

Module: M2. Optimization and inference techniques for CV Final exam

Date: December 3rd, 2013

Teachers: Nicolas Duchateau, Marcelo Bertalmío, Joan Serrat, Oriol Ramos Time: 2,5h

■ Books, lecture notes, calculators, phones, etc. are not allowed.

- All sheets of paper should have your name on them.
- Each answer must be a provided in a separate sheet of paper.
- All results must be demonstrated or justified.

BLOCK I

Problem 1: 0.75 Points

Iterative methods for solving systems like: A x = b.

- a) Explain what is the difference between the Jacobi iteration method and the Gauss-Seidel iteration method.
- b) Which one is expected to converge faster?

Problem 2: 1 Point

We perform linear regression on a set of N known pairs $\{(x_i, y_i)\}_{(i=1...N)}$. This means that the function to be fit to the data is of the form: $f(x) = a_0 + a_1 x$.

- a) Write down the expression of the energy $E(a_0, a_1)$ to be minimized (no smoothing term involved).
- b) Compute the 2 normal equations associated to this problem.
- c) What would change in the energy formulation in case of multivariate data $\{(x_i, y_i, z_i)\}_{(i=1...N)}$? (still linear regression)

Problem 3: 0.75 Points

Gradient descent.

- a) What is the main property the gradients $g^{(k)}$ and $g^{(k+1)}$ (gradients at iteration k and k+1) should verify when going for the optimal step in the gradient descent? Where does this come from?
- b) Without entering in the computational details, what is the main difference between the search direction $g^{(k+1)}$ for the gradient descent with optimal step and the search direction $d^{(k+1)}$ the conjugate gradient method?

BLOCK II

Problem 4: 1 Point

- a) What is the difference between these two denoising methods: Gaussian blurring, Total Variation minimization?
- b) Explain these two approaches to representing a curve: explicit, implicit. Which is the one originally used in early active contours segmentation algorithms? Which is the one used in current variational methods for segmentation? Why?
- c) What are the main limitations of the original "Snakes" active contours segmentation algorithm of Kass et al. (1988)?
- d) What are the main limitations of the geodesic active contours segmentation algorithm of Caselles et al. (1997)?

Problem 5: 1.5 Points

- a) How can we take into account texture information in a statistical, region-based segmentation method?
- b) What can we gain if we also take into account motion field (optical flow) information?
- c) What is the general structure of a variational optical flow method, assuming small displacements?
- d) What is the *optic flow constraint*?
- e) Explain what are the most common constancy assumptions in optic flow estimation.
- f) Explain what are the most common smoothness terms used in optic flow estimation.

BLOCK III

Problem 6: 1 Point

Recall the problem of noise filtering in a $N \times N$ binary image. How can it be posed as maximum a posteriori inference? Specifically:

- a) What were the possible labels for pixels and the expressions for the likelihood, prior (with) and posterior terms? Which was the meaning of the unary and pairwise potentials of the prior?
- b) After taking logarithms, what was the final expression to minimize?

Problem 7: 1 Point

With regard to inference with belief propagation,

- a) What does sum-product estimate? And max-sum?
- b) The complexity of belief propagation is $O(N K^c)$, what's N, K, c? Does it provide an exact or an approximate solution to what it intends to estimate?

Problem 8: 1 Point

As you know, belief propagation (BP) and graph-cuts (GC) are widely spread inference algorithms.

- a) What are the limitations of GC with respect BP? Can you run GC on any graphical model?
- b) What's the min-cut / max-flow algorithm? What's the relationship with GC?

BLOCK IV

Problem 9: 1 Point

Explain (briefly) the link between log-linear models and the Maximum Entropy optimization problem

Problem 10: 1 Point

We have a set of images, all of the same scene, but each focusing on a different part as explained in class and you can see in the figure (a) below. The goal is to generate a new image, like (b), taking into account the information provided by the initial set of images. Propose a GM that can be used for this task. In particular, the following is expected:

- a) (0.25 Points) Define the set of random variables and their domain (i.e. the values they can take)
- b) (0.25 Points) Define the model factors and draw the corresponding factor graph.
- c) (0.50 Points) For each kind of factor and variable, define and explain the respective feature functions.



(a) Set of images



(b) Expected result