已知 Beta Distribution = $f(\theta, \alpha, \beta)$

$$= \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{\int_0^1 \theta^{\alpha-1}(1-\theta)^{\beta-1}\, d\theta}$$

$$= \theta^{\alpha-1}\cdot(1-\theta)^{\beta-1}\cdot\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)}$$

Bayesian Inference :

$$\underset{\text{posterior}}{p(\theta|x)} = \frac{\overset{\text{likelihood}}{p(x|\theta)}\cdot \underset{\text{prior}}{p(\theta)}}{\underbrace{\int_\theta p(x|\theta)\cdot p(\theta)\, d\theta}_{\text{marginal.}}}$$

$$= \frac{C_m^n \theta^m (1-\theta)^{n-m}\cdot \theta^{\alpha-1}(1-\theta)^{\beta-1}\cdot\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)}}{\int_0^1 C_m^n \theta^m (1-\theta)^{n-m}\cdot \theta^{\alpha-1}(1-\theta)^{\beta-1}\cdot\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)}\, d\theta}$$

$$= \frac{C_m^n\cdot\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)}\cdot\theta^{m+\alpha-1}\cdot(1-\theta)^{n-m+\beta-1}}{C_m^n\cdot\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)}\cdot\int_0^1 \theta^{m+\alpha-1}(1-\theta)^{n-m+\beta-1}\, d\theta}$$

$$= \frac{\theta^{m+\alpha-1}\cdot(1-\theta)^{n-m+\beta-1}}{\int_0^1 \theta^{m+\alpha-1}(1-\theta)^{n-m+\beta-1}\, d\theta}$$

$$= \theta^{m+\alpha-1}\cdot(1-\theta)^{n-m+\beta-1}\cdot\frac{\Gamma(n+\alpha+\beta)}{\Gamma(m+\alpha)\,\Gamma(n+\beta-m)} = f(\theta, m+\alpha, n+\beta-m).$$