

IDA Assignment 3

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Q1.

Consider the `nhanes` dataset in `mice`. For more information please type `help(nhanes)` in the R console.

a)

(2 marks) What percentage of the cases is incomplete?

Answer :

```
cat("The percentage of incomplete cases is",  
    (nrow(nhanes)-nrow(cc(nhanes)))*100/nrow(nhanes))
```

```
## The percentage of incomplete cases is 48
```

b)

(4 marks) Impute the data with `mice` using the defaults with `seed=1`, in step 2 predict `bmi` from `age`, `hyp`, and `chl` by the normal linear regression model, and then pool the results. What are the proportions of variance due to the missing data for each parameter? Which parameters appear to be most affected by the nonresponse?

Answer :

```
pool1 <- pool(with(mice(nhanes, printFlag = F, seed = 1),
  lm(bmi ~ as.factor(age) + hyp + chl)))
kable(pool1$pooled[,c(1,3,7,8,10)], caption = "Imputation with seed=1") %>%
  kable_styling(latex_options = "hold_position")
```

Table 1: Imputation with seed=1

| term | estimate | dfcom | df | lambda |
|-----------------|------------|-------|----------|-----------|
| (Intercept) | 16.0426979 | 20 | 9.743911 | 0.3071661 |
| as.factor(age)2 | -5.2508182 | 20 | 5.496624 | 0.5204870 |
| as.factor(age)3 | -6.8779315 | 20 | 3.466705 | 0.6819661 |
| hyp | 2.4662556 | 20 | 8.463472 | 0.3606031 |
| chl | 0.0544513 | 20 | 8.530893 | 0.3576185 |

c)

(4 marks) Repeat the analysis for $\text{seed} \in \{2, 3, 4, 5, 6\}$. Do the conclusions remain the same?

Answer :

```
pool2 <- pool(with(mice(nhanes, printFlag = F, seed = 2), lm(bmi ~ age + hyp + chl)))
pool3 <- pool(with(mice(nhanes, printFlag = F, seed = 3), lm(bmi ~ age + hyp + chl)))
pool4 <- pool(with(mice(nhanes, printFlag = F, seed = 4), lm(bmi ~ age + hyp + chl)))
pool5 <- pool(with(mice(nhanes, printFlag = F, seed = 5), lm(bmi ~ age + hyp + chl)))
pool6 <- pool(with(mice(nhanes, printFlag = F, seed = 6), lm(bmi ~ age + hyp + chl)))

parameters <- c("(Intercept)", "age", "hyp", "chl")
df <- data.frame(parameters, pool2$pooled[,10], pool3$pooled[,10],
                 pool4$pooled[,10], pool5$pooled[,10], pool6$pooled[,10])
colnames(df) <- c("parameters", "seed=2", "seed=3", "seed=4", "seed=5", "seed=6")
kable(df, caption="Imputation with seed=2,3,4,5,6") %>%
  kable_styling(latex_options = "hold_position")
```

Table 2: Imputation with seed=2,3,4,5,6

| parameters | seed=2 | seed=3 | seed=4 | seed=5 | seed=6 |
|-------------|-----------|-----------|-----------|-----------|-----------|
| (Intercept) | 0.4144454 | 0.2772900 | 0.1315114 | 0.4855733 | 0.4168136 |
| age | 0.4033924 | 0.5895051 | 0.2189333 | 0.4511896 | 0.6549523 |
| hyp | 0.1430995 | 0.4101152 | 0.1961083 | 0.5942866 | 0.2960364 |
| chl | 0.2959966 | 0.5621346 | 0.3305334 | 0.2346065 | 0.5196295 |

d)

(4 marks) Repeat the analysis with $M = 100$ with the same seeds. Would you prefer these analyses over those with $M = 5$? Explain why.

Answer :

```
pool1 <- pool(with(mice(nhanes, m = 100, printFlag = F, seed = 1),
  lm(bmi ~ age + hyp + chl)))
pool2 <- pool(with(mice(nhanes, m = 100, printFlag = F, seed = 2),
  lm(bmi ~ age + hyp + chl)))
pool3 <- pool(with(mice(nhanes, m = 100, printFlag = F, seed = 3),
  lm(bmi ~ age + hyp + chl)))
pool4 <- pool(with(mice(nhanes, m = 100, printFlag = F, seed = 4),
  lm(bmi ~ age + hyp + chl)))
pool5 <- pool(with(mice(nhanes, m = 100, printFlag = F, seed = 5),
  lm(bmi ~ age + hyp + chl)))
pool6 <- pool(with(mice(nhanes, m = 100, printFlag = F, seed = 6),
  lm(bmi ~ age + hyp + chl)))

parameters <- c("(Intercept)", "age", "hyp", "chl")
df <- data.frame(parameters, pool1$pooled[,10], pool2$pooled[,10],
  pool3$pooled[,10], pool4$pooled[,10],
  pool5$pooled[,10], pool6$pooled[,10])
colnames(df) <- c("parameters", "seed=1", "seed=2", "seed=3", "seed=4",
  "seed=5", "seed=6")
kable(df, caption="Imputation with seed=1,2,3,4,5,6 and M=100") %>%
  kable_styling(latex_options = "hold_position")
```

Table 3: Imputation with seed=1,2,3,4,5,6 and M=100

| parameters | seed=1 | seed=2 | seed=3 | seed=4 | seed=5 | seed=6 |
|-------------|-----------|-----------|-----------|-----------|-----------|-----------|
| (Intercept) | 0.2290445 | 0.1882474 | 0.2199607 | 0.2144722 | 0.2294356 | 0.2472607 |
| age | 0.4324680 | 0.4031077 | 0.3093072 | 0.3943223 | 0.3322570 | 0.4430300 |
| hyp | 0.2915346 | 0.2825108 | 0.2425105 | 0.2565132 | 0.2893046 | 0.2860700 |
| chl | 0.3217837 | 0.2939693 | 0.3281911 | 0.2835232 | 0.2461956 | 0.3113085 |

The pooled estimates is more stable when the value of M is higher. Thus, we would prefer $M = 100$ than $M = 5$

Q2.

(15 marks) Each of the 100 datasets contained in the file dataex2.Rdata was generated in the following way

$$y_i|x_i \stackrel{\text{ind.}}{\sim} N(\beta_0 + \beta_1 x_i, 1), \quad x_i \stackrel{\text{ind.}}{\sim} \text{Unif}(-1, 1), \quad \beta_0 = 1, \quad \beta_1 = 3$$

for $i = 1, \dots, 100$. Additionally, some of the responses were set to be missing using a MAR mechanism. The goal of this exercise is to study the effect that acknowledging/not acknowledging parameter uncertainty when performing step 1 of multiple imputation might have on the coverage of the corresponding confidence intervals. Further suppose that the analysis of interest in step 2 is to fit the regression model that was used to generate the data, i.e., a normal linear regression model where the response is y and the covariate is x . With the aid of the `mice` package, calculate the empirical coverage probability of the 95% confidence intervals for β_1 under the following two approaches: stochastic regression imputation and the corresponding bootstrap based version. Comment. For both approaches, please consider $m = 20$ and `seed=1`. **NOTE 1:** In order to calculate the empirical coverage probability, you only need to compute the proportion of the time (over the 100 intervals) that the interval contains the true value of the parameter. **NOTE 2:** The data are stored in an array structure such that, for instance, `dataex2[, , 1]`, corresponds to the first dataset (which has 100 rows and 2 columns, with the first column containing the values of x and the second the values of y).

Answer :

```
# initialize a counter
count <- 0
for (i in 1:nrow(dataex2)) {
  #impute values for the ith dataset using M=20
  impute.sri <- mice(dataex2[, , i], m = 20, method = "norm.nob", printFlag = F, seed = 1)
  fit.sri <- with(impute.sri, lm(Y ~ X)) #step 2
  pool.sri <- pool(fit.sri) # step 3
  summary.sri <- summary(pool.sri, conf.int = TRUE)
  if (summary.sri[2, 7] <= 3 & summary.sri[2, 8] >= 3) {
    count <- count + 1 #add to the counter if the the value of beta1 is contained in the
  }
}
cat("the proportion of the time for Stochastic Imputation is", count/nrow(dataex2))
```

the proportion of the time for Stochastic Imputation is 0.88

```
# initialize a counter
count <- 0
for (i in 1:nrow(dataex2)) {
  #impute values for the ith dataset, using m=20
  impute.bootstrap <- mice(dataex2[, , i], m = 20, method = "norm.boot",
                           printFlag = FALSE, seed = 1)
  fit.bootstrap <- with(impute.bootstrap, lm(Y ~ X)) #step 2
  pool.bootstrap <- pool(fit.bootstrap) # step 3
  summary.bootstrap <- summary(pool.bootstrap, conf.int = TRUE)
  if (summary.bootstrap[2, c(7)] <= 3 & summary.bootstrap[2, c(8)] >= 3) {
    count = count + 1 #add to the counter if the true value of beta1 is contained in the
  }
}
cat("the proportion of the time for Bootstrap is", count/nrow(dataex2))
```

the proportion of the time for Bootstrap is 0.95

Q3.

(9 marks) Show that for a linear (in the coefficients) regression model, the following two strategies coincide:

- (i) Computing the predicted values (point estimates) from each fitted model in step 2 and then pooling them according to Rubin's rule for point estimates (i.e., averaging the predicted values across the imputed datasets).
- (ii) Pooling the regression coefficients from each fitted model in step 2 using Rubin's rule for point estimates and then computing the predicted values afterwards

Answer :

We consider a linear regression model given a dataset as $\{y_i, x_{1i}, \dots, x_{ni}\}$

$$y_i = \beta_0 + \beta_1 x_{1i} + \dots + \beta_n x_{ni} + \varepsilon_i, \quad \varepsilon_i \sim N(0, \sigma^2)$$

Now we look into Case (i), we compute the predicted values for each fitted model from step 2. Then we obtain as below,

$$\hat{y}_i^{(m)} = \hat{\beta}_0^{(m)} + \hat{\beta}_1^{(m)} x_{1i} + \dots + \hat{\beta}_n^{(m)} x_{ni}$$

Then we pool them according to Rubin's rule for point estimates.

$$\begin{aligned} \bar{y}_i &= \frac{1}{M} \sum_{m=1}^M \hat{y}_i^{(m)} \\ &= \frac{1}{M} \sum_{m=1}^M \left(\hat{\beta}_0^{(m)} + \hat{\beta}_1^{(m)} x_{1i} + \dots + \hat{\beta}_n^{(m)} x_{ni} \right) \\ &= \frac{1}{M} \sum_{m=1}^M \hat{\beta}_0^{(m)} + \frac{1}{M} \sum_{m=1}^M \hat{\beta}_1^{(m)} x_{1i} + \dots + \frac{1}{M} \sum_{m=1}^M \hat{\beta}_n^{(m)} x_{ni} \\ &= \bar{\beta}_0 + \bar{\beta}_1 x_{1i} + \dots + \bar{\beta}_n x_{ni} \end{aligned}$$

Now, let us consider Case (ii) to validate if they coincide. We pool the regression coefficients from each fitted model in step 2 using Rubin's rule for point estimates.

$$\begin{aligned} \bar{\beta}_0 &= \frac{1}{M} \sum_{m=1}^M \hat{\beta}_0^{(m)} \\ &\vdots \\ \bar{\beta}_n &= \frac{1}{M} \sum_{m=1}^M \hat{\beta}_n^{(m)} \end{aligned}$$

Then we can compute the predicted values as follow

$$\bar{y}_i = \bar{\beta}_0 + \bar{\beta}_1 x_{1i} + \dots + \bar{\beta}_n x_{ni}$$

As shown above, the order of the computation of predicted values for each fitted model in step 2 and pooling according to Rubin's rule for point estimates do not matter mathematically. Therefore, we conclude here by saying that both cases coincide.

Q4.

The goal of this exercise is to study different ways of using `mice` when the analysis model of interest/substantive model involves an interaction term between incomplete variables. The model used to generate the data (available in `dataex4.Rdata`), which corresponds to our model of interest in step 2, was the following one:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{1i} x_{2i} + \varepsilon_i, \quad x_{1i} \stackrel{\text{iid}}{\sim} N(0, 1), \quad x_{2i} \stackrel{\text{iid}}{\sim} N(1.5, 1), \quad \varepsilon_i \stackrel{\text{iid}}{\sim} N(0, 1)$$

for $i = 1, \dots, 1000$, $\beta_0 = 1.5$, $\beta_1 = 1$, $\beta_2 = 2$ and $\beta_3 = 1$. Additionally, missingness was imposed on y and x_1 and so the interaction variable $x_1 x_2$ also has missing values, although the missingness in this interaction variable is induced by the missing in the covariate x_1 . In the following, please use $M = 50$ and `seed=1`.

```
kable(head(dataex4), caption="dataex4 observation") %>%
  kable_styling(latex_options = "hold_position")
```

Table 4: dataex4 observation

| y | x1 | x2 |
|----------|------------|-----------|
| NA | NA | 1.0983213 |
| 4.609476 | 0.0265944 | 1.1290673 |
| NA | -1.5165531 | 1.0748538 |
| 1.718231 | -1.3626533 | 1.9411515 |
| NA | 1.1784892 | 0.8272496 |
| 1.820674 | -0.9341513 | 1.9166941 |

a)

(6 marks) By only imputing the y and x_1 variables in step 1, provide the estimates of β_1, β_2 , and β_3 along with 95% confidence intervals. Comment. Note that this approach where the interaction variable is left outside the imputation process and calculated afterwards in the analysis model, is known as *Impute, then transform*

Answer :

```
impute.sri <- mice(dataex4, m = 50, seed = 1, printFlag = FALSE)
fit.sri <- with(impute.sri, lm(y ~ x1 + x2 + x1*x2))
pool.sri <- pool(fit.sri)
kable(summary(pool.sri, conf.int = TRUE)[, c(1,2,3,7,8)],
  caption = "Summary Statistics of Imputation of $y$ and $x_1$") %>%
  kable_styling(latex_options = "hold_position")
```

Table 5: Summary Statistics of Imputation of y and x_1

| term | estimate | std.error | 2.5 % | 97.5 % |
|-------------|-----------|-----------|----------|-----------|
| (Intercept) | 1.5929831 | 0.0954133 | 1.404501 | 1.7814655 |
| x1 | 1.4112333 | 0.0973291 | 1.219397 | 1.6030697 |
| x2 | 1.9658191 | 0.0532322 | 1.860657 | 2.0709812 |
| x1:x2 | 0.7550367 | 0.0570146 | 0.642302 | 0.8677715 |

b)

(10 marks) Now, start by calculating the interaction variable in the incomplete data and append it as a variable to your dataset. Then, use *passive imputation* to impute the interaction variable. Provide the estimates of β_1 , β_2 , and β_3 along with 95% confidence intervals. Comment.

Answer :

```
x1 <- dataex4$x1; x2 <- dataex4$x2; dataex4$x1x2 <- x1*x2
impute.null <- mice(dataex4, maxit = 0)
method <- impute.null$method
method["x1x2"] <- "~I(x1*x2)"
pred <- impute.null$predictorMatrix
pred[c("x1", "x2"), "x1x2"] <- 0
visit.seq <- impute.null$visitSequence
visit.seq

## [1] "y"      "x1"      "x2"      "x1x2"

impute.passive <- mice(dataex4, method = method, predictorMatrix = pred,
                        visitSequence = visit.seq, m = 50, seed = 1, printFlag = FALSE)
pool.passive <- pool(with(impute.passive, lm(y ~ x1 + x2 + x1*x2)))
kable(summary(pool.passive, conf.int=TRUE)[,c(1,2,3,7,8)],
       caption = "Summary Statistics of Imputation of $y$ and $x_1$") %>%
  kable_styling(latex_options = "hold_position")
```

Table 6: Summary Statistics of Imputation of y and x_1

| term | estimate | std.error | 2.5 % | 97.5 % |
|-------------|-----------|-----------|-----------|-----------|
| (Intercept) | 1.5534782 | 0.0884221 | 1.3788626 | 1.7280939 |
| x1 | 1.1926170 | 0.0958435 | 1.0034980 | 1.3817360 |
| x2 | 1.9964402 | 0.0493658 | 1.8989468 | 2.0939336 |
| x1:x2 | 0.8740573 | 0.0567852 | 0.7615712 | 0.9865434 |

c)

(10 marks) Now that you have already appended the interaction variable to the dataset, impute it as it was *just another variable* (or like any other variable) in the dataset and use this variable for the interaction term in step 2. Provide the estimates of β_1, β_2 and β_3 along with 95% confidence intervals. Comment.

Answer :

```
impute.jav <- mice(dataex4, m = 50, seed = 1, printFlag = FALSE)
fit.jav <- with(impute.jav, lm(y ~ x1 + x2 + x1x2))
pool.jav <- pool(fit.jav)
kable( summary(pool.jav, conf.int=TRUE)[, c(1,2,3,7,8)],
       caption = "Summary Statistics of Imputation of $y$ and $x_1$") %>%
  kable_styling(latex_options = "hold_position")
```

Table 7: Summary Statistics of Imputation of y and x_1

| term | estimate | std.error | 2.5 % | 97.5 % |
|-------------|----------|-----------|-----------|----------|
| (Intercept) | 1.499714 | 0.0782144 | 1.3452011 | 1.654227 |
| x1 | 1.003930 | 0.0822837 | 0.8414967 | 1.166363 |
| x2 | 2.026180 | 0.0437161 | 1.9398113 | 2.112548 |
| x1x2 | 1.017793 | 0.0442807 | 0.9303479 | 1.105238 |

d)

(6 marks) What is the obvious conceptual drawback of the *just another variable* approach for imputing interactions?

Answer :

The conceptual drawback of the *just another variable* approach for imputing interaction is the imputation on x_1x_2 using the x_1 and x_2 from the observed dataset. As a result, this will result in the biasness as the unbiased estimator for the parameters from the regression will no longer hold.

Q5

(30 marks) The file `NHANES2.Rdata` contains a subset of data from the *National Health and Nutrition Examination Survey* (NHANES), whose goal is to assess the health and nutritional status of adults and children in the United States. The variables in the dataset are the following:

- `wgt`: weight in kg,
- `gender`: male vs female,
- `bili`: bilirubin concentration in mg/dL,
- `age`: in years,
- `chol`: total serum cholestrol in mg/dL,
- `HDL`: High-density lipoprotein cholestrol in mg/dL,
- `hgt`: height in metres,
- `educ`: educational status; 5 ordered categories,
- `race`: 5 unordered categories,
- `SBP`: systolic blood pressure in mmHg,
- `hypten`: hyptertensive status; binary,
- `WC`: waist circumference in cm.

The analysis of interest is the following:

$$\text{wgt} = \beta_0 + \beta_1 \text{gender} + \beta_2 \text{age} + \beta_3 \text{hgt} + \beta_4 \text{WC} + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2).$$

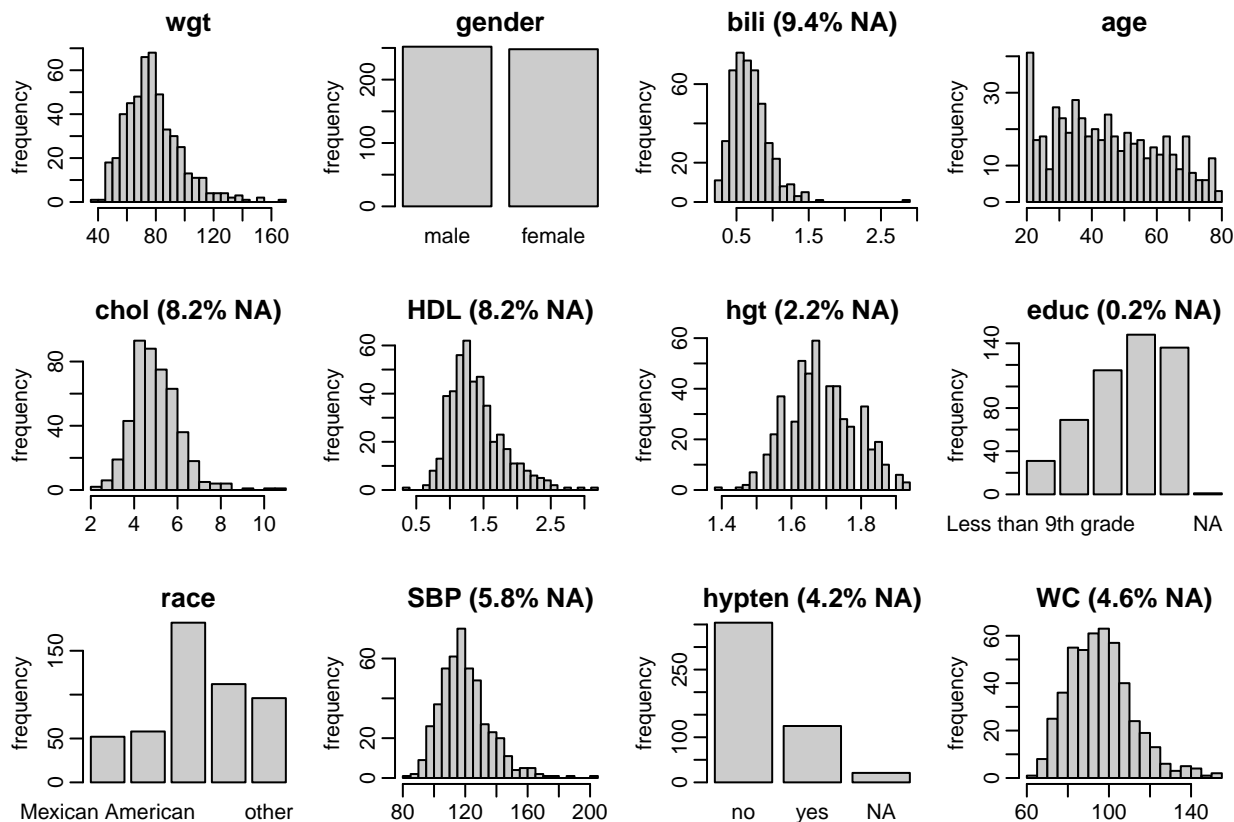
Using multiple imputation and conducting all necessary checks, report your findings.

Answer :

```
nhanes2 <- NHANES2
md_pattern(nhanes2, pattern = FALSE, color = c('#34111b', '#e30f41'))
```



```
par(mar = c(3,3,2,1), mgp = c(2,0.6,0))
plot_all(nhanes2, breaks = 30, ncol = 4)
```



```
impute.null <- mice(nhanes2, maxit = 0)
meth <- impute.null$method
meth["hgt"] <- "norm"
meth
```

```
##      wgt      gender      bili      age      chol      HDL      hgt      educ
##      ""      ""      "pmm"      ""      "pmm"      "pmm"      "norm"      "polr"
##      race      SBP      hypten      WC
##      ""      "pmm" "logreg"      "pmm"
```

```
post <- impute.null$post
post["hgt"] <- "imp[[j]][,i] <- squeeze(imp[[j]][,i], c(1.397, 1.9304))"
# visSeq <- impute.null$visitSequence
```

While inspecting the missing data pattern, we found 411 observations with observed values on all 12 variables. Also, 10 observations for which on WC is missing, 6 observations for which on hgt is missing. Interestingly, we can observe that there are 29 observations for which on chol, HDL and bili are missing.

We can visualise the variables' distributions using `plot_all()`. In the plot above, depicting the distribution of the observed data for the different variables, we could appreciate that hgt following a normal distribution is possibly not a completely unreasonable idea. Let us then change the default from predictive mean matching method (pmm) to norm for the variable hgt.

However, we need to be careful, because we do not want to risk imputing a negative value for the height. With the below syntax all imputed values of hgt that are outside the interval `(min(nhanes2$hgt), max(nhanes2$hgt))` will be set to those limiting values.

Since our model is limited to this, $wgt = \beta_0 + \beta_1 \text{gender} + \beta_2 \text{age} + \beta_3 \text{hgt} + \beta_4 \text{WC} + \varepsilon$, $\varepsilon \sim N(0, \sigma^2)$. We will end our data preprocessing and proceed to the next them in tuning the hyperparameters for imputation

```

seed1.5 <- pool(with(mice(nhanes2, methods = meth, post = post,
                        m = 5, seed = 1, printFlag = FALSE),
                    lm(wgt ~ gender + age + hgt + WC)))
seed1.10 <- pool(with(mice(nhanes2, methods = meth, post = post,
                           m = 10, seed = 1, printFlag = FALSE),
                       lm(wgt ~ gender + age + hgt + WC)))
seed1.20 <- pool(with(mice(nhanes2, methods = meth, post = post,
                           m = 20, seed = 1, printFlag = FALSE),
                       lm(wgt ~ gender + age + hgt + WC)))
seed1.25 <- pool(with(mice(nhanes2, methods = meth, post = post,
                           m = 25, seed = 1, printFlag = FALSE),
                       lm(wgt ~ gender + age + hgt + WC)))

seed2.5 <- pool(with(mice(nhanes2, methods = meth, post = post, maxit = 20,
                           m = 5, seed = 2, printFlag = FALSE),
                       lm(wgt ~ gender + age + hgt + WC)))
seed2.10 <- pool(with(mice(nhanes2, methods = meth, post = post, maxit = 20,
                           m = 10, seed = 2, printFlag = FALSE),
                       lm(wgt ~ gender + age + hgt + WC)))
seed2.20 <- pool(with(mice(nhanes2, methods = meth, post = post, maxit = 20,
                           m = 20, seed = 2, printFlag = FALSE),
                       lm(wgt ~ gender + age + hgt + WC)))
seed2.25 <- pool(with(mice(nhanes2, methods = meth, post = post, maxit = 20,
                           m = 25, seed = 2, printFlag = FALSE),
                       lm(wgt ~ gender + age + hgt + WC)))

# parameters <- c("(Intercept)", "gender", "age", "hgt", "WC")
# df <- data.frame(parameters, seed1.5$pooled[,10], seed1.10$pooled[,10],
#                  seed1.20$pooled[,10], seed1.25$pooled[,10],
#                  seed2.5$pooled[,10], seed2.10$pooled[,10],
#                  seed2.20$pooled[,10], seed2.25$pooled[,10])
# colnames(df) <- c("parameters", "seed1.5", "seed1.10", "seed1.20", "seed1.25",
#                  "seed2.5", "seed2.10", "seed2.20", "seed2.25")
# kable(df, caption="Imputation with various seed and m values") %>%
#   kable_styling(latex_options = "hold_position")

kable(data.frame(summary(seed1.5, conf.int = TRUE)[, c(1, 2, 3, 7, 8)],
                  lambda = seed1.5$pooled[,10]), caption = "Seed=1 and m=5") %>%
  kable_styling(latex_options = "hold_position")

```

Table 8: Seed=1 and m=5

| term | estimate | std.error | X2.5.. | X97.5.. | lambda |
|--------------|--------------|-----------|--------------|-------------|-----------|
| (Intercept) | -101.4588664 | 7.5938549 | -116.3816321 | -86.5361006 | 0.0195179 |
| genderfemale | -1.4195844 | 0.8425729 | -3.0758469 | 0.2366781 | 0.0355672 |
| age | -0.1589451 | 0.0213354 | -0.2008787 | -0.1170115 | 0.0293297 |
| hgt | 53.0413585 | 4.3496144 | 44.4934595 | 61.5892575 | 0.0227495 |
| WC | 1.0225600 | 0.0223080 | 0.9787286 | 1.0663914 | 0.0049737 |

```

kable(data.frame(summary(seed1.10, conf.int = TRUE)[, c(1, 2, 3, 7, 8)],
                  lambda = seed1.10$pooled[,10]), caption = "Seed=1 and m=10") %>%
  kable_styling(latex_options = "hold_position")

```

Table 9: Seed=1 and m=10

| term | estimate | std.error | X2.5.. | X97.5.. | lambda |
|--------------|--------------|-----------|--------------|-------------|-----------|
| (Intercept) | -101.6700795 | 7.8551715 | -117.1260787 | -86.2140802 | 0.0937749 |
| genderfemale | -1.3085502 | 0.8493953 | -2.9787291 | 0.3616288 | 0.0667687 |
| age | -0.1576161 | 0.0215356 | -0.1999575 | -0.1152746 | 0.0616516 |
| hgt | 52.9908125 | 4.4813711 | 44.1741136 | 61.8075113 | 0.0899366 |
| WC | 1.0244107 | 0.0222726 | 0.9806462 | 1.0681752 | 0.0168680 |

```
kable(data.frame(summary(seed1.20, conf.int = TRUE)[, c(1, 2, 3, 7, 8)],
  lambda = seed1.20$pooled[,10]), caption = "Seed=1 and m=20") %>%
  kable_styling(latex_options = "hold_position")
```

Table 10: Seed=1 and m=20

| term | estimate | std.error | X2.5.. | X97.5.. | lambda |
|--------------|--------------|-----------|--------------|-------------|-----------|
| (Intercept) | -101.2897304 | 7.8011807 | -116.6273935 | -85.9520673 | 0.0822178 |
| genderfemale | -1.3626948 | 0.8438271 | -3.0211578 | 0.2957682 | 0.0527619 |
| age | -0.1602819 | 0.0216544 | -0.2028488 | -0.1177150 | 0.0690126 |
| hgt | 52.7103557 | 4.4457568 | 43.9703249 | 61.4503865 | 0.0766459 |
| WC | 1.0265327 | 0.0222736 | 0.9827681 | 1.0702973 | 0.0122374 |

```
kable(data.frame(summary(seed1.25, conf.int = TRUE)[, c(1, 2, 3, 7, 8)],
  lambda = seed1.25$pooled[,10]), caption = "Seed=1 and m=25") %>%
  kable_styling(latex_options = "hold_position")
```

Table 11: Seed=1 and m=25

| term | estimate | std.error | X2.5.. | X97.5.. | lambda |
|--------------|--------------|-----------|--------------|-------------|-----------|
| (Intercept) | -101.0547825 | 7.7120024 | -116.2127029 | -85.8968621 | 0.0624659 |
| genderfemale | -1.3203389 | 0.8335918 | -2.9583505 | 0.3176727 | 0.0279959 |
| age | -0.1593655 | 0.0214497 | -0.2015208 | -0.1172102 | 0.0512832 |
| hgt | 52.5781406 | 4.4232119 | 43.8838452 | 61.2724359 | 0.0683664 |
| WC | 1.0259679 | 0.0224005 | 0.9819518 | 1.0699840 | 0.0233236 |

```
kable(data.frame(summary(seed2.5, conf.int = TRUE)[, c(1, 2, 3, 7, 8)],
  lambda = seed2.5$pooled[,10]), caption = "Seed=2 and m=5") %>%
  kable_styling(latex_options = "hold_position")
```

Table 12: Seed=2 and m=5

| term | estimate | std.error | X2.5.. | X97.5.. | lambda |
|--------------|--------------|-----------|--------------|-------------|-----------|
| (Intercept) | -100.9966596 | 7.7300251 | -116.2136672 | -85.7796520 | 0.0752096 |
| genderfemale | -1.3775163 | 0.8404887 | -3.0302117 | 0.2751791 | 0.0471154 |
| age | -0.1580193 | 0.0213131 | -0.1999240 | -0.1161147 | 0.0436862 |
| hgt | 52.5424942 | 4.4106181 | 43.8613258 | 61.2236627 | 0.0718380 |
| WC | 1.0259230 | 0.0222557 | 0.9821911 | 1.0696550 | 0.0142010 |

```
kable(data.frame(summary(seed2.10, conf.int = TRUE)[, c(1, 2, 3, 7, 8)],
  lambda = seed2.10$pooled[,10]), caption = "Seed=2 and m=10") %>%
  kable_styling(latex_options = "hold_position")
```

Table 13: Seed=2 and m=10

| term | estimate | std.error | X2.5.. | X97.5.. | lambda |
|--------------|--------------|-----------|--------------|-------------|-----------|
| (Intercept) | -101.5186858 | 7.5571364 | -116.3699779 | -86.6673938 | 0.0311981 |
| genderfemale | -1.3215513 | 0.8256067 | -2.9438439 | 0.3007412 | 0.0185686 |
| age | -0.1592563 | 0.0213802 | -0.2012842 | -0.1172285 | 0.0510297 |
| hgt | 52.8833997 | 4.3363259 | 44.3609058 | 61.4058936 | 0.0384335 |
| WC | 1.0255032 | 0.0222773 | 0.9817285 | 1.0692778 | 0.0197263 |

```
kable(data.frame(summary(seed2.20, conf.int = TRUE)[, c(1, 2, 3, 7, 8)],
  lambda = seed2.20$pooled[,10]), caption = "Seed=2 and m=20") %>%
  kable_styling(latex_options = "hold_position")
```

Table 14: Seed=2 and m=20

| term | estimate | std.error | X2.5.. | X97.5.. | lambda |
|--------------|--------------|-----------|--------------|------------|-----------|
| (Intercept) | -101.6993777 | 7.6823878 | -116.7986779 | -86.600078 | 0.0547441 |
| genderfemale | -1.3663151 | 0.8370945 | -3.0113631 | 0.278733 | 0.0398821 |
| age | -0.1576975 | 0.0214614 | -0.1998791 | -0.115516 | 0.0557808 |
| hgt | 53.0055071 | 4.4023585 | 44.3525473 | 61.658467 | 0.0590317 |
| WC | 1.0246288 | 0.0223533 | 0.9807051 | 1.068553 | 0.0239163 |

```
kable(data.frame(summary(seed2.25, conf.int = TRUE)[, c(1, 2, 3, 7, 8)],
  lambda = seed2.25$pooled[,10]), caption = "Seed=2 and m=25") %>%
  kable_styling(latex_options = "hold_position")
```

Table 15: Seed=2 and m=25

| term | estimate | std.error | X2.5.. | X97.5.. | lambda |
|--------------|--------------|-----------|--------------|-------------|-----------|
| (Intercept) | -101.4521949 | 7.6290783 | -116.4452338 | -86.4591561 | 0.0472501 |
| genderfemale | -1.3125011 | 0.8317349 | -2.9468852 | 0.3218831 | 0.0303398 |
| age | -0.1585538 | 0.0213320 | -0.2004759 | -0.1166316 | 0.0456505 |
| hgt | 52.8201428 | 4.3724292 | 44.2269381 | 61.4133475 | 0.0516546 |
| WC | 1.0255459 | 0.0223605 | 0.9816083 | 1.0694836 | 0.0243072 |

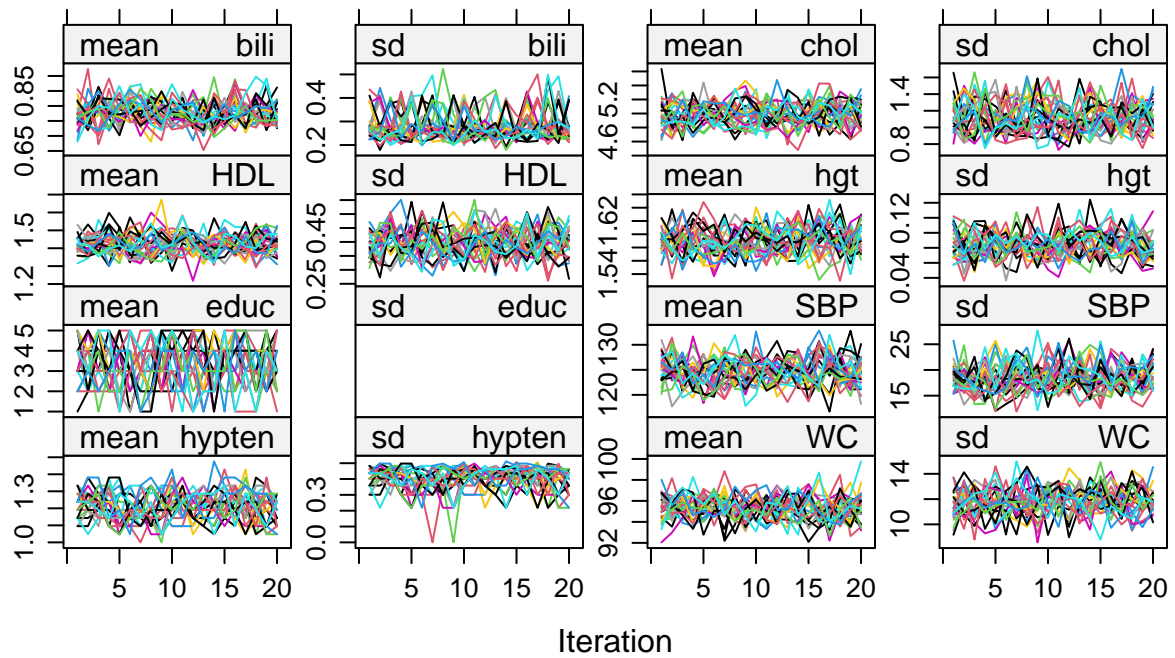
Looking at Tables above, we have the (pooled) estimates, standard errors, the confident intervals and lambda which get more stable as M increases and we can be more confident in any one specific run. Since we run the multiple imputation with a sufficiently large M, the results will with high probability only differ by a small amount. Thus we will choose seed=1 with M=25 and proceed to the multiple imputation.

```
imp <- mice(nhanes2, method = meth, post = post, maxit = 20,
  m = 25, seed = 1, printFlag = FALSE)
imp$loggedEvents
```

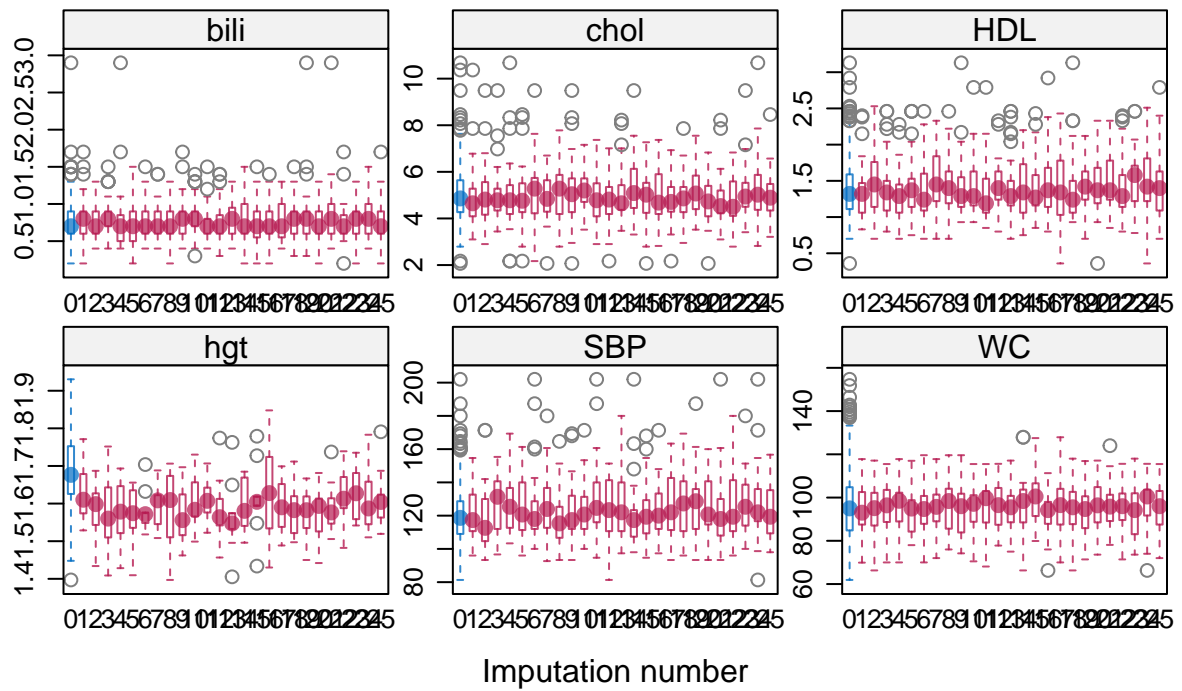
```
## NULL
```

We can also confirm through `loggedEvents` that no problems occurred during the imputation.

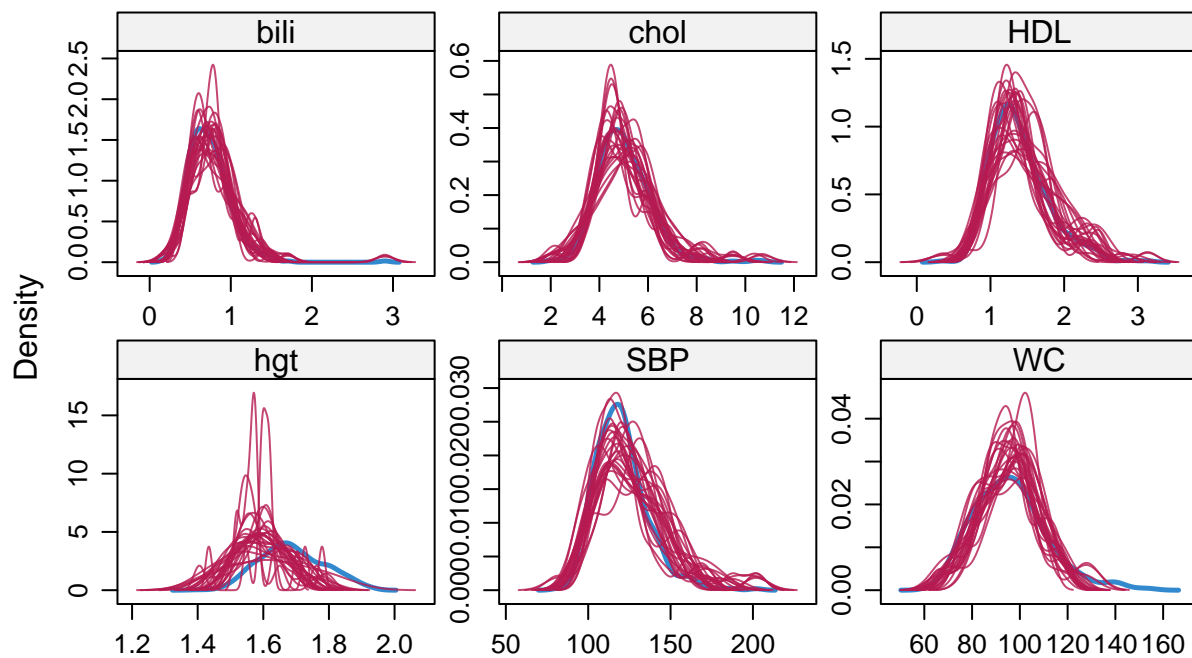

```
plot(imp, layout=c(4,4))
```



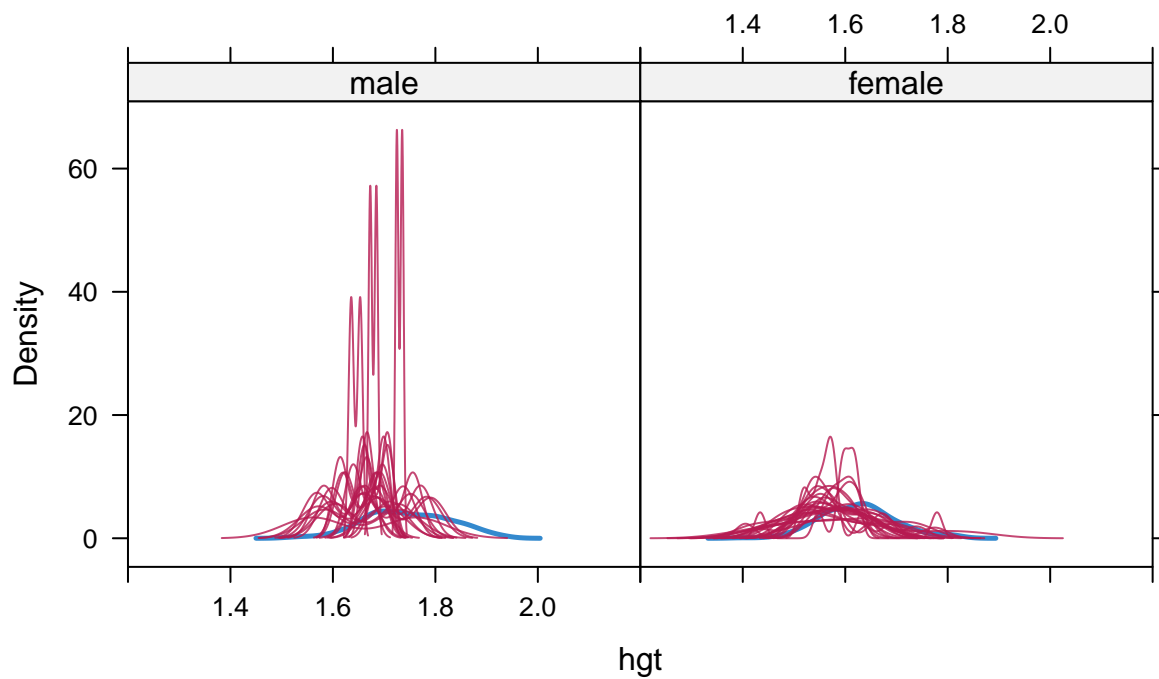
```
bwplot(imp) [c(2,4,5,6,7,8)]
```



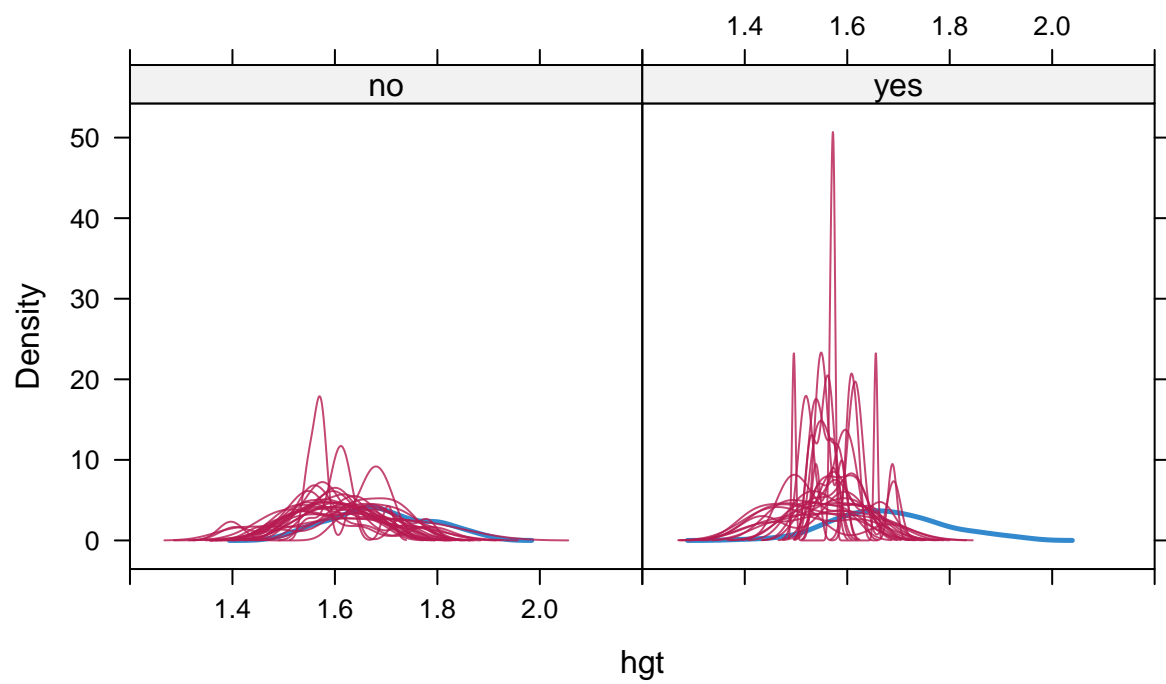
```
densityplot(imp)
```



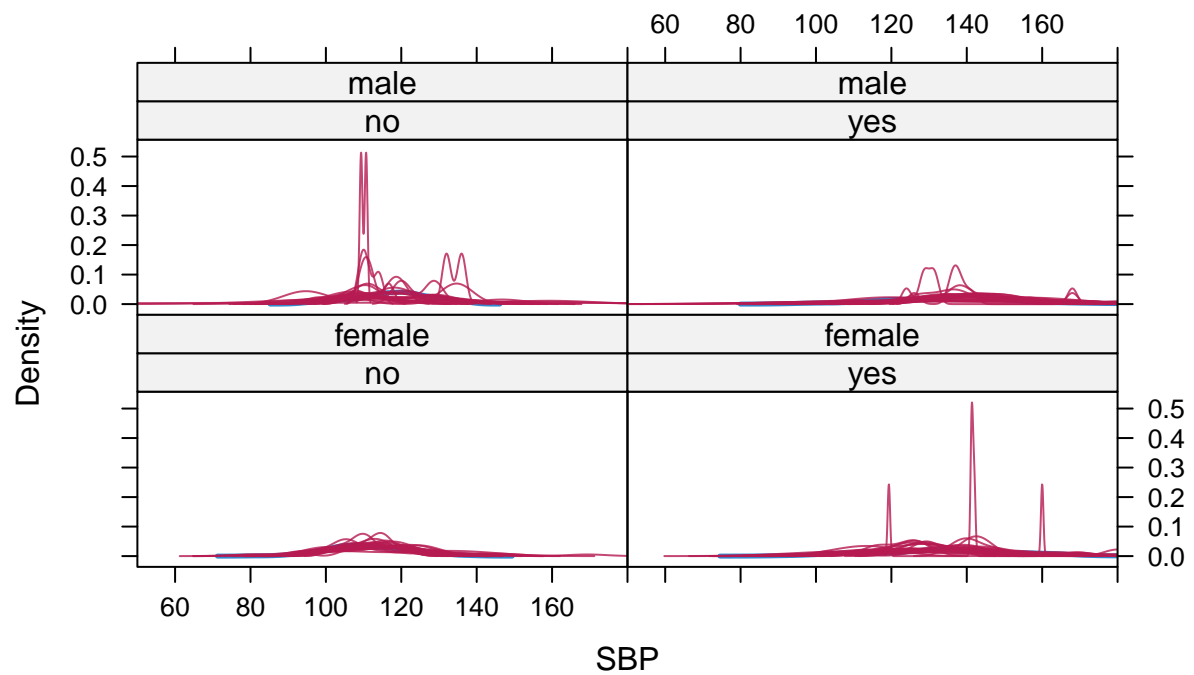
```
densityplot(imp, ~hgt | gender, xlim = c(1.2, 2.2))
```



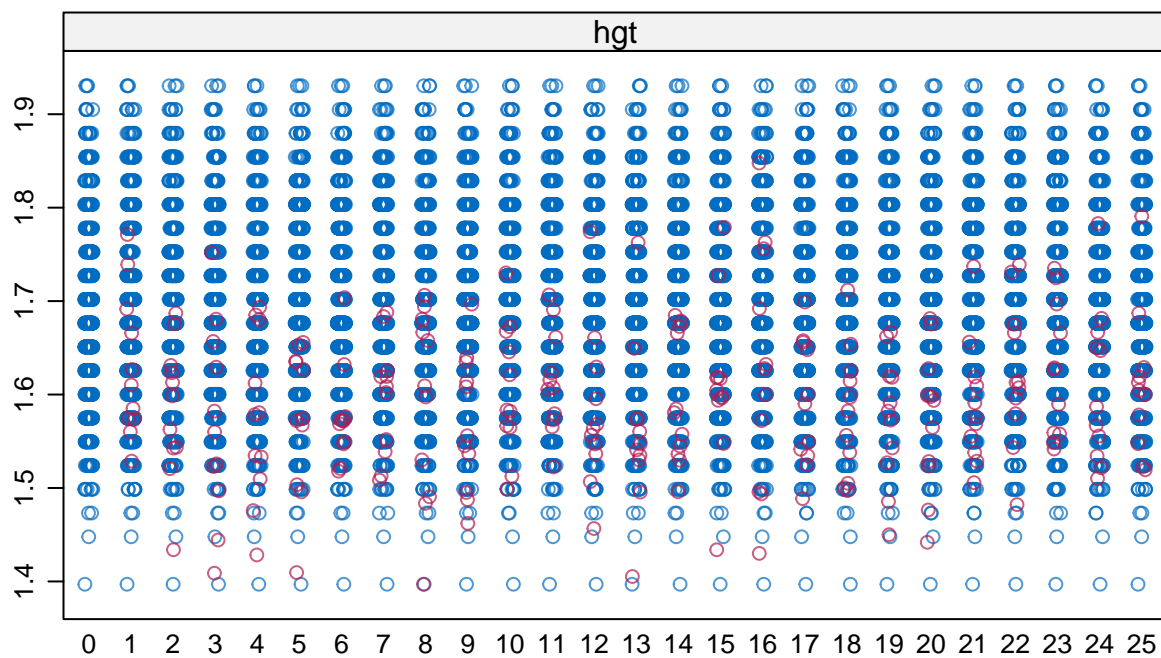
```
densityplot(imp,~hgt|hypten, xlim = c(1.2, 2.2))
```



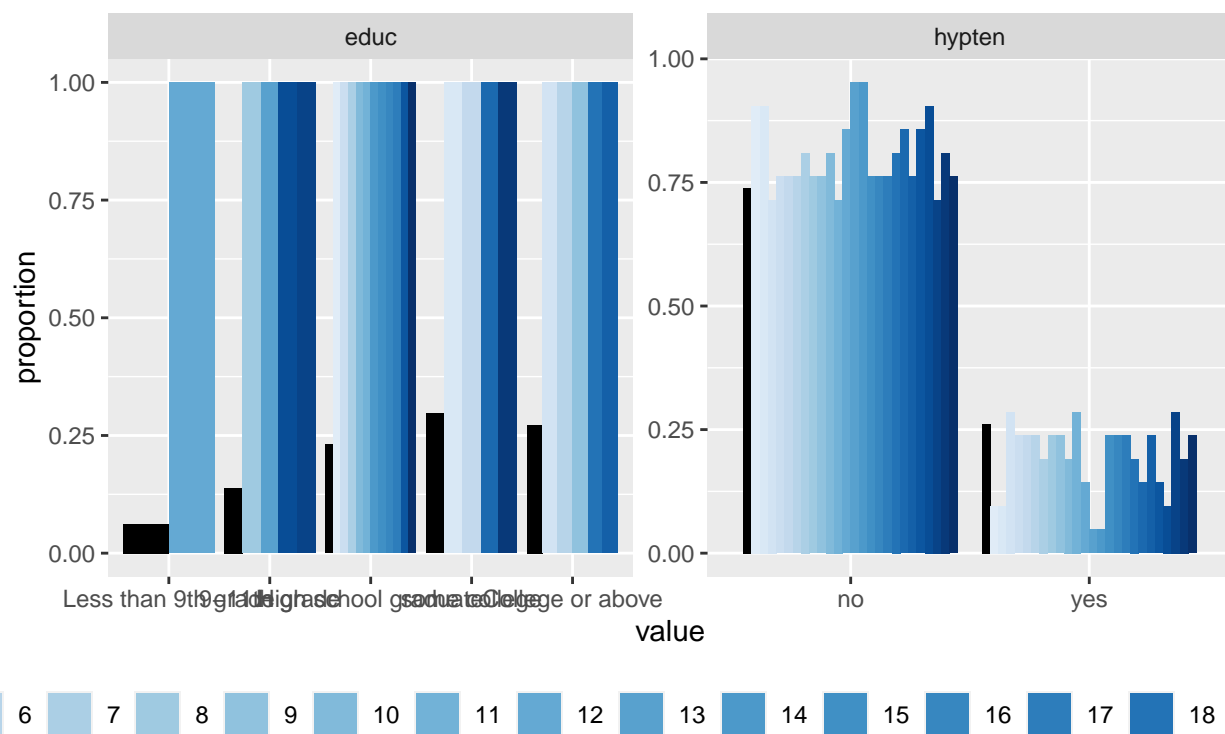
```
densityplot(imp,~SBP|hypten + gender, xlim=c(50, 180))
```



```
stripplot(imp)[6]
```



```
propplot(imp)
```



Now let us process the plots one by one to check if the `mice()` converged. Except for `educ`, rest of the variables are continuous are we proceed to the first plot. The traceplot indicates that the iterative algorithm appears to have converged for all variables that were imputed. The next plots are boxplots and density plots for the observed and imputed ones where we label them as blue and red respectively.

Note that we are using `M=25` and because the density of the observed data is possibly plotted first, we can barely see it. However, we can generally confirm that most imputed datasets follow a similar distribution. The most outstanding plots are the ones from `SBP` and `hgt` as there is a shift towards lower values.

Specifically, we investigated with `hgt` conditional on the `gender` and `hgt` conditional on `hypoten`. We can see that to a certain extent, both plots explain the differences between the observed and imputed values for `hgt`. This is because `male` and `positive` have much narrower distribution compared to wider distribution for `female` and `negative`. For `SBP` we could observe similarity. Using `stripplot()` we can double confirm that the imputation seems reasonable.

We observe an abnormal imputation pattern for the `educ` variable however this is **not** a cause for concern since we are only imputing a single missing value out of 500 cases. Meanwhile, for the `hypoten` variable, we observe a reasonable amount between imputation variance but not enough such that the general distribution of this variable is lost. In summary, all imputations for variables with missing data have been performed successfully.

we can compare the distribution of the imputed values against the distribution of the observed categorical variables' values using `propplot()`. This compares the proportion of values in each category. This shows a large discrepancy between the observed and imputed data distributions for the `educ`. However `educ` only contains 1 missing variable and we can conclude that there is insufficient evidence to say that the imputation `educ` has problem. For the `hypoten`, there is a reasonable proportion between the imputation variances.

All together, we can confirm that the multiple imputation is successful and proceed to analysis of the imputed data.

```
fit <- with(imp, lm(wgt ~ gender + age + hgt + WC))
summary(fit$analyses[[1]])

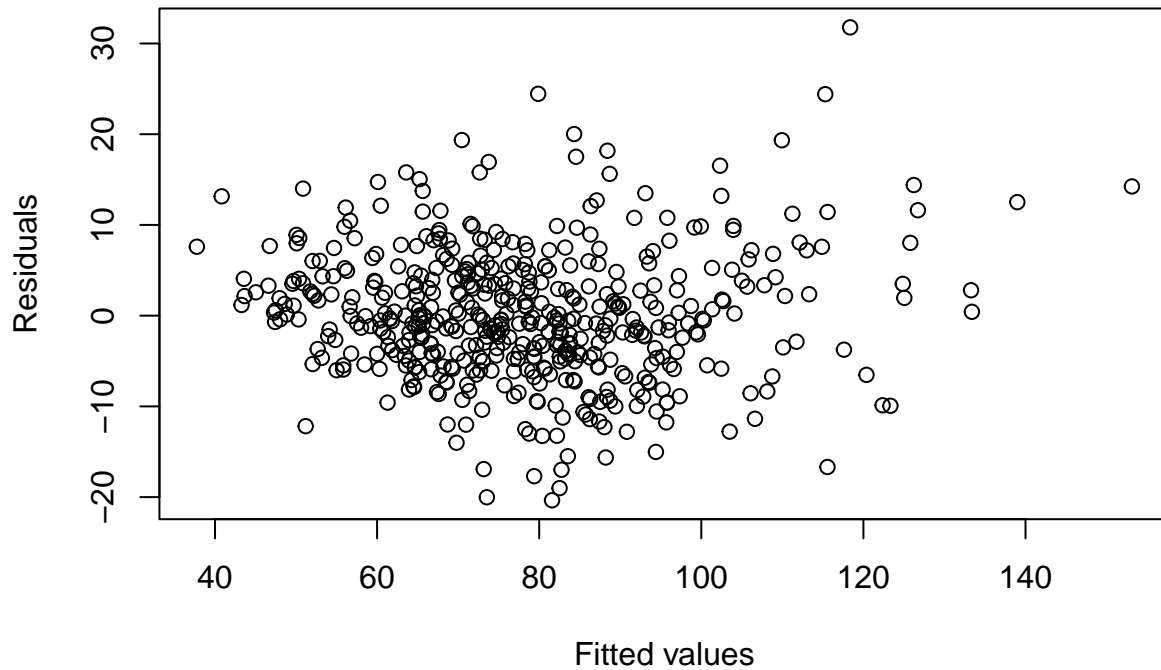
##
## Call:
## lm(formula = wgt ~ gender + age + hgt + WC)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -20.349  -4.568  -0.428   4.068  31.774
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -100.83781    7.51251  -13.423  < 2e-16 ***
## genderfemale  -1.36542    0.82182   -1.661   0.0973 .
## age          -0.15556    0.02085   -7.460 3.91e-13 ***
## hgt           52.42865    4.29633   12.203  < 2e-16 ***
## WC            1.02524    0.02210   46.387  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.192 on 495 degrees of freedom
## Multiple R-squared:  0.857, Adjusted R-squared:  0.8558
## F-statistic: 741.4 on 4 and 495 DF, p-value: < 2.2e-16
```

Looking at the summary we obtained the estimates as

$$\text{wgt} = -100.83781 - 1.36542 \times \text{gender} - 0.15556 \times \text{age} + 52.42865 \times \text{hgt} + 1.02524 \times \text{WC}$$

We can also check that only the coefficient for `genderfemale` has a p-value greater than 0.05. To this, we will check using Wald Test later for the importance of it.

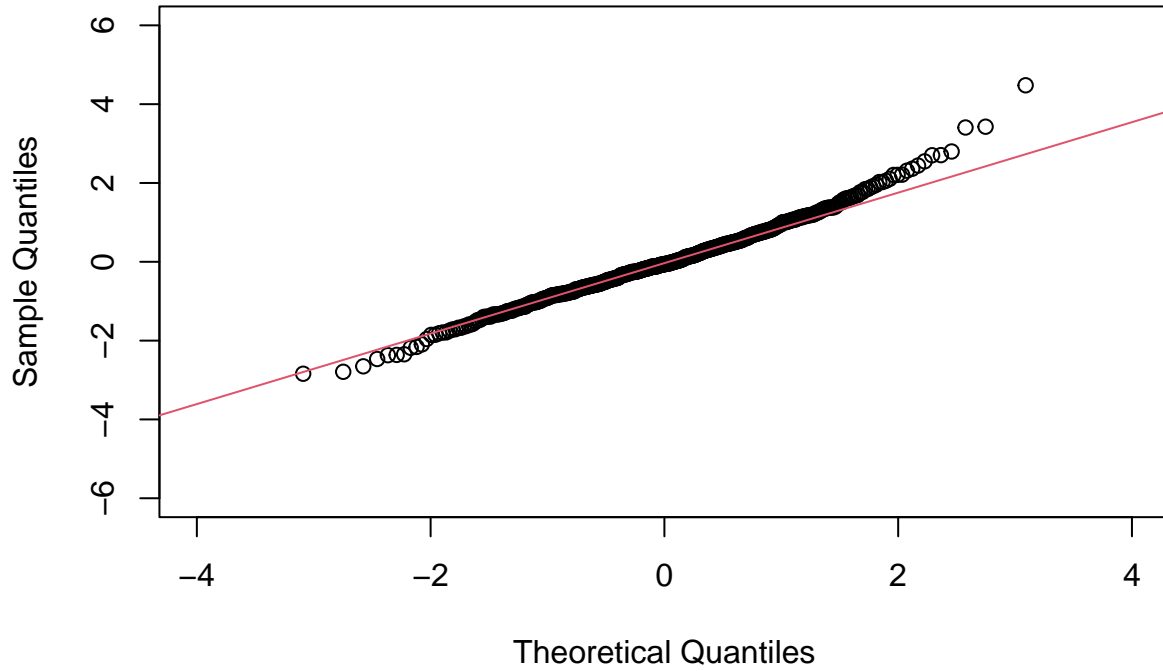
```
comp1 <- complete(imp, 1)
plot(fit$analyses[[1]]$fitted.values, residuals(fit$analyses[[1]]),
     xlab = "Fitted values", ylab = "Residuals")
```



In this fitted values versus residuals plot we can observe two clusters. The residuals are symmetric about zero, and we can suspect that maybe the homoscedastic assumption is slightly violated, but nothing that we cannot live with. I was however curious and decided to plot the response variable, SBP, against the other variables. They seem to indicate that the major cause of the two clusters is the hypertensive status.

```
qqnorm(rstandard(fit$analyses[[1]]), xlim = c(-4, 4), ylim = c(-6, 6))
qqline(rstandard(fit$analyses[[1]]), col = 2)
```

Normal Q-Q Plot



By observing the normal Q-Q plot, we do not see the deviance from the red dotted base line. Therefore, we can conclude that the data after imputation follows normal distribution and the normality assumption holds. Lastly, we will proceed to pooling the estimates with our imputed dataset.

```
pooled_ests <- pool(fit)
kable(summary(pooled_ests, conf.int = TRUE)[c(1,2,6,7,8)],
       caption = "Summary Statistics") %>%
  kable_styling(latex_options = "hold_position")
```

Table 16: Summary Statistics

| term | estimate | p.value | 2.5 % | 97.5 % |
|--------------|--------------|-----------|--------------|-------------|
| (Intercept) | -101.1180859 | 0.0000000 | -116.0326895 | -86.2034823 |
| genderfemale | -1.3477467 | 0.1063107 | -2.9844502 | 0.2889568 |
| age | -0.1582103 | 0.0000000 | -0.2004609 | -0.1159597 |
| hgt | 52.6066777 | 0.0000000 | 44.0729374 | 61.1404180 |
| WC | 1.0256687 | 0.0000000 | 0.9818758 | 1.0694617 |

```
kable(pool.r.squared(pooled_ests, adjusted = TRUE), caption = "") %>%
  kable_styling(latex_options = "hold_position")
```

Table 17:

| | est | lo 95 | hi 95 | fmi |
|--------------------|-----------|-----------|-----------|-----------|
| adj R ² | 0.8563534 | 0.8308076 | 0.8783204 | 0.0261765 |

Looking at the Table, the adjusted R^2 values indicates good fit.

Now we conduct Wald Test to check the importance of the feature.

```
fit.no.WC <- with(imp, lm(wgt ~ gender + age + hgt ))
fit.no.hgt<- with(imp, lm(wgt ~ gender + age + WC))
fit.no.age <- with(imp, lm(wgt ~ gender + hgt + WC))
fit.no.gender <- with(imp, lm(wgt ~ age + hgt + WC))
wald.stats <- rbind(D1(fit, fit.no.WC)$result, D1(fit, fit.no.hgt)$result,
  D1(fit, fit.no.age)$result, D1(fit, fit.no.gender)$result)
excluded.features <- c("WC", "hgt", "age", "gender")
df <- data.frame(excluded.features, wald.stats)
kable(df, caption = "Wald Test on each features") %>%
  kable_styling(latex_options = "hold_position")
```

Table 18: Wald Test on each features

| excluded.features | F.value | df1 | df2 | P..F. | RIV |
|-------------------|-------------|-----|----------|-----------|-----------|
| WC | 2117.811020 | 1 | 488.6612 | 0.0000000 | 0.0174882 |
| hgt | 146.751120 | 1 | 472.7845 | 0.0000000 | 0.0394115 |
| age | 54.164973 | 1 | 447.6375 | 0.0000000 | 0.0621865 |
| gender | 2.618292 | 1 | 479.2870 | 0.1062959 | 0.0319663 |

We conducted the Wald test by excluding each feature one by one and used `D1()`. From the Table, we can see that **gender** is the only feature having p-value greater than 0.05. Although it is above 0.05, we can still consider its important but limiting it into 0.1 significance level. Thus, **WC**, **hgt** and **age** have more influence than **gender** here.