Master in
Computer Vision
Barcelona

Module: M1 – Introduction to Human and CV

Morphological and nonlinear processing

Lecturer: Philippe Salembier, UPC

Morphological and nonlinear processing

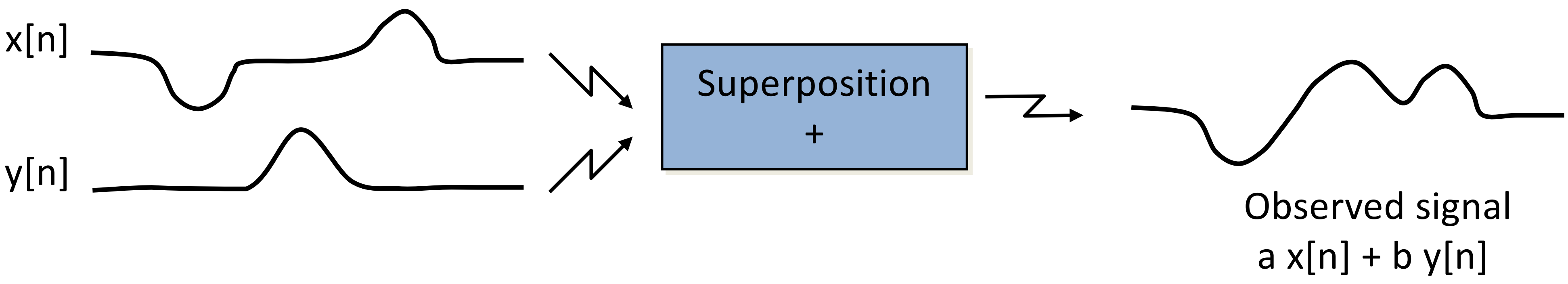
- **Mathematical Morphology and Lattice**

- Basic Operators: Dilation and Erosion
  - Definition and structuring element
  - 1D, 2D examples and Properties
  - Practical use

- Opening, Closing
  - Definition and properties
  - Geometric interpretation
  - Practical use: Object elimination, Top Hat

- Morphological filters

Linear superposition and convolution



Mathematical structure: Vector space

- E = {x[n], n∈]-∞,∞[}, Set of vectors
- K = scalars
- + : vector addition (sequences)
- . : Scalar product

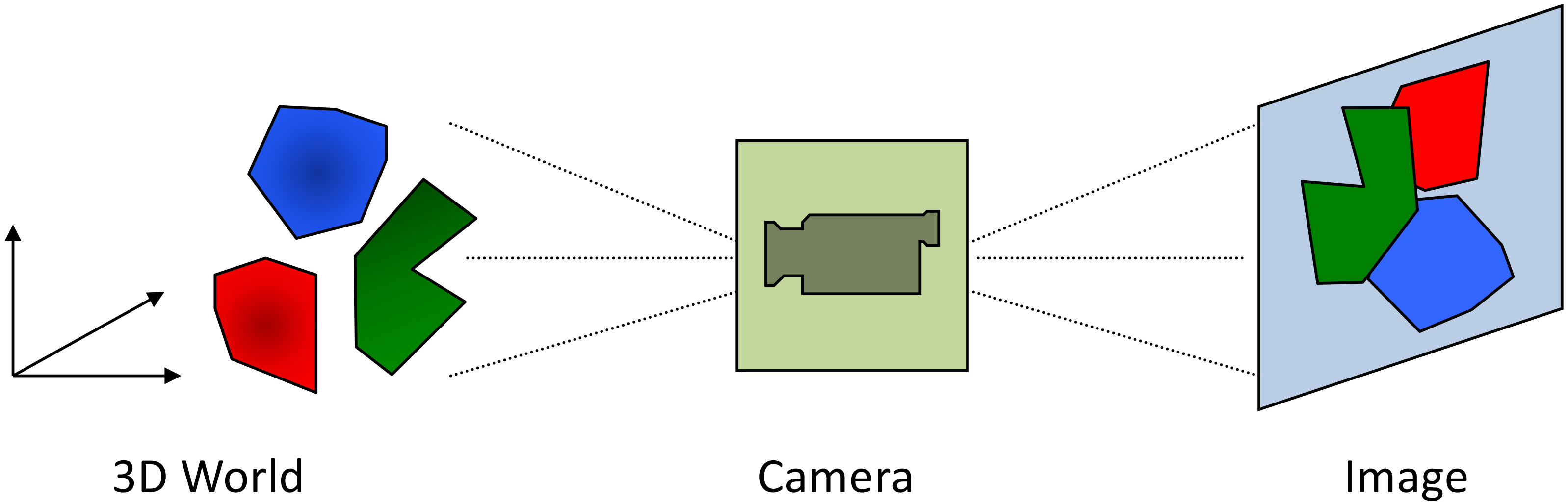
In this context, the natural operator is characterized by an "impulse response". The output can be computed through convolution.

Mathematical Morphology and Lattice

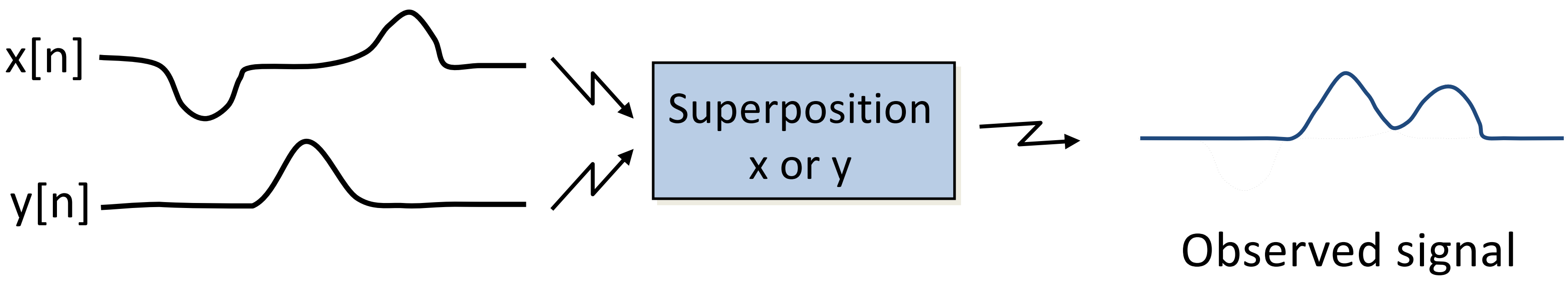
Is the linear superposition principle good for images?

¿+?

=



Mathematical Morphology and Lattice



Mathematical structure: Lattice

- Order
- Supremum and Infimum

Mathematical Morphology and Lattice

Mathematical structure: lattice

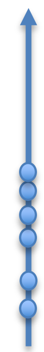
cat:	reticle
es:	retículo
fr:	treillis
de:	verband

- R : Set of elements (sequences)

- Partial order relation: ≤

  $\forall$ x, y, z $\in$ R
  - x ≤ x
  - x ≤ y, y ≤ x ⇒ x = y
  - x ≤ y, y ≤ z ⇒ x ≤ z

Ex: $\bigvee_n \frac{-1}{n}=0$



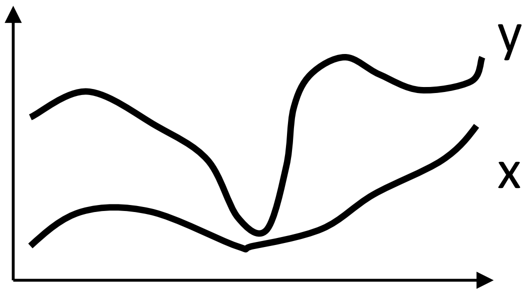
- Two dual operations:
  - "Supremum": $\vee$: Least upper bound (max)
  - "Infimum": $\wedge$: Greatest lower bound (min)

Mathematical Morphology and Lattice

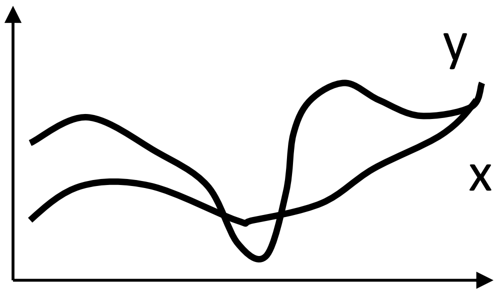
For image processing: Lattice of functions

- $R = \{x[n]\ ,\ n \in]-\infty,\infty[\}$ or $R = \{x[m,n]\}$
- Partial order: "≤"

$$x \leq y \quad \Rightarrow \quad x[n] \leq y[n],\ \forall\ n$$



$x \leq y$



x not bigger
nor smaller than y

Mathematical Morphology and Lattice

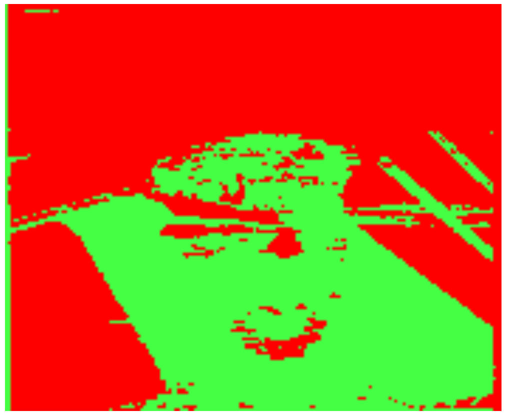
Image comparison

A

B

C

A ≤ B



A ? C

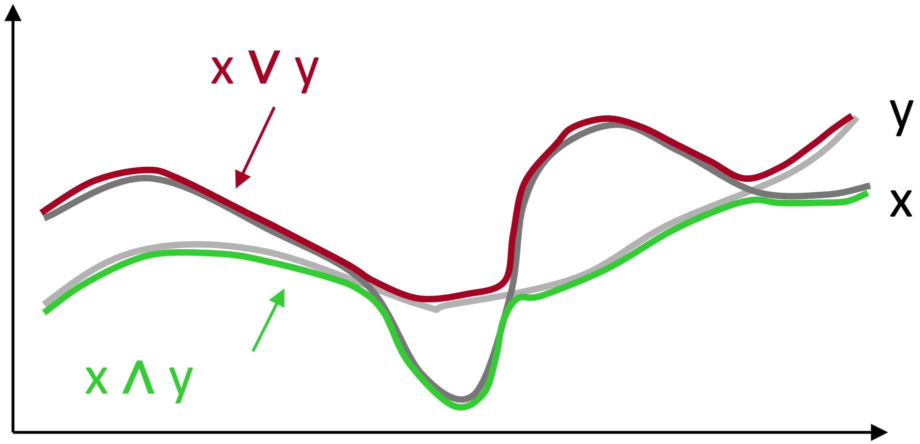
Mathematical Morphology and Lattice

Supremum and infimum in the function lattice

Operators: $\vee$ and $\wedge$

$z = x \vee y$ with $z[n] = \mathrm{Max}\{x[n], y[n]\}, \forall n$

$z = x \wedge y$ with $z[n] = \mathrm{Min}\,\{x[n], y[n]\}, \forall n$



Mathematical Morphology and Lattice

Supremum and infimum in the function lattice

Duality between ∧ and ∨ : f ↔ - f

$$f \leftrightarrow -f \qquad f \wedge g = -((-f) \vee (-g))$$

$$f \leftrightarrow 255 - f \qquad f \wedge g = 255 - ((255 - f) \vee (255 - g))$$

⇒ all morphological operators will appear in pairs

Mathematical Morphology and Lattice

Supremum and Infimum of images

X

Y

$X \vee Y$

$X \wedge Y$

Morphological and nonlinear processing

- Mathematical Morphology and Lattice

- **Basic Operators: Dilation and Erosion**
  - Definition and structuring element
  - 1D, 2D examples and Properties
  - Practical use

- Opening, Closing
  - Definition and properties
  - Geometric interpretation
  - Practical use: Object elimination, Top Hat
- Morphological filters

Dilation and erosions

Dilation

$$\delta_b\{x[n]\} = \bigvee_{k=-\infty}^{\infty} (x[k] + b[n-k])$$

$$= x[n] \oplus b[n]$$

Similar to (nonlinear) convolution

Erosion

$$\varepsilon_b\{x[n]\} = \bigwedge_{k=-\infty}^{\infty} (x[k] - b[k-n])$$

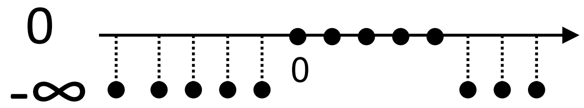
$$= x[n] \Theta b[n]$$

Similar to (nonlinear) correlation

$$\text{Convolution: } y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

Dilation and erosions

Flat structuring element: **b[n] ∈ {0,-∞}**

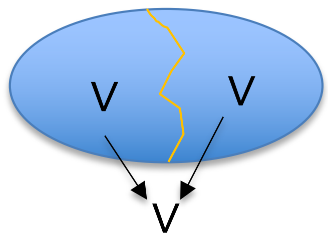


Dilation $$y[n] = \bigvee_{k=-\infty}^{\infty} \left\{ x[k] + b[n-k] \right\}$$

Change of variable $k \rightarrow n - k$

$$= \bigvee_{k=-\infty}^{\infty} \left\{ x[n-k] + b[k] \right\} \quad \text{but } b[k] \in \{0, -\infty\}$$

$$= \vee \left\{ \bigvee_{k/b[k]=0} \left\{ x[n-k] + 0 \right\}, \bigvee_{k/b[k]=-\infty} \left\{ x[n-k] - \infty \right\} \right\}$$



$$= \vee \left\{ \bigvee_{k/b[k]=0} \left\{ x[n-k] \right\}, -\infty \right\}$$

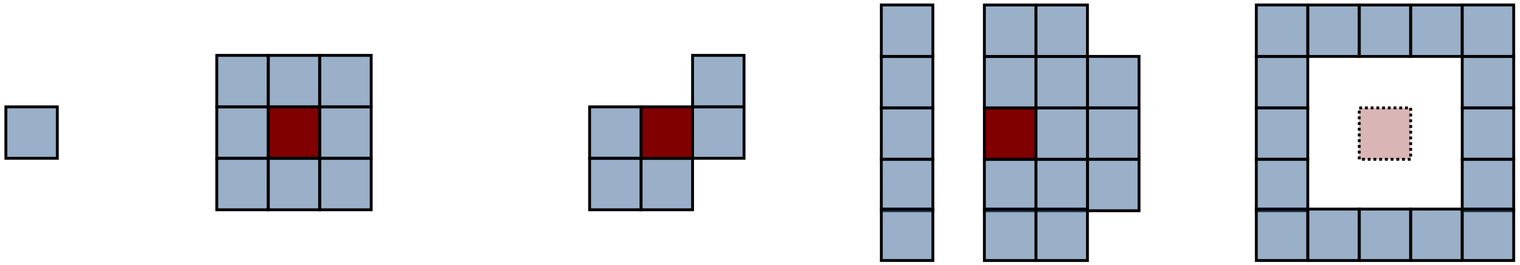
$$= \bigvee_{k/b[k]=0} \left\{ x[n-k] \right\}$$

The locations where b[n]=0 define a window and the dilation consists in computing the maximum of the gray level values below the window

Dilation and erosions

Examples of flat structuring elements:

Pixel =

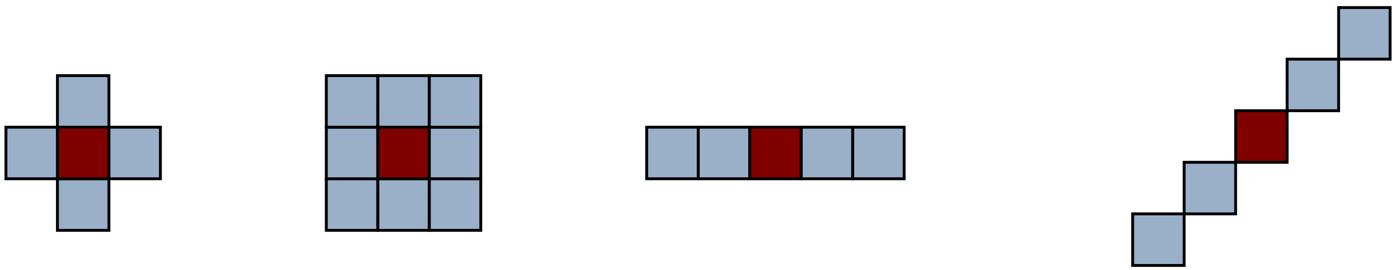


Often, symmetrical structuring elements are used

B is symmetrical if B(m, n) = B(-m, -n)

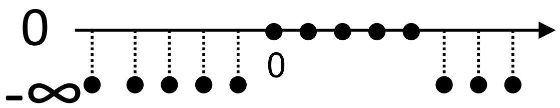
-> No difference between B and its transpose

Examples of symmetrical structuring element:

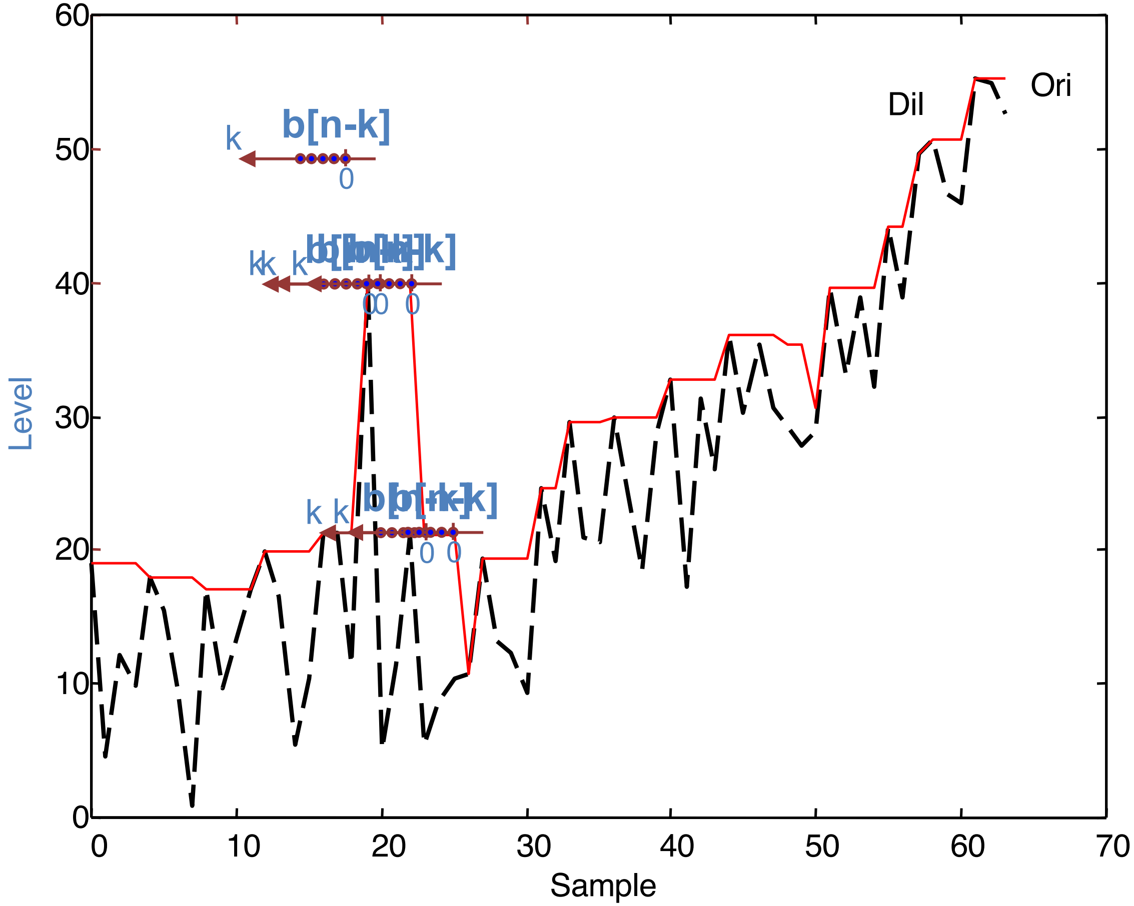


Dilation and erosions

1D signal Dilation



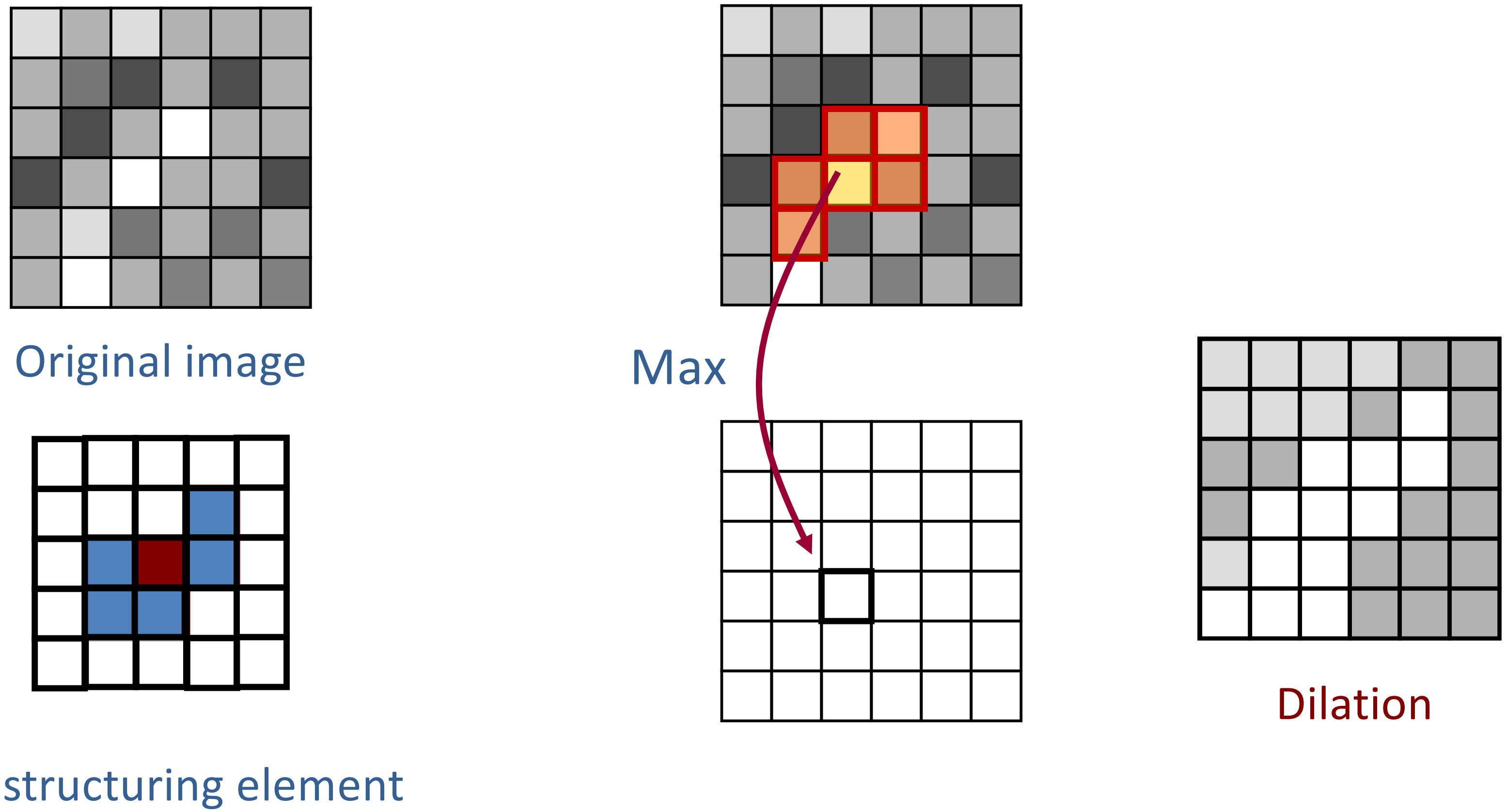
Flat structuring element of length 5



For each location, compute the maximum of the gray levels below the time-reverted structuring element

Dilation and erosions

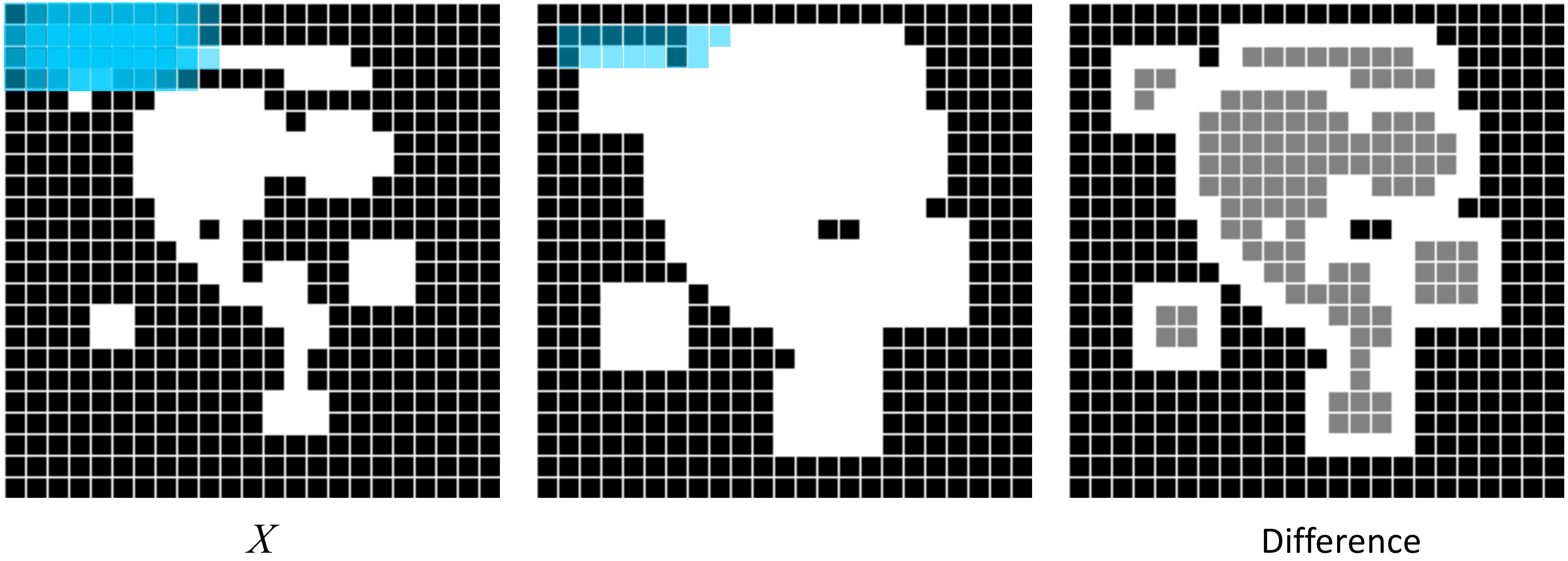
2D signal Dilation (I)

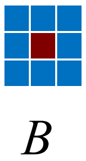


For each location, compute the maximum of the gray levels below the transposed structuring element

Dilation and erosions

2D signal Dilation (II)

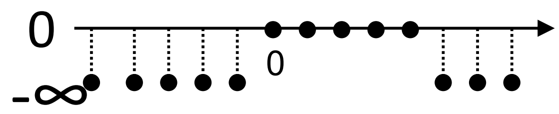




Square structuring element

Dilation and erosions

Flat structuring element: b[n] ∈ {0,-∞}



$$\text{Erosion} \quad y[n] = \bigwedge_{k=-\infty}^{\infty} \left\{ x[k] - b[k-n] \right\}$$

$$= \bigwedge_{k=-\infty}^{\infty} \left\{ x[k+n] - b[k] \right\} \quad \text{but } b[k] \in \{0, -\infty\}$$

$$= \wedge \left\{ \bigwedge_{k/b[k]=0} \left\{ x[k+n] + 0 \right\}, \bigwedge_{k/b[k]=-\infty} \left\{ x[k+n] + \infty \right\} \right\}$$

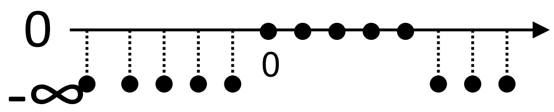
$$= \wedge \left\{ \bigwedge_{k/b[k]=0} \left\{ x[k+n] \right\}, \infty \right\}$$

$$= \bigwedge_{k/b[k]=0} \left\{ x[k+n] \right\}$$

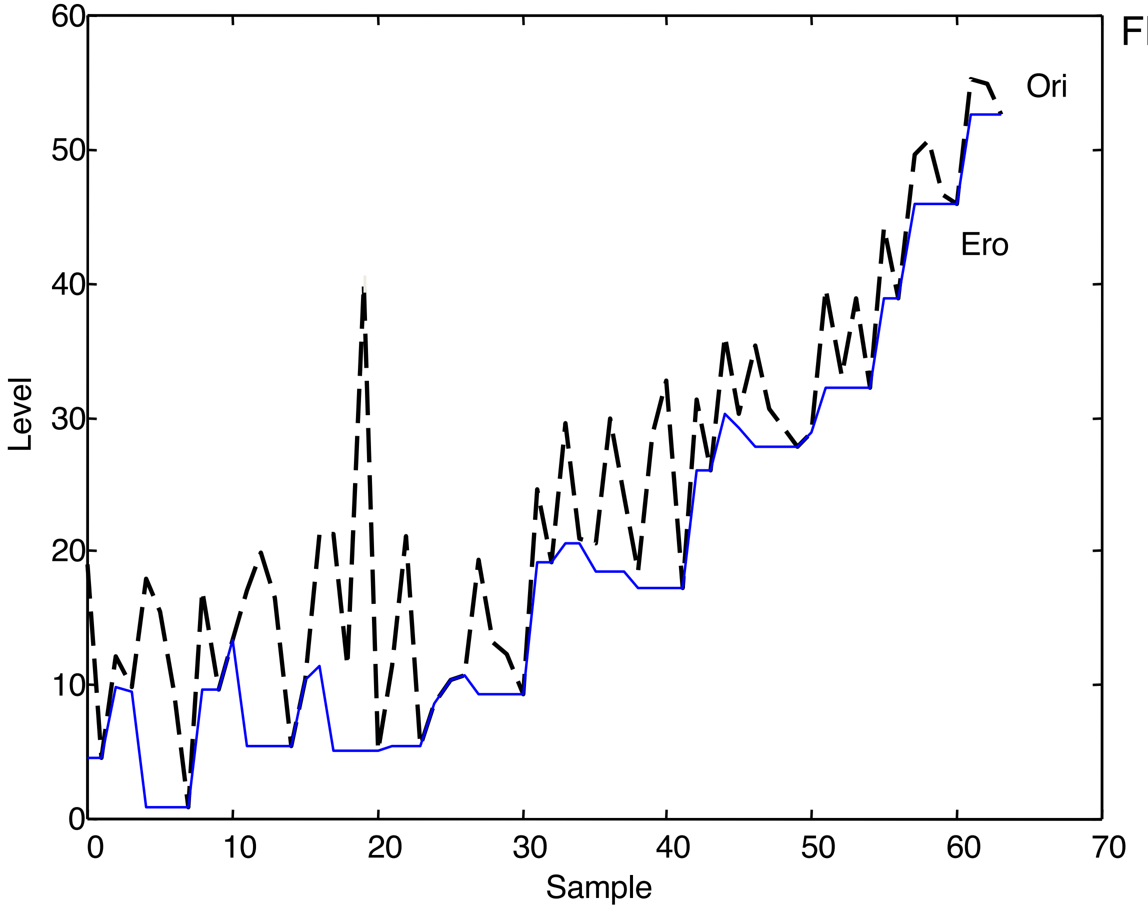
The locations where b[n]=0 define a window and the erosion consists in computing the minimum of gray level values below the window (in the erosion, no need to use the transposed Structuring Element)

Dilation and erosions

1D signal Erosion



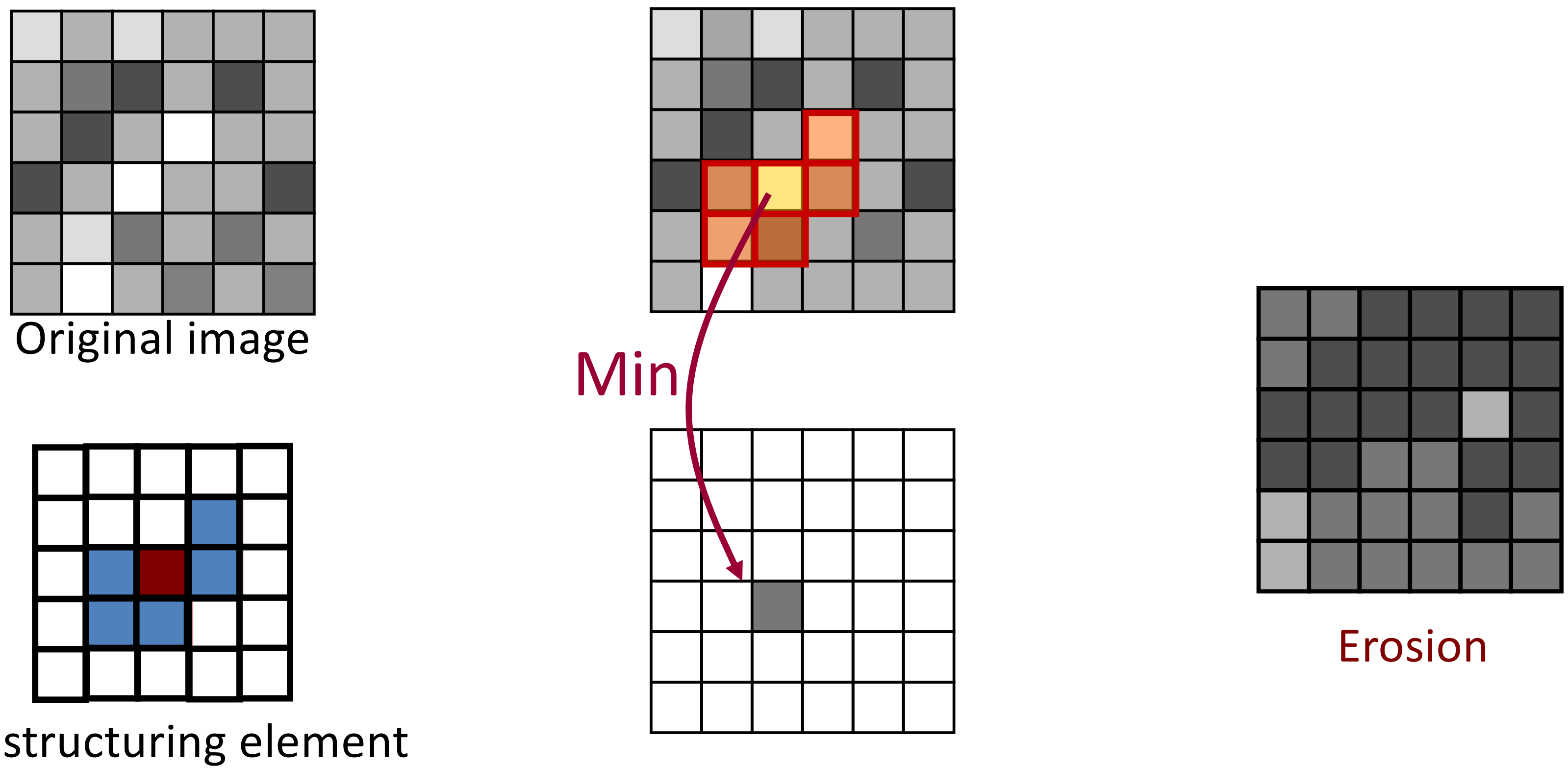
Flat structuring element of length 5



For each location, compute the minimum of the gray levels below the structuring element

Dilation and erosions

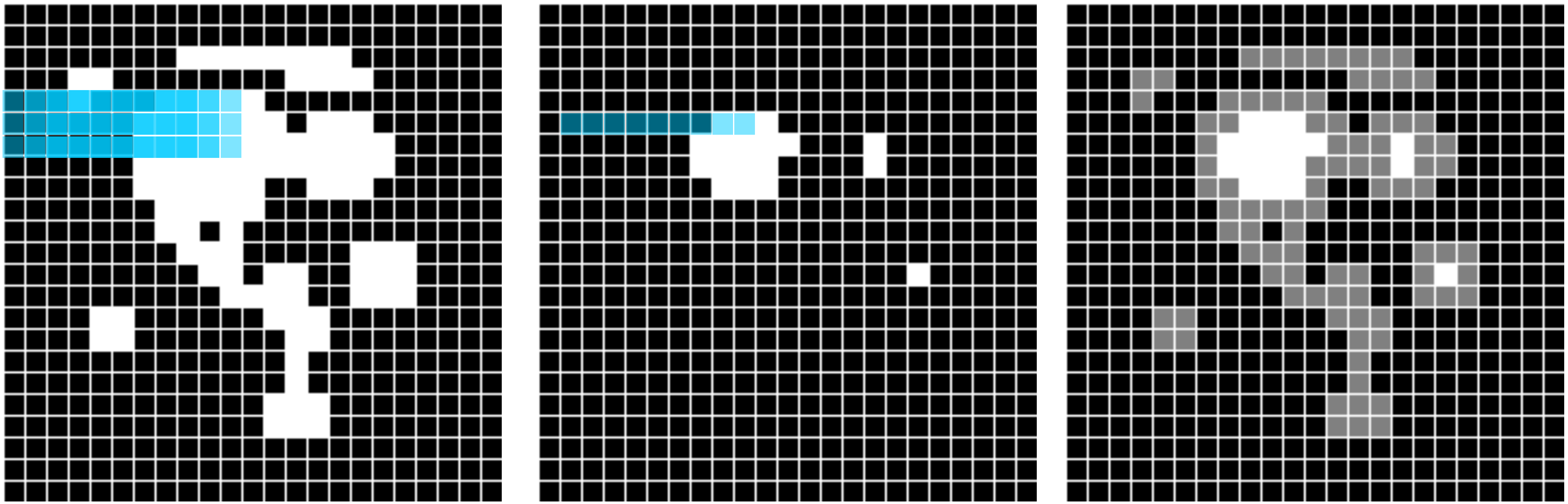
2D signal Erosion (I)



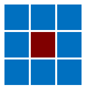
For each location, compute the minimum of the gray levels below the structuring element

Dilation and erosions

2D signal Erosion (II)



Difference



Square structuring element

B

Dilation and erosions

Example of dilation / erosion on binary images

Original

Dilation

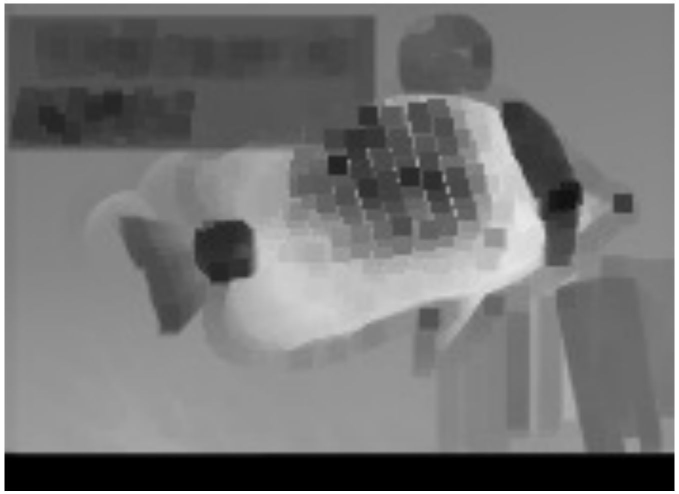
Erosion

Dilation and erosions

Example of dilation / erosion on gray level images

Original

Dilation



Erosion

Dilation and erosions

Properties (I)

- Increasing:

  $x \leq y \Rightarrow x \oplus b \leq y \oplus b$

  $x \leq y \Rightarrow x \ominus b \leq y \ominus b$

  Preserve the lattice structure

- Distributive:

  $(x \vee y) \oplus b = (x \oplus b) \vee (y \oplus b)$

  $(x \wedge y) \ominus b = (x \ominus b) \wedge (y \ominus b)$

- Composition:

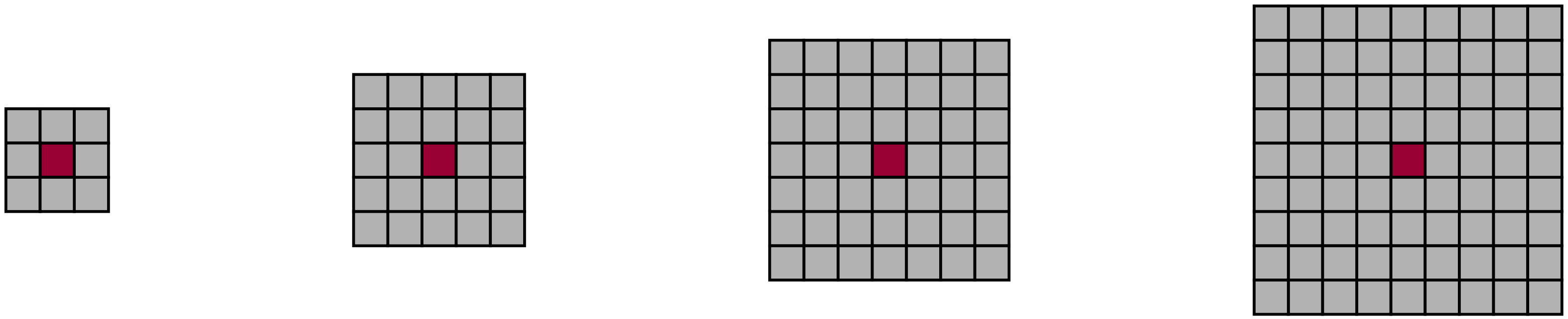
  $x \oplus b1 \oplus b2 = x \oplus b, \quad b = b1 \oplus b2$

  $x \ominus b1 \ominus b2 = x \ominus b, \quad b = b1 \oplus b2$ ($\neq$ associativity)

Dilation and erosions

Properties (II)

Composition: Iteration of erosion or dilation is equivalent to use larger structuring elements



b1 $\quad$ $b2=b1\oplus b1$ $\quad$ $b3=b2\oplus b1$ $\quad$ $b4=b3\oplus b1$

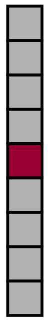
$$x \oplus b1 \oplus b1 \oplus b1 \oplus b1 = x \oplus b4$$

Dilation and erosions

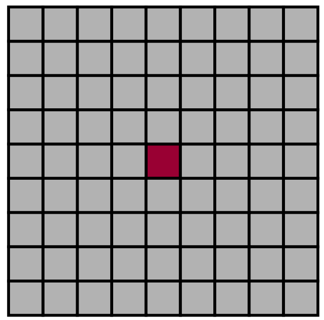
Properties (II)

Composition: Iteration of erosion or dilation is equivalent to use larger structuring elements

b1



b2



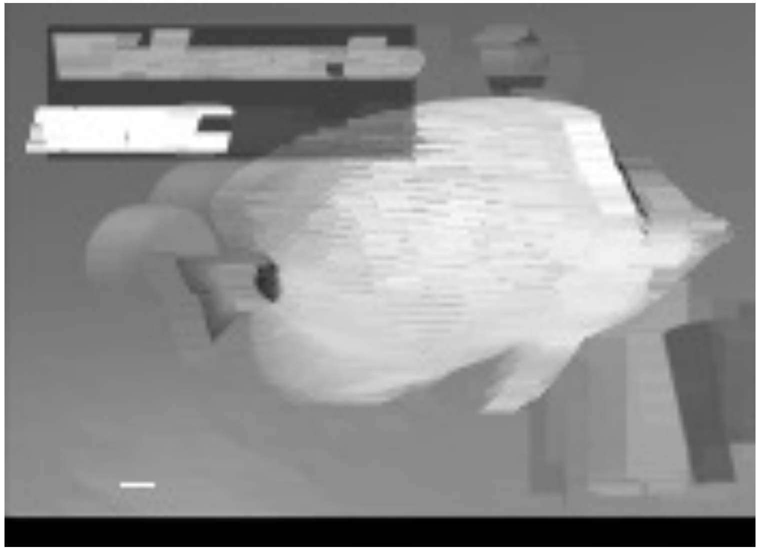
$b3=b2\oplus b1$

Number of comparison: NxN Square: N^2 comparisons

Concatention of two segments: 2N comparisons

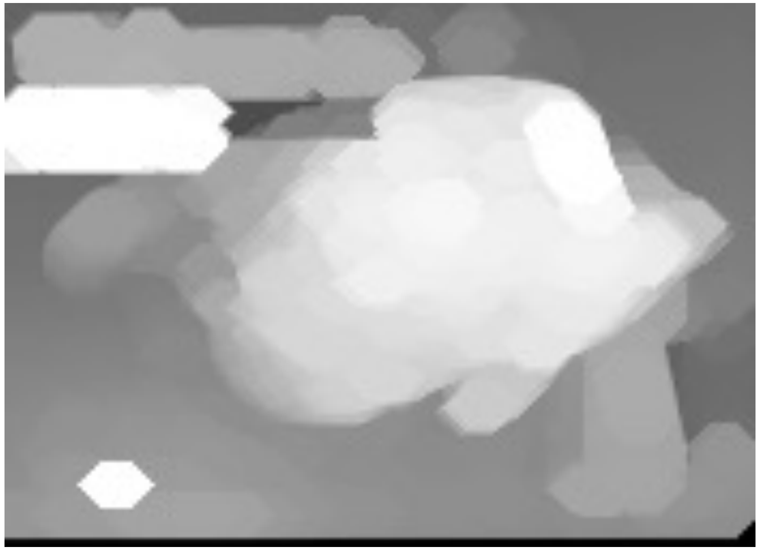
Dilation and erosions

x



$x \oplus b1$

$x \oplus b1 \oplus b2$



$x \oplus b1 \oplus b2 \oplus b3$

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Dilation and erosions

Properties (III)

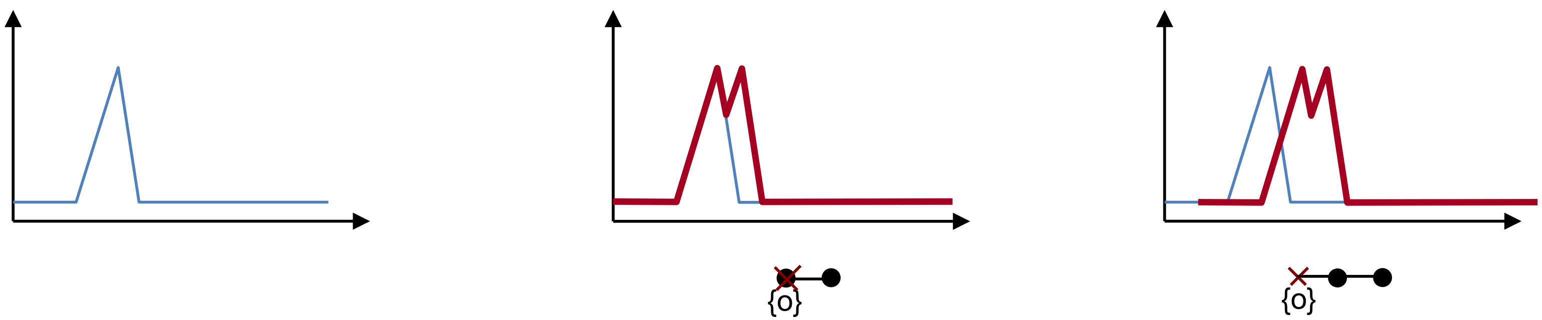
- Extensive / anti-extensive:

  Definition:

  Operator : extensive $x \leq \Psi(x)$ for all x

  anti-extensive $\Psi(x) \leq x$ for all x

> If the space origin belongs to the structuring element then the dilation is extensive, erosion anti-extensive



Dilation and erosions

Practical use: Image simplification

Allow part of the information to be removed:

- dilation: remove dark components (local minima)
- erosion: remove bright components (local maxima)

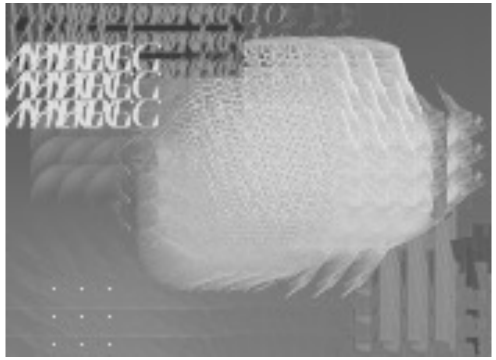
But

- Shift contours
- The simplification effect depends on shape of the structuring element

Dilation and erosions

Original

SE: square



SE: 9 points

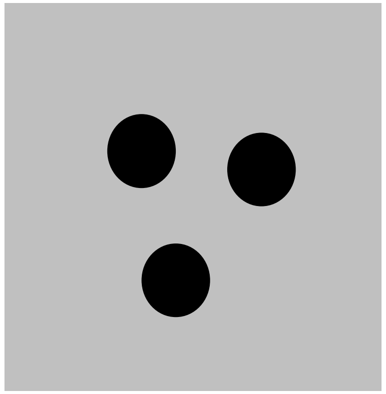
(dilations)

Dilation and erosions

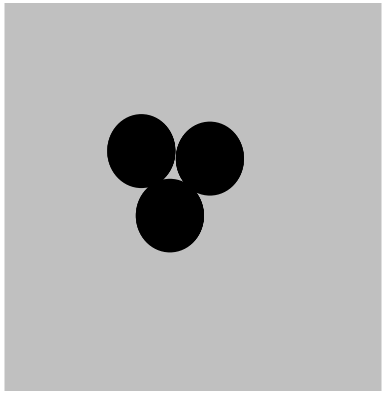
Object separation

How many objects?

3 separated objects

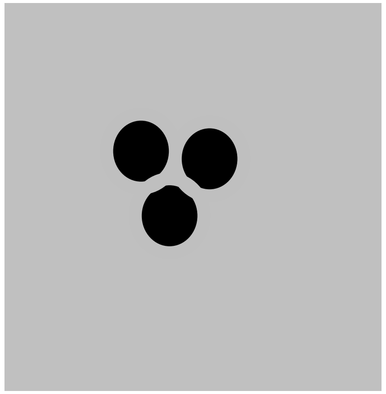


3 overlapping objects



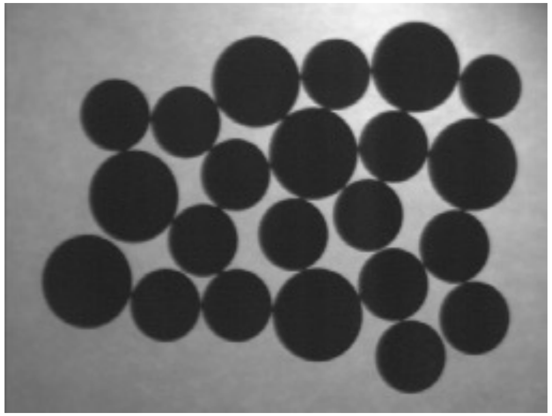
Object separation

with dilation

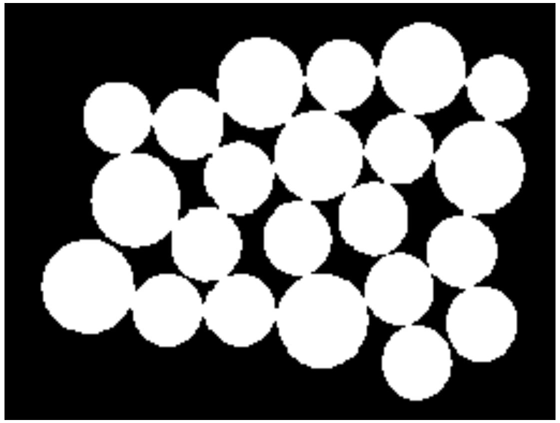


Dilation and erosions

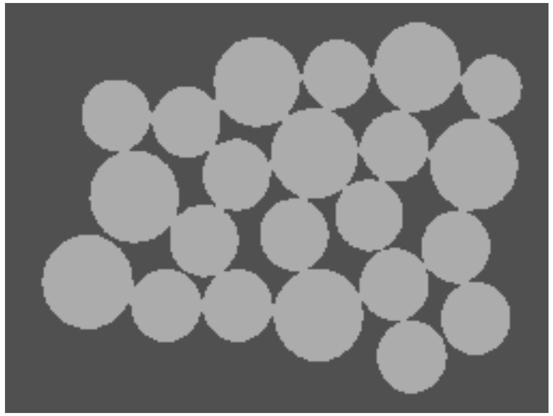
Counting coins



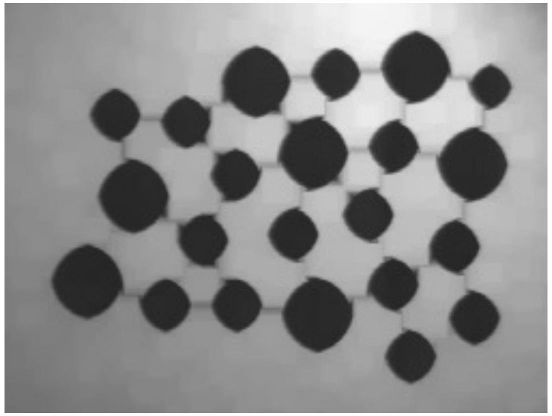
Original: X



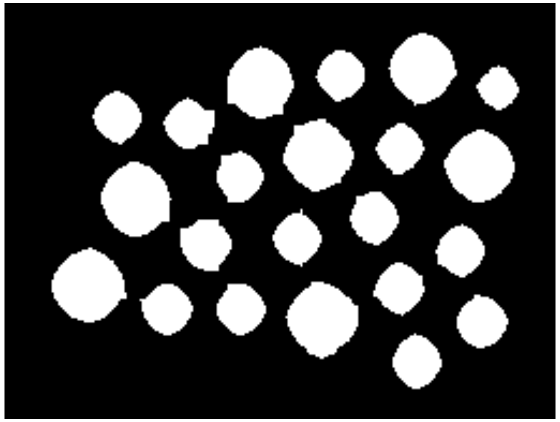
Binarization



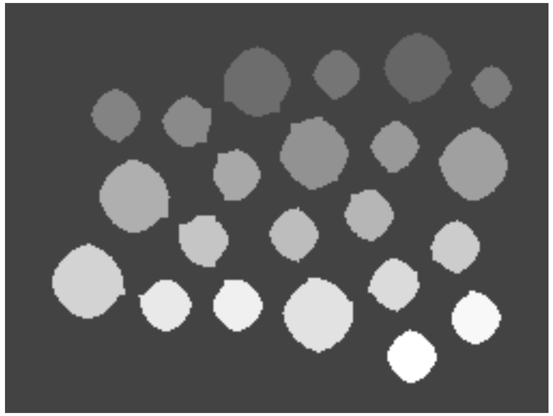
1 connected component



Dilation: $X \oplus B$



Binarization of $X \oplus B$



22 connected components

Dilation and erosions

Morphological gradients (I)

Goal: highlight transitions (contours)

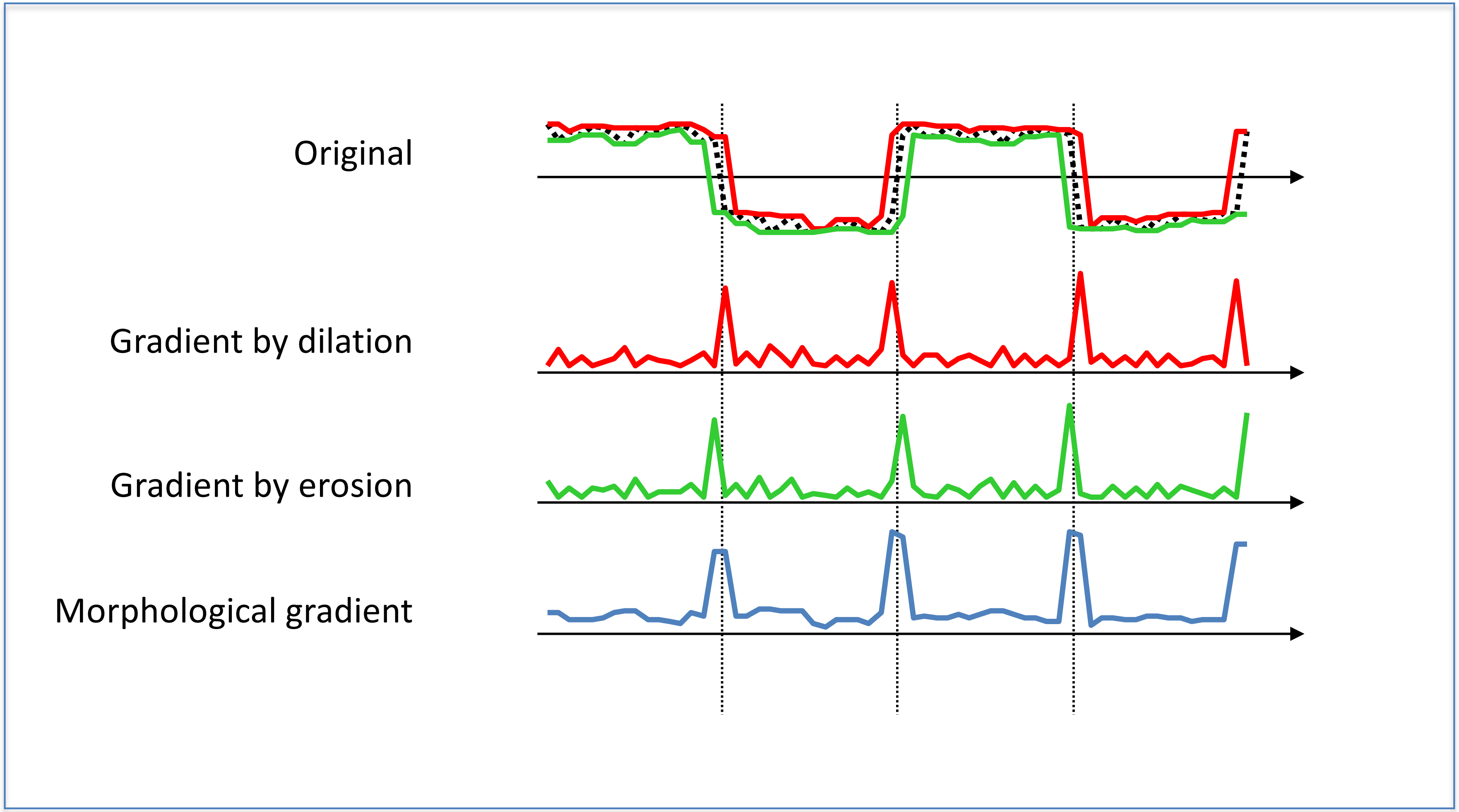
Definition:

gradient	by dilation:	$(x \oplus b) - x$
	by erosion:	$x - (x \ominus b)$
	morphological:	$(x \oplus b) - (x \ominus b)$

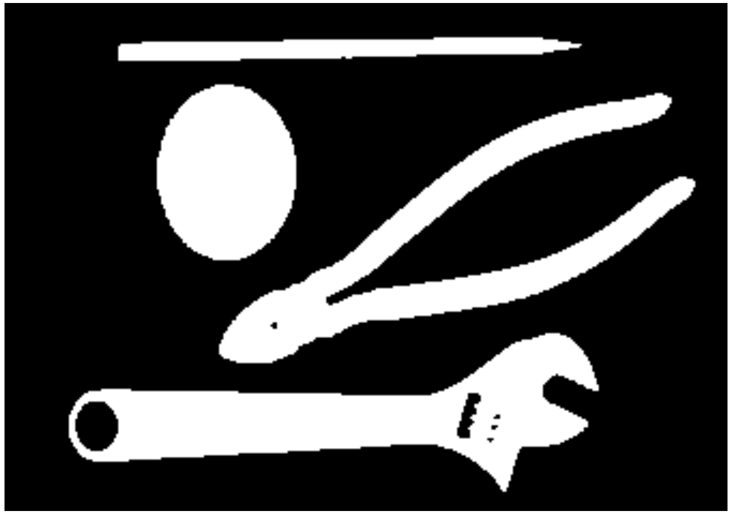
Smallest possible structuring element:

Dilation and erosions

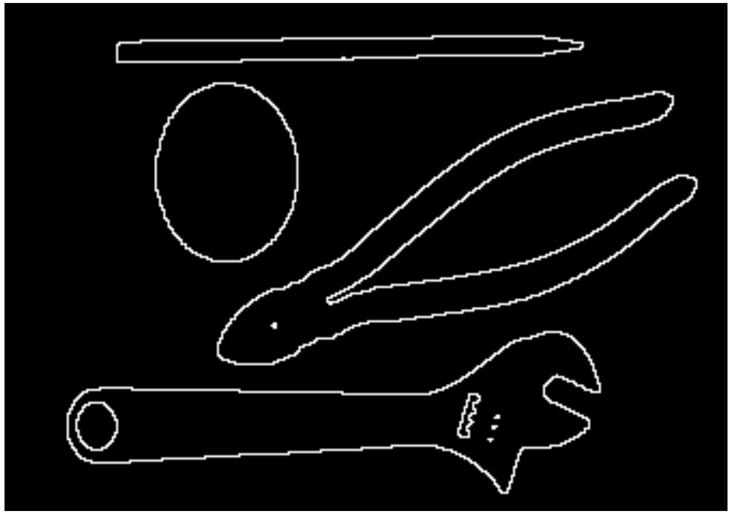
Morphological gradients (II)



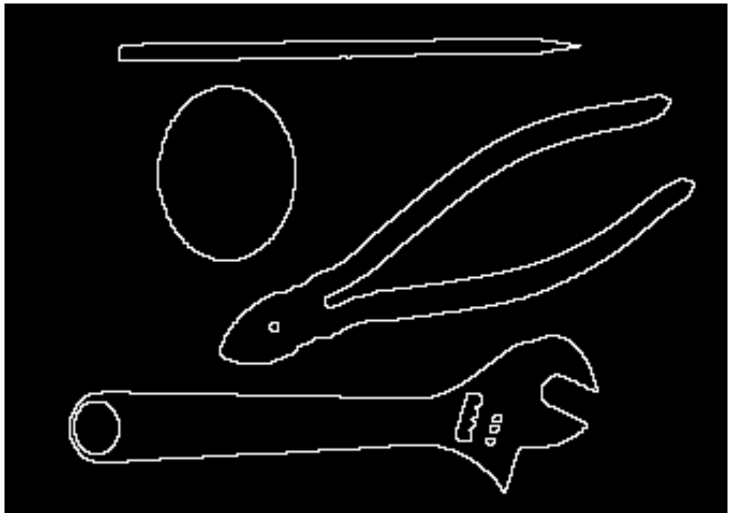
Morphological gradients (III)



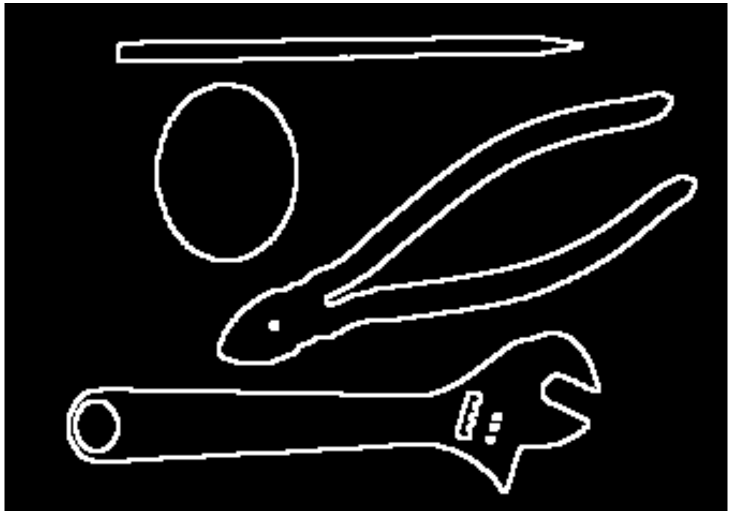
X



$(X \oplus B) - X$



$X - (X \ominus B)$



$(X \oplus B) - (X \ominus B)$

Dilation and erosions

Contour extraction with morphological gradient

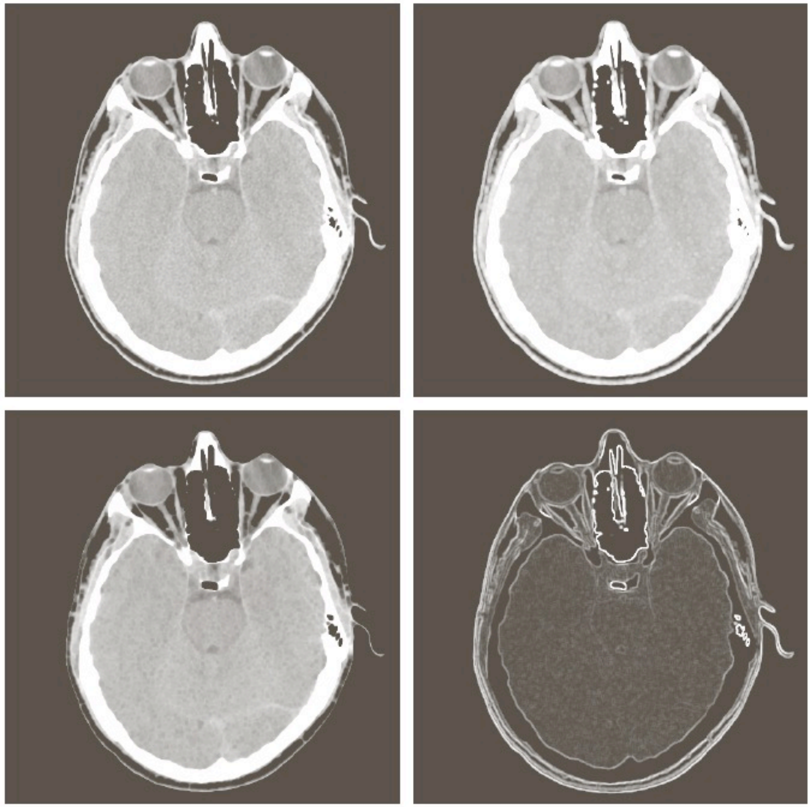


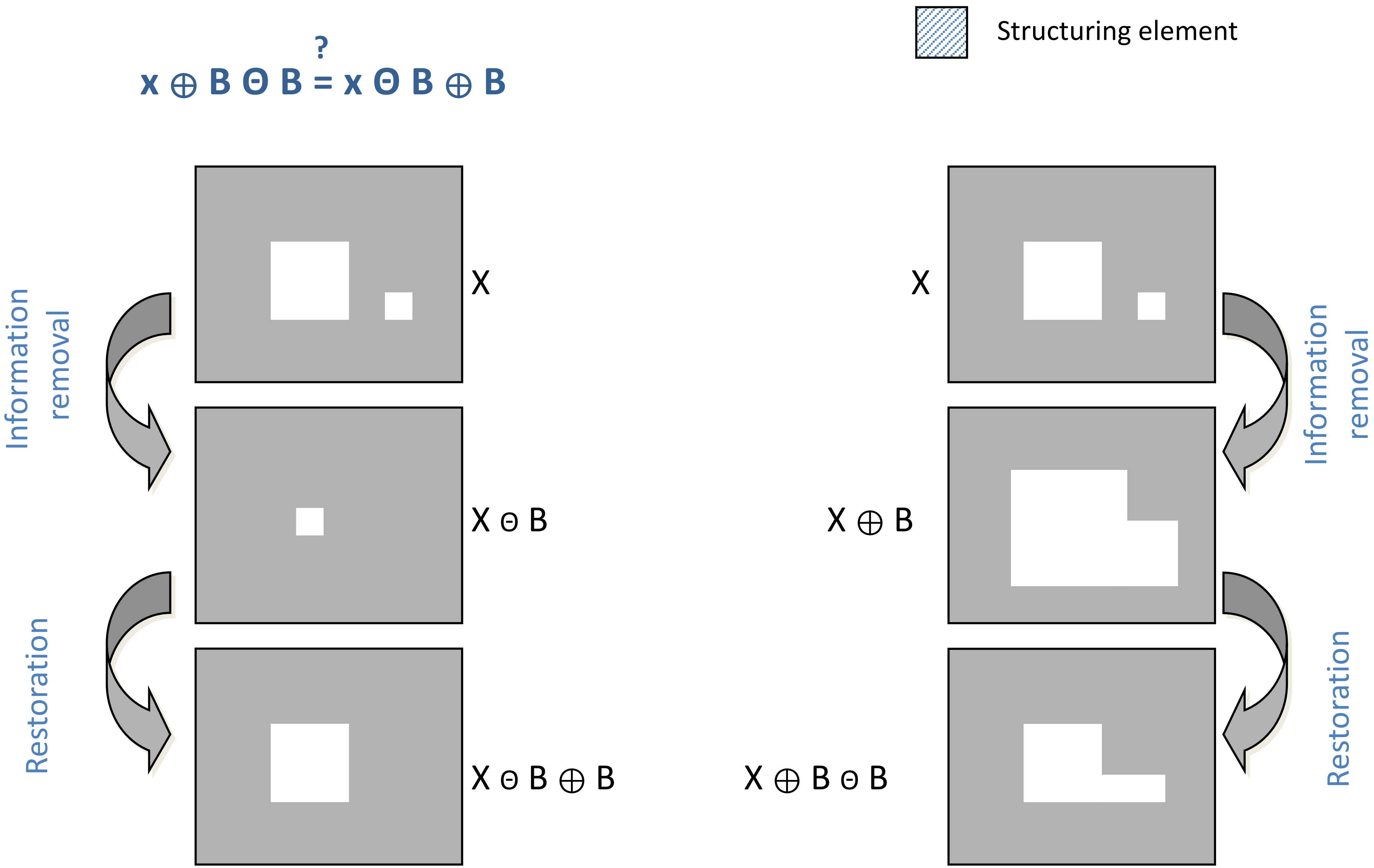
Illustration from Gonzalez and Woods: Digital image processing

Morphological and nonlinear processing

- Mathematical Morphology and Lattice
- Basic Operators: Dilation and Erosion
  - Definition and structuring element
  - 1D, 2D examples and Properties
  - Practical use
- **Opening, Closing**
  - Definition and properties
  - Geometric interpretation
  - Practical use: Object elimination, Top Hat
- Morphological filters

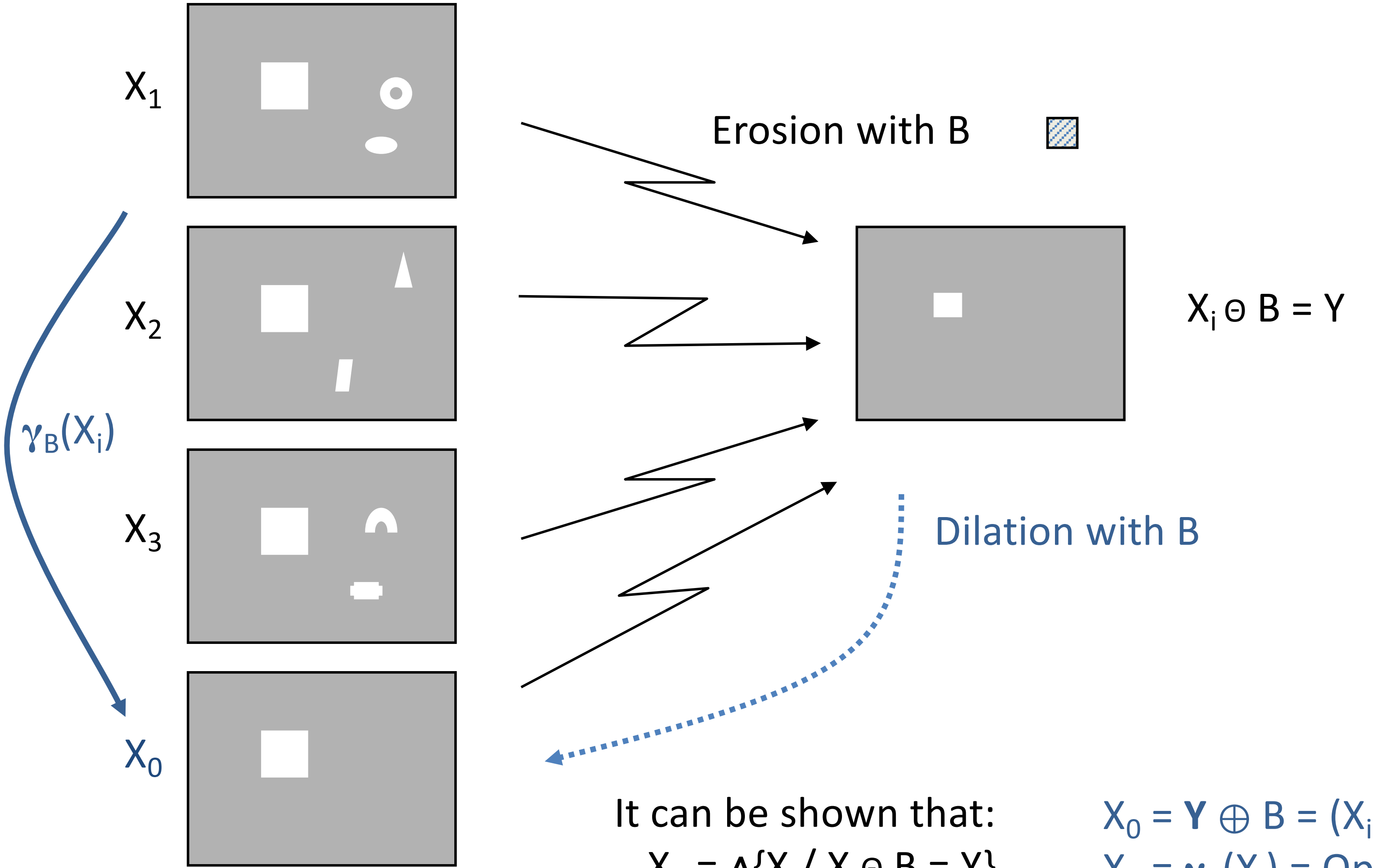
Opening and closing

$$x \oplus B \ominus B \overset{?}{=} x \ominus B \oplus B$$



Opening and closing

"Inverse operator" issue



It can be shown that:

$X_0 = \wedge\{X \,/\, X \ominus B = Y\}$

$X_0 = \mathbf{Y} \oplus B = (X_i \ominus B) \oplus B$

$X_0 = \gamma_B(X_i) = \text{Opening}$

Opening and closing

Morphological Opening: $y = (x \ominus b) \oplus b = \gamma_b(x)$
Morphological Closing: $y = (x \oplus b) \ominus b = \phi_b(x)$

Properties:

Increasing: if $x \leq y$ => $\gamma_b(x) \leq \gamma_b(y)$
=> $\phi_b(x) \leq \phi_b(y)$
(composition of increasing operators)

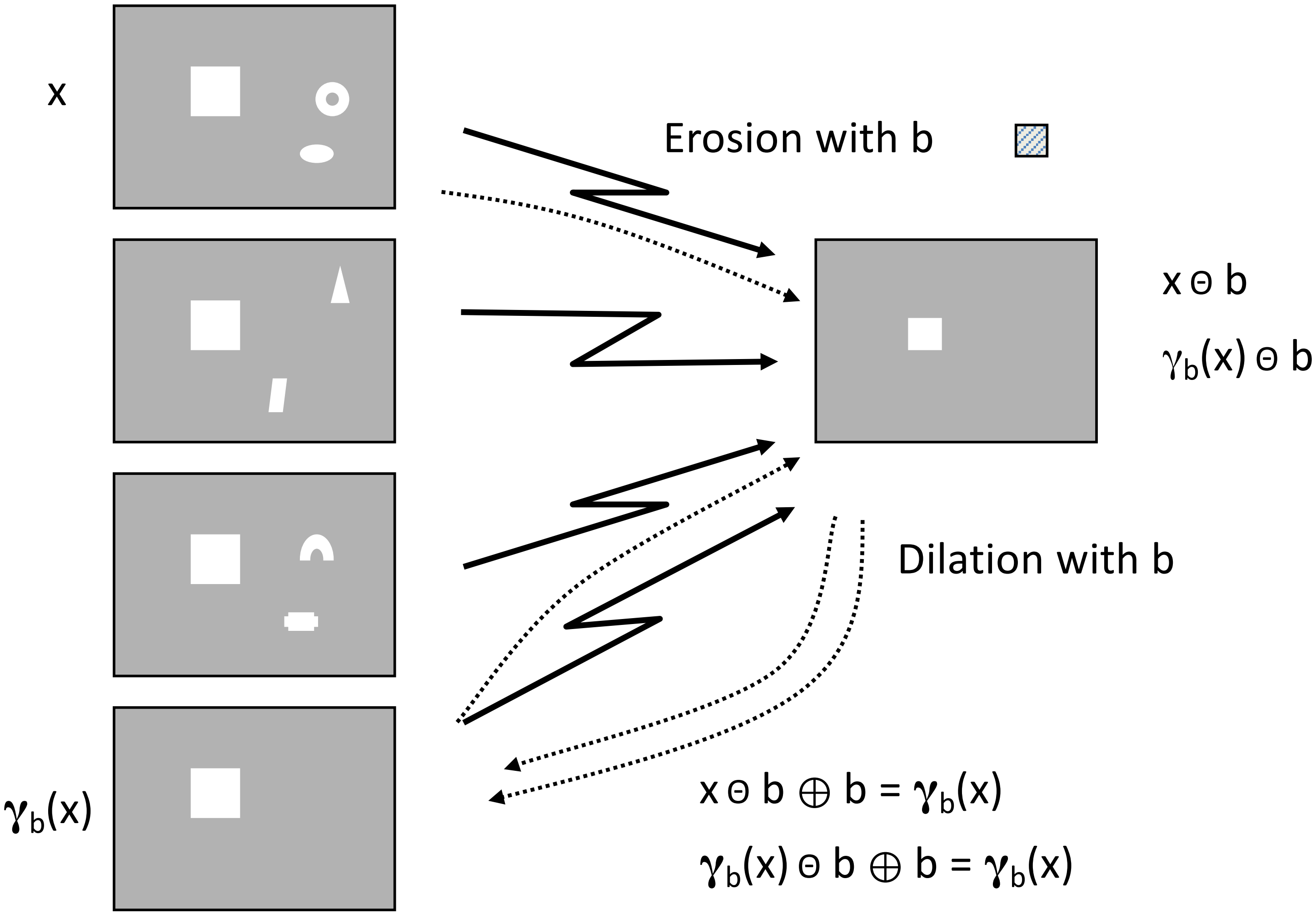
$x \leq y$
$\epsilon(x) \leq \epsilon(y)$
$\delta(\epsilon(x)) \leq \delta(\epsilon(y))$

Extensive:
Opening: **anti-**extensive $\gamma_b(x) \leq x$
Closing: extensive $x \leq \phi_b(x)$

Idempotent: $\psi(\psi(x)) =? \psi(x)$

Opening and closing

Idempotence of opening and closing (I)



$x \ominus b \oplus b = \gamma_b(x)$

$\gamma_b(x) \ominus b \oplus b = \gamma_b(x)$

$\gamma_b(\gamma_b(x)) = \gamma_b(x)$ **(idempotent)**

Opening and closing

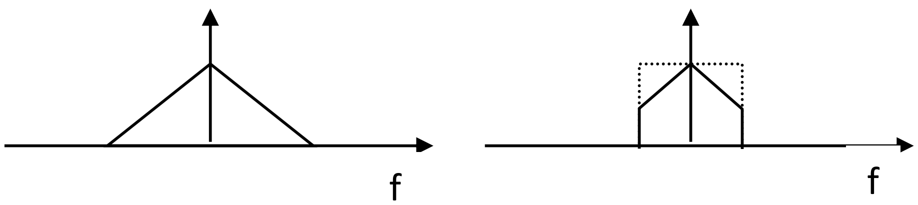
Idempotence of openning and closing (II)

Opening and closing => idempotent

"$\gamma\gamma = \gamma$" and "$\phi\phi = \phi$"

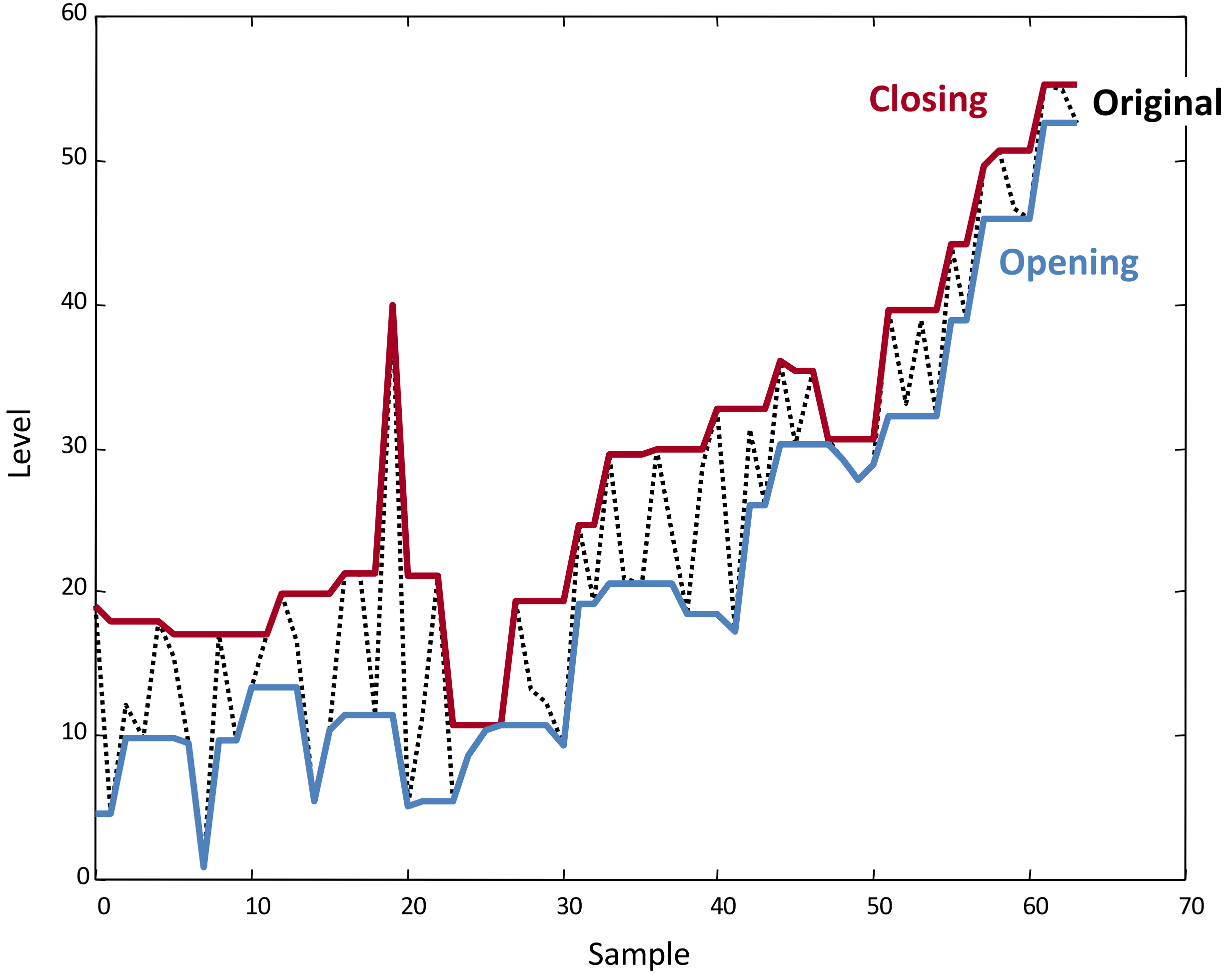
- It is not useful to iterate the filter

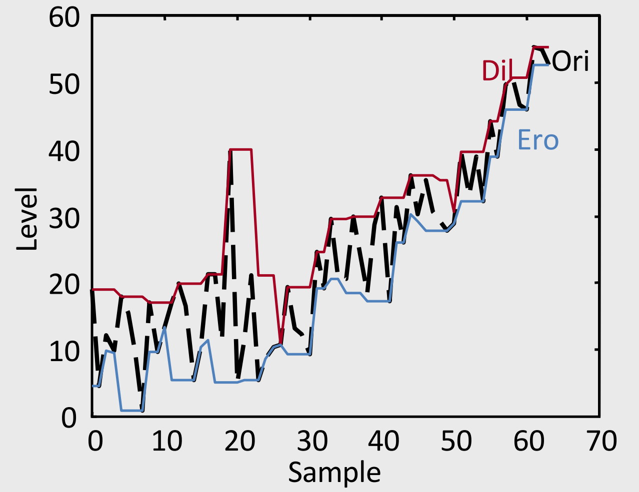
Idea ≈ ideal lineal filters



Opening and closing

1D example





Erosion / dilation

Upper and lower envelop:
- Extensivity

Pic (Max/Min) removal:
- Duality

Opening and closing

Example (II)

Original

Erosion

Dilation

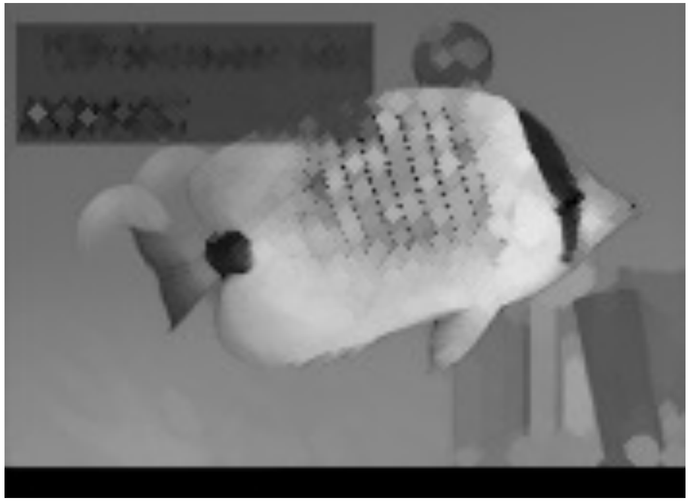
Opening

Closing

Opening and closing

Example (III)

Original



Opening

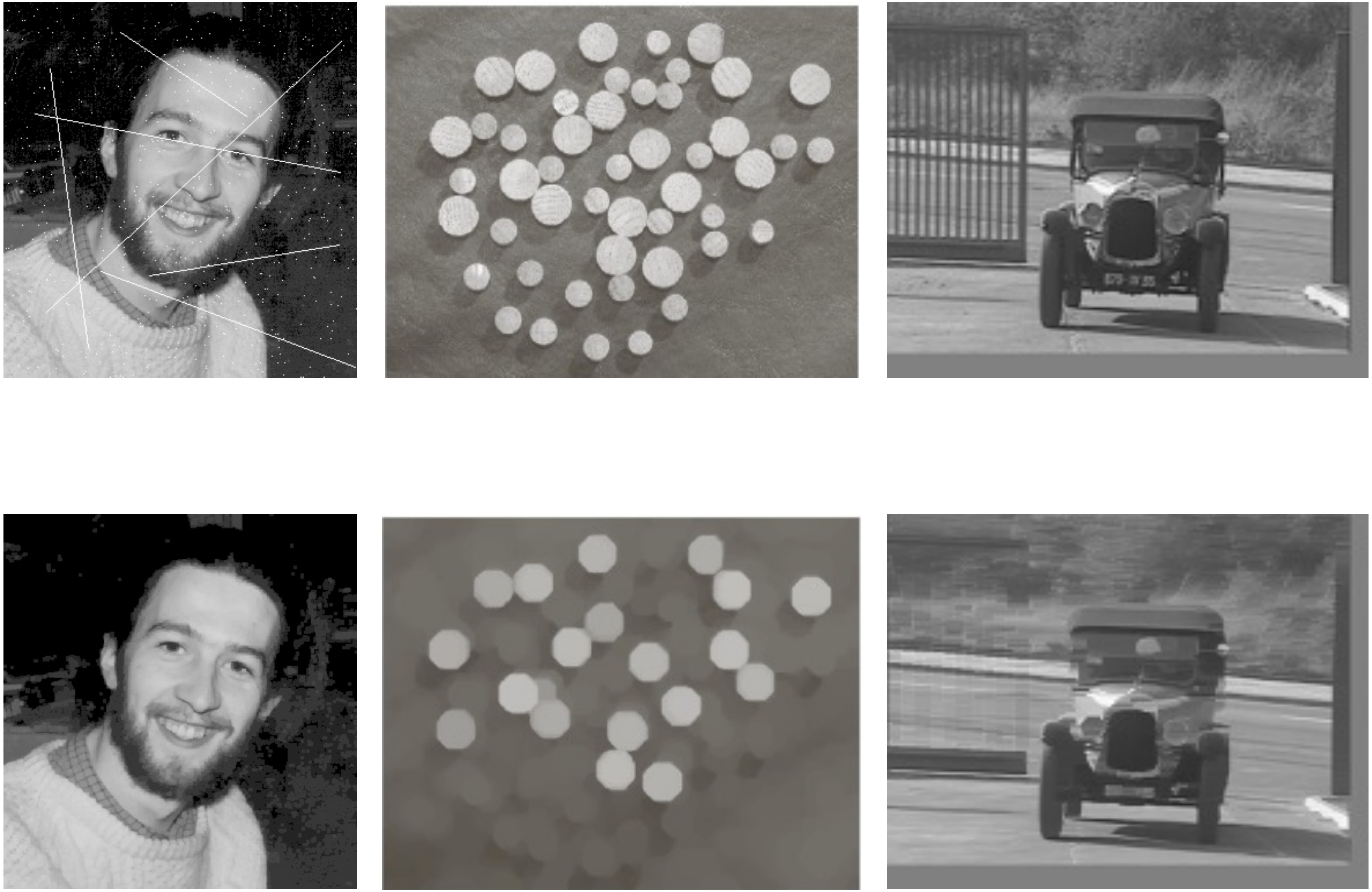
Closing

Opening and closing: Applications

Object characterization: **- Gray level**
- Geometry
=> Operator

- Object: - white → opening
  - black → closing
- Geometry: Structuring element larger than the object to remove

Opening and closing: Applications

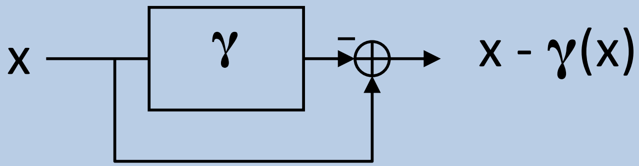


Opening and closing: Tophat

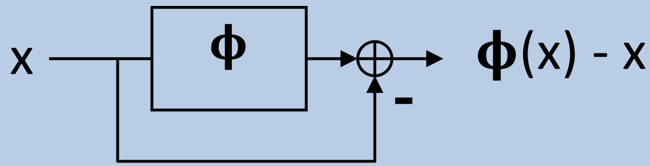
Opening: remove objects of small size: $\gamma(x)$

Top-hat: detect objects of small size: $x - \gamma(x)$

Top-hat

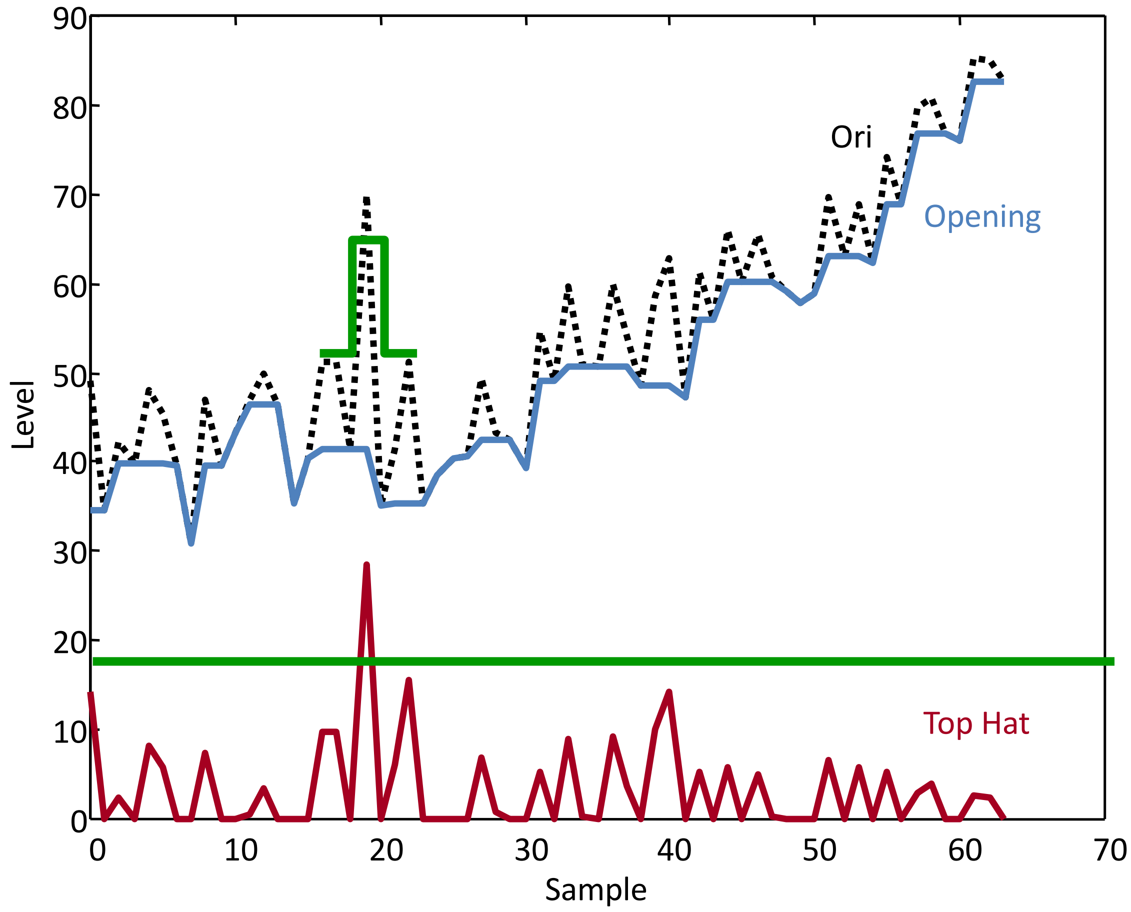


Dual top-hat



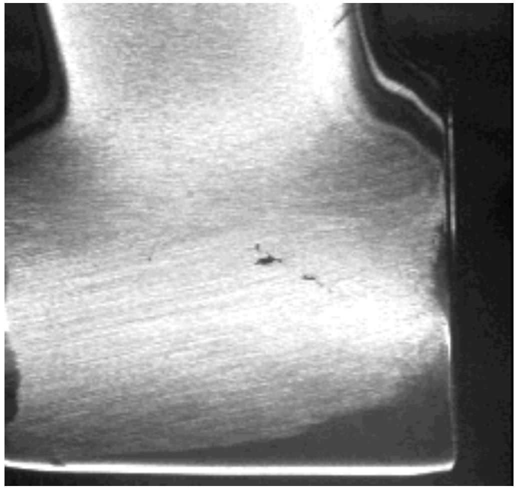
Opening and closing: Tophat

1D example

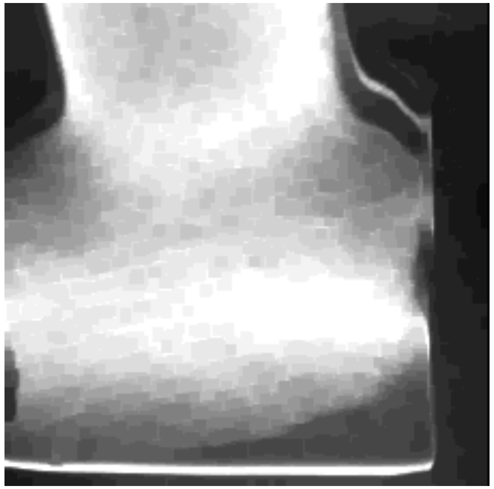


Opening and closing: Tophat

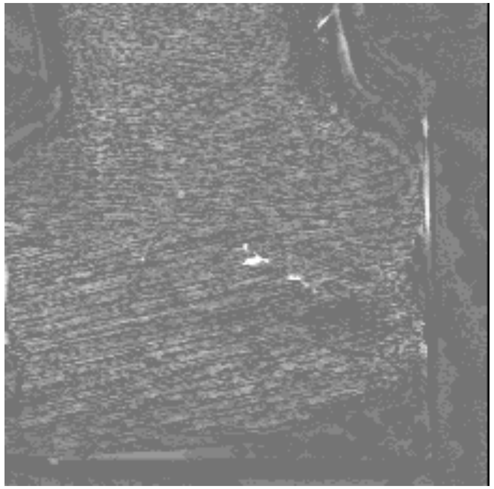
Example of defect detection with Top-hat



Original



Closing



Top-hat

Morphological and nonlinear processing

- Mathematical Morphology and Lattice

- Basic Operators: Dilation and Erosion
  - Definition and structuring element
  - 1D, 2D examples and Properties
  - Practical use

- Opening, Closing
  - Definition and properties
  - Geometric interpretation
  - Practical use: Object elimination, Top Hat

- **Morphological filters**

Morphological filters

In mathematical morphology:

A (morphological) filter

=

operator

increasing and idempotent

Preserve the lattice structure:
$x \leq y \Rightarrow \psi(x) \leq \psi(y)$

Control the filtering effect:
Filter just once

Examples:

Operators	*Filters*
Erosion	Opening
Dilation	Closing

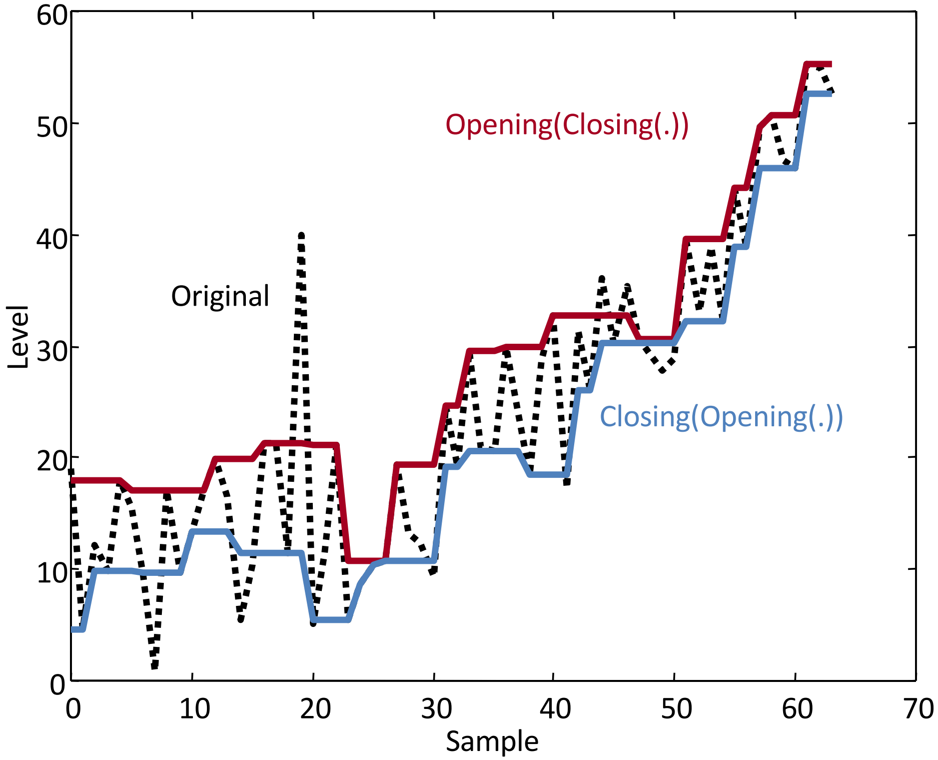
Morphological filters

Alternating filters

- <u>Two filters</u>: θ and ψ such that θ ≤ ψ (∀x, θ(x) ≤ ψ (x))

Classical example: θ: opening, ψ: closing $\gamma(x) \leq x \leq \varphi(x)$

=> θψ, ψθ
θψθ, ψθψ → are morphological filters



Morphological filters

Example of use: smoothing & noise cleaning

X

$\varphi_B\left(\gamma_B(X)\right)$

Morphological filters

Example of use:

•"Almost" self dual simplification (ψ is self-dual $\Leftrightarrow$ $\psi(-x) = -\psi(x), \forall x$)

Original

Opening: γ (.)

$\phi\gamma$ (.)

Closing: ϕ (.)

$\gamma\phi$ (.)

Morphological and nonlinear processing

- Mathematical Morphology and Lattice
  - **Lattice framework** for non-linear processing
  - Emphasis on geometrical features, maxima & minima

- Basic morphological operators
  - **Dilation** and **erosion**
  - Similar to convolution and correlation
  - Simplification: ability to remove maxima or minima

- Basic morphological filters
  - **Opening** and **closing**
  - Practical use: Object elimination, Top Hat
  - Can be combined to form new morphological filters