INTRODUCTION TO NLP

Session 4

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BUILDING A BASIC SPELLCHECKER

- Based on two notions that we have already seen: The edit distance and a Language model, we can build an spellchecker.
- Let us consider that we want to correct misspelled words:
  - That is: words that are not in the vocabulary
- Base algorithm: Let x be a sentence and V the vocabulary.

```
For $w$ in $x$:
 If $w$ not in $V$:
    Find the closest words to w (candidate search)
    Evaluate each of the candidate words (candidate evaluation)
    Return the most probable candidate
```

FINDING THE CLOSEST ITEMS TO A QUERY

- Efficient search of similar values in a dataset is a very challenging problem. In particular, computing distances between a word and a huge vocabulary can be computationally expensive.
- Let w be a string that is out of the vocabulary.
- Let us consider $W_k(w; X) = \{w \,|\, w \in V, d(w, w_j) < k\}$
- Finding $W_k(w; X)$ can be done in two different ways:

  I) compute $d(w, w_j)$ for all w_j then select the elements that are at distance at most k

  II) Use a data structure to avoid computing $d(w, w_j)$ for all w_j in X.

TREE INTUITION

- We can build a tree to do efficient search of similar words. This will allow us to prune a lot of the search space, with the objective of avoiding many distance computations on a big part of the vocabulary.
- Example: consider w = pleistation

ana	d(pleistation, ana)
playstation	d(pleistation, playstation)
house	d(pleistation, house)

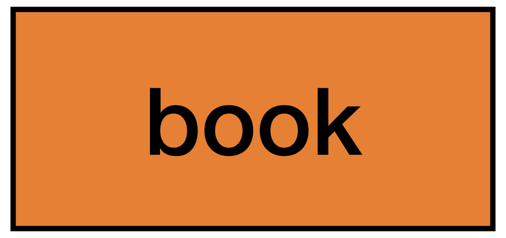
- If pleistation has 11 characters and we want all candidates to bet at most at distance k=3, is there any need to compute d(pleistation, ana) ?
    - Ana has 3 characters!

BK-TREE

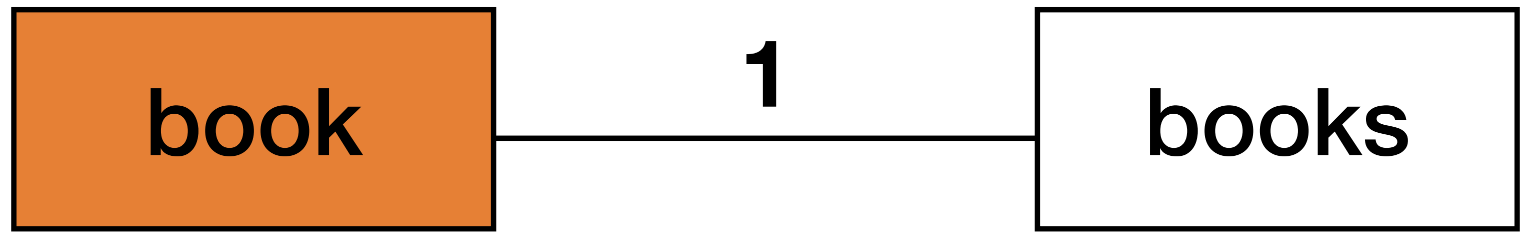
- To create a BK-tree we will follow this approach.
  - Select any word from the vocabulary and use it in as the root node.
  - Keep adding words until all vocabulary is the tree.
    - Each time we add a word the distance between the word and the root node is computed, let us assume this distance is d.
    - If no node from the root node is at distance d we add a new leave as a descendant of the rood node with edge value equal to d.
    - If there exist another node at distance d then… we repeat this process redefining the root node as the node that produced the collisiond(pleistation, ana)

BK TREE CONSTRUCTION EXAMPLE

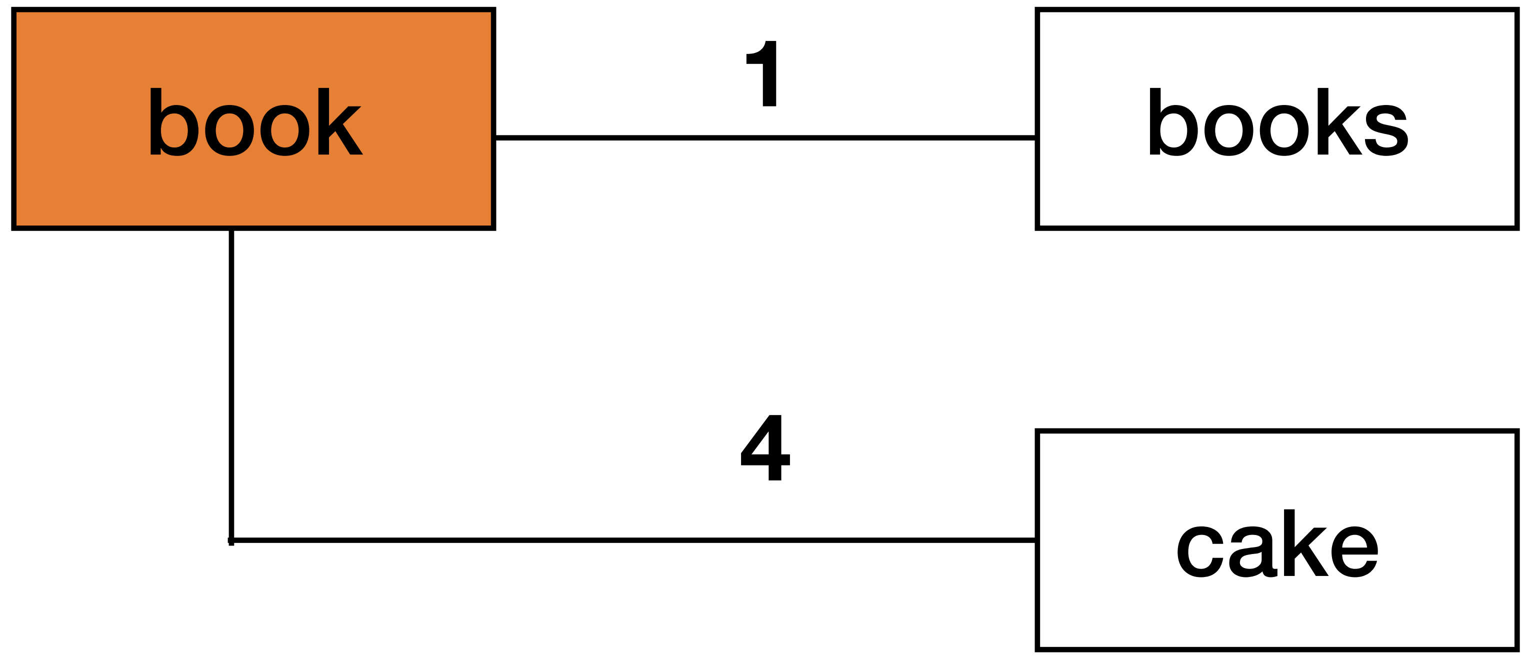
- Let us consider the data [book, books, cake], we start from **book (which becomes root node)**



- [book, <u>books</u>, cake]: **books** comes and we compute d(book, books) = 1

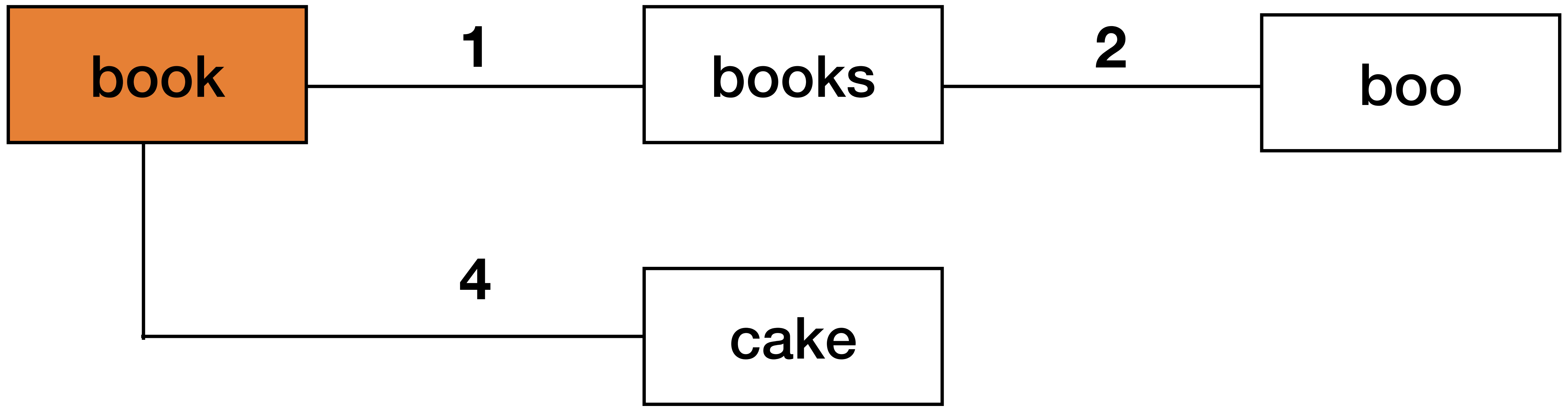


- [book, books, <u>cake</u>]: **cake** comes and we compute d(book, cake) = 4



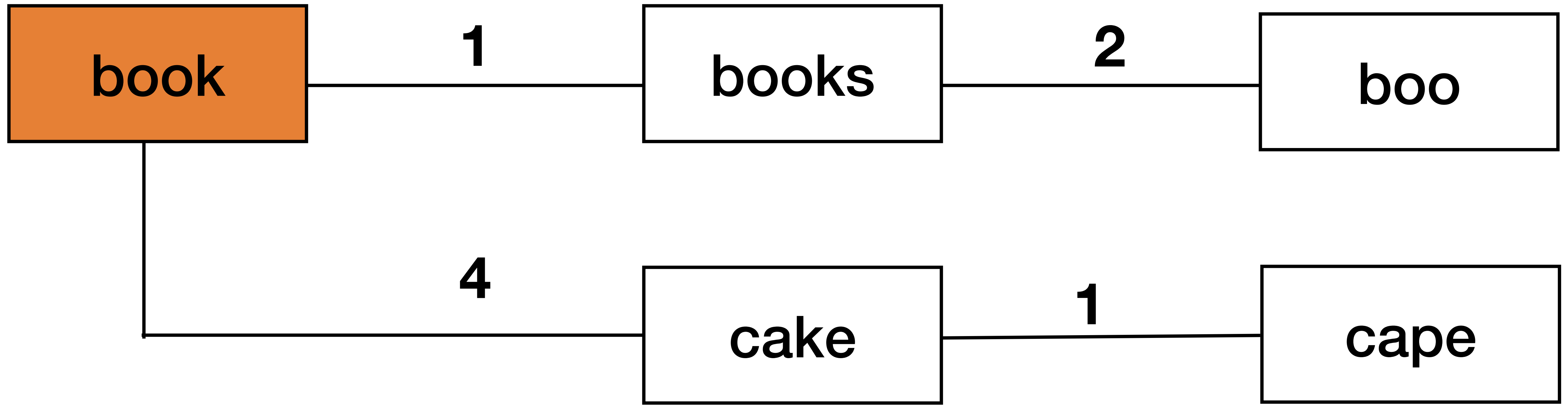
BK-TREE EXAMPLE: [BOOK, BOOKS, CAKE, BOO, CAPE, CART, BOON, COOK]

- [book, books, cake, boo]: **boo** comes and we compute d(book, boo)=1.
  Note that there is already **books** at distance 1.
- The BK tree has to respect that every node have all children with different distances, **since there is already a word at the same edit distance 1** we go to the branch of words at distance 1.
- **If there is a collision** (like we have now) **the new word must become a children of the collisioned word**. In this case, a children of book.
- The new weight from books to boo has to be distance(books, boo)=2.



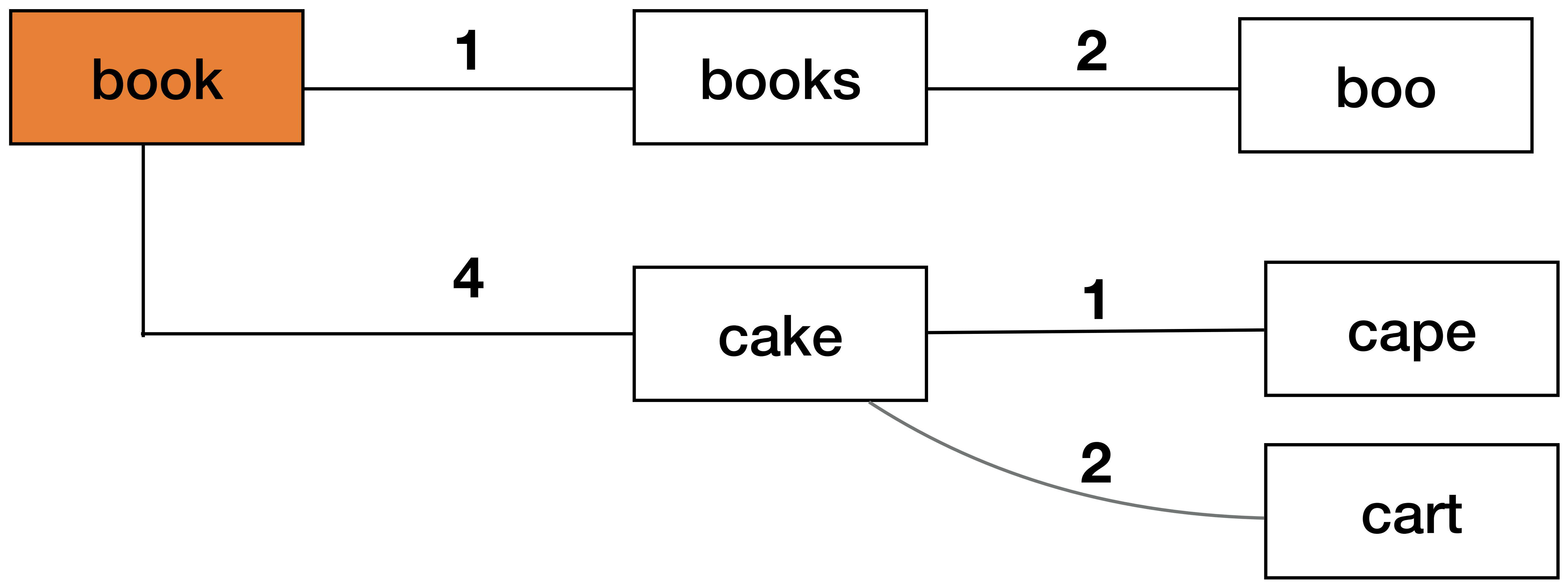
BK-TREE EXAMPLE: [BOOK, BOOKS, CAKE, BOO, CAPE, CART, BOON, COOK]

- Root=book: Compute d(book, cape)=4
  - Collision! There is already cake at distance 4 from book
  - Root node is now cake
  - Root=cake: Compute d(cake, cape)=1
  - There is no descendant from cake at distance 1 => we can add it



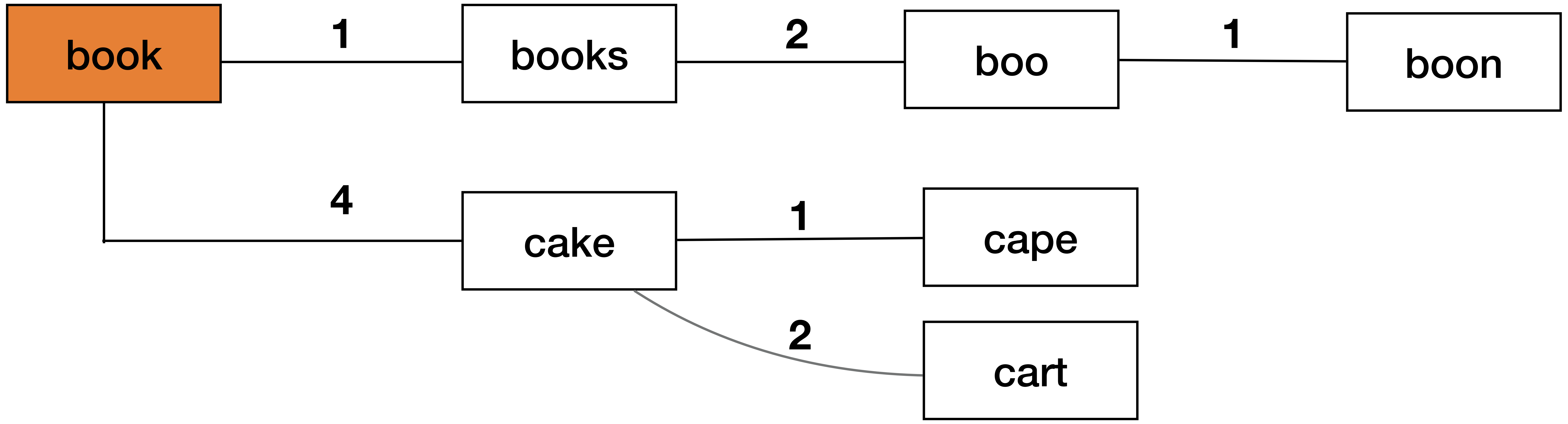
BK-TREE EXAMPLE: [BOOK, BOOKS, CAKE, BOO, CAPE, CART, BOON, COOK]

- Root=book: Compute d(book, cart)=4
  - Collision! There is already cake at distance 4 from book
  - Root node is now cake
  - Root=cake: Compute d(cake, cart)=2
  - There is no descendant from cake at distance 2 => we can add it



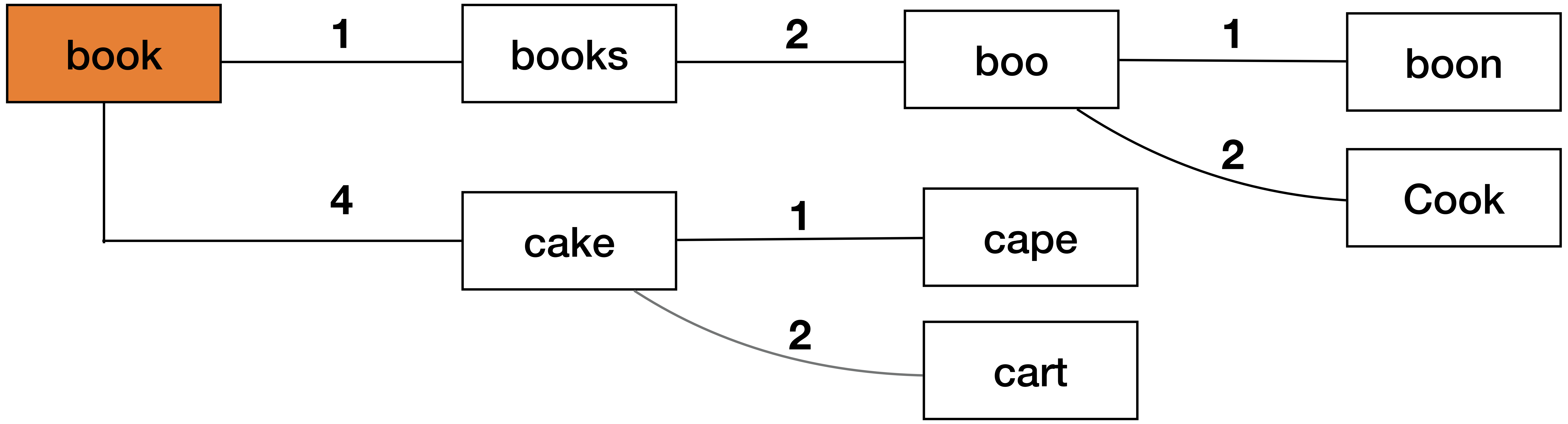
BK-TREE EXAMPLE: [BOOK, BOOKS, CAKE, BOO, CAPE, CART, BOON, COOK]

- Root=book: Compute d(book, boon)=1
  - Collision! There is already books at distance 1 from book
  - Root node is now books
  - Root=books: Compute d(books, boon)=2
  - Collision! There is already boo at distance 2 from books
  - Root=boo: Compute d(books, boon)=1
  - There is no descendant from boo at distance 1 => we can add it



BK-TREE EXAMPLE: [BOOK, BOOKS, CAKE, BOO, CAPE, CART, BOON, COOK]

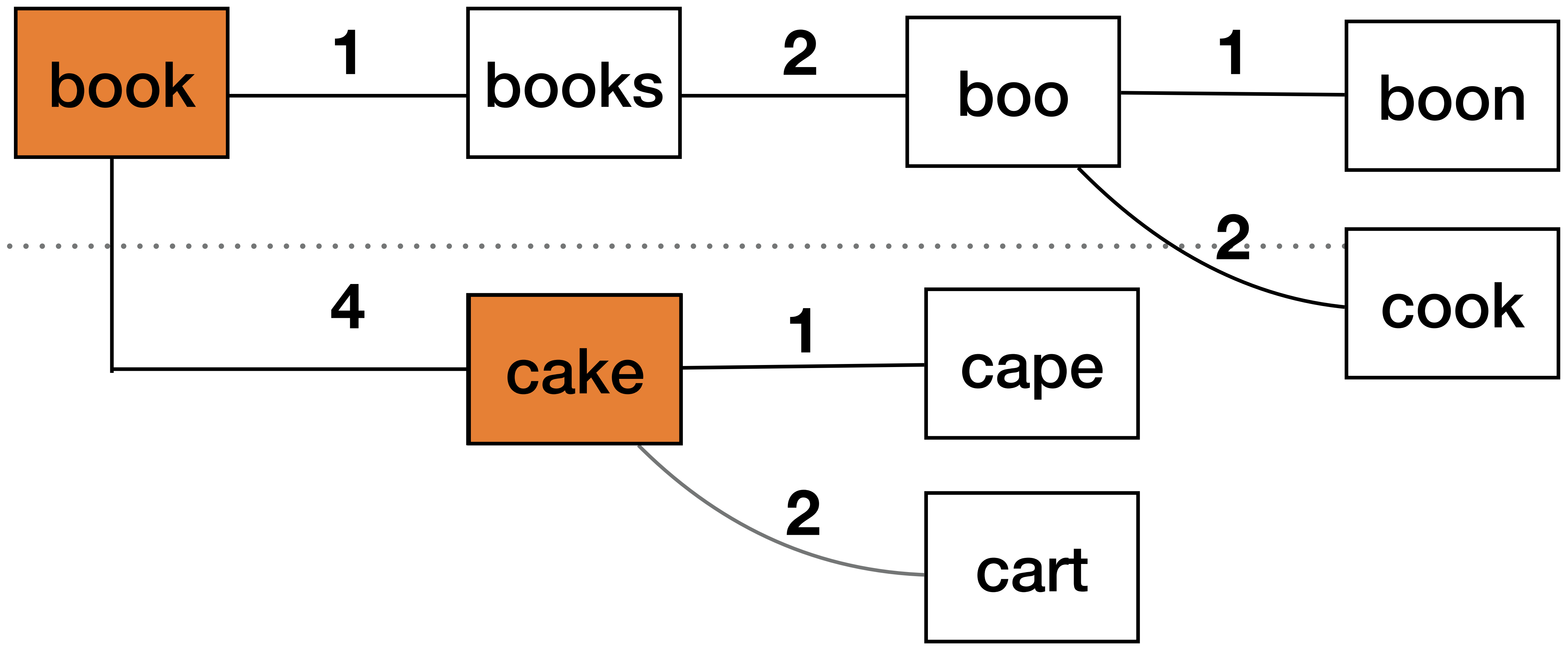
- Root=book: Compute d(book, cook)=1
    - Collision! There is already books at distance 1 from book
    - Root node is now books
    - Root=books: Compute d(books, cook)=2
    - Collision! There is already boo at distance 2 from books
    - Root=boo: Compute d(boo, cook)=2
    - There is no descendant from boo at distance 2 => we can add it



SEARCHING IN A BK-TREE

- Problem: Search all words that appear at distance less or equal than a tolerance T form a query word q .
- Bad solution: Compute all edit distances between q and w for w in the Vocabulary.
- Key idea: Visit all words w that are at distance [d(w,q)-T, d(w,q)+T].

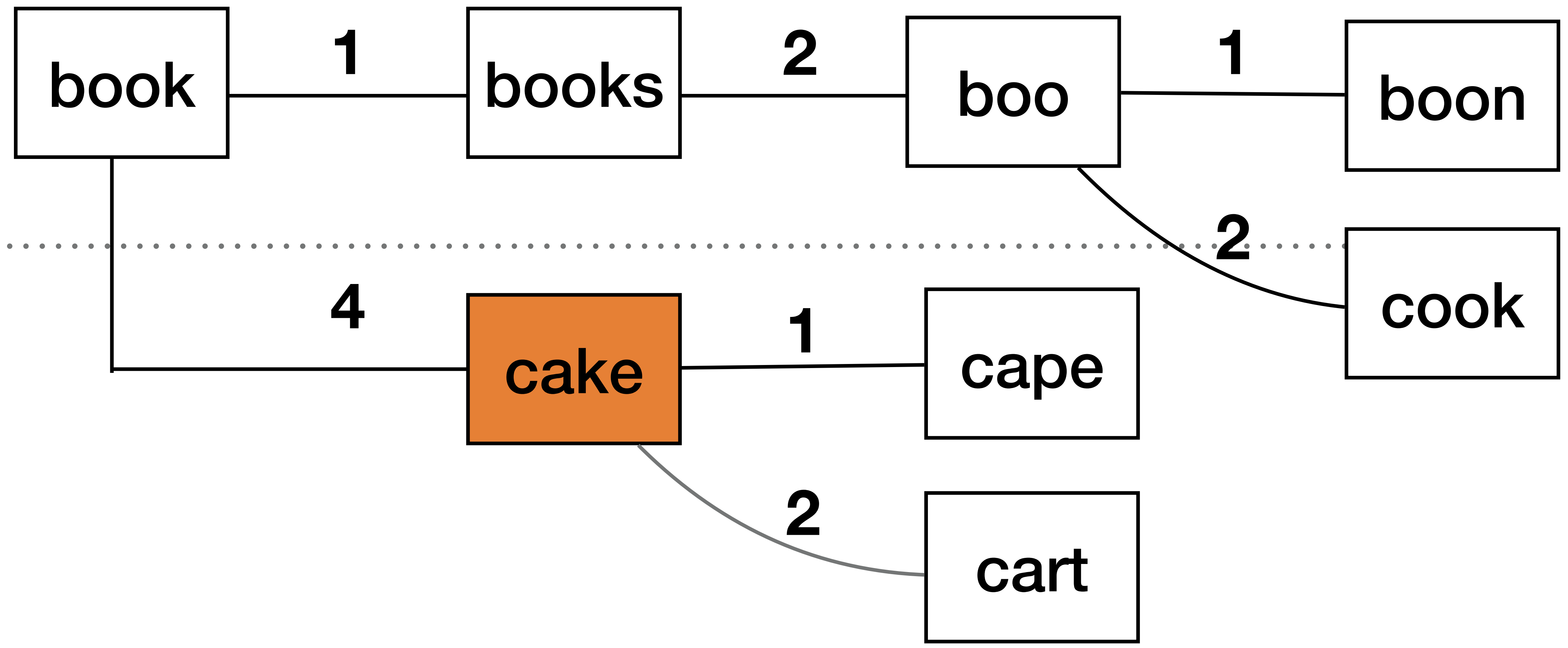
BK-TREE EXAMPLE: SEARCHING



- Let us consider
- q=caqe, T=1, candidates = []

- Select candidate **book** from search=[book]
  - d(book, caqe) = 4 => candidates is not updated
    - Crawl all children of book at distance I=[4-1,4+1]=[3,5]
    - Only node cake is connected to book and with distance in I=[3,5]
    - search = [book, cake]\book = [cake]

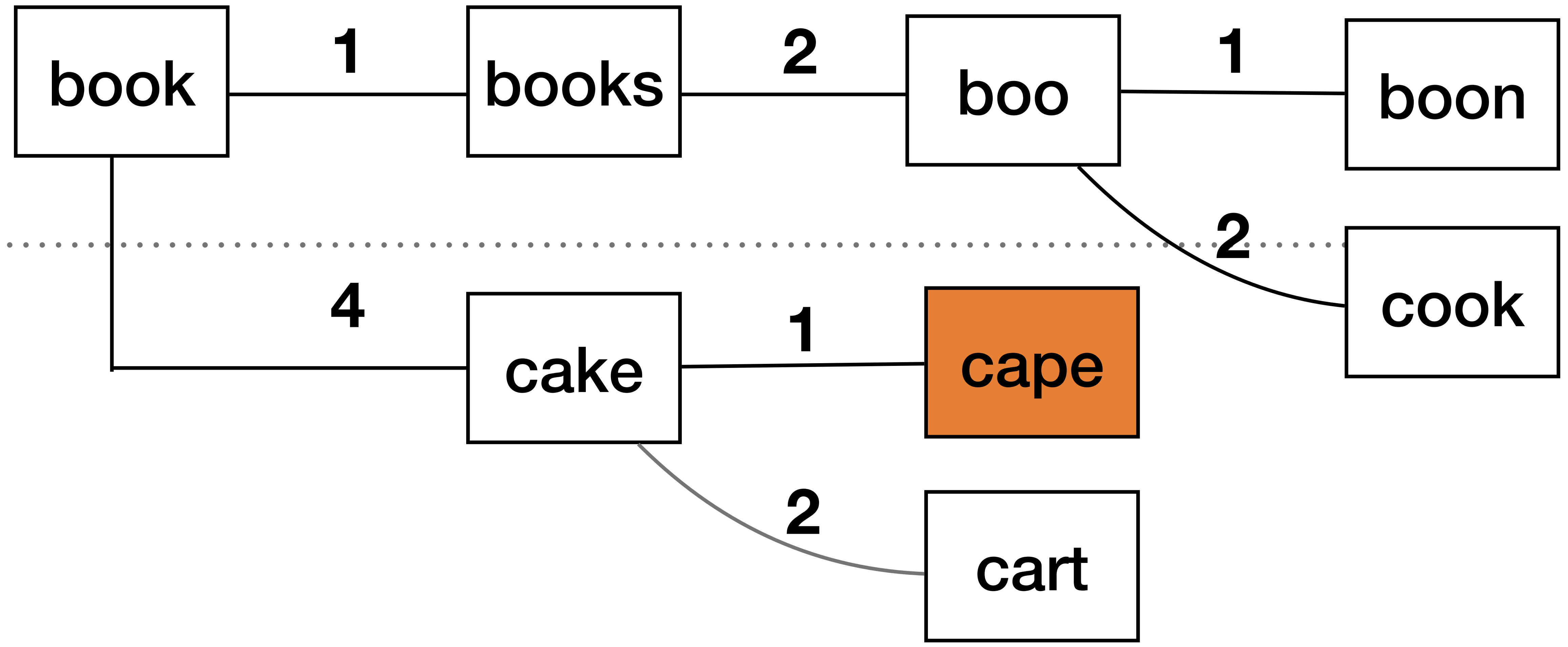
BK-TREE EXAMPLE: SEARCHING



- Let us consider
- q=caqe, T=1, candidates = []

- Select candidate **cake** from search=[cake]
  - d(cake, caqe) = 1 => candidates += [cake]
    - Crawl all children of cake at distance I=[1-1,1+1]=[0,2]
    - Only are 2 possible nodes, search=[cape, cart]

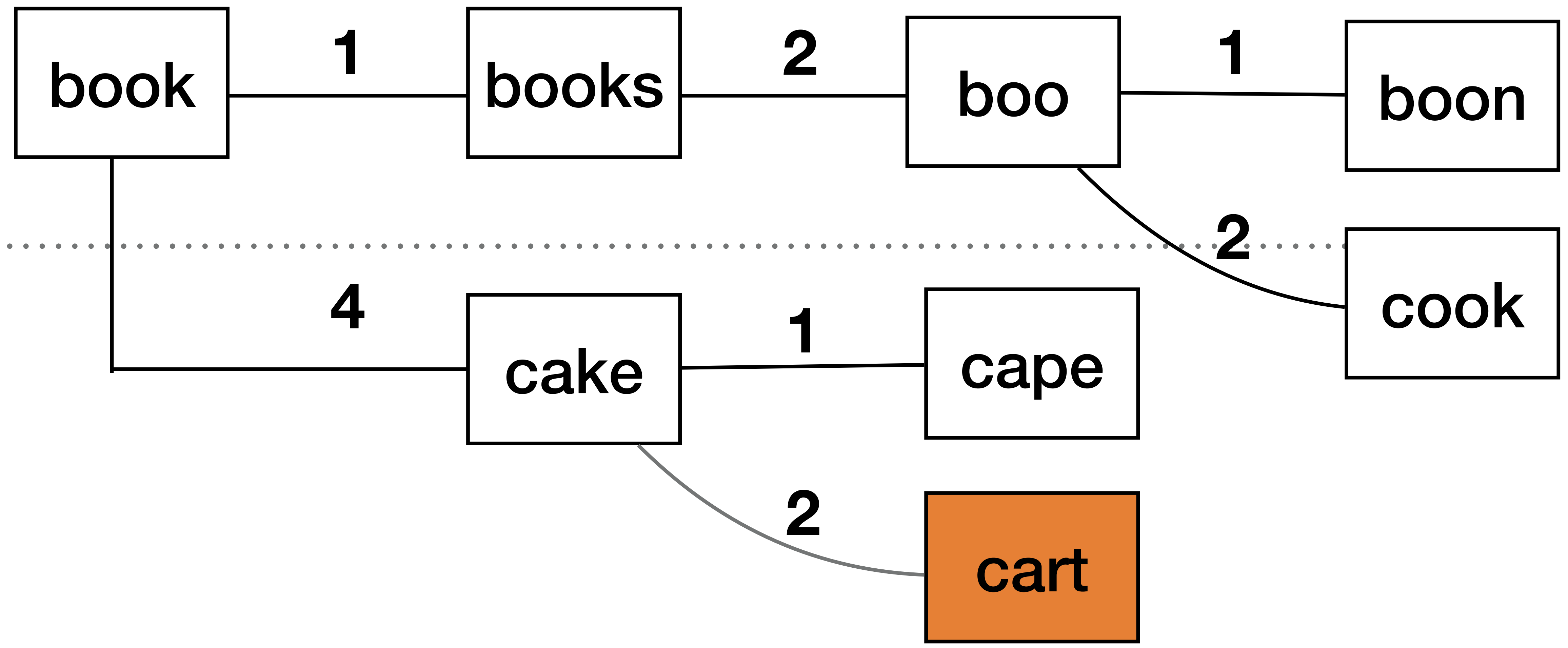
BK-TREE EXAMPLE: SEARCHING



- Let us consider
- q=caqe, T=1, candidates = [cake]

- Select candidate **cape** from search=[cape, cart]
  - d(cape, caqe) = 1 => candidates += [cape]
    - Crawl all children of cape at distance I=[1-1,1+1]=[0,2]
    - cape has no children
    - search = [cape, cart]\cape = [cart]

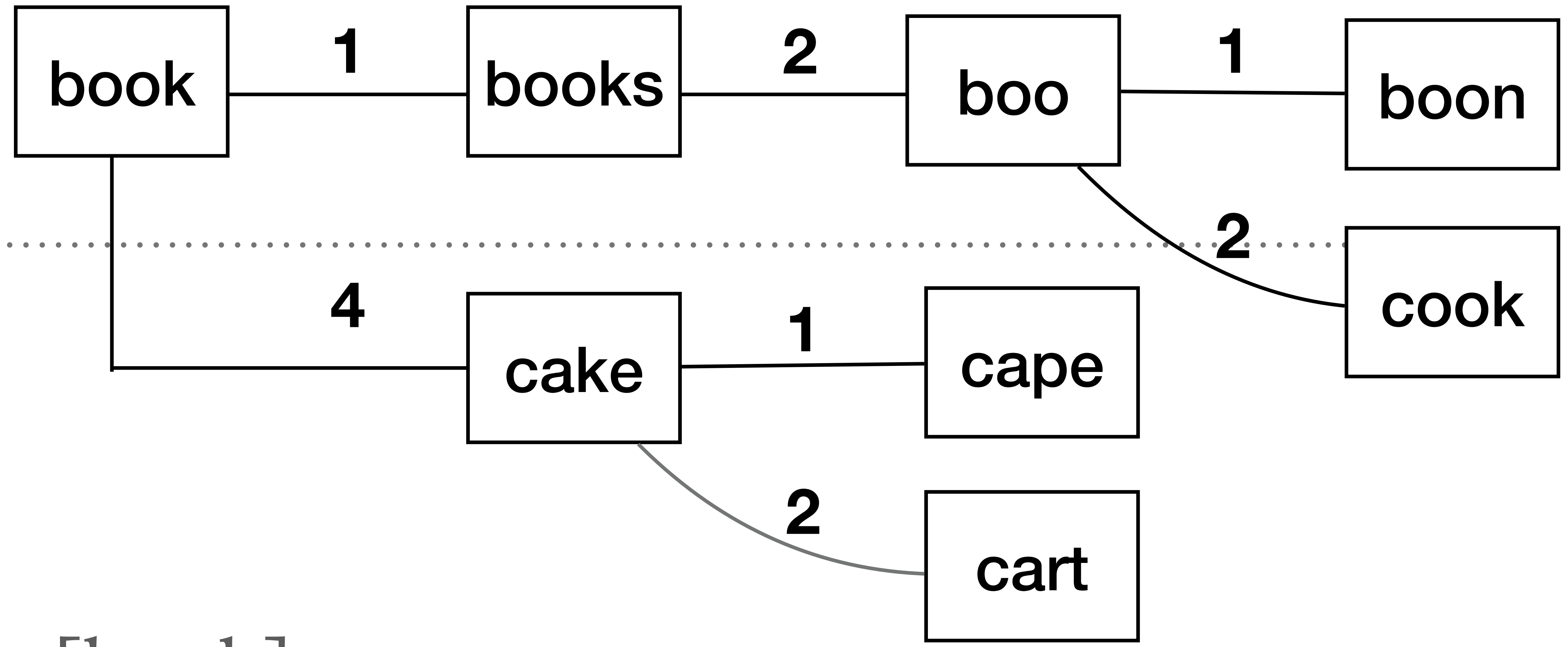
BK-TREE EXAMPLE: SEARCHING



- Let us consider
- q=caqe, T=1, candidates = [cake, cape]

- Select candidate **cart** from search=[cart]
  - d(cart,caqe) = 2 => candidates is not updated
    - Crawl all children of cape at distance I=[1-1,1+1]=[0,2]
    - Caqe has no children
    - search = [cart]\cart = [] =>Search space is empty, stop search
- The resulting set of possible candidates at distance 1 are: [cape,cake]

BK-TREE EXAMPLE: SEARCHING

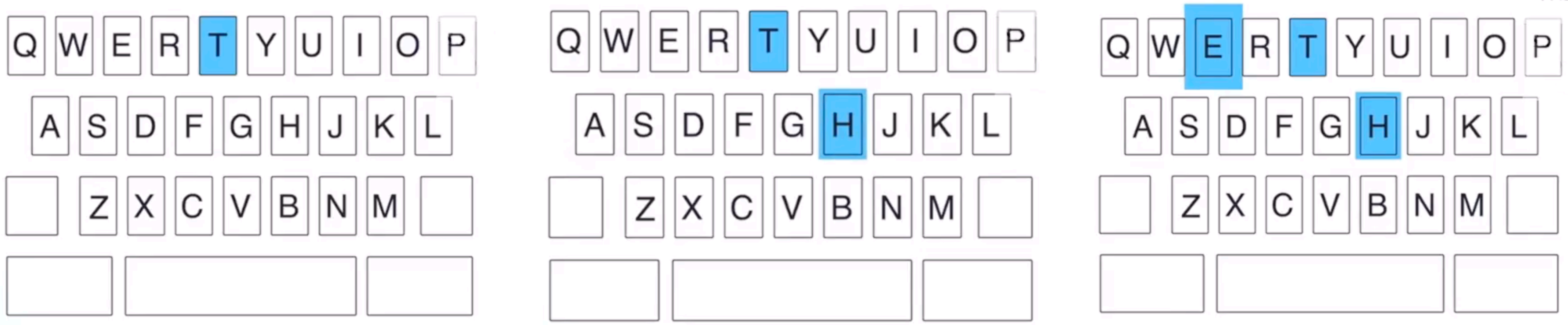


- To sum up:
  - We started from:
    - q=caqe, T=1, candidates = [], search=[book]
  - After searching in the BK-Tree we know
    - The set of possible candidates at distance 1 are: [cape, cake].
- Observation:we ended up computing 4 edit distances yet we have 8 nodes.

A VERY IMPORTANT APPLICATION OF LANGUAGE MODELS

- Most tactile keyboards before the iPhone were bad.
- Users could make mistakes by simply hitting a region nearby a letter but outside the key that contains a letter.

- The iphone introduced dynamically adjusted hit regions in the keyboard.



CONSTRUCTING A LANGUAGE MODEL

- Let us consider a toy corpus from:
  https://en.wikipedia.org/wiki/This_Is_the_House_That_Jack_Built

  This is the house that Jack built
  This is the malt
  That lay in the house that Jack built
  This is the rat
  That ate the malt
  That lay in the house that Jack built
  This is the cat
  That killed the rat
  That ate the malt
  That lay in the house that Jack build

CONSTRUCTING A LANGUAGE MODEL

- We want to compute a bi-gram language model, which assumes $p(w_n \mid w_1, \ldots, w_{n-1}) \approx p(w_n \mid w_{n-1}) = p(w_n, w_{n-1})/p(w_{n-1})$

This is the house that Jack built
This is the malt
That lay in the house that Jack built
This is the rat
That ate the malt
That lay in the house that Jack built
This is the cat
That killed the rat
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That lay in the house that Jack build

What could be a sensible way to estimate $p(w_i, w_j)$?

MARKOV ASSUMPTION

- N-gram language models make the Markov assumption, which we already saw in the hidden states of the HMM.
- Let us consider $x = (w_1, w_2, w_3, \ldots, w_N)$
- Using the Chain rule of probabilities:
  - $p(x) = p(w_1)p(w_2 \mid w1)p(w_3 \mid w_2, w_1)\ldots p(w_N \mid w_{N-1}, \ldots, w_2, w_1)$
- A k-Markov assumption assumes:
  - $p(w_n \mid w_1, \ldots, w_{n-1}) = p(w_n \mid w_{n-k+1}, \ldots, w_{n-1})$

EXAMPLES OF MARKOV ASSUMPTIONS

- A k-Markov assumption assumes:
  - $p(w_n \mid w_1, \ldots, w_{n-1}) = p(w_n \mid w_{n-k+1}, \ldots, w_{n-1})$
- Example: if k=2 and n=4
  - $p(w_4 \mid w_1, w_2, w_3) = p(w_4 \mid w_{n-k+1}, \ldots, w_{n-1}) = p(w_4 \mid w_{4-2+1}, \ldots, w_{4-1}) = p(w_4 \mid w_3)$
- Example: k=3 and n=4
  - $p(w_4 \mid w_1, w_2, w_3) = p(w_4 \mid w_{n-k+1}, \ldots, w_{n-1}) = p(w_4 \mid w_{4-3+1}, \ldots, w_{4-1}) = p(w_4 \mid w_2, w_3)$

A NOTE ON STORING COUNTS FOR AN N-GRAM LANGUAGE MODEL

- An expression of the form $p(w_n \mid w_{n-1}) = \dfrac{C(w_{n-1}w_n)}{C(w_{n-1})}$ requires us to store the counts of pairs of words and single words.
- There is an alternative:
  - $p(w_n \mid w_{n-1}) = \dfrac{C(w_{n-1}w_n)}{C(w_{n-1})} = \dfrac{C(w_{n-1}w_n)}{\sum_{\tilde{w}} C(w_{n-1}\tilde{w})}$
  - The expression that marginalises over $\tilde{w}$ has the advantage that needs less memory but the disadvantage that it requires summing over $\tilde{w}$ at runtime.

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Count how many times (w_i, w_j) appears in the corpus and divide by the counts of (w_j)

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What could be a sensible way to estimate $p(w_i \mid w_j)$?

Count how many times (w_i, w_j) appears in the corpus and divide by the counts of (w_j)

$$p(\text{house} \mid \text{the}) = \frac{c(\text{the house})}{c(\text{the})} = ?$$

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What could be a sensible way to estimate $p(w_i \mid w_j)$?

Count how many times (w_i, w_j) appears in the corpus and divide by the counts of (w_j)

$$p(\text{house} \mid \text{the}) = \frac{c(\text{the house})}{c(\text{the})} = \frac{4}{10}$$

CONSTRUCTING A LANGUAGE MODEL

- We want to compute a 4-gram language model, which assumes $p(w_n \,|\, w_1, \ldots, w_{n-1}) \approx p(w_n \,|\, w_{n-2}, w_{n-1}) = p(w_{n-3}, w_{n-2}, w_{n-1}, w_n) / p(w_{n-3}, w_{n-2}, w_{n-1})$

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  That lay in the house that Jack built
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  That killed the rat
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$$p(\text{house} \,|\, \text{this is the}) = \frac{c(\text{this is the house})}{c(\text{this is the})} = ?$$

CONSTRUCTING A LANGUAGE MODEL

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$$p(\text{house} \mid \text{this is the}) = \frac{c(\text{this is the house})}{c(\text{this is the})} = ?$$

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$$p(\text{house} \mid \text{this is the}) = \frac{c(\text{this is the house})}{c(\text{this is the})} = \frac{1}{4}$$

LANGUAGE MODELS WITH <S> AND </S>

- In order to deal with n-grams that affect the beginning or the end of a sentence the special words <s> and </s> are used to denote the start and stop of a sentence.
- Using this special words will affect the model probabilities.
- Consider the corpus:
  <s> This is the malt </s>
  <s> That lay in the house that Jack built </s>
- A 2-gram language model without <s> and </s> will assign to 'this is the house':

$$p(\text{this is the house}) = p(\text{this})p(\text{is}\,|\,\text{this})p(\text{the}\,|\,\text{is})p(\text{house}\,|\,\text{the}) = \frac{1}{12}\frac{1}{1}\frac{1}{1}\frac{1}{2} = \frac{1}{24}$$

- A 2-gram language model with <s> and </s> will assign to 'this is the house':

$$p(<\text{s}>\text{this is the house}</\text{s}>) = p(\text{this}\,|\,\text{start})p(\text{is}\,|\,\text{this})p(\text{the}\,|\,\text{is})p(\text{house}\,|\,\text{the})p(\text{end}\,|\,\text{house}) = ?$$

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$$\frac{1}{2} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{2} \cdot 0 = 0$$

DEALING WITH UNSEEN N-GRAMS: LAPLACE SMOOTHING

- We want to allow unseen grams to have non zero probability. We will pretend we saw each word one more time than we did.
- A common technique is called "Laplace Smoothing" or "add 1 smoothing"
    - Consist on using the following formulas:

$$p_{add_1}(w_n \mid w_{n-1}) = \frac{c(w_{n-1}w_n) + 1}{c(w_{n-1}) + V} \qquad p_{add_1}(w_i) = \frac{c(w_i) + 1}{c() + V} = \frac{c(w_i) + 1}{n_{tokens} + V}$$

- Consider the corpus:
  <s> This is the malt </s>
  <s> That lay in the house that Jack built </s>
- With V = 13 = #{ This, is, the, malt, That, lay, in, house, that, Jack, build, start, stop}
- $p(< \text{s} > \text{this is the house} < /\text{s} >) = p(\text{this} \mid < \text{s} >)p(\text{is} \mid \text{this})p(\text{the} \mid \text{is})p(\text{house} \mid \text{the})p(< /\text{s} > \mid \text{house}) =$
  …

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$$\frac{1+1}{2+13} \cdot \frac{1+1}{1+13} \cdot \frac{1+1}{1+13} \cdot \frac{1+1}{2+13} \cdot \frac{0+1}{1+13}$$

DEALING WITH UNSEEN N-GRAMS: LIDSTONE SMOOTHING

- We want to allow unseen grams to have non zero probability. We will pretend we saw each word one more time than we did.
- A common technique is called “Lidstone Smoothing” .
  - Consist on using the following formulas:

$$p_{lindstone}(w_i | wj) = \frac{c(w_j w_i) + \epsilon}{c(w_j) + V \cdot \epsilon} \qquad p_{lindstone}(w_i) = \frac{c(w_i) + \epsilon}{c() + V \cdot \epsilon} = \frac{c(w_i) + \epsilon}{n_{tokens} + V \cdot \epsilon}$$

- Consider the corpus:
  <s> This is the malt </s>
  <s> That lay in the house that Jack built </s>
- With V = 13 = #{ This, is, the, malt, That, lay, in, house, that, Jack, build, start, stop}
- $p(< \text{s} > \text{this is the house} < /\text{s} >) = p(\text{this} | < \text{s} >)p(\text{is} | \text{this})p(\text{the} | \text{is})p(\text{house} | \text{the})p(< /\text{s} > | \text{house}) = \ldots$

DEALING WITH UNSEEN N-GRAMS: LIDSTONE SMOOTHING

- We want to allow unseen grams to have non zero probability. We will pretend we saw each word one more time than we did.
- A common technique is called "Lidstone Smoothing" or add-k smoothing
  - Consist on using the following formulas:

$$p_{lindstone}(w_i | wj) = \frac{c(w_j w_i) + k}{c(w_j) + V \cdot k} \qquad p_{lindstone}(w_i) = \frac{c(w_i) + k}{c() + V \cdot k} = \frac{c(w_i) + k}{n_{tokens} + V \cdot k}$$

- Consider the corpus:
  <s> This is the malt </s>
  <s> That lay in the house that Jack built </s>
- With V = 13 = #{ This, is, the, malt, That, lay, in, house, that, Jack, build, start, stop}
- $p(< \text{s} > \text{this is the house} < /\text{s} >) = p(\text{this} | < \text{s} >)p(\text{is} | \text{this})p(\text{the} | \text{is})p(\text{house} | \text{the})p(< /\text{s} > | \text{house}) = \ldots$

$$\frac{1+3}{2+13\cdot 3} \cdot \frac{1+3}{1+13\cdot 3} \cdot \frac{1+3}{1+13\cdot 3} \cdot \frac{1+3}{2+13\cdot 3} \cdot \frac{0+3}{1+13\cdot 3} \text{ if we use } k = 3$$

DEALING WITH UNSEEN N-GRAMS: LAPLACE AND LIDSTONE SMOOTHING

- Note that if we add a 1 to each of the counts of all the n-grams we would get:

$$p_{add_1}(w_n \mid w_{n-1}) = \frac{c(w_{n-1}w_n) + 1}{\sum_w (c(w_{n-1}w) + 1)} = \frac{c(w_{n-1}w_n) + 1}{\sum_w c(w_{n-1}w) + V} = \frac{c(w_{n-1}w_n) + 1}{c(w_{n-1}) + V}$$

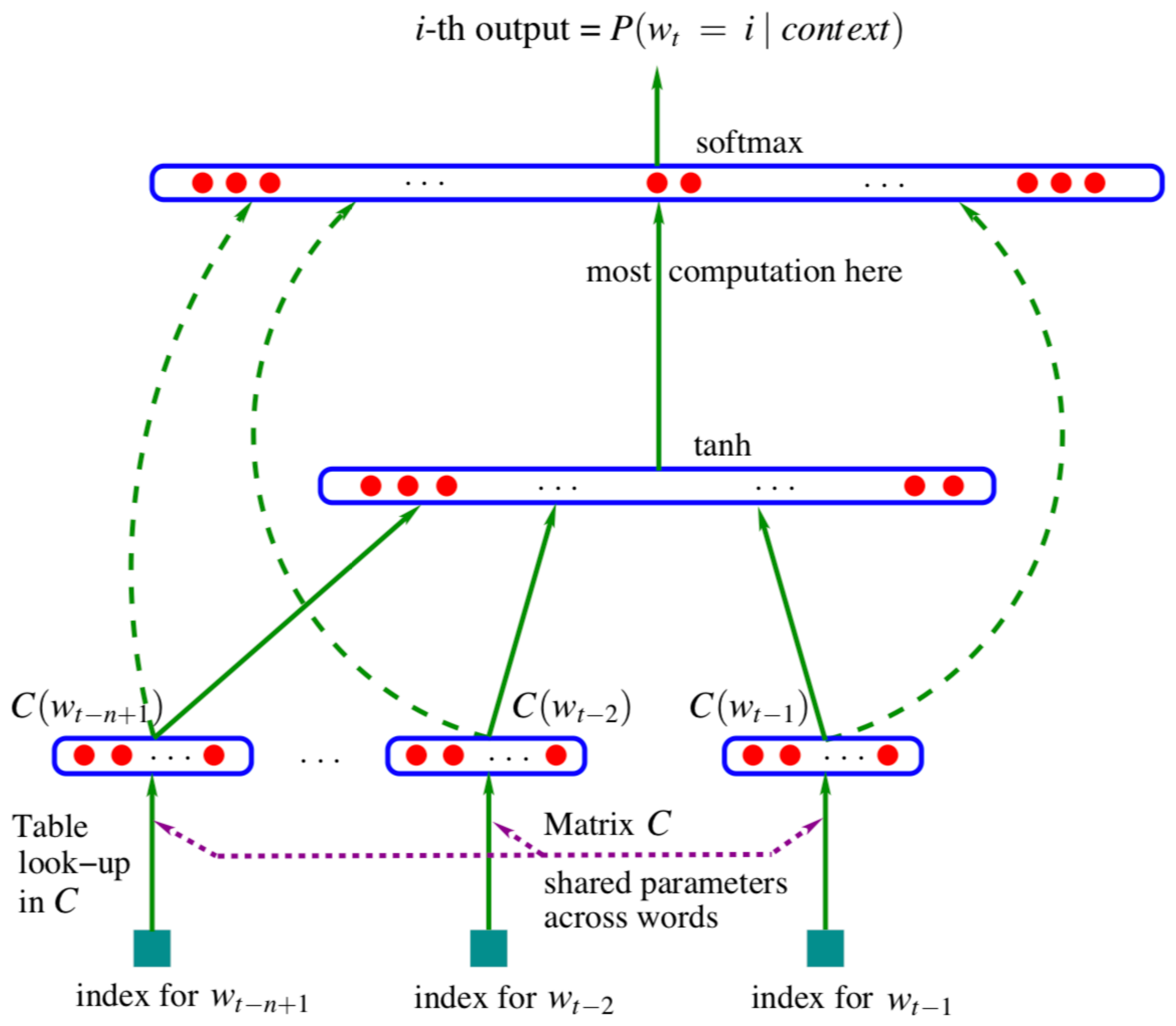
- In a similar way, if we add k to each of the n-gram counts we would get:

$$p_{add_k}(w_n \mid w_{n-1}) = \frac{c(w_{n-1}w_n) + k}{\sum_w (c(w_{n-1}w) + k)} = \frac{c(w_{n-1}w_n) + k}{\sum_w c(w_{n-1}w) + k \cdot V} = \frac{c(w_{n-1}w_n) + k}{c(w_{n-1}) + k \cdot V}$$

Add-k will make the probabilities smoother.

PROBLEMS WITH N-GRAM LANGUAGE MODELS

- They suffer from scalability issues:
    - Storing all counts for n-grams might require a lot of memory if N is big
    - 140 GB of ram for https://kheafield.com/papers/edinburgh/estimate_paper.pdf
- Neural Networks offer an alternative where RAM requirements scale with the number of words.



EXERCISE

- Given the corpus
  <s> I am Sam </s>
  <s> Sam I am </s>
  <s> I do not like green eggs and ham </s>

- Compute:
  P(I | <s>) =
  P(Sam | <s>) =
  P(am | I) =
  P(</s> | Sam) =
  P(Sam | am) =
  P(do| I) =

EXAMPLE

- Given the corpus
  <s> I am Sam </s>
  <s> Sam I am </s>
  <s> I do not like green eggs and ham </s>

- Compute:

- P(I | <s>) = 2/3
  P(Sam | <s>) = 1/3
  P(am | I) = 2/3
  P(</s> | Sam) = 1/2
  P(Sam | am) = 1/2
  P(do| I) = 1/3

NUMERICALL STABILITY WITH N-GRAM MODELS

- When computing the probability for a long sentence you end up multiplying a lot of probabilities over the n-grams. Each product of two small numbers becomes an even smaller number.

$$p(<\text{s}> \text{this is the house} </\text{s}>) = p(\text{this}\,|\,\text{start})p(\text{is}\,|\,\text{this})p(\text{the}\,|\,\text{is})p(\text{house}\,|\,\text{the})p(\text{end}\,|\,\text{house})$$

- This can yield to underflow:
  - 0.0001 * 0.0003 * 0.0003 * 0.0003 * 0.0005 = 1.349 e-18

COMPUTING IN LOG DOMAIN

- Computing in log domain allows us to deal with products of probabilities in a smart approach that can avoid numerical problems.
- If a number a is positive then $a = \exp(\log(a)) = \log(\exp(a))$
- Therefore we can compute:

$$p_1 \cdot p_2 \cdot p_3 = \exp(\log(p_1 \cdot p_2 \cdot p_3)) = \exp\left(\log(p_1) + \log(p_2) + \log(p_3)\right)$$

- Yeah but … if the previous equality is an equality why do we care??
  - Because summing $\log(p_i)$ will not result to numerical underflow but multiplying the p_i might.

EVALUATING LANGUAGE MODELS

- One of the most common metrics to evaluate a language model is perplexity.
- The perplexity of a language model on a test set is defined as the inverse probability of the test set, normalized by the number of words.
- Note that the higher the probability the lower the perplexity (thus we want low perplexity)
- For a test set $X_{te} = w_1 \ldots w_N$ the perplexity of a model h on X_{te} is defined as:

$$PP(h; X_{te}) = P_h(w_1 \ldots w_N)^{-\frac{1}{N}} = \sqrt[N]{\frac{1}{P_h(w_1 \ldots w_N)}}$$

- If we use the chain rule to compute $P_h(w_1 \ldots w_N)$ we have

$$PP(h; X_{te}) = \sqrt[N]{\frac{1}{P_h(w_1 \ldots w_N)}} = \sqrt[N]{\frac{1}{\prod_{n=1}^{N} P_h(w_i \mid w_1 \ldots w_{i-1})}} = \sqrt[N]{\prod_{n=1}^{N} \frac{1}{P_h(w_i \mid w_1 \ldots w_{i-1})}}$$

EVALUATING LANGUAGE MODELS

- Note that in the definition of perplexity we used $X_{te} = w_1 \ldots w_N$
- Here we mean the concatenation of all the lines in the test set. Since the test set will contain many sentence boundaries we need to include them in the probability computation.
- Given the test set
  <s> I am Sam </s>
  <s> Sam I am </s>
  <s> I do not like green eggs and ham </s>
- We would create "a single line test set":
- <s> I am Sam </s> <s> Sam I am </s><s> I do not like green eggs and ham </s>