Master in
Computer Vision
Barcelona

Module: Introduction to human and computer vision

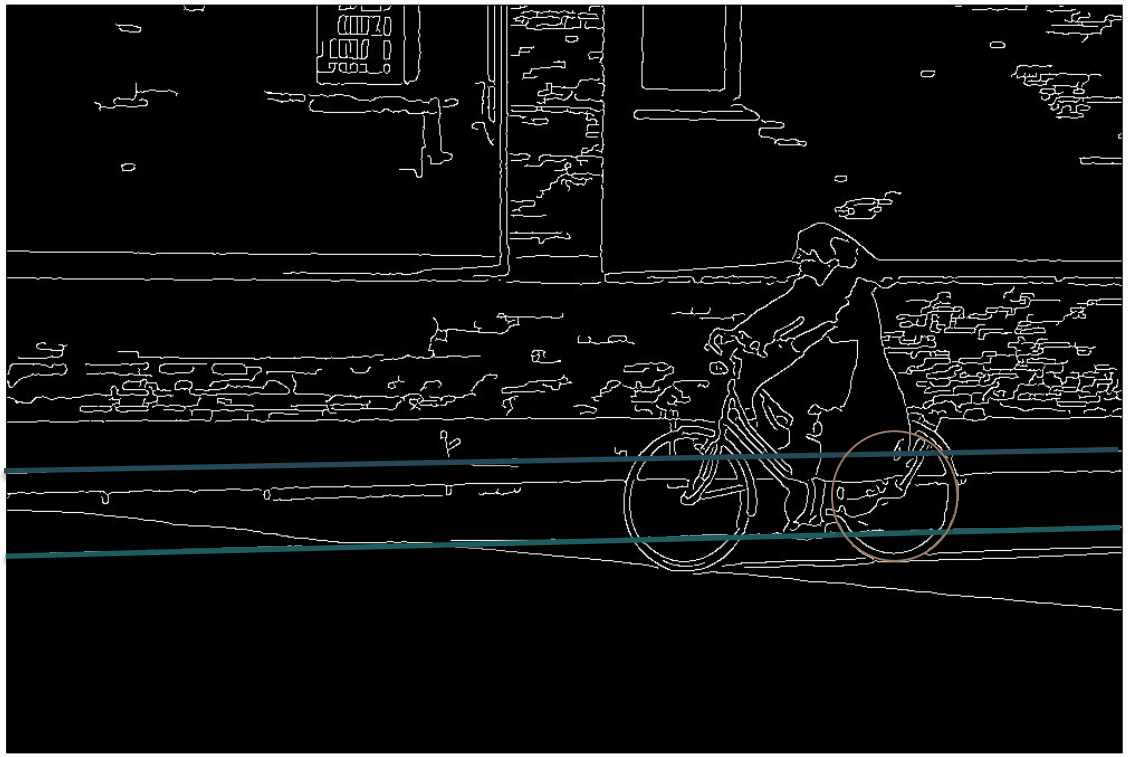
Lecture 9: Grouping, segmentation and classification (I)

Lecturer: Ramon Morros

GROUPING & MODELING: SHAPE

Shape

- Objective: find simple shapes in images (lines, circles, etc.)



Shape

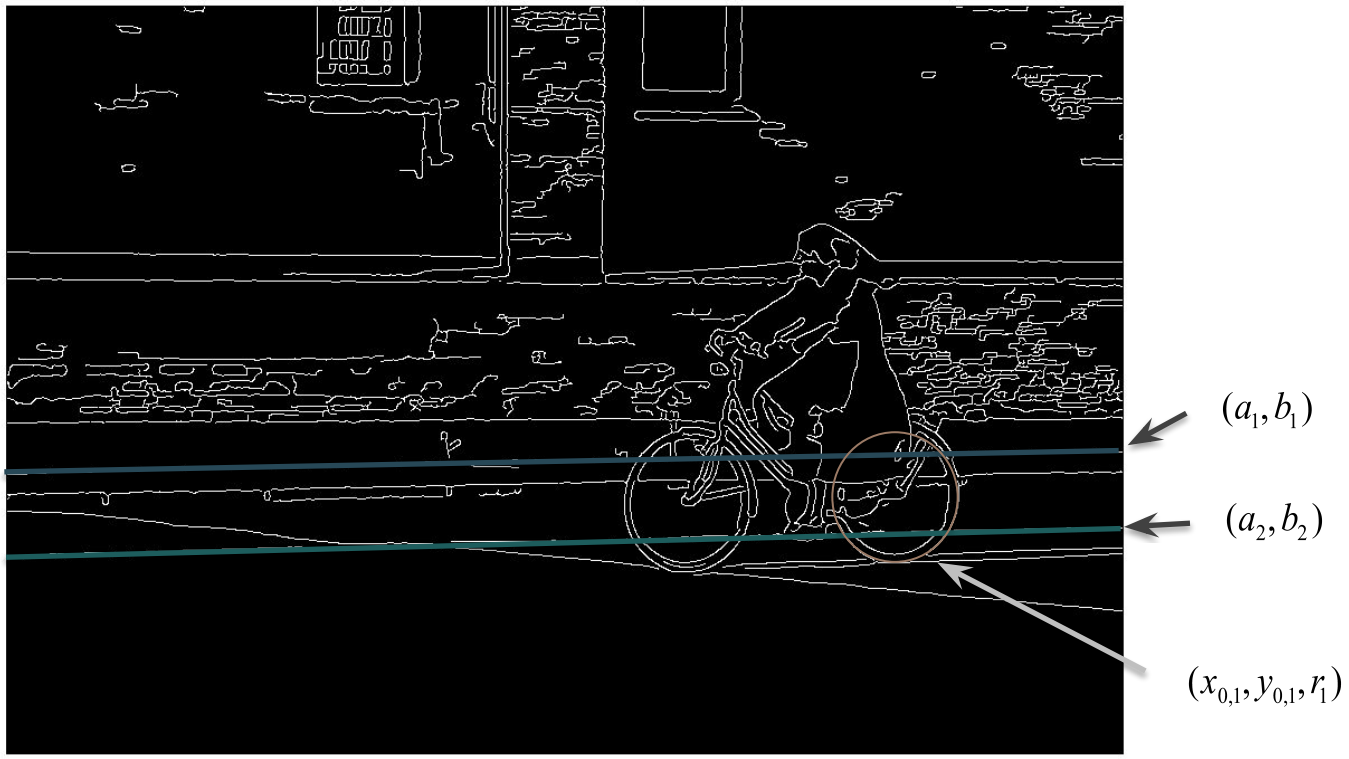
- Objective: find simple shapes in images (lines, circles, etc.)
- Shapes that can be parametrized

Line

$y = ax + b$

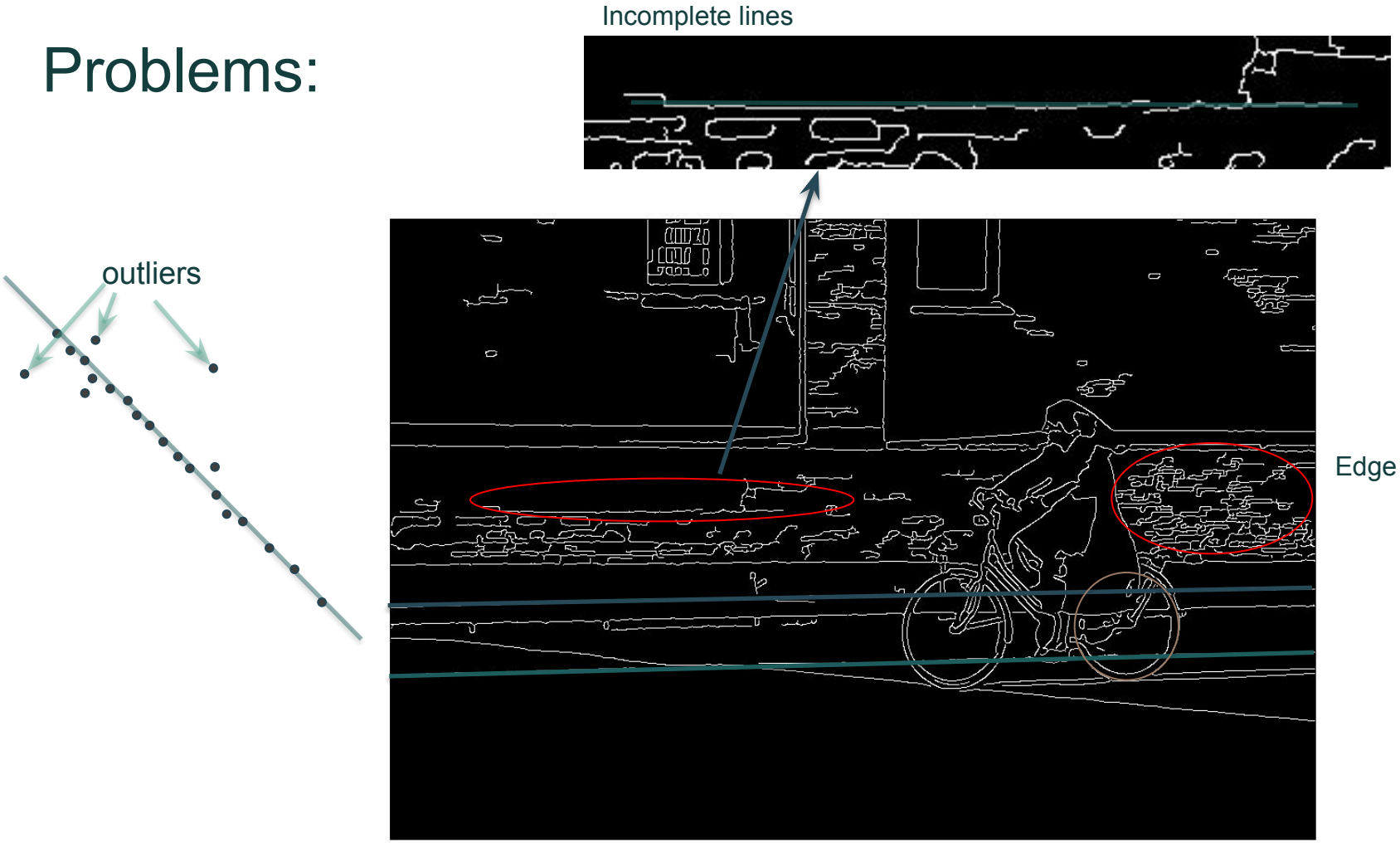
Circle

$(x - x_0)^2 + (y - y_0)^2 = r^2$



Shape

Problems:



Outliers: points that do not fit to the model within a given small error

Shape

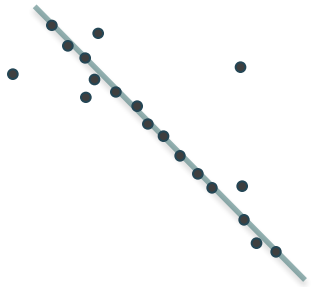
Canny, Pb, … (L8)

- Method
  - Obtain the data: edge detection, or another method to generate shape from images
  - Adjust data to a model / group the features
- Techniques
  - Linear regression: least squares
  - Hough transform : voting in parameter space
  - RANSAC : try randomly subsets of points to adjust the shape

Shape: Least Squares

- Goal: find the parameters that best fit a set of data to a model
- Over-determined system : more equations than parameters!

Example: line



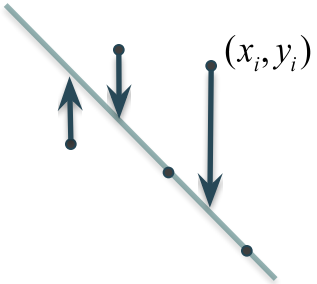
Line equation:

$$y = ax + b$$

$$a?, b?$$

- Method: minimize the sum of squared residuals

$$E = \sum_{i=0}^{N} (y_i - ax_i + b)^2$$



Shape: Least Squares

- Minimization: $\mathbf{p} = \underset{p \in \Re^2}{\operatorname{argmin}}\{E\}$ $\mathbf{p} = [a\ b]$

Energy:

$$E = \sum_{i=0}^{N} (y_i - ax_i - b)^2$$

coordinates of contour points

In matrix form:

$$E = \sum_{i=0}^{N} \left([x_i\ 1] \begin{bmatrix} -a \\ -b \end{bmatrix} + y_i \right)^2 = \left\| \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{bmatrix} \begin{bmatrix} -a \\ -b \end{bmatrix} - \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} \right\|^2 = \|\mathbf{A}p - \mathbf{y}\|^2$$

$$= \mathbf{y}^T\mathbf{y} - 2(\mathbf{A}\mathbf{p})^T\mathbf{y} + (\mathbf{A}\mathbf{p})^T(\mathbf{A}\mathbf{p})$$

Minimization:

$$\frac{dE}{d\mathbf{p}} = -2\mathbf{A}^T\mathbf{y} + 2\mathbf{A}^T\mathbf{A}\mathbf{p} = 0$$

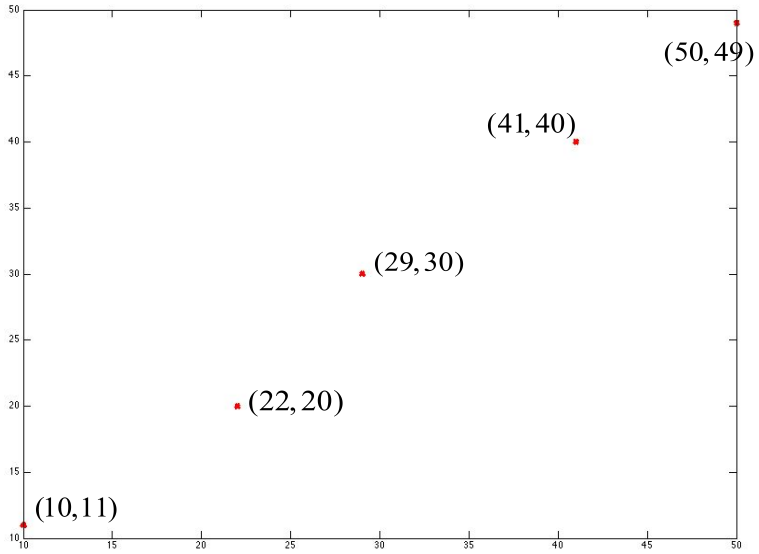
$$\mathbf{p} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{y} = \mathbf{A}^{-1}\mathbf{y}$$

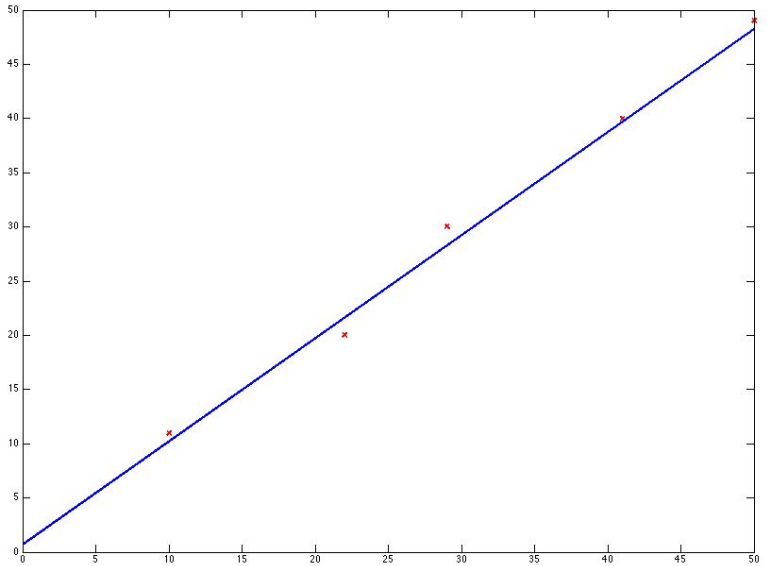
Python:
p = np.linalg.pinv(A)*y

Source: S. Lazebnik, J. Hays

Shape: Least Squares

- Example: Line

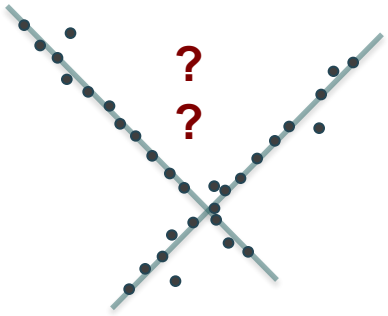




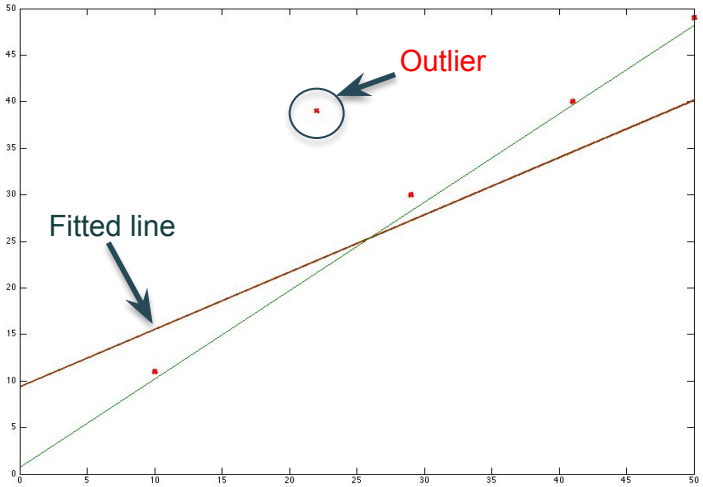
$$\mathbf{A} = \begin{bmatrix} 10 & 1 \\ 22 & 1 \\ 29 & 1 \\ 41 & 1 \\ 50 & 1 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 11 \\ 20 \\ 30 \\ 40 \\ 49 \end{bmatrix}$$

$$p = A^{-1}y = [0.96,\ 0.69]$$

Shape: Least Squares



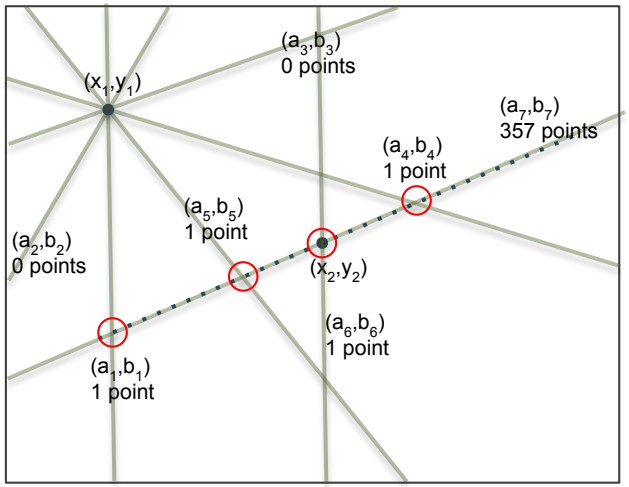
- Strengths:
  - Simple, closed form
  - Fast!
- Problems:
  - Do not allow multiple fits
  - Not robust: Very sensible to outliers



- Common applications
  - Computing a homography (e.g., image stitching)
  - Estimating fundamental matrix (relating two views)
  - Modeling depth in depth estimation

Shape: Hough transform

- Method: generate shape candidates by varying shape parameters, try to match with data and vote best one(s)
- Select maximum points in parameter space
  - Naïve approach: for each possible line in the image, count the contour pixels that are compatible with this line



(a_1,b_1) 1 points
(a_2,b_2) 1 points
(a_3,b_3) 1 points
(a_4,b_4) 1 points
(a_5,b_5) 1 points
(a_6,b_6) 1 points
(a_7,b_7) 357 points
(a_8,b_8) 1 points
…
(a_N,b_N) 22 points

Shape: Hough Transform

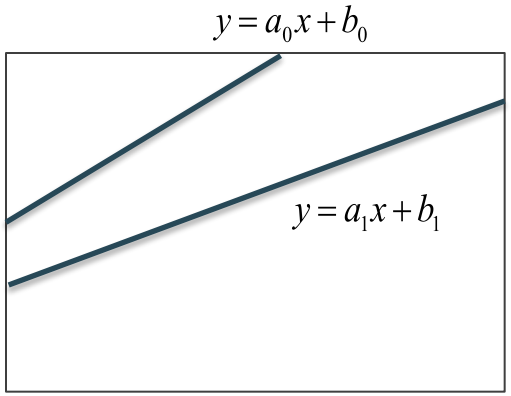
- Method: generate shape candidates by varying shape parameters, try to match with data and vote best one(s)
- Select maximum points in parameter space
- Problem: computational complexity!
  - Infeasible to generate all combinations of parameters and check on data
- The features (for instance, edge pixels) vote for the model instances that are compatible to them
  - Cycle through features, cast votes for model parameters.
  - Select model parameters that receive a lot of votes.
  - Noise & clutter do not follow the model and receive few votes

Shape: Hough Transform

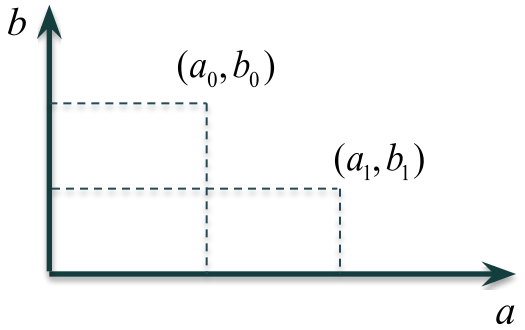
Line equation:

$$y = ax + b$$

Image space



Parameter space

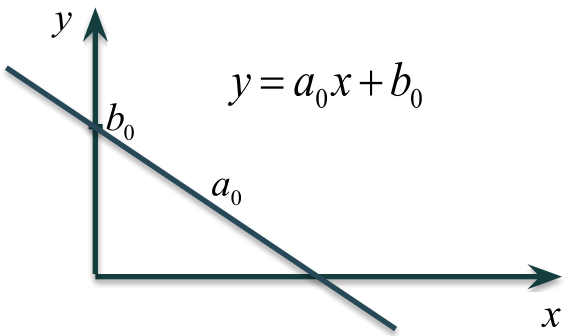


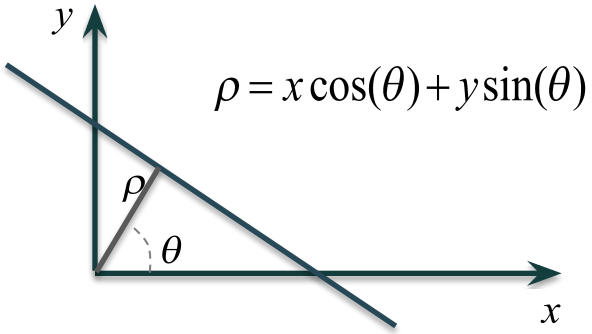
A line in image space corresponds to a point in parameter space

Shape: Hough Transform

- Vertical lines → parameter *a* (line slope) goes to infinity!!
- Polar representation is better in this case

$$\rho = x\cos(\theta) + y\sin(\theta)$$



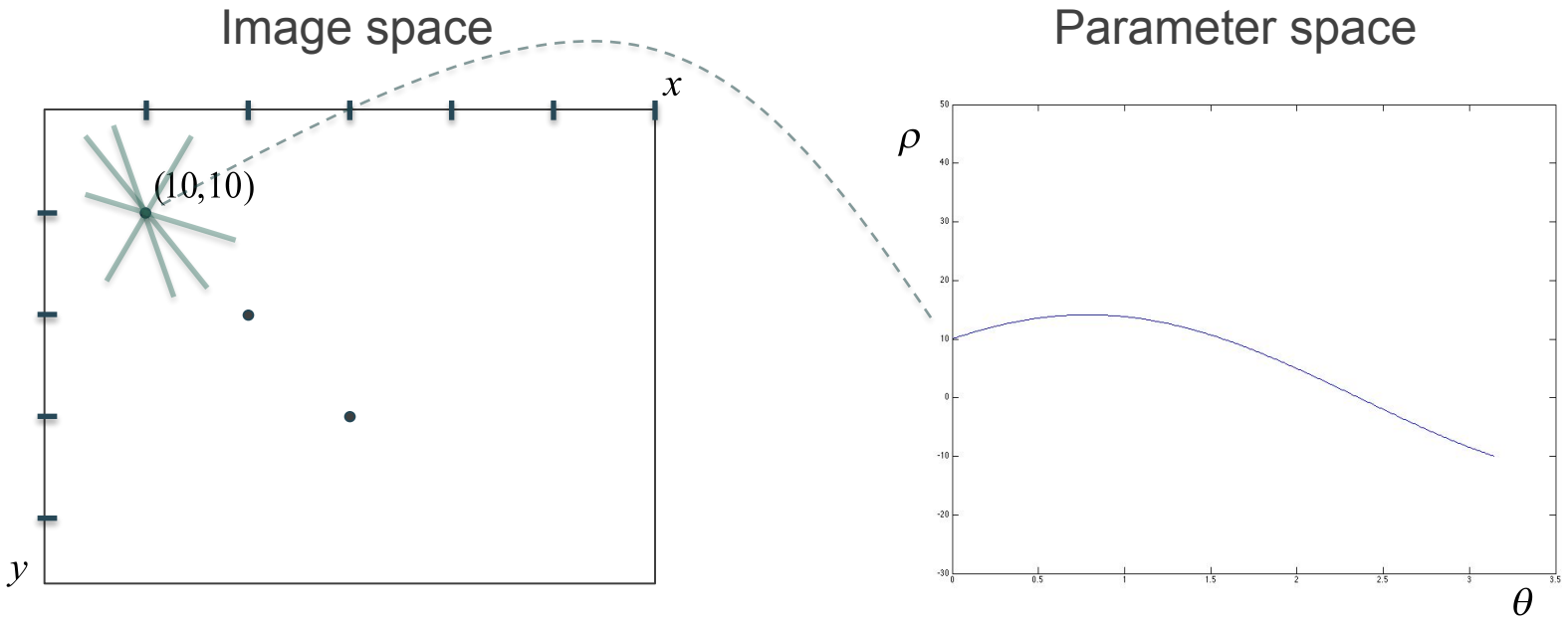


Shape: Hough Transform

- Example: lines

Coordinates of contour point

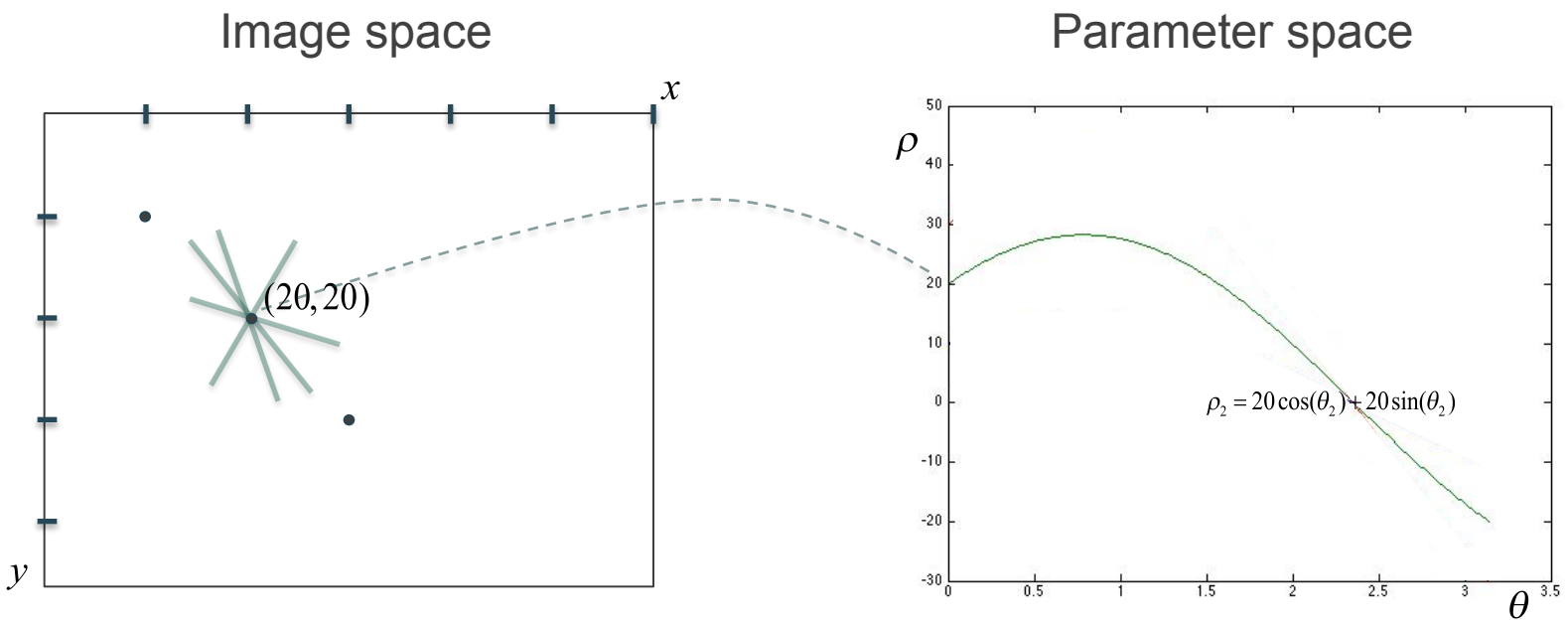
$$\rho = x\cos(\theta) + y\sin(\theta)$$



Each possible line passing through a point reflects into a 'sinusoidal' curve in parameter space

Shape: Hough Transform

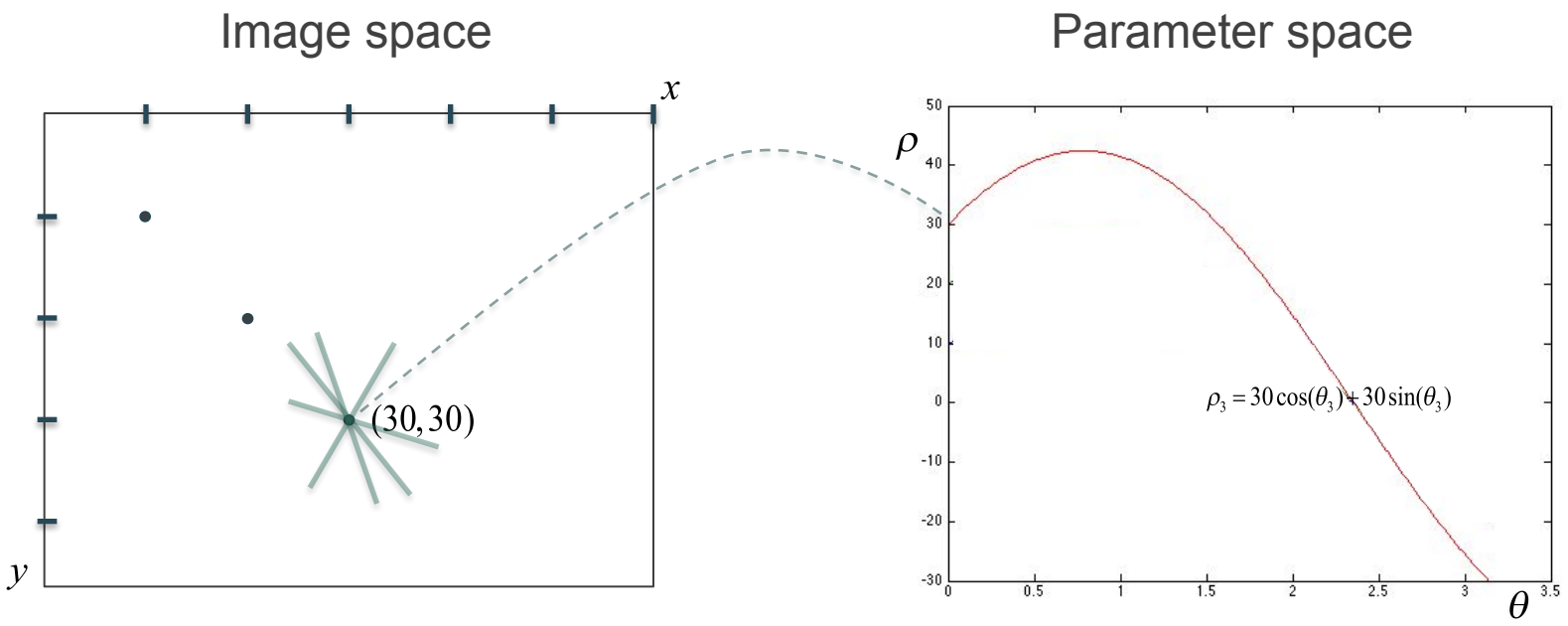
- Example: lines



Each possible line passing through a point reflects into a 'sinusoidal' curve in
parameter space

Shape: Hough Transform

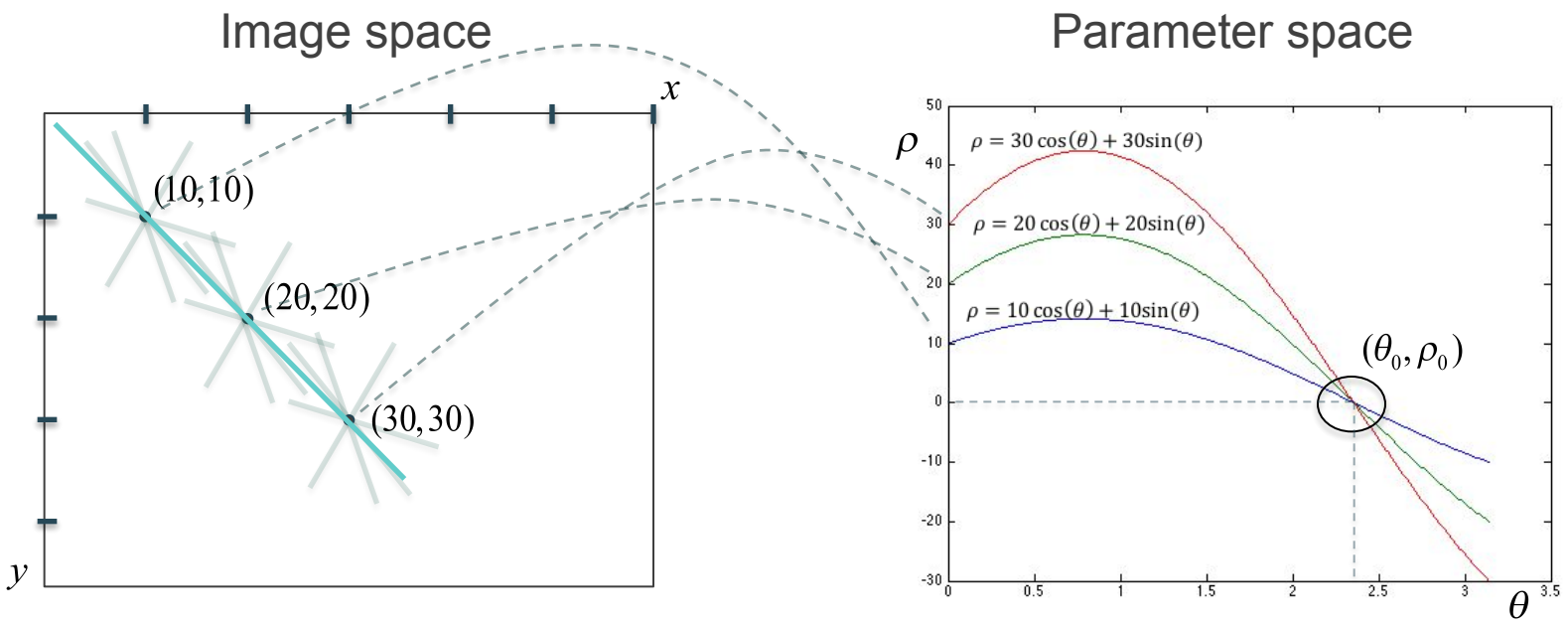
- Example: lines



Each possible line passing through a point reflects into a 'sinusoidal' curve in
parameter space

Shape: Hough Transform

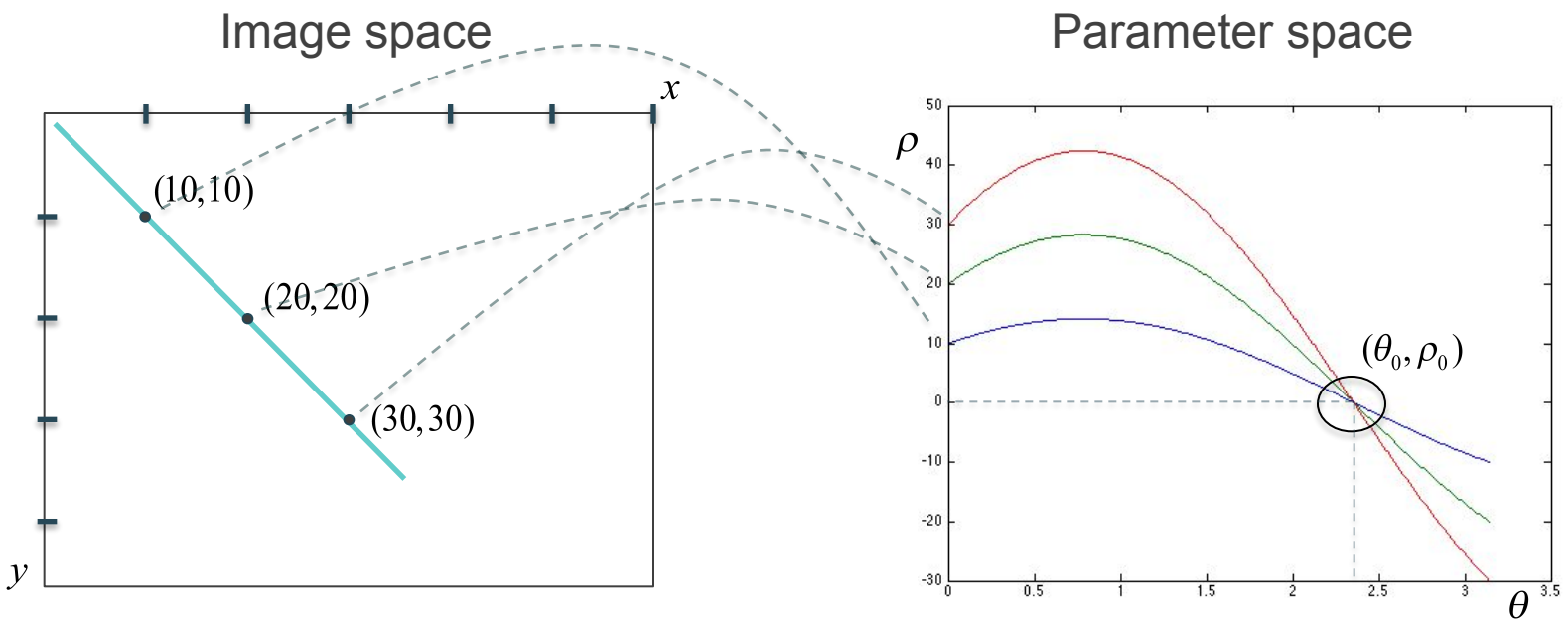
- Example: lines



Points on the same line in image space define curves in the parameter space that intersect at a single point.

Shape: Hough Transform

- Example: lines



Main idea: points in image space are represented by curves in parameter space. Find the points in parameter space that have many curves passing through.

Shape: Hough Transform

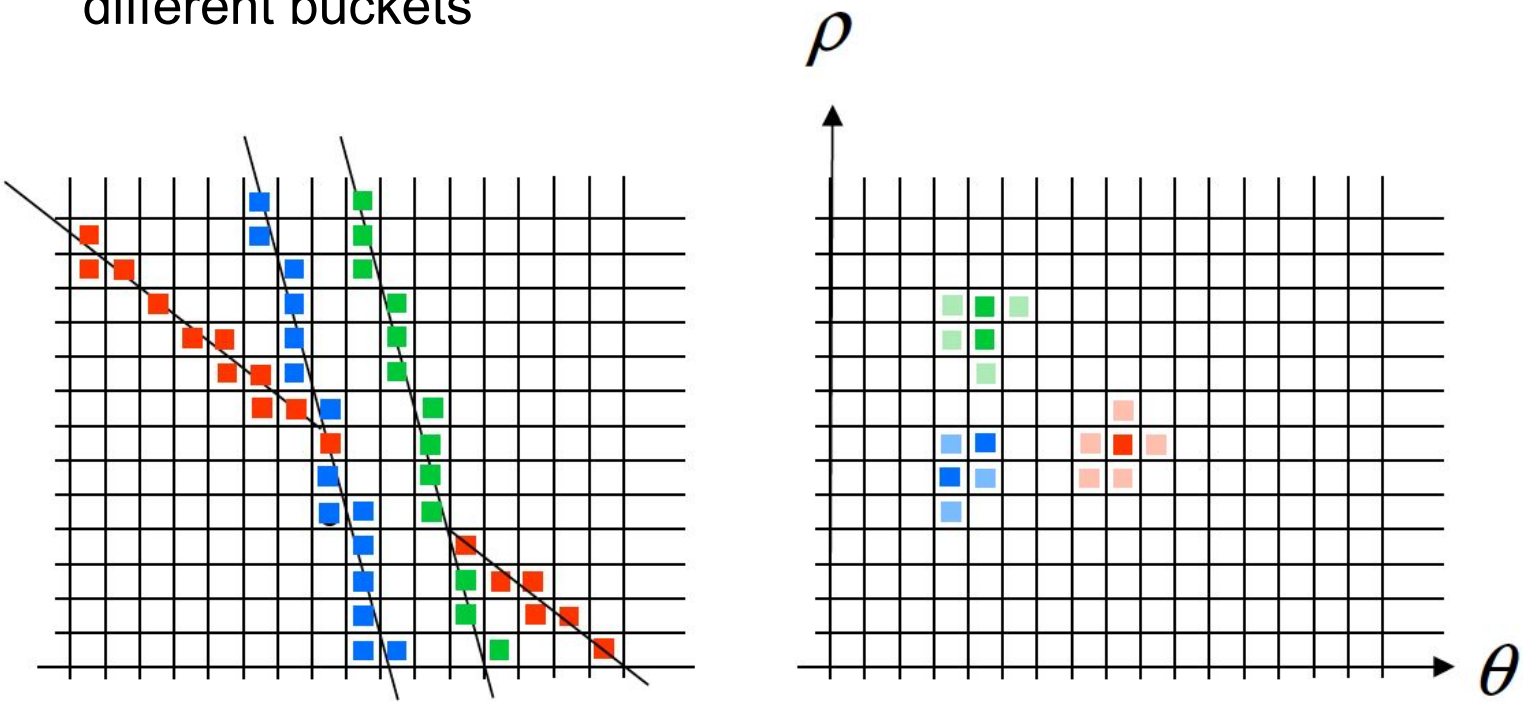
1. Quantize the parameter space
   int $P[\rho_{max}][\theta_{max}]$; // accumulators
   P = 0
2. Loop for all edge points
   for each edge point (x, y) {
      // Compute parameters
      for ($\theta = 0$; $\theta <= \theta_{max}$; $\theta = \theta + \Delta\theta$) {
         $\rho = x \cdot \sin(\theta) + y \cdot \cos(\theta)$; // round off to integer (quantization)
         $(P[\rho][\theta])$++;
      }
   }
3. Find the peaks in $P[\rho][\theta]$.

Quantized angle

Source: Chang Shu

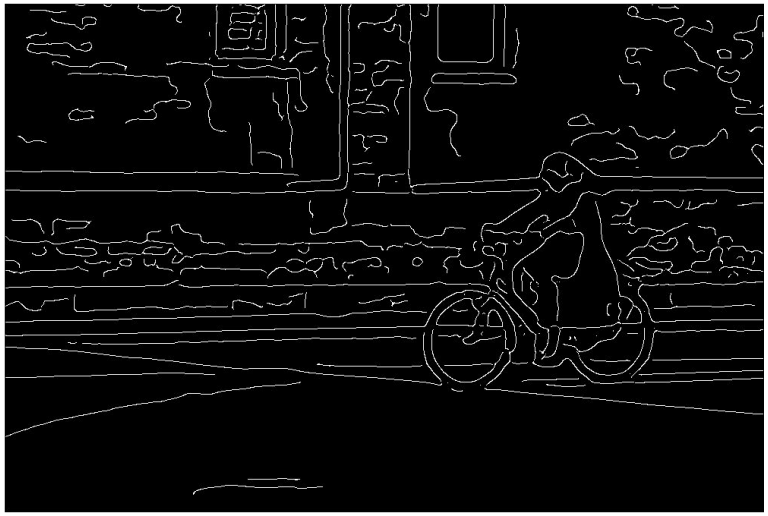
Shape: Hough Transform

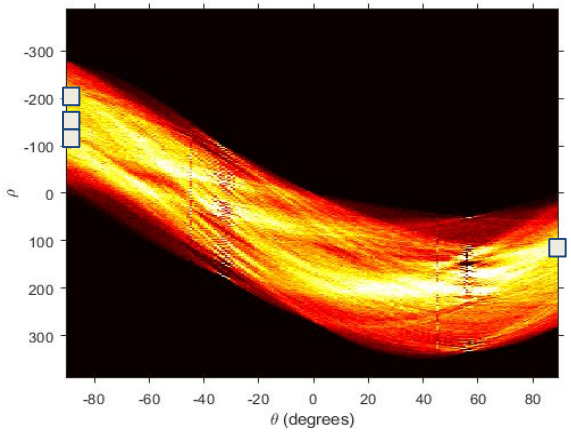
- Select quantization of parameter space
  - Too coarse: Too many different lines correspond to a single bucket
  - Too fine: miss lines because points not exactly collinear vote for different buckets



Source: Chang Shu

Shape: Hough Transform





Matlab:
- hough()
- houghpeaks()
- houghlines()

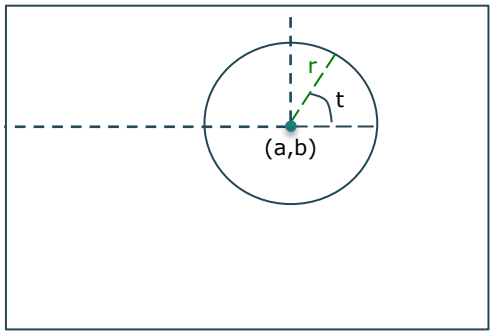
Python:
- scikit-image
- OpenCV

Shape: Hough Transform

Example: circumference:

$$a = x - r\cos(t)$$

$$b = y - r\sin(t)$$

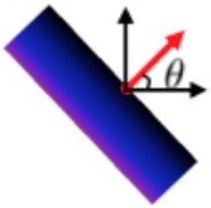



```
int P[amax][bmax][rmax]; // accumulators
for each edge point (x, y) {
    // Compute parameters
    for (r = rmin; r <= rmax; r = r+Δr) {
        for (t = 0; t <= tmax; t = t+Δt) {
            a = x - r·cos(t);
            b = y - r·sint(t);
            P[a][b][r]++;
        }
    }
}
```

Shape: Hough Transform

Incorporating gradients:

- When detecting edges, gradient direction provides the orientation of the edge
- No need to loop over all possible angles, line is determined
- Faster computation!



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

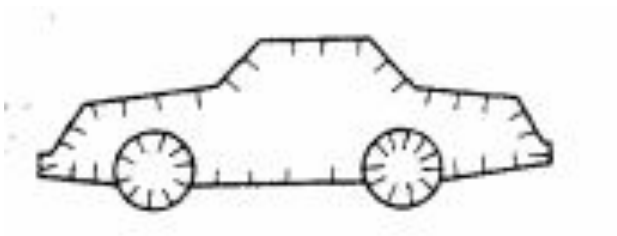
```
for each edge point (x, y) {
        // Compute parameters
        θ = gradient_orientation (x,y)
        ρ = x·sin(θ)+y·cos(θ)
        (P[ρ][θ])++
}
```

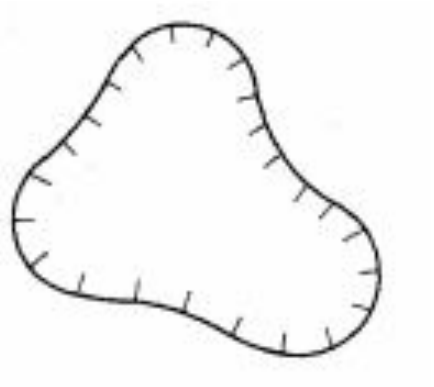
Shape: Hough Transform

- Pros
  - All points are processed independently, so it can cope with occlusion
  - Moderate robustness to noise: noise points unlikely to contribute consistently to any single bin
  - Detection of multiple instances of a model in a single pass
- Cons
  - Search time increases exponentially with the number of model parameters
  - Non-target shapes can produce spurious peaks in parameter space
  - Quantization: hard to pick a good grid size

Shape: Generalized Hough Transform

- The generalized Hough transform can be used to detect arbitrary shapes (i.e., shapes having no simple analytical form).
- It requires the complete specification of the exact shape of the target object.
- Uses edge direction information





D.H. Ballard, **Generalizing the Hough transform to detect arbitrary shapes** Pattern Recognition, Volume 13, Issue 2, 1981, Pages 111–122

DEMO: http://www.rob.cs.tu-bs.de/content/04-teaching/06-interactive/GeneralHoughTransformation.html

Shape: Generalized Hough Transform

- The generalized Hough transform can be used to detect arbitrary shapes (i.e., shapes having no simple analytical form).
- **It requires the complete specification of the exact shape of the target object.**

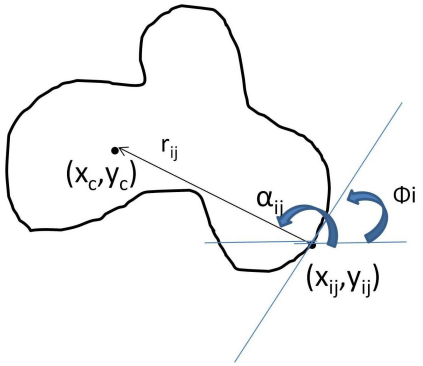
$$a = \{y, s, \theta\}$$

y is a reference origin for the shape, θ is its orientation, and s = (s_x, s_y) describes two orthogonal scale factors

- Shape is described in terms of a template table called the **R table** of possible edge pixel orientations
- Computation of parameters s and θ is accomplished by straightforward transformations to this table

Shape: Generalized Hough Transform

- R table uses edge direction information
  - Choose a reference point y for the shape (typically chosen inside the shape).
  - For each boundary point x, compute $\phi(x)$, the gradient direction and r = y – x as shown in the image. Store r as a function of ϕ
  - Each index of ϕ may have many values of r



i	ϕ_i	R_{ϕ_i}
1	0	$(r_{11}, \alpha_{11})\ (r_{12}, \alpha_{12}) \ldots\ (r_{1n}, \alpha_{1n})$
2	$\Delta\phi$	$(r_{21}, \alpha_{21})\ (r_{22}, \alpha_{22}) \ldots\ (r_{1m}, \alpha_{1m})$
3	$2\Delta\phi$	$(r_{31}, \alpha_{31})\ (r_{32}, \alpha_{32}) \ldots\ (r_{3k}, \alpha_{3k})$
...	...	...

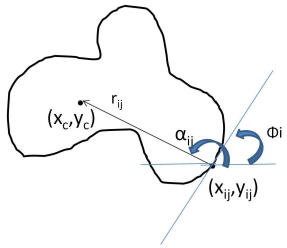
Shape: Generalized Hough Transform

- **Code:**

```
def build_r_table(image, origin):
  '''
  R-table from the given shape image and a ref point
  '''
    edges = canny(image,
                  low_threshold=MIN_CANNY_THRESHOLD,
                  high_threshold=MAX_CANNY_THRESHOLD)
    gradient = gradient_orientation(edges)

    r_table = defaultdict(list)
    for (i,j),value in np.ndenumerate(edges):
        if value:
            r_table[gradient[i,j]].append((origin[0]-i,
origin[1]-j))

    return r_table
```



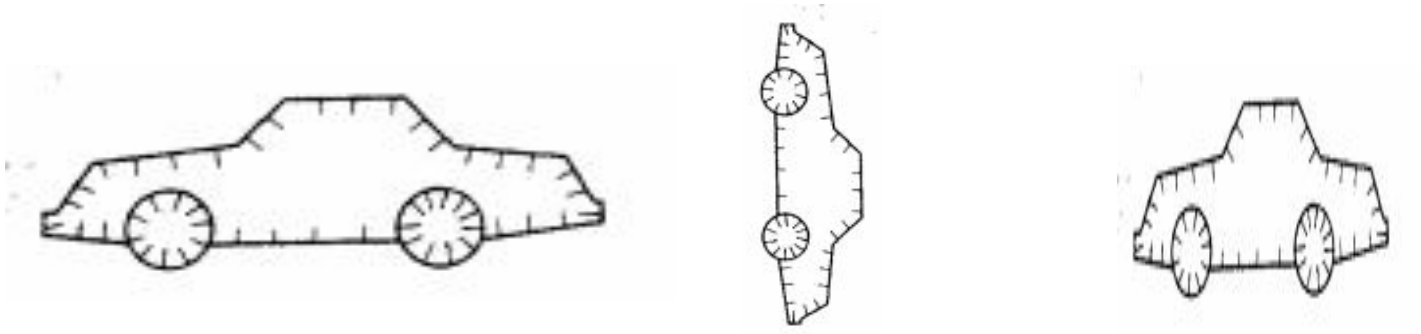
i	ϕ_i	R_{ϕ_i}
1	0	$(r_{11}, a_{11})\ (r_{12}, a_{12}) \ldots (r_{1n}, a_{1n})$
2	$\Delta\phi$	$(r_{21}, a_{21})\ (r_{22}, a_{22}) \ldots (r_{1m}, a_{1m})$
3	$2\Delta\phi$	$(r_{31}, a_{31})\ (r_{32}, a_{32}) \ldots (r_{3k}, a_{3k})$
...	...	...

```
def accumulate_gradients(r_table, grayImage):
  '''
  Perform a General Hough Transform with the given image and R-table
  '''
  edges = canny(grayImage, low_threshold=MIN_CANNY_THRESHOLD,
                high_threshold=MAX_CANNY_THRESHOLD)
  gradient = gradient_orientation(edges)

  accumulator = np.zeros(grayImage.shape)
  for (i,j),value in np.ndenumerate(edges):
     if value:
        for r in r_table[gradient[i,j]]:
           accum_i, accum_j = i+r[0], j+r[1]
           if accum_i < accumulator.shape[0] and accum_j <
accumulator.shape[1]:
              accumulator[accum_i, accum_j] += 1
  return accumulator
```

Shape: Generalized Hough Transform

- **Advantages**
  - Robust to partial or slightly deformed shapes (i.e., robust to recognition under occlusion).
  - Robust to the presence of additional structures in the image.
  - Tolerant to noise.
  - Can find multiple occurrences of a shape in one pass.
- **Disadvantages**
  - Has substantial computational and storage requirements



Shape: RANSAC

- Goal: adjust models to noisy data
- Shapes with a mathematical model (lines, planes, circles, etc.)
- Idea:
  - Obtain random subsets of points and adjust to model
  - Store the parameters that minimize residual error by using discriminant function:

$$\min \sum_i f(r_i)$$

$$where \quad f(r_i) = \begin{cases} 1, & r_i > t \\ 0, & r_i \leq t \end{cases}$$

r_i : residual of i-th sample

t: threshold

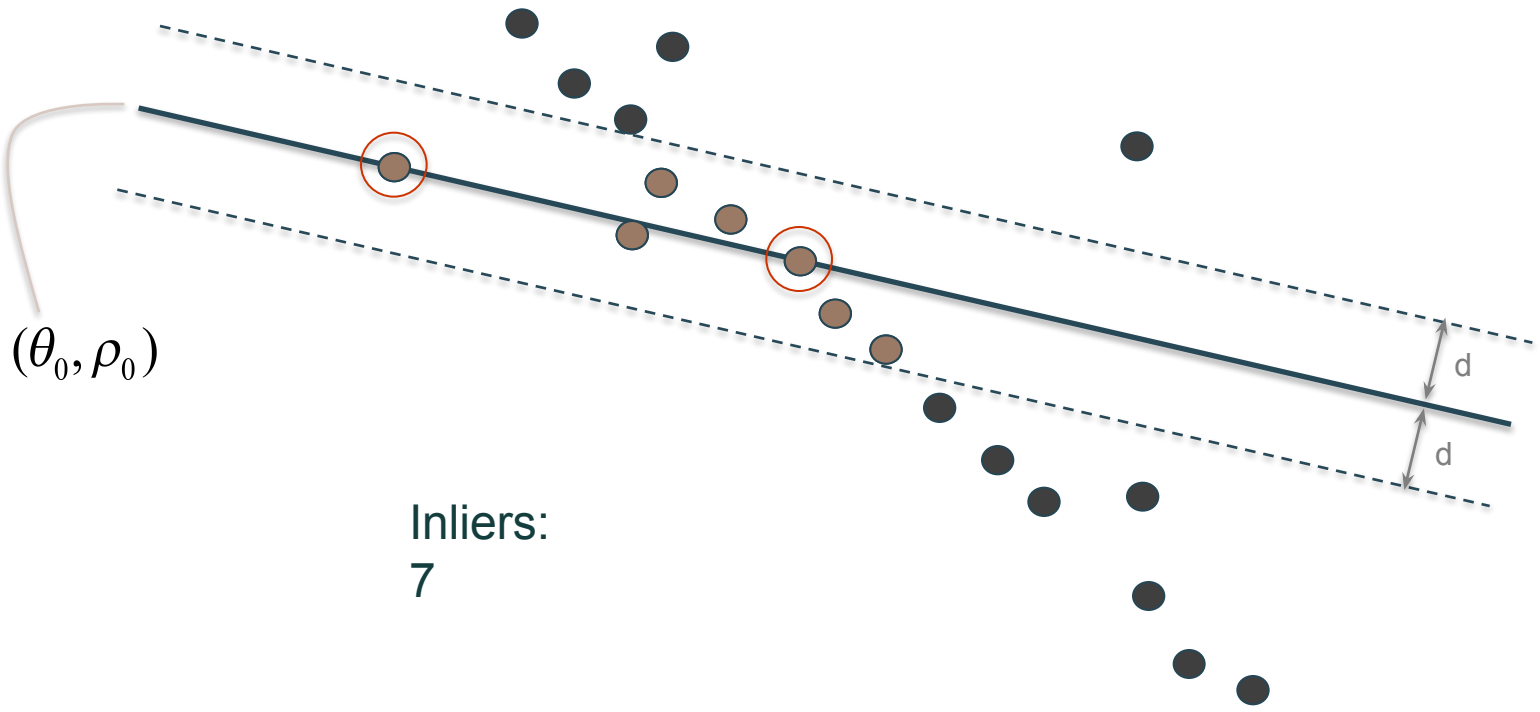
Shape: RANSAC

- Process:
  - Randomly select a minimal set of points necessary to estimate an instance of the model
    - For instance, two points in the case of a line, three for a circle, etc.
  - Compute the parameters and evaluate the model, measuring the number of inliers.
  - Re-compute the model using only inliers using LS (optional)
  - Iterate procedure K times and select the model that better represent the set of data (for instance, the one that gives the maximum number of inliers).

Inliers: points which approximately can be fitted to the model within a given small error

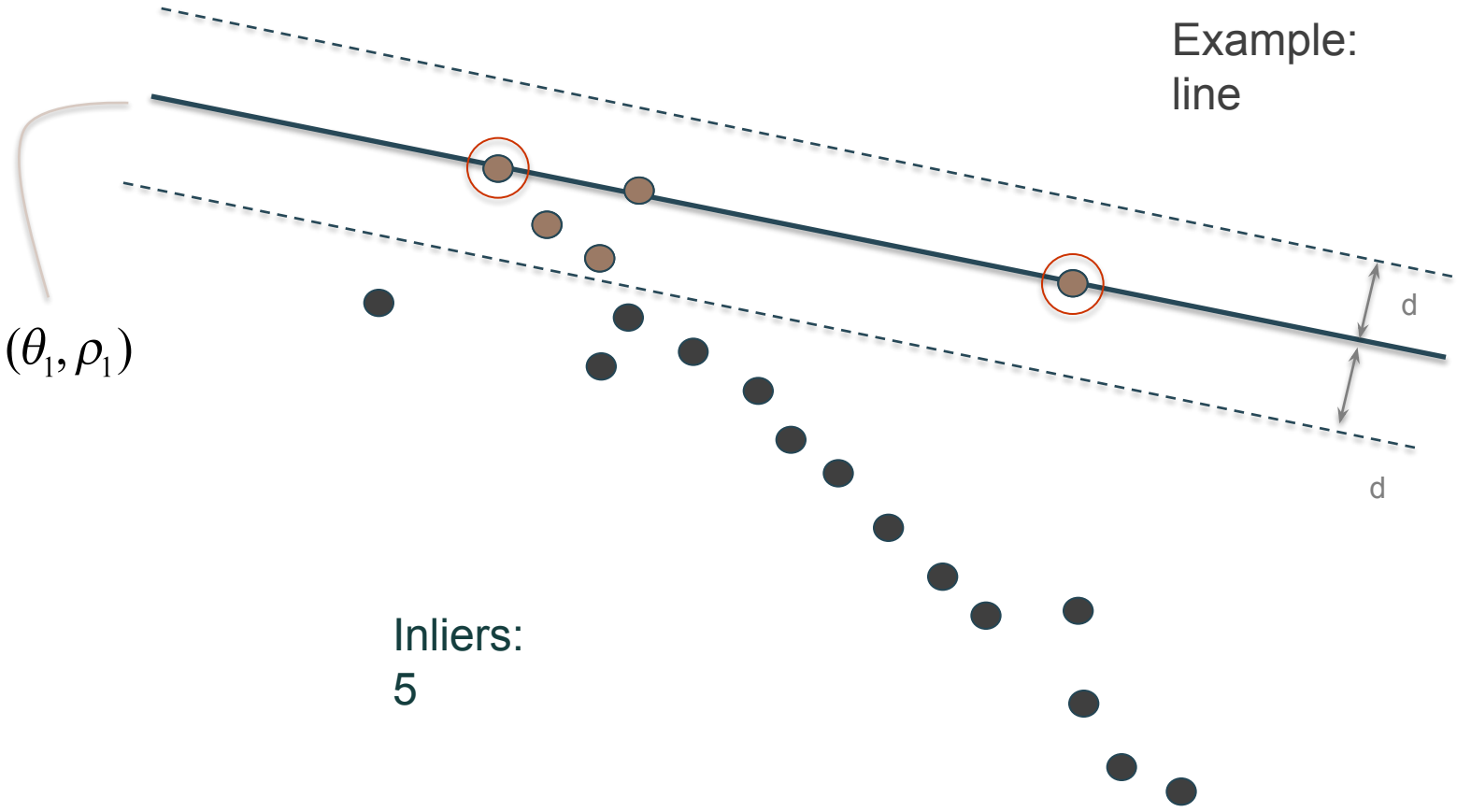
Shape: RANSAC

Example:
line

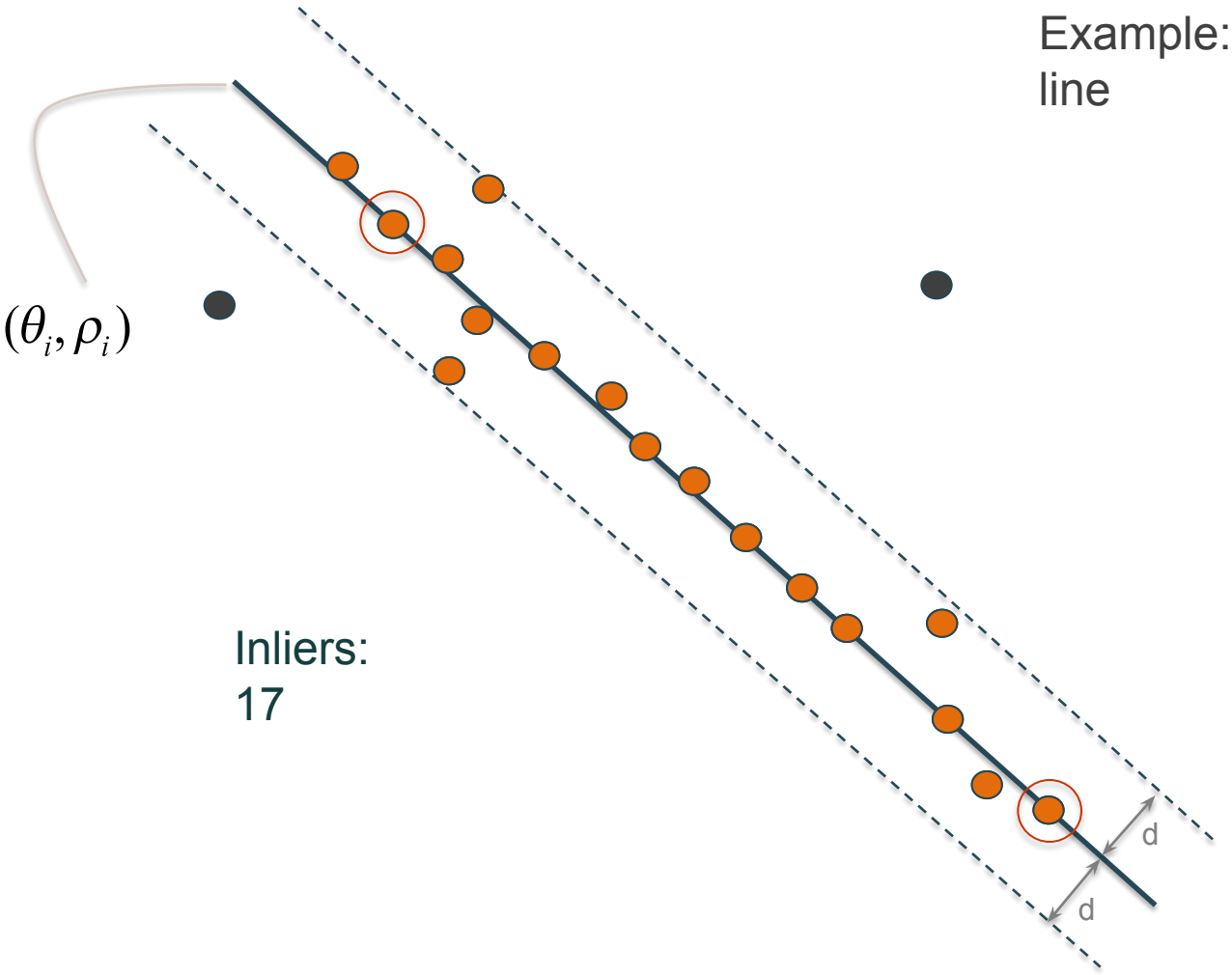


Inliers:
7

Shape: RANSAC

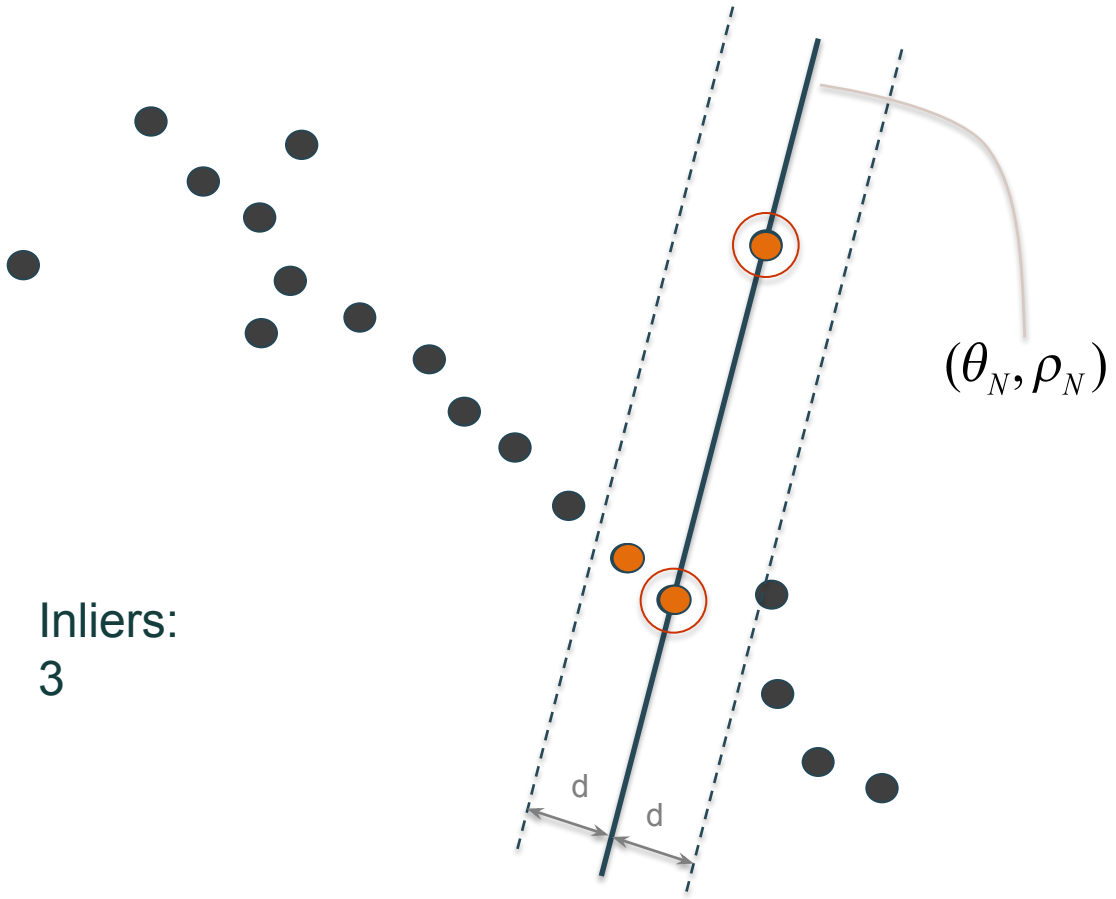


Shape: RANSAC



Shape: RANSAC

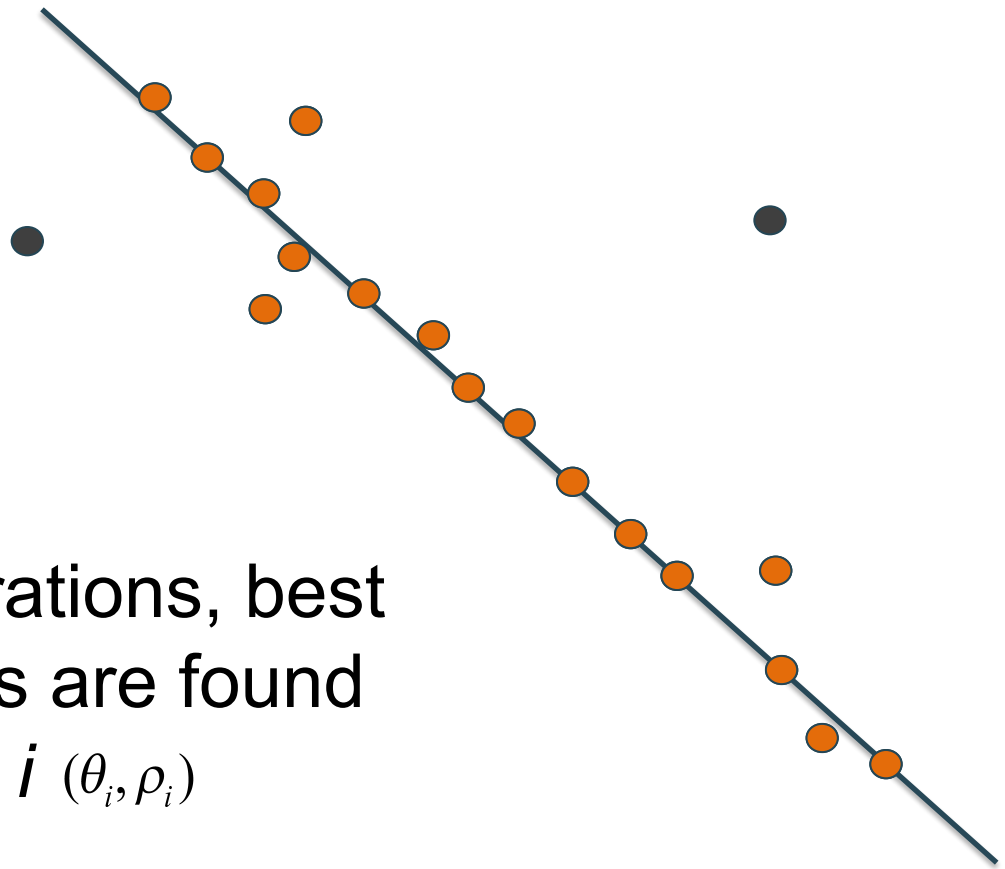
Example:
line



Inliers:
3

Shape: RANSAC

Example:
line



After K iterations, best parameters are found in iteration *i* (θ_i, ρ_i)

Shape: RANSAC

- Variants:
  - Define a threshold on the measure ands stop iterating if the measure is reached
    - Saves computational time at the expense of another parameter
  - Change model evaluation: use fitting error instead of inliers
    - Randomly select a minimal set of points necessary to estimate an instance of the model
    - Compute the parameters and evaluate the number of inliers $N_{inliers}$
    - If $N_{inliers} < Th_{inliers}$, discard model. Otherwise re-compute model using LS and compute fitting error on the set on inliers. Store the fitting error.
    - Iterate procedure many times and select the model with minimal error

Shape: RANSAC

- RANSAC parameters:
  - Number of model parameters (*n*), number of iterations (k), threshold to consider a point an inlier (*d*)
- Selecting number of iterations (k):
  - Suppose we know the percentage of inliers (w)
  - Let *p* be the probability that RANSAC in some iteration selects only inliers (i.e. the probability to produce a useful result)

$$k = \frac{\log(1-p)}{\log(1-w^n)}$$

- Selecting the threshold for inliers/outliers (*d*)
  - Application/model dependent

Shape: RANSAC

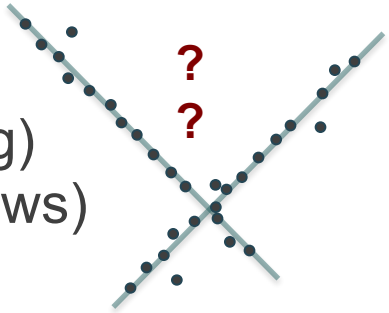
- Selecting number of iterations :
  - If we do not know the percentage of inliers: compute an estimate at each iteration:

```
if ninliers > bestscore    % Largest set of inliers so far...
    bestscore = ninliers;  % Record data for this model

    % Update estimate of N, the number of trials to ensure we pick,
    % with probability p, a data set with no outliers.
    fracinliers =  ninliers/npts;
    k = log(1 - p)/log(1 -  fracinliers^n);
end
```

Shape: RANSAC

- Strengths:
  - Robust to outliers
  - Applicable for larger number of parameters than Hough transform
  - Parameters are easier to choose than Hough transform
- Weaknesses:
  - Computational time (iterations) grows quickly with fraction of outliers and number of parameters
  - Do not allow multiple fits
- Common applications
  - Computing a homography (e.g., image stitching)
  - Estimating fundamental matrix (relating two views)
  - Modeling depth in depth estimation



Shape: RANSAC alternatives

- LMedS:
  - Similar to MSAC: Minimize the median of the residuals
  - The threshold is determined automatically as a value proportional to the median of the residuals produced for each subset.
  - Typically produces better results than RANSAC/MSAC when the inliers contain some noise because in such cases RANSAC or MSAC might have thresholds set too large.

$$f(r_i) = \begin{cases} r_i, & r_i \leq t \\ t, & r_i > t \end{cases}$$

Shape: RANSAC alternatives

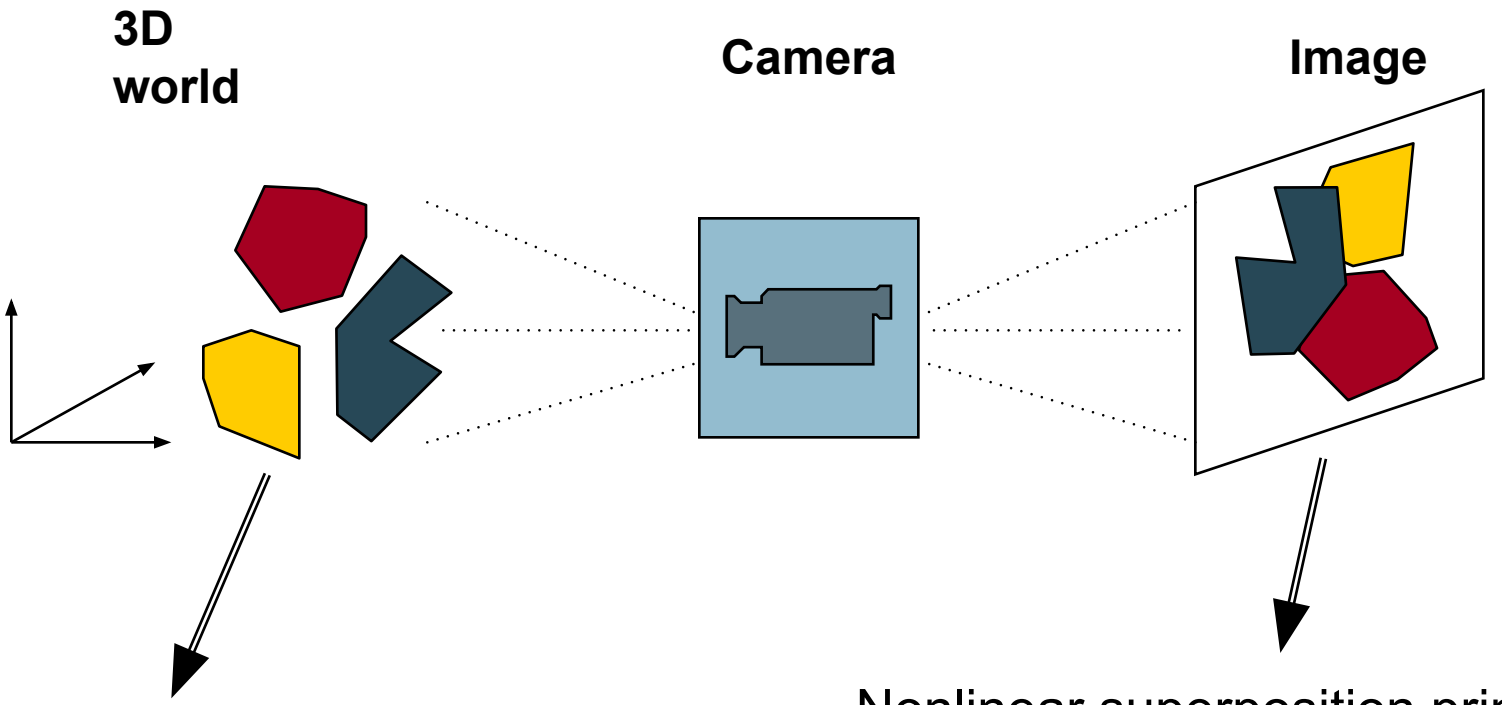
- PROMEDS:
  - Takes into account additional information of the **quality scores** of the points
  - Points are ordered from best to worse quality
  - The algorithm starts picking subsets of points among the points having the largest quality.
  - If a suitable solution having a minimum number of inliers is not found, the algorithm selects matches with lower quality.
  - Usually, solutions are found with a few iterations. Much faster than RANSAC, with similar quality
  - An application dependent threshold is required as in RANSAC
  - Quality scores may not be available in most applications

Shape: Conclusions

	LS	RANSAC	HOUGH
Multiple fits	No	No	Yes
Computational complexity	Low, closed form	High. Depends on % outliers	High
Robustness	Low	High	Medium/High
Scalability (model dimensionality)	Good	Good	Bad
Applications	Homography, fund. Matrix, modeling depth	Homography, fund. Matrix, modeling depth	Lines, circles, simple forms

SEGMENTATION & CLASSIFICATION

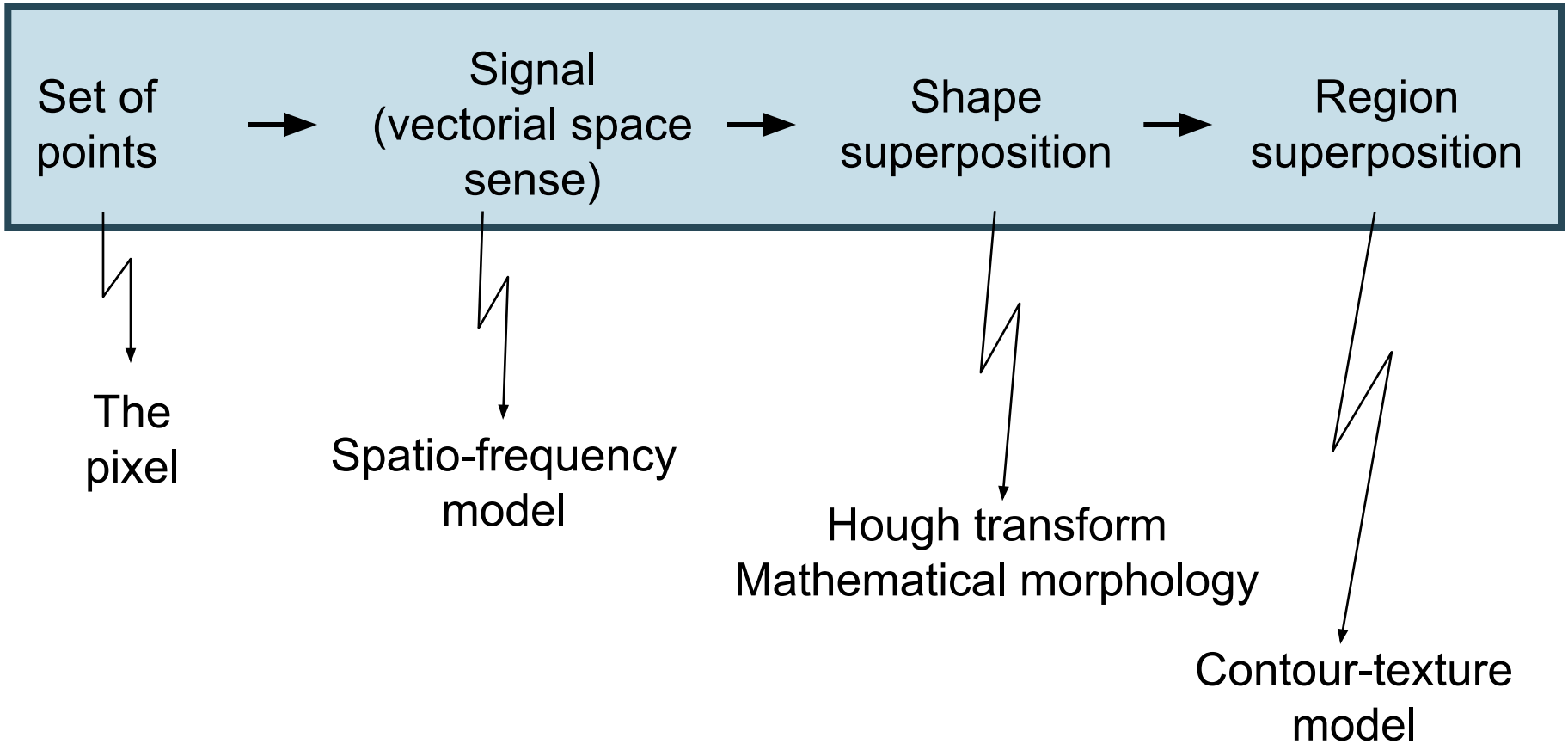
Segmentation: objects & superposition model



Object: Shape
Texture (Gray level, color)

Nonlinear superposition principle
↓
Previous sections: Shapes
↓
But ultimately, objects are represented as **regions**

Segmentation: Image interpretation

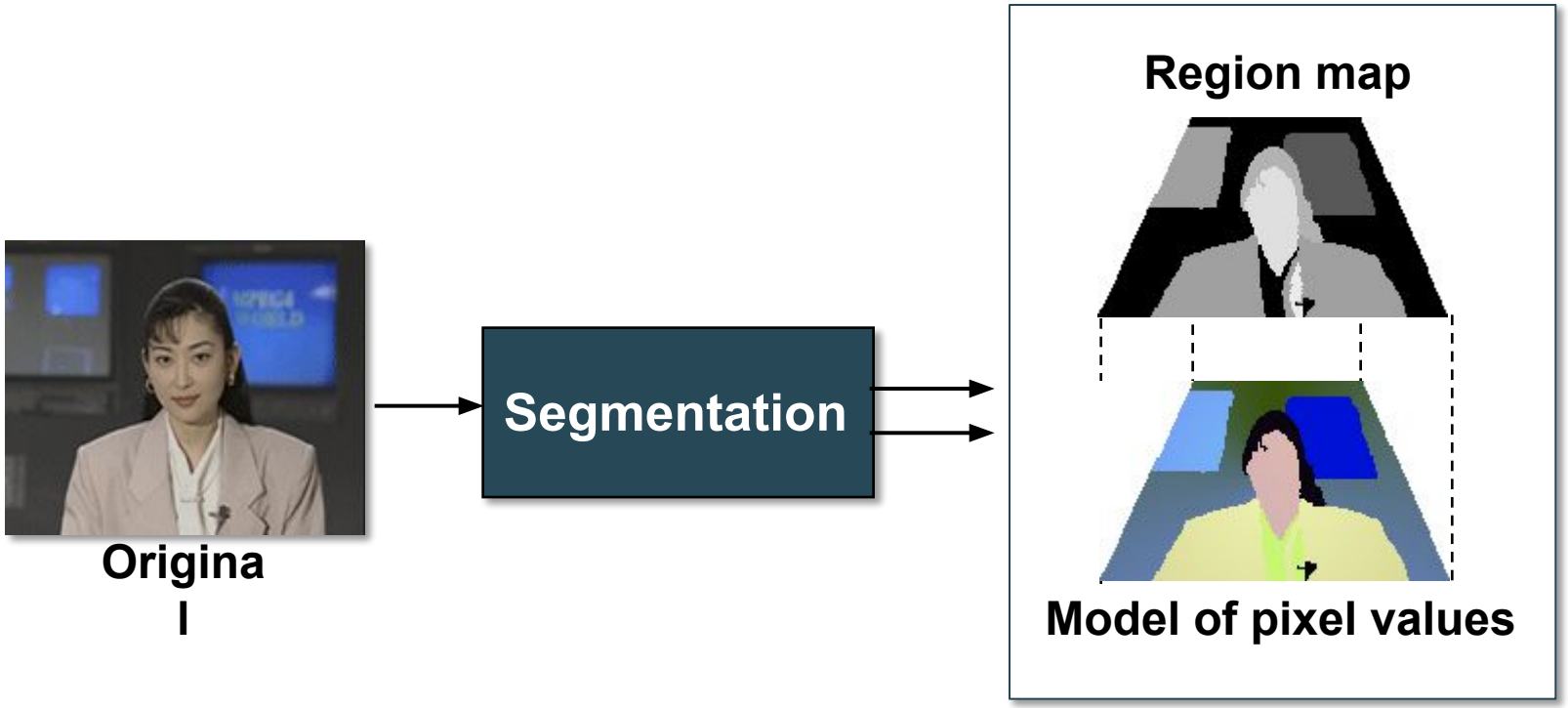


Segmentation

- The contour-texture model
- Segmentation definition
- General segmentation scheme
- Segmentation approaches:
  - Transition-based
  - Region-based

Segmentation: The contour-texture model

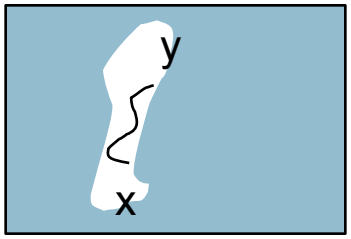
- Images can be understood as a set of patches forming a **region map** (**contour information**), where the interior of each region can be represented by a **pixel model** (**texture information**):



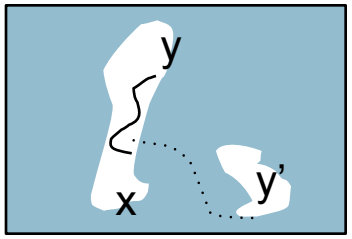
Segmentation: Regions

- A region is a **connected component** of the map:
  - Map value indicates the pixel membership (label image)
  - Regions have closed boundaries

- **Connected component**:
  - A set of pixels with the same label such that there exists a path, that is, a succession of **neighboring pixels**, **connecting them**.



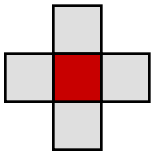
1(2) connected component(s)

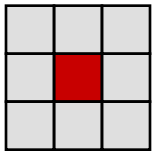


2(3) connected components

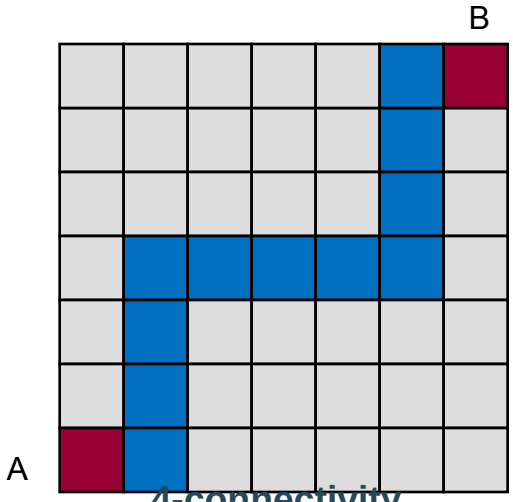
Segmentation: Regions

- What is a neighboring pixel in a discrete image?
  - **4-connectivity** **8-connectivity**



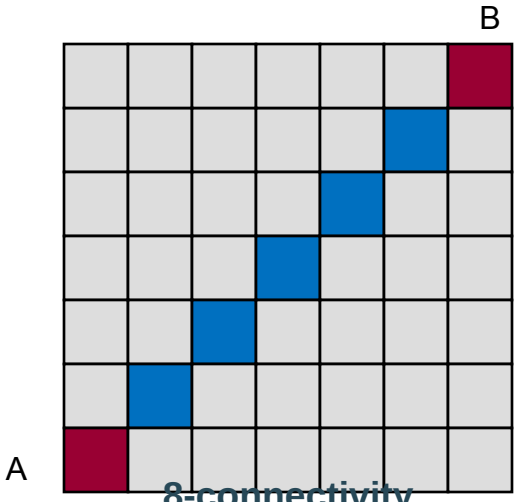


- Examples of connected paths:



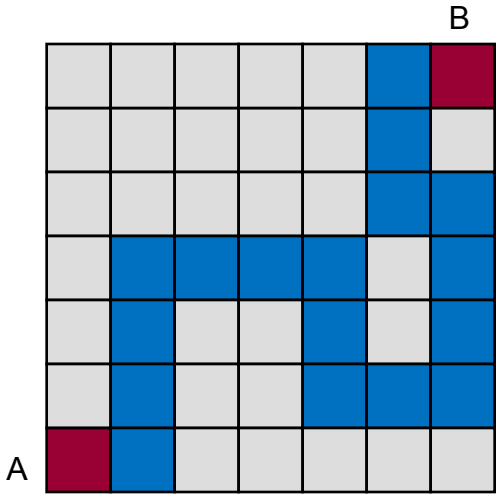
4-connectivity

Also valid as 8-connected path



8-connectivity

Not valid as 4-connected path



4-connectivity: 1 path

8-connectivity: various possible paths

Segmentation: Definition

- Create a **partition** of image I: $\{R_i\}$

$$\begin{cases} \bigcup R_i = \text{Image} \\ R_i \bigcap R_j = \varnothing, \qquad \forall i, j \end{cases}$$

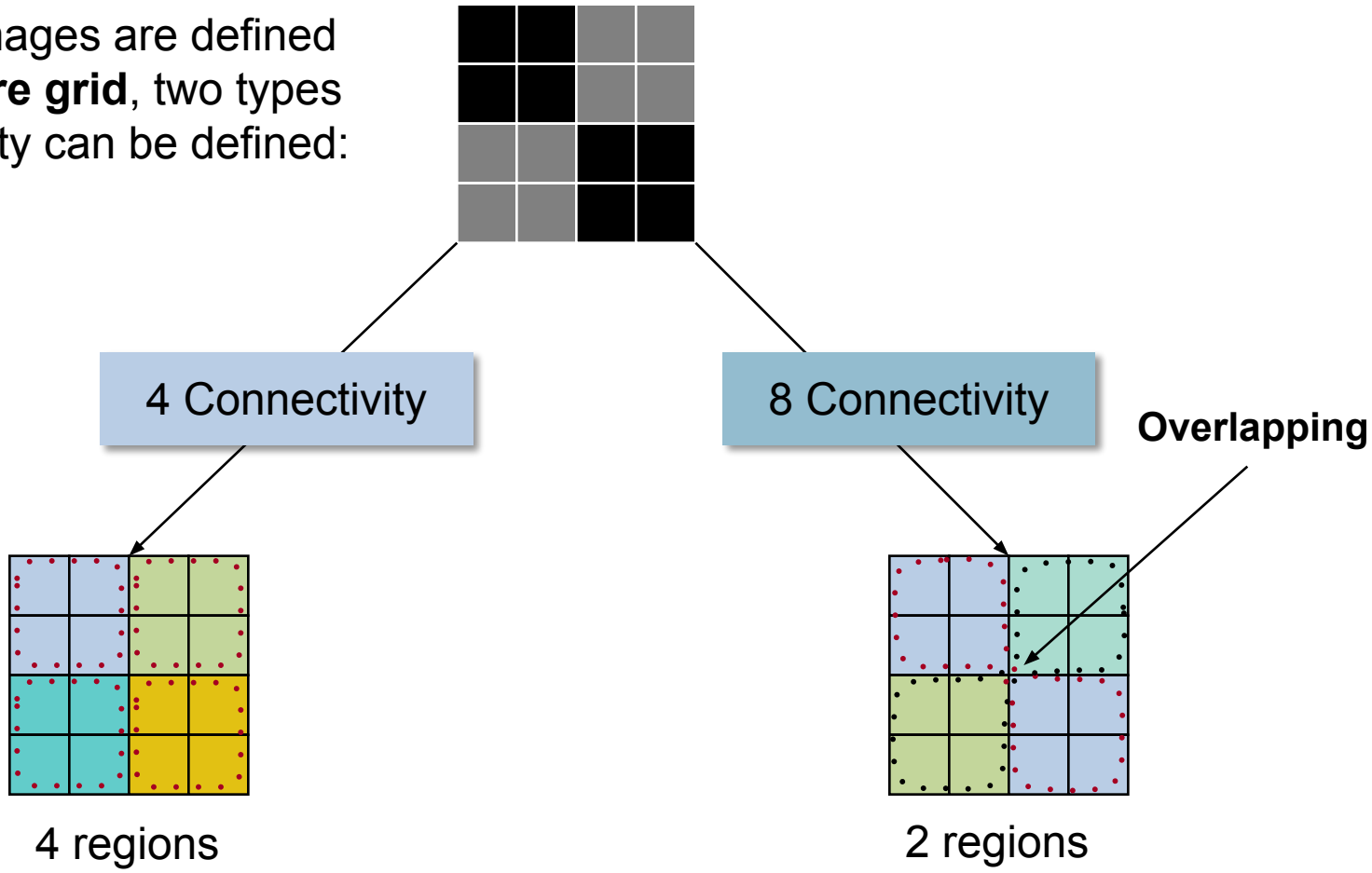
... of **connected regions** R_i, R_j that optimize a **criterion** C:

$$\begin{cases} C(R_i) = \text{True}, \quad \forall i \\ C(R_i \cup R_j) = \text{False}, \qquad \forall i, j \end{cases}$$

- What is a **good criterion** for segmentation
- How to **optimize** it?

Segmentation: Definition

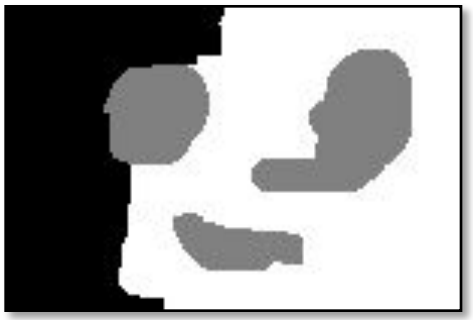
Assuming images are defined over a **square grid**, two types of connectivity can be defined:



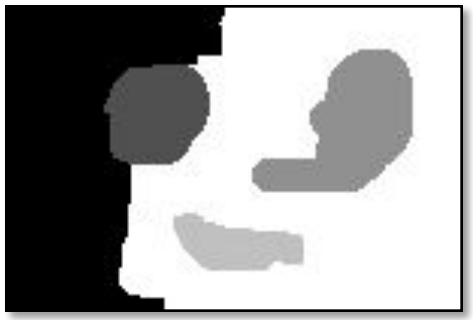
➔ **For segmentation**, in general: 4 connectivity

Segmentation vs. classification

Classification: Segmentation with regions, called **classes**, that are not necessarily connected.



Classification



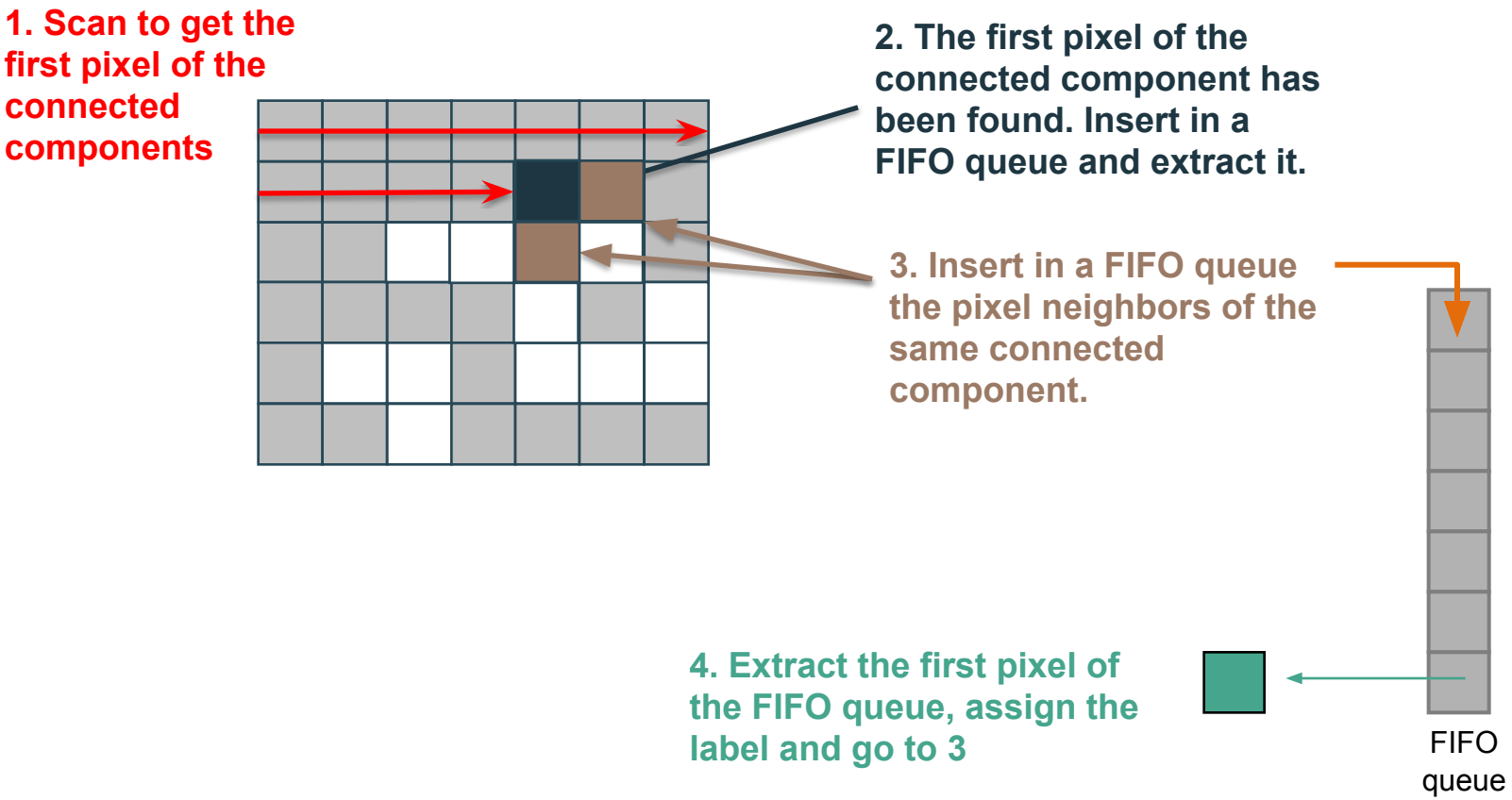
Segmentation
(image partition)

Labeling

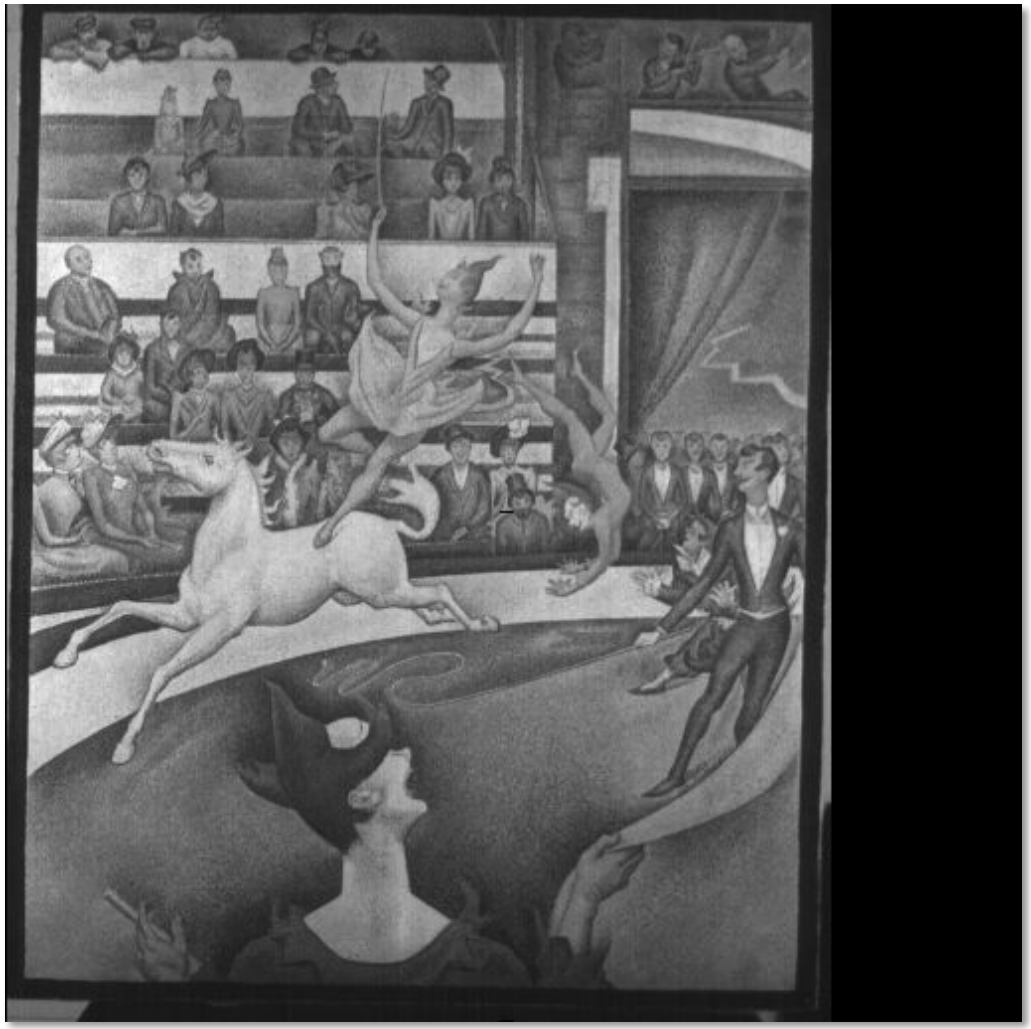
Segmentation

- **Labeling process**: Assign a different label to **each connected component**

    1. Search for a connected component
    2. Completely label the connected component

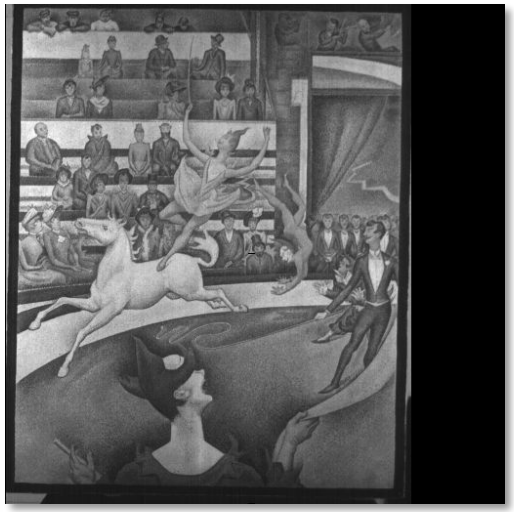


Segmentation: Evaluation



- (Extremely) difficult
- Application dependant
- Often deceiving

Segmentation: Evaluation



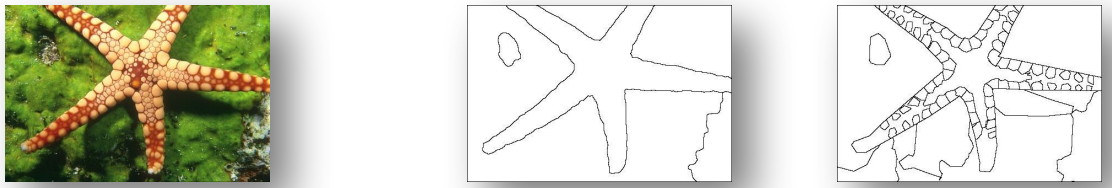
Best segmentation?

Segmentation

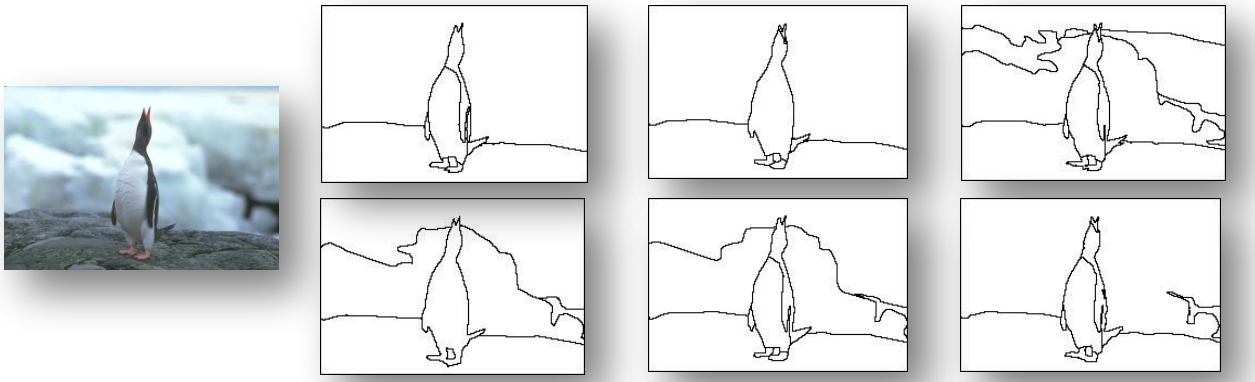
Segmentation is **an ill posed problem**:

- A unique solution does not exist and the partition definition depends on the final application

Different levels of detail



Same level of detail



Segmentation: General Scheme

- Remove useless or annoying information
- Preserve shape information

- Feature space:
  - Gray level
  - Color
  - Variance
  - Frequency component
- Pixels belonging to the same regions should correspond to homogeneous features

- Partition definition:
  - Transition
  - Homogeneity

Segmentation: Simplification

Image with impulsive noise

Image with artifacts

Median

Opening

Classical filters:

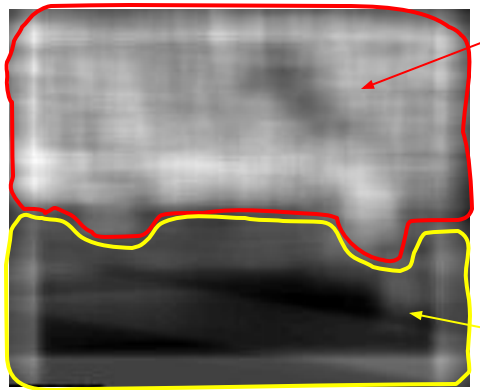
- Low-pass
- Median
- Opening / closing
- Op. reconstruction
- Alternating filters

Segmentation: Feature extraction

Feature space:

- Gray level
- Color
- Variance
- Frequency
- Activity

Original



Variance

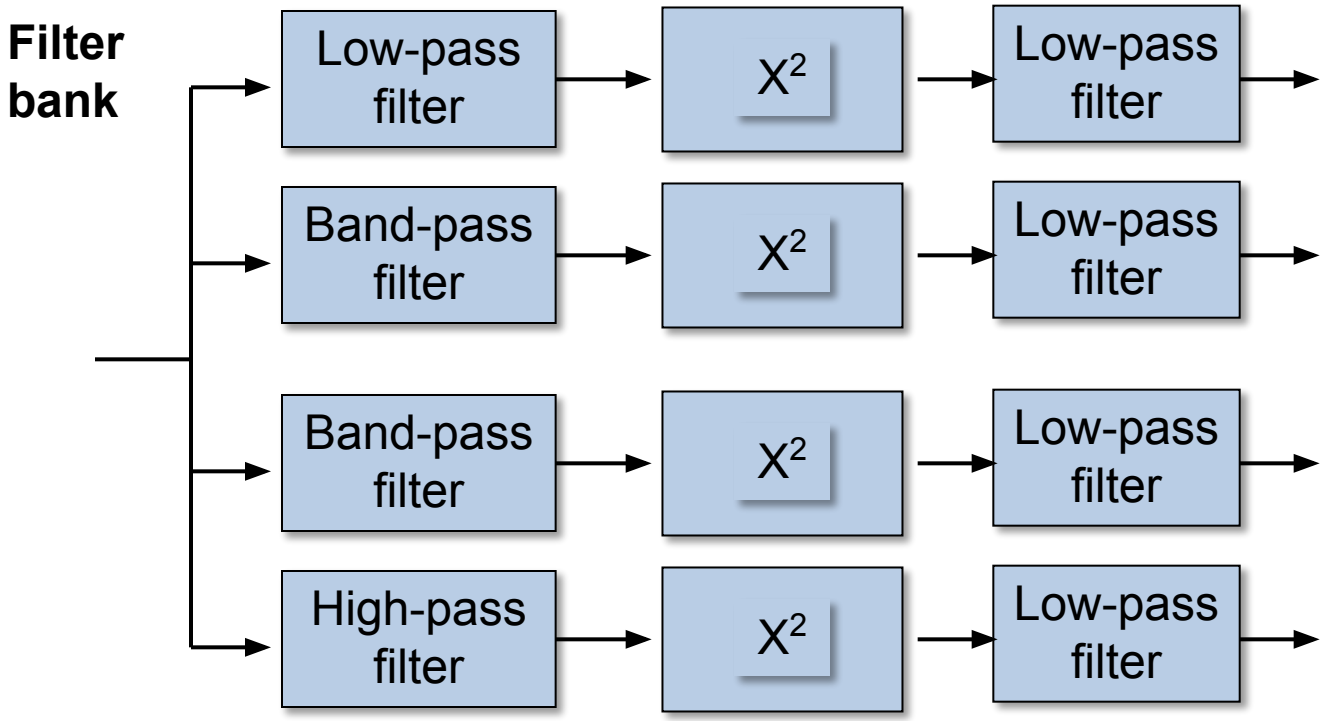
Spectators

Ground

Segmentation: Feature extraction

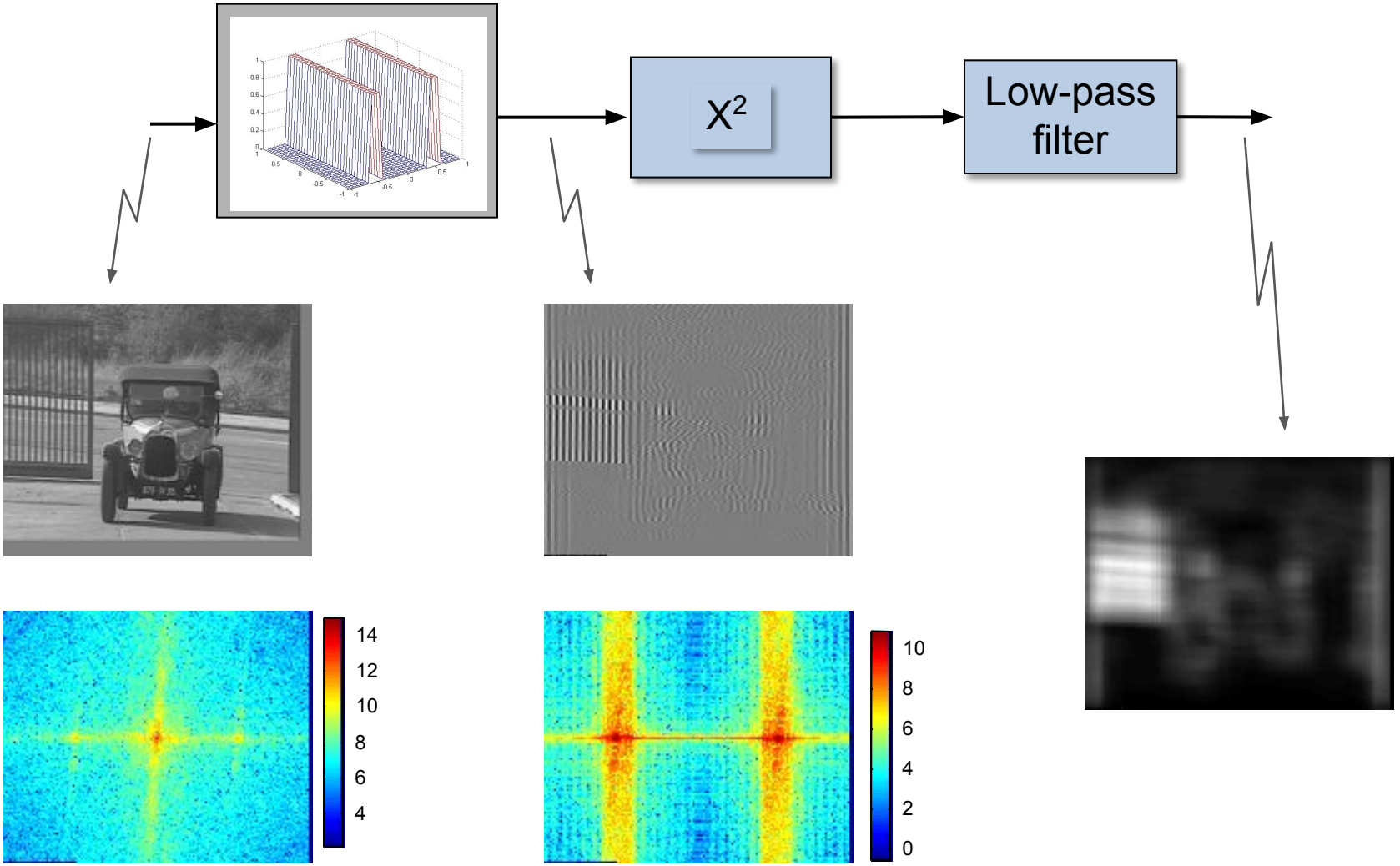
Frequency analysis

- Estimate the **energy** in a given frequency band



Segmentation: Feature extraction

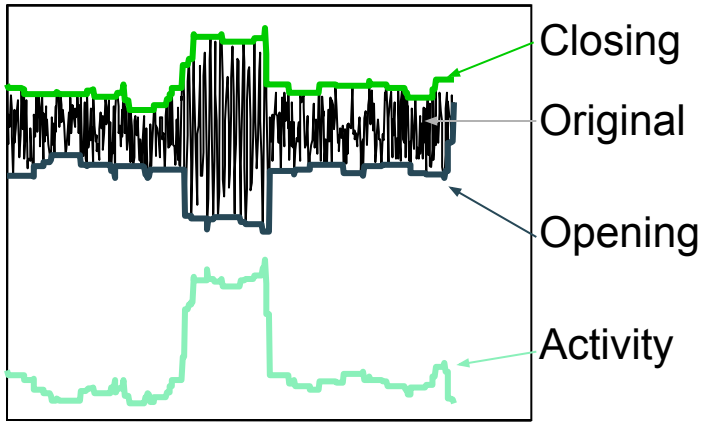
Frequency analysis



Segmentation: Feature extraction

Activity

- Gray level fluctuation
- Difference between opening and closing



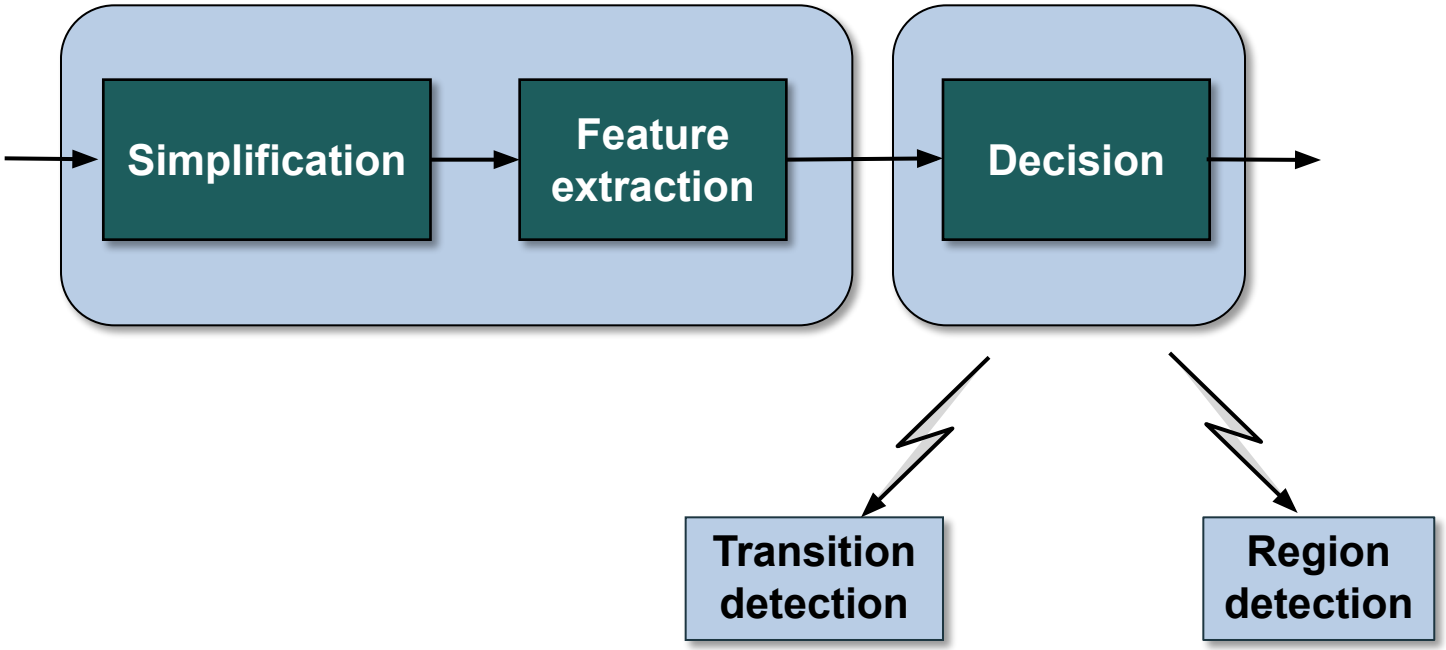
Original

Closing

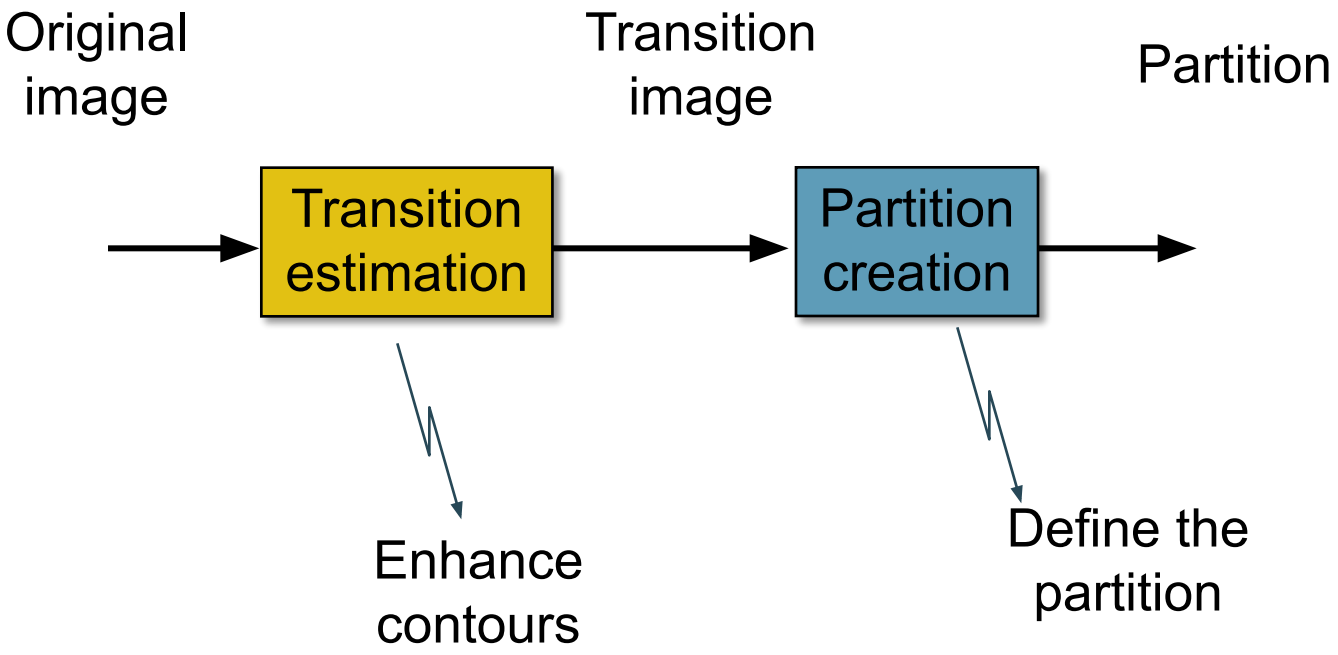
Opening

Activity

Segmentation: Overview



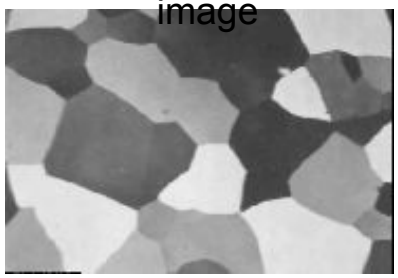
Segmentation: Transition based



Segmentation: Transition based

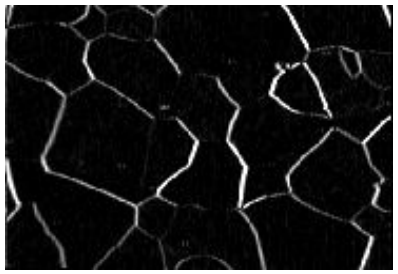
Example: Sobel

Original image



−1	0	1
−2	0	2
−1	0	1

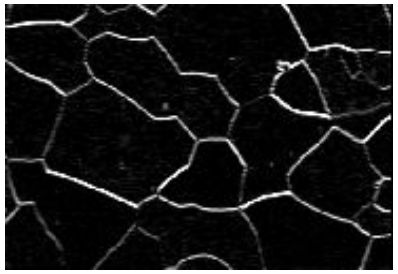
Vertical contours



$| h_1 * f |$

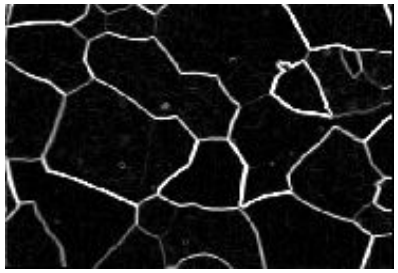
−1	−2	−1
0	0	0
1	2	1

Horizontal contours



$| h_2 * f |$

Combined information: gradient

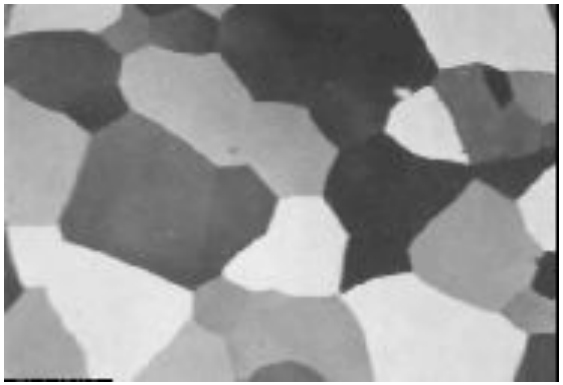


$| h_1 * f | \vee | h_2 * f |$

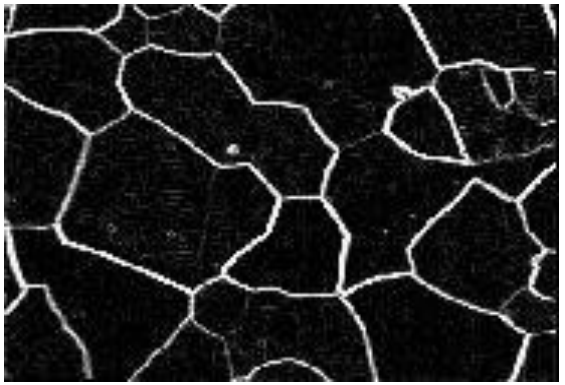
$\mathrm{Sqrt}(| h_1 * f |^2 + | h_2 * f |^2)$

Segmentation: Transition based

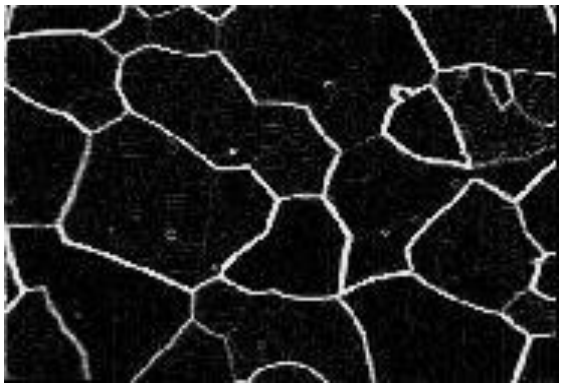
Example: Morphological gradient



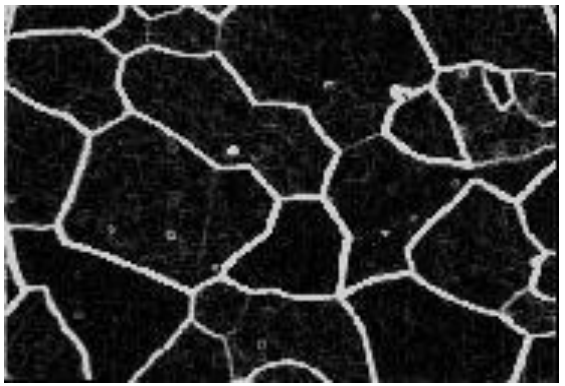
x



x - (x Θ b)

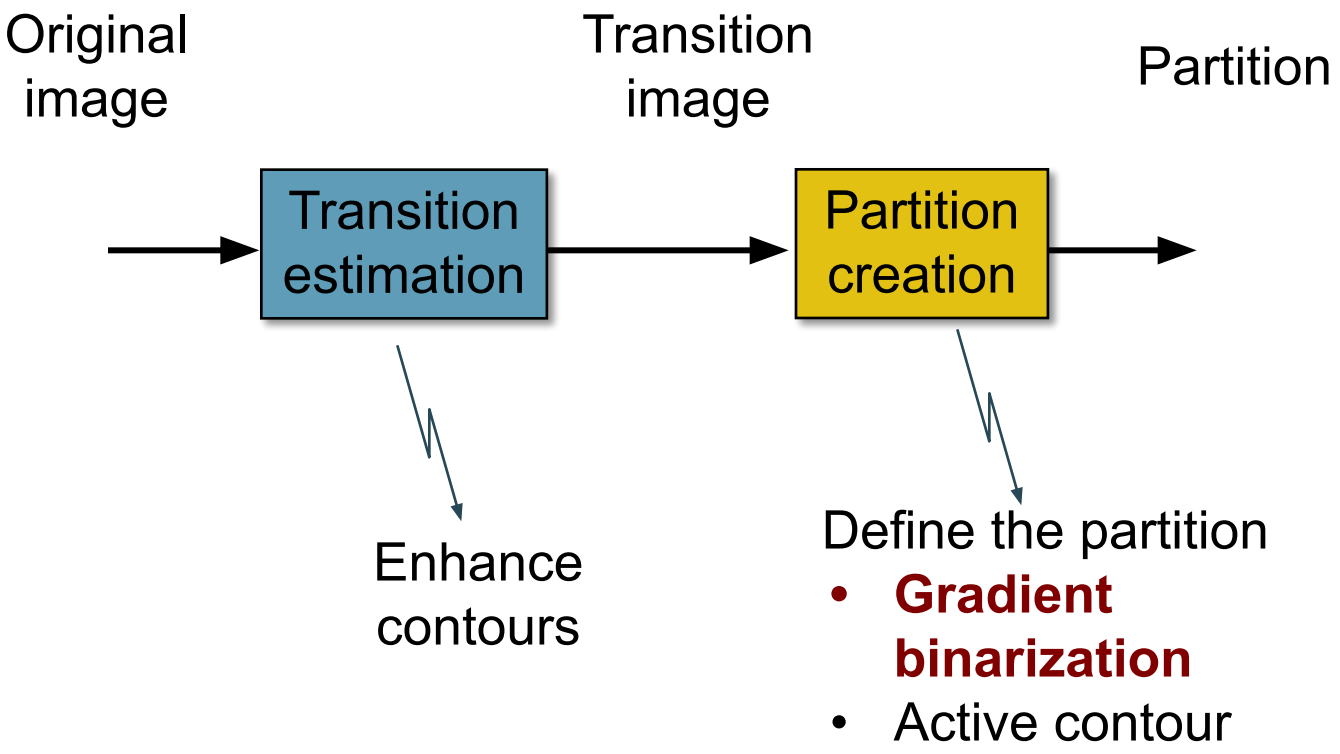


(x ⊕ b) - x



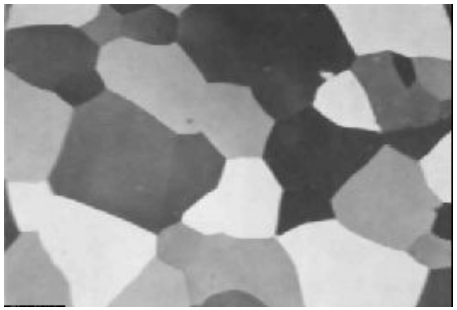
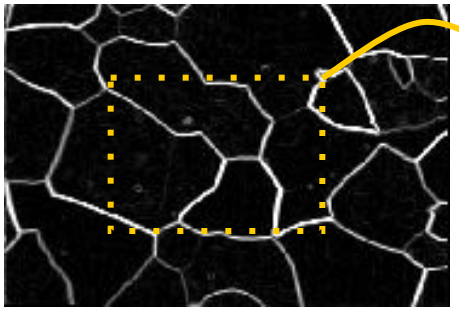
(x ⊕ b) - (x Θ b)

Segmentation: Transition based



Segmentation: Transition based

- Straightforward strategy: Gradient binarization

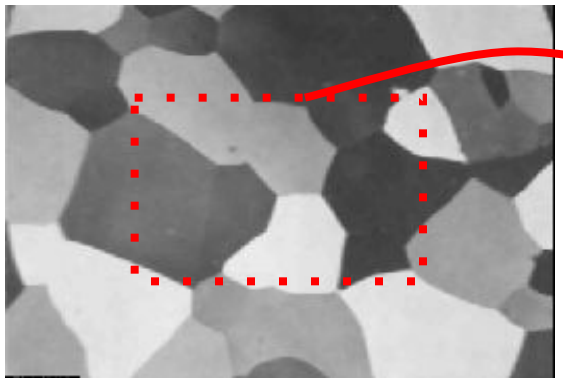




Issues to consider:

- Difficulty to define the threshold
- Contours may not be closed
- Noise may appear
- Contours may be thick

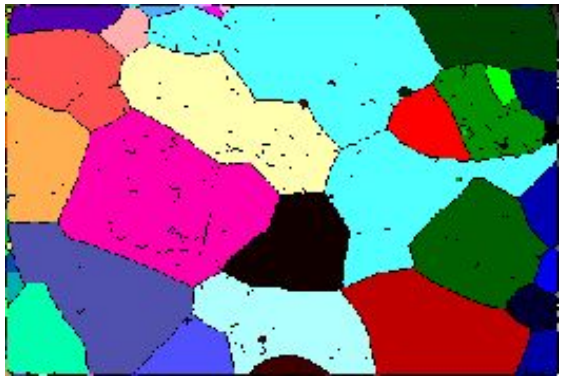
Segmentation: Transition based

Gradient binarization



Original

Gradient binarization



Labeling of regions

Closing + Thinning

Useful in simple cases, but lack of robustness