Master in Computer Vision Barcelona

UAB UOC UPC upf.

Module: M2. Optimization and inference techniques for Computer Vision **Final exam**
Date: December 5th, 2019
Teachers: Juan F. Garamendi, Coloma Ballester, Karim Lekadir, Oriol Ramos, Joan Serrat **Time: 2h30min**

- Books, lecture notes, calculators, phones, etc. are not allowed.
- All sheets of paper should have your name.
- Answer each problem in a separate sheet of paper.
- All results should be demonstrated or justified.

Problem 1 *Juan F. Garamendi, 2 Points*

Consider the following minimization problem:
Given the gray scale image $f \in L^{\infty}(\Omega)$ such that $u : \Omega \to \mathbb{R}$, solve

$$u = \underset{u \in W^{1,2}(\Omega)}{\arg\min} \left\{ \frac{1}{2} \int_{\Omega} |\nabla u|^2 + \lambda |u - f|^2 dx dy \right\}$$

Where

- $\Omega \in \mathbb{R}^2$ is a bounded open domain of the 2 dimensional euclidean space $\mathbb{R}^2$.
- $W^{1,2}(\Omega) = \{u \in L^2(\Omega);\ \nabla u \in L^2(\Omega)^2\}$
- $\lambda > 0$ is a given (i.e. known) parameter. $\lambda \in \mathbb{R}$
- $\mathbf{x} \in \Omega$ such that $\mathbf{x} = (x_1, x_2)$ is the spatial variable and ∇ is the gradient operator such that $\nabla u(\mathbf{x}) = (u_{x_1}, .., u_{x_d})$

(a) (0.25 points) Say in a few words which is the image processing solved.

(b) (0.25 points) Describe in a few (but concise) words the role of parameter λ. How λ affects to the solution if we decrease its value?

(c) Knowing that

$$\frac{d\,J}{d\,u} = \lambda(u - f) - \Delta u$$

where Δu denotes the laplacian of u

- (0.5 points) Write a gradient descent scheme at the function level. Use f as initial image to the scheme.

- (0.5 points) Discretize the previous gradient descent and write a explicit gradient descent at the pixel level using the following discretization for the second derivative

$$\frac{\partial^2 u}{\partial x^2} \approx u_{i+1,j} - 2u_{i,j} + u_{i-1,j}$$

$$\frac{\partial^2 u}{\partial y^2} \approx u_{ij+1} - 2u_{ij} + u_{ij-1}$$

Use homogeneous Von-Neumann boundary conditions.

- (0.5 points) What happens if instead of taking f as initial image to the gradient descent scheme we take a black image? How this affects to the resolution of the problem? How will be the final image u compared with the previous scheme? (remember, we model the black color as 0, so the initial image is an image with all pixels values equal to zero).

Problem 2 — *Juan F. Garamendi, 1 Point*

(a) (0.7 points) Compute the gradient at point $\bar{x} = (1, 1, 2)^T$ of the function $F(\bar{x}) = (2x_1 - x_2)^2 x_3$ using back-propagation. Write the flow graph and intermediate values for the forward passing as well as the back-propagation passing.

(b) (0.15 points) What happens if we try to compute the gradient at point $\bar{x} = (1, 2, 2)^T$ of the function $F(\bar{x}) = |2x_1 - x_2|x_3$ using back-propagation? Will be any difference if we compute the gradient by hand or using some numerical method? (just if it is needed, remember that for $y = |x|$ the derivative is $y' = \frac{x}{|x|}$)

(c) (0.15 points) Related with the question (b), What if for function from question (b) we try to compute the gradient at point from question (a)? Can we compute the derivative? (just say in a few words, you do not have to compute the gradient using back-propagation).

Problem 3 — Coloma Ballester *0.75 Points*

Consider the constrained minimization problem

$$\begin{array}{ll} \min\limits_{x_1,x_2} & -x_1 + 3x_2 \\ \text{subject to} & \frac{1}{2}x_1 - x_2 + 2 \geq 0 \\ & x_1 - 3 \leq 0, \\ & x_1 + x_2 + 3 \geq 0. \end{array}$$

(a) Sketch the set of constraints of the problem. Is it a convex set? (0.2 *points*)

(b) Write the KKT optimality conditions for the problem. (0.4 *points*)

(c) Check if any of the points $(x_1, x_2) = (0, -3)$ and $(x_1, x_2) = (3, -6)$ could be the solution of the problem using the KKT conditions. (0.15 *points*)

Problem 4

Coloma Ballester *1.25 Points*

Let us consider the vectors $\mathbf{b}, \mathbf{d} \in \mathbb{R}^n$, $\mathbf{c} \in \mathbb{R}^m$, the $m \times n$ real matrix A, the real constants $\lambda, \mu > 0$, and the following minimization problem:

$$\min_{\mathbf{x}} \|\mathbf{x} - \mathbf{b}\| + \lambda \|\mathbf{A}\ \mathbf{x} - \mathbf{c}\|^2 + \mu \langle \mathbf{x}, \mathbf{d} \rangle$$

for $\mathbf{x} \in \mathbb{R}^n$.

(a) Is this a convex function? Why? (0.2 *points*)

(b) Describe why we can not use a gradient descent method to solve it. (0.15 *points*)

(c) Write an equivalent min-max problem and the resulting iterations of a primal-dual algorithm to solve it. (0.5 *points*)

(d) How the dual function is defined? Outline the dual algorithm to solve this problem? (0.4 *points*)

Problem 5

Karim Lekadir *0.5 Points*

(a) Why is the Chan-Vese segmentation method also called “Active Contours Without Edges”?

(b) What is an appropriate choice for the initial level set function? Justify.

Problem 6 J.Serrat *0.5 Points*

We saw that binary image denoising could be modeled as a problem of maximum a posteriori inference over the graphical model of figure 1. The goal then was

$$\arg\max_{\mathbf{x}} p(\mathbf{x}|\mathbf{y}) = \arg\max_{\mathbf{x}} p(\mathbf{y}|\mathbf{x})\, p(\mathbf{x})$$

Which of the following statements are *false* ? (0.5 points if answer is exact)

(a) $p(\mathbf{y}|\mathbf{x})$ is the prior, $p(\mathbf{x})$ the likelihood

(b) $\mathbf{x}$ is the clean image (the pixels) and $\mathbf{y}$ the noisy one

(c) the order of the model is 2, also called pairwise

(d) the solution for the former equation is called the "*max-marginals*"

(e) $p(\mathbf{y})$ is the evidence

(f) in order to get a tractable expression for $p(\mathbf{x})$, we want the noise model to be independent (uncorrelated) and identically distributed

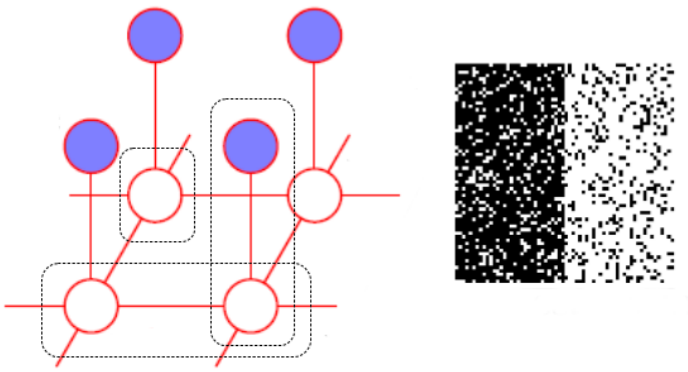


Figure 1: graphical model for binary image denoising

Problem 7 J.Serrat *0.5 Points*

The problem of dense disparity estimation from a pair of stero images I_L, I_R can be posed also as inference on a graphical model. We arrived at the following formulation:

$$\hat{D} = \arg\max_{D} p(D|I_L, I_R) = \arg\max_{D} p(I_L, I_R|D)\, p(D)$$

with for instance

$$\begin{aligned} p(I_L, I_R|D) &= \prod_i \exp[-(I_L(i) - I_R(i - D(i))] \\ p(D) &\propto \exp[-\sum_{i,j \in \mathrm{Ne}_i} |D(i) - D(j)|] \end{aligned}$$

Then, what is *false* again ? (one or more choices, 0.5 points if answer is exact)

(a) the likelihood expresses our preference for planar surfaces perpendicular to the viewing direction

(b) the prior says corresponding pixels have similar intensity

(c) $I_L(i) - I_R(i - D(i))$ will work worse than averaging differences over windows centered at i and $i - D(i)$ in I_L, I_R respectively

(d) it is convenient that prior and likelihood factorize and this actually happens

(e) $|D(i) - D(j)|$ may penalize too much the disparity differences at object borders

(f) one solution to the previous problem is to lower bound this value

Problem 8 J.Serrat *0.5 Points if (a) and (b) are correct*

Consider the graphical model of figure 2 where observations are binary images $16 \times 8 = 128$ pixels, that is, $x_i \in \{0,1\}^{128}$, $y_i \in Y = \{\mathrm{a}, \mathrm{b} \ldots \mathrm{z}\}$ (26 lowercase letters), $x = (x_1 \ldots x_9), y = (y_1 \ldots y_9)$.

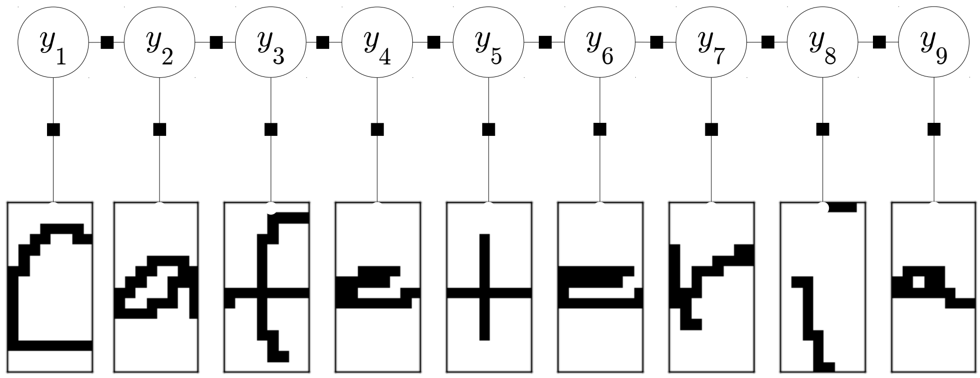


Figure 2

We want to learn w to later infer a word from a series of binary images of letters as

$$
\begin{aligned}
y^\star &= \underset{y \in \mathcal{Y}}{\arg\max} \ \langle\, w, \psi(x,y)\,\rangle \\
&= \underset{y \in Y^9}{\arg\max} \sum_{i=} \sum_{p=} \sum_{j=} \sum_{k=} w_{pjk}\, x_{ijk} \ + \ \sum_{l=} \sum_{p=} \sum_{q=} w_{pq}\, \mathbf{1}_{y_l = \mathrm{p},\, y_{l+1} = \mathrm{q}}
\end{aligned}
$$

where $\mathbf{1}_{y_i = \mathrm{p},\, y_{i+1} = \mathrm{q}}$ evaluates to 1 if $y_i = \mathrm{p}$ and $y_{i+1} = \mathrm{q}$. In this context,

(a) What are the initial and final values of each index in the summatories ?

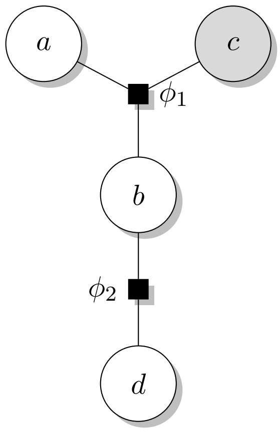
(b) What would be if we apply the technique of two-stage training ?

Answer right here :

	i	j	k	l	p	q
(a)						
(b)						

Problem 9 *Oriol Ramos Terrades, 1 Points*

Given the following factor graph:



$$\phi_1(a, b=0, c) = \begin{pmatrix} 10 & 5 \\ 20 & 3 \end{pmatrix}$$

$$\phi_1(a, b=1, c) = \begin{pmatrix} 4 & 7 \\ 2 & 5 \end{pmatrix}$$

$$\phi_2(b, d) = \begin{pmatrix} 3 & 8 \\ 4 & 9 \end{pmatrix}$$

with factors ϕ_1 and ϕ_2 defined above, compute:

a) The message sent by factor ϕ_2 to variable b: $m_{b\leftarrow 2}(b)$. Assume that d has not been observed [0.25 points].

b) The message sent by factor ϕ_1 to variable b: $m_{b\leftarrow 1}(b)$. Assume that c has been observed as $c = 1$ but a, not [0.5 points].

c) The belief of b: $b(b)$ [0.25 points].

Problem 10 *Oriol Ramos Terrades, 0.5 Points*

Say whether the next statements are true (**T**) or false (**F**) [Correct: +1/8, Incorrect: -1/8, unanswered: 0 points].

a) Belief Propagation (BP) infers exact marginals in undirected graphs.

b) An initial distribution is said to be *stationary* if, in the Markov chain specified by this initial distribution, the conditional distribution of x_{n+1} given x_n depend on n.

c) The Importance sampling method generates unbiased estimators.

d) Be $u \sim U(0,1)$, a random value uniformly drawn from the interval $[0,1]$, in the Metropolis-Hasting algorithm, we accept a new sample y if $u \leq \min\left\{1, \frac{h(y)q(y,x)}{h(x)q(x,y)}\right\}$.