously with Haile.

In Beat Detection mode, Haile uses the Max/MSP object `beat~` to track the tempo and beat of the input rhythm. Although `beat~` can be effective for pre-recorded audio, in a live setting human players naturally adjust to the robot's tempo, which leads to an unsteady input tempo that is difficult for `beat~` to follow. Haile therefore uses `beat~` to listen for a short period (5–10 sec) and then locks the tempo before joining in.

Figure 10. *A combination of centralized, decentralized, synchronous, and sequential musical actions in an asymmetric topology with weighted gates of influence (after Weinberg 2005).*



A simpler yet effective synchronous interaction mode is Simple Accompaniment, where Haile plays pre-recorded MIDI files so that players can interact with it by entering their own rhythms or by modifying elements such as drumhead pressure to modulate and transform Haile's timbres in real time. This synchronous centralized mode allows composers to feature their structured compositions in a manner that is not susceptible to algorithmic transformation or significant user input. The Simple Accompaniment mode is also useful for sections of synchronized unisons where human players and Haile play together.

Perhaps the most advanced mode of interaction is the Perceptual Accompaniment mode, which combines synchronous, sequential, centralized, and decentralized operations. Here, Haile plays simultaneously with human players while listening to and analyzing their input. It then creates local call-and-response interactions with different players based on its perceptual analysis. In this mode, we employ the amplitude and density perceptual modules described previously. While Haile plays short looped sequences (captured during the Imitation and Stochastic Transformation modes), it also listens to and analyzes the amplitude and density curves of human playing. It then modifies its looped sequence based on the amplitude and density coefficients of the human players. When the rhythmic input from the human players is dense, Haile plays sparsely, providing only the strong beats and allowing humans to perform denser solos. When humans play sparsely, on the other hand, Haile then improvises using dense rhythms that are based on stochastic and perceptual transformations. Haile also responds in direct relationship to the amplitude of the human players so that the louder humans play, the stronger Haile plays to accommodate the human dynamics, and vice versa.

Compositions

The authors wrote two compositions for the system, each using a different set of perceptual and interaction modules. The first composition, titled *Pow,* premiered at the Eyedrum Gallery in Atlanta as part of the Listening Machines concert in January 2005. The second piece, titled *Jam'aa,* was commissioned by the Hamaabada Performing Art Center and premiered in Jerusalem, Israel, in March 2006. Video clips of these compositions are available online at www.cc.gatech.edu/~gilwein/Haile.htm. [Editor's note: Also see the DVD accompanying this issue.]

Pow

The composition *Pow,* written for one or two human players and a one-armed robotic percussionist playing a powwow drum, served as test case for some of Haile's early mechanical, perceptual, and interaction modules (see Figure 11). The piece begins with call-and-response routines, featuring Haile's onset, amplitude, and pitch detection in Imitation mode. It develops into an improvisatory section where Haile invokes the Stochastic Improvisation mode. Here analyzed pitch, amplitude, and rhythmic data are used to generate simple stochastic manipulations such as hit division and pitch averaging. A structured section then follows, using the Simple Accompaniment mode, where human players interact with Haile in a synchronous manner, taking turns as soloists based on a prerecorded 7/4-time MIDI file. The piece ends with a short showcase of Haile's mechanical abilities, featuring a fast trill that slides back and forth along the drum's radius.

Jam'aa

The composition *Jam'aa* ("gathering" in Arabic) builds on the unique communal nature of the

Figure 11. The composition Pow *for the first one-arm prototype of Haile as performed in concert at the Eyedrum, Atlanta, Georgia.*

Middle-Eastern percussion ensemble, attempting to enrich its improvisational nature, call-and-response routines, and virtuosic solos with algorithmic transformation and human-robotic interactions (see Figure 12). In *Jam'aa,* we added to the sonic variety of the piece by incorporating two robotic arms and by including other percussive instruments such as darbukas (goblet-shaped hand drums), djembes, and tambourines. Here, Haile listens to audio input via directional microphones installed inside two darbuka drums played by humans. It responds by playing a powwow drum using two arms while other human players join the drum circle, supporting the beat without interacting directly with the robot.

In some sections of the piece, the left arm merely provides the beat, whereas in other sections it participates in the algorithmic interaction. *Jam'aa* uses interaction modes that were not included in *Pow* such as Perceptual Transformation, in which Haile responds based on its cognition of rhythmic stability and similarity, and Perceptual Accompaniment, in which Haile modifies its drumming in real time based on the perceived density and loudness of human playing. We also developed a new response algorithm for *Jam'aa* called Morphing, where Haile combines elements from two or more of the motifs played by humans based on a number of integration functions. A more detailed description of the piece can be found in Weinberg, Driscoll, and Thatcher (2006).

User Study

To evaluate our approaches in design, mechanics, perception, and interaction, we conducted a user

Figure 12. The composition Jam'aa *for two darbuka players and robotic percussionist as performed in concert at Hamaabada Art Center, Jerusalem, Israel.*

study where subjects were asked to interact with Haile, to participate in a perceptual experiment, and to complete a questionnaire regarding their experience. The 14 undergraduate students who participated in the study were enrolled in the percussion ensemble class at the Georgia Institute of Technology in Spring 2006 and had at least eight years of experience each in playing percussion instruments. This level of experience was required to support the musical interaction with Haile as well as to support a meaningful discussion about subjects' experiences.

Each subject spent about 20 minutes experimenting with four different interaction modes: Imitation, Stochastic Transformation, Perceptual Accompaniment, and Perceptual Transformation. Subjects were then asked to compare their notion of rhythmic stability with Haile's algorithmic implementation. As part of the perceptual experiment on stability, subjects were asked to improvise a one-measure rhythmic phrase while Haile provided a 4/4-time beat at 90 beats per minute. Subjects were then randomly presented with three transformations of their phrase: a less stable version, a version with similar stability, and a more stable version. The transformed measures were generated by our Max/MSP stability external using stability ratings of 0.1, 0.5, and 0.9 for less, similar, and more stability, respectively.

The original phrase and the three transformations were played by Haile's right arm while its left arm provided a metronomic beat. All phrases were played twice to familiarize subjects with the materials. Students were then asked to indicate which phrase, in their opinion, was less stable, similarly stable, or more stable in comparison to the original input. Stability was explained as representing the "predictability of" or "ease of tapping one's foot along with" a particular rhythm. [Editor's note: For a video clip from the experiment, see www.coa.gatech.edu/~gil/HaileUserStudy.mov or the DVD accompanying this issue.] The goal of this experiment was not to reach a definite, well-controlled conclusion regarding the rhythmic stability model we used, but rather to obtain a preliminary notion about the correlation between our algorithmic implementation and a number of human subjects' perceptions in an interactive setting.

The next section of the user study involved a written survey where subjects were asked to answer questions describing their impression of Haile's physical design, mechanical operation, the different perceptual and interaction modes, as well as a number of general questions about human-robot interaction and "robotic musicianship." The survey included 39 questions such as "What aspects of the design and mechanical operation make Haile compelling to play with?" "What design aspects are problematic and require improvements?" "What musical aspects were captured by Haile in a satisfactory manner?" "What aspects were not captured well?" "Did Haile's response make musical sense?" "Did the responses encourage you to play differently than usual and in what ways?" "Did the interaction with Haile encourage you to come up with new musical ideas?" and "Do you think that new musical experiences, and new music, can evolve from musical human-robot interaction?"

Results

Most subjects addressed Haile's physical design in positive terms, using descriptors such as "unique," "artistic," "stylized," "organic," and "functional." Other opinions included "the design offered a feeling of comfort," "the design was pleasing and inviting," and "if Haile was not anthropomorphic, it would not have been as encouraging to play with." When asked about caveats in the design, several subjects mentioned "too many visible electronics" and "exposed cabling" and suggested that future designs should be "less cluttered." Another critique was that "the design did not appear to be versatile for use with other varieties of drums."

Regarding Haile's mechanical operation, subjects provided positive comments regarding the steadiness and accuracy of the left hand and the speed and "smoothness" of the right hand. The main mechanical caveats mentioned were Haile's limited timbre and volume control as well as the lack of larger and more visual movements. Only one respondent complained about the mechanical noise Haile produces.

In the perceptual rhythmic stability study, half of the respondents (7 out of 14) correctly identified the three transformations. (By comparison, a random response would choose 2.3 out of 14 correctly, on average.) The majority of confusions were between similar and more-stable transformations and between similar and less-stable transformations. Only three responses out of the total 42 decisions confused a more-stable version for a less-stable version, implying that larger differences in algorithmic stability ratings made differentiation easier. Only one subject labeled all three generated rhythms incorrectly.

Subjects' responses to the four interaction modes were varied. In Imitation Mode, respondents mentioned Haile's "accuracy" and "good timing and speed" as positive traits and its lack of volume control as a caveat. Responses to the question "How well did Haile imitate your playing?" ranged from "pretty well" to "amazingly well." Some differences between the interaction modes became apparent. For example, in Stochastic Transformation Mode (STM), about 85% of the subjects provided a clear positive response to the question "Was Haile responsive to your playing?" Only about 40% gave such a clear positive response to this question in Perceptual Accompaniment Mode (PAM). Respondents refer to the delay between user input and robotic response in PAM as the main cause for the "less responsive feel." To the question "Did Haile's responses encourage you to play differently than usual?" 50% of the subjects provided a positive response in STM whereas only 30% gave a positive response to this question in PAM.

When asked to describe how different than usual their playing was in STM, subjects focused on two contradicting motivations. Some mentioned that they played simpler rhythms than usual so Haile could transform them easily and in an identifiable manner. Others made an effort to play complex rhythms to challenge and test Haile's abilities. These behaviors were less apparent in PAM. Although only 40% (across all interaction modes) provided a positive answer to the question "Did Haile's responses encourage you to come up with new musical ideas?", more than 90% of participants answered positively to the question "Do you think that new musical experiences and new music can evolve from human-machine musical interactions?", strengthening their answers with terms such as "definitely," "certainly," and "without question."

Discussion

Based on the experiment and survey, we feel that our preliminary attempt at robotic musicianship provided promising results. The most encouraging survey outcome, in our opinion, was that subjects felt that the human–machine collaboration established with Haile did on occasion lead to novel musical experiences and new musical ideas that would not have been conceived by other means. It is clear, though, that further work in mechanics, perception, and interaction design is required to create a robot that can truly demonstrate "musicianship." Nearly all subjects addressed Haile's design in positive terms, strengthening our assumption that the wood and the organic look function well in a drum-circle context. Our decision to complement the organic look with exposed electronics was criticized by some subjects, although we feel that this hybrid design conveys the robotic functionalities and reflects the electroacoustic nature of the project.

Mechanically, most subjects were impressed with the speed and smoothness of Haile's operation. Only one subject complained about the noise produced by the robot, which suggests that most players were able to either mask the noise or to accept it as an inherent and acceptable aspect of human–robot interaction. Several subjects, however, indicated that Haile's right arm, which was responsible for playing back the transformations, did not provide satisfactory visual cues and could not produce adequate variety of loudness and timbre. The control mechanism that we later developed for the left arm was designed to address this problem, and it can now provide a wide dynamic range and large visual cues. We therefore plan to use the left arm as the playing device in future user studies. We also intend to improve its control in an effort to increase timbral variety through various techniques such as applying damping briefly after hits.

The user study and survey also provided encouraging results in regard to Haile's perceptual and interaction modules. The high percentage of positive responses regarding Imitation Mode indicates that our low-level onset and pitch-detection algorithms were effective. In general, a large majority of the respondents indicated that Haile was responsive to their playing. Perceptual Accompaniment Mode (PAM), however, was an exception to this rule, as subjects felt Haile was not responding to their actions with acceptable timing. PAM was also unique in the high percentage of subjects who reported that they did not play differently in comparison to playing with humans. We explain these results by the synchronous accompaniment nature of PAM, which is more familiar to most percussion students. Most subjects, on the other hand, felt compelled to play differently than usual in sequential call-and-response modes such as Stochastic Transformation Mode (STM). Here, subjects changed their usual drumming behaviors either by simplifying their rhythms to better follow Haile's responses or by playing complex rhythms in an effort to challenge the robot's perceptual and mechanical abilities. We believe that these behaviors were caused by the novelty effect as players attempted to explore Haile's physical and cognitive boundaries. We assume that subjects may develop more complex interaction behaviors if given longer play times.

Given the high level of variance in the notion of rhythmic stability in human perception, we feel that our rhythm stability experiment performed better than expected. Some problems in our method may have also hindered the results. For example, misalignment of subject drumming with the metronome during recording led to misaligned transformations, which may have been unjustifiably perceived as unstable. Also, because the transformed rhythms were generated based on subjects' input, the relative difference between the output stabilities in some cases became minimal and difficult to identify. For example, when a subject's original phrase was extremely stable, the algorithm would not be able to produce an identifiably "more stable" phrase. Asking subjects to play a unified mid-stability rhythm as input could have solved this problem, although we were specifically interested in evaluating Haile's perception in a live improvisatory context.

As indicated earlier, the most encouraging results were that 40% of subjects stated that the interaction with Haile encouraged them to come up with new musical ideas, and more than 90% claimed that they believe that new musical experiences and new music can evolve from such human–machine interaction. This may indicate that although the potential for creating novel musical experiences between humans and robots was not fully realized in our current implementation, the experience led a large majority of the subjects to feel optimistic about the prospect of achieving such novel musical experiences in the future.

Future Work

We identify several directions for immediate and long-term future work. Mechanically, we intend to further investigate the workings of hand drumming in an effort to improve Haile's timbral control. Drum timbre is highly dependent on multiple factors such as hand shape, contact area, contact location, contact duration, and pressure on the skin (Taylor 2004). A wide sonic variation can be produced by playing a hand drum using different stroke motions, contact areas (e.g., fingers vs. palm), leaving the hand momentarily on the skin to briefly dampen it, stretching the skin with the other hand, etc. Most mechanical instruments (player pianos, drums, mallets, etc.) produce only quick bounces off the surface and avoid "human finesse" during the hit. But it is this finesse that makes a human player's expression and intonation interesting and colorful. We believe that current technology cannot support the creation of a robot with such dexterity that would compare with a human's expressivity and virtuosity, but we do believe that significant advancements are possible.

We would also like to expand on what is humanly possible by experimenting with alternative striker shapes, materials, and mechanisms that do not reflect traditional percussion instruments. In light of our plan to continue improving Haile's mechanical stroke variety and timbre control, we also plan to explore better approaches for the perception of timbre and stroke variety. To this end, we are examining a number of neural-network and machine-learning approaches (e.g., Chordia 2004; Tindale et al. 2005).

In the longer term, we also hope to expand the promise of robotic musicianship to other instruments, such as pitched membranophones, idiophones, chordophones, and aerophones. This direction would call for further research into perceptual modeling of pitch and tonality, allowing Haile to listen to and respond to pitch-based monophonic (and ultimately polyphonic) musical instruments. Some of the percepts that we have started to investigate in that regard are local attractions, melodic similarity (Hewlett and Selfridge-Field 1998), and melodic complexity (Narmour 1992). We are also considering implementing models for melodic attraction (Lerdahl and Jackendoff 1983), melodic tension (Narmour 1990), and contour directionality (Trehub, Bull, and Thorpe 1984). The choice of percepts and modeling schemes will be integral to the definition of future interaction and response algorithms. We plan to continue to evaluate our current interaction design with in-depth user studies and to adapt future interaction modes to Haile's new capabilities. In particular, we are interested in designing new interaction schemes that would take advantage of Haile's ability to listen to and interact with multiple players in a group.

In addition to expanding our research in mechanics, perception, and interaction design, we also plan to investigate the use of Haile for educational purposes. Our educational pedagogy is informed by the theory of constructionism, which demonstrates how learning is most effective when students construct personally meaningful technological artifacts (Papert 1980). The theory emphasizes the unique ability of computers to provide personal and configurable learning experiences to a wide variety of learners. Recent research elaborated on Papert's ideas, showing how interaction with digital physical objects enhances children's and adults' learning (Resnick et al. 1996). In the field of music, however, little has been done to develop physical constructionist systems that can provide a compelling interdisciplinary education, not only in music, but also in mathematics, the sciences, and computer programming. For our educational work with Haile, therefore, we hope to capitalize on the beneficial effect of music education on learning in domains of knowledge beyond that of music (Schillinger 1976; Bamberger 2000; Rauscher, Shaw, and Ky 1993).

We plan to build on our previous work in this area (Weinberg, Lackner, and Jay 2000) by developing a constructionist educational application for Haile that would allow learners to translate abstract musical ideas into symbolic representations and physical gestures. The mathematical and scientific aspects of the project will be guided and motivated by learners' drive to rhythmically compose acoustic sounds, creating personal interactive musical compositions. The educational environment will allow learners to develop their intuitions by emphasizing shared structures in music and mathematics such as hierarchies, periodicity, units, ratio-proportion, symmetries, and patterns. Students will also be able to experiment with creating perceptual social behaviors by programming rule-based responses in an effort to make Haile an expressive, responsive, and intriguing playing companion.

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References

Baginsky, N. A. 2004. "The Three Sirens: A Self-Learning Robotic Rock Band." Available online at www.the-three-sirens.info (accessed May 1, 2006).

Bamberger, J. 2000. "Music, Math, and Science: Towards an Integrated Curriculum." *Journal for Learning Through Music* Spring:32–35.

Brooks, A.G., et al. 2004. "Robot's Play: Interactive Games with Sociable Machines." *Computer Entertainment* 2(3):10–20.

Chordia, P. 2004. *Automatic Tabla Transcription.* Ph.D. Thesis, Stanford University.

Cope, D. 1996. *Experiments in Music Intelligence.* Madison, Wisconsin: A-R Editions.

Coren, S., and L. M. Ward. 1984. *Sensation and Perception.* San Diego, California: Harcourt.

Coyle, E. J., and I. A. Shmulevich. 1998. "System for Machine Recognition of Music Patterns." Paper presented at the 1998 IEEE International Conference on Acoustics, Speech, and Signal Processing, Seattle, 12–15 May.

Crick, C., M. Munz, and B. Scassellati, 2006. "Robotic Drumming: Synchronization in Social Tasks." Presented at the 15th IEEE International Symposium on Robot and Human Interactive Communication. University of Hertfordshire, Hatfield, United Kingdom, 6–8 September 2006.

Dannenberg, R. 1984. "An Online Algorithm for Real-Time Accompaniment." *Proceedings of the 1984 International Computer Music Conference.* San Francisco, California: International Computer Music Association, pp. 193–198.

Dannenberg, R. B., B. Brown, G. Zeglin, and R. Lupish. 2005. "McBlare: A Robotic Bagpipe Player." *Proceedings of the 2005 International Conference on New Interface for Musical Expression.* Vancouver, Canada: University of British Columbia, pp. 80–84.

Desain, P., and H. J. Honing. 2002. "Rhythmic Stability as Explanation of Category Size." Paper presented at the 2002 International Conference on Music Perception and Cognition, University of New South Wales, Sydney, July 17–21.

Dorsenn, M. V. 2006. "The Cell." Available online at www.cell.org.au (accessed May 2006).

Foote, J., and S. Uchihashi. 2001. "The Beat Spectrum: A New Approach to Rhythmic Analysis." Paper presented at the 2001 International Conference on Multimedia and Expo, Tokyo, 22–25 August.

Hewlett, B., and E. Selfridge-Field. 1998. *Melodic Similarity : Concepts, Procedures, and Applications.* Cambridge, Massachusetts: MIT Press.

Johnson-Laird, P. N. 2002. "How Jazz Musicians Improvise." *Music Perception* 19(3):415–442.

Jordà, S. 2002. "Afasia: The Ultimate Homeric One-Man Multimedia Band." Paper presented at the 2002 International Conference on New Interface for Musical Expression, Dublin, 24–26 May.

Kapur, A. 2005. "A History of Robotic Musical Instruments." *Proceedings of the 2005 International Computer Music Conference.* San Francisco, California: International Computer Music Association, pp. 21–28.

Lerdahl, F., and R. Jackendoff. 1983. *A Generative Theory of Tonal Music.* Cambridge, Massachusetts: MIT Press.

Lewis, G. 2000. "Too Many Notes: Computers, Complexity, and Culture in Voyager." *Leonardo Music Journal* 10:33–39.

Narmour, E. 1990. *The Analysis and Cognition of Basic Melodic Structures.* Chicago: University of Chicago Press.

Narmour, E. 1992. *The Analysis and Cognition of Melodic Complexity: The Implication-Realization Model.* Chicago: University of Chicago Press.

Orio, N., S. Lemouton, and D. Schwarz. 2003. "Score Following: State of the Art and New Developments." *Proceedings of the 2003 International Conference on New Interfaces for Musical Expression.* Montreal: McGill University, pp. 36–41.

Pachet, F. 2002. "The Continuator: Musical Interaction with Style." *Proceedings of the 2002 International Computer Music Conference.* San Francisco, California: International Computer Music Association, pp. 211–218.

Papert, S. 1980. *Mindstorm.* New York: Basic Books.

Paulus, J., and A. Klapuri. 2002. "Measuring the Similarity of Rhythmic Patterns." Paper presented at the 2002 International Conference on Music Information Retrieval, Paris, France, 13–17 October.

Pongas, D., A. Billard, and S. Schaal. 2006. "Rapid Synchronization and Accurate Phase-Locking of Rhythmic Motor Primitives." *Proceedings of the 2005 International Conference on Intelligent Robots and Systems (IROS 2005).* New York: Institute for Electrical and Electronics Engineers, pp. 2911–2916.

Pressing, J. 1994. *Compositions for Improvisers: An Australian Perspective.* Melbourne: La Trobe University Press.

Puckette, M., T. Apel, and D. D. Zicarelli. 1998. "Real-Time Audio Analysis Tools for Pd and MSP." *Proceedings of the 1998 International Computer Music Conference.* San Francisco, California: International Computer Music Association, pp. 109–112.

Rae, G. W. 2005. Robotic Instruments. Available online at logosfoundation.org/instrum_gwr/automatons.html (accessed May 1, 2005).

Rauscher, F. H., G. L. Shaw, and K. N. Ky. 1993. "Music and Spatial Task Performance." *Nature* 365:611.

Resnick, M., F. Martin, R. Sargent, and B. Silverman. 1996. "Programmable Bricks: Toys to Think With." *IBM Systems Journal* 35:3–4.

Rosheim, M. 1994. *Robot Evolution: The Development of Anthrobotics.* New York: Wiley.

Rowe, R. 1992. *Interactive Music Systems: Machine Listening and Composing.* Cambridge, Massachusetts: MIT Press.

Rowe, R. 2004. *Machine Musicianship.* Cambridge, Massachusetts: MIT Press.

Scheirer, E. 1998. "Tempo and Beat Analysis of Acoustic Musical Signals." *Journal of the Acoustical Society of America* 103(1):588–601.

Schillinger, J. 1976. *The Mathematical Basis of the Arts.* New York: Da Capo Press.

Singer, E., J. Feddersen, C. Redmon, and B. Bowen. 2004. "LEMUR's Musical Robots." *Proceedings of the 2004 International Conference on New Interfaces for Musical Expression.* Hamamatsu, Japan: Shizuoka University of Art and Culture, pp. 181–184.

Singer, E., K. Larke, and D. Bianciardi. 2003. "LEMUR GuitarBot: MIDI Robotic String Instrument." *Proceedings of the 2003 International Conference on New Interfaces for Musical Expression.* Montreal: McGill University, pp. 188–191.

Solis, J., et al. 2004. "The Anthropomorphic Flutist Robot WF-4 Teaching Flute Playing to Beginner Students." *Proceedings of the 2004 IEEE International Conference on Robotics & Automation.* New York: Institute for Electrical and Electronics Engineers, pp. 146–151.

Sony. 2003. "QRIO Conductor Robot." Available online at www.sony.net/SonyInfo/QRIO/works/20040325e.html (accessed May 1, 2006)

Takanishi, A., and M. Maeda. 1998. "Development of Anthropomorphic Flutist Robot WF-3RIV." *Proceedings of the 1998 International Computer Music Conference.* San Francisco, California: International Computer Music Association, pp. 328–331.

Tanguiane, A. 1993. *Artificial Perception and Music Recognition.* New York: Springer.

Taylor, M. 2004. *Remembering How to Drum: Djembe Technique.* DVD. Blue 7 Media B0001GNB86.

Tindale, A. R., A. Kapur, W. A. Schloss, and G. Tzanetakis. 2005. "Indirect Acquisition of Percussion Gestures Using Timbre Recognition." Paper presented at the 2005 Colloque Interdisciplinaire de Musicologie, Montreal, 10 March.

Toyota. 2004. "Trumpet Robot." Available online at www.toyota.co.jp/en/special/robot (accessed May 2006).

Trehub, S. E., D. Bull, and L. A. Thorpe. 1984. "Infants' Perception of Melodies: The Role of Melodic Contour." *Child Development* 55(1):821–830.

Trimpin. 2000. *SoundSculptures: Five Examples.* Munich: MGM MediaGruppe Munchen.

Vercoe, B. 1984. "The Synthetic Performer in the Context of Live Performance." *Proceedings of the 1984 International Computer Music Conference.* San Francisco, California: International Computer Music Association, pp. 199–200.

Weinberg, G. 2005. "Interconnected Musical Networks: Toward a Theoretical Framework." *Computer Music Journal* 29(2):23–39.

Weinberg, G., S. Driscoll, and M. Parry. 2005. "Musical Interactions with a Perceptual Robotic Percussionist." *Proceedings of the 2005 of IEEE International Workshop on Robot and Human Interactive Communication.* New York: Institute for Electrical and Electronics Engineers, pp. 456–461.

Weinberg, G., S. Driscoll, and T. Thatcher. 2006. "*Jam'aa* for Haile." *Proceedings of the 2006 International Conference and Exhibition on Computer Graphics and Interactive Techniques (SIGGRPAH 2006).* New York: Association for Computing Machinery, p. 124.

Weinberg, G., T. Lackner, and J. Jay. 2000. "The Musical Fireflies: Learning About Mathematical Patterns in Music Through Expression and Play." *Proceedings of the 2000 Colloquium on Musical Informatics.* L'Aquila, Italy: Instituto Gramma, pp. 146–149.

Winkler, T. 2001. *Composing Interactive Music: Techniques and Ideas Using Max.* Cambridge, Massachusetts: MIT Press.