Complex Networks

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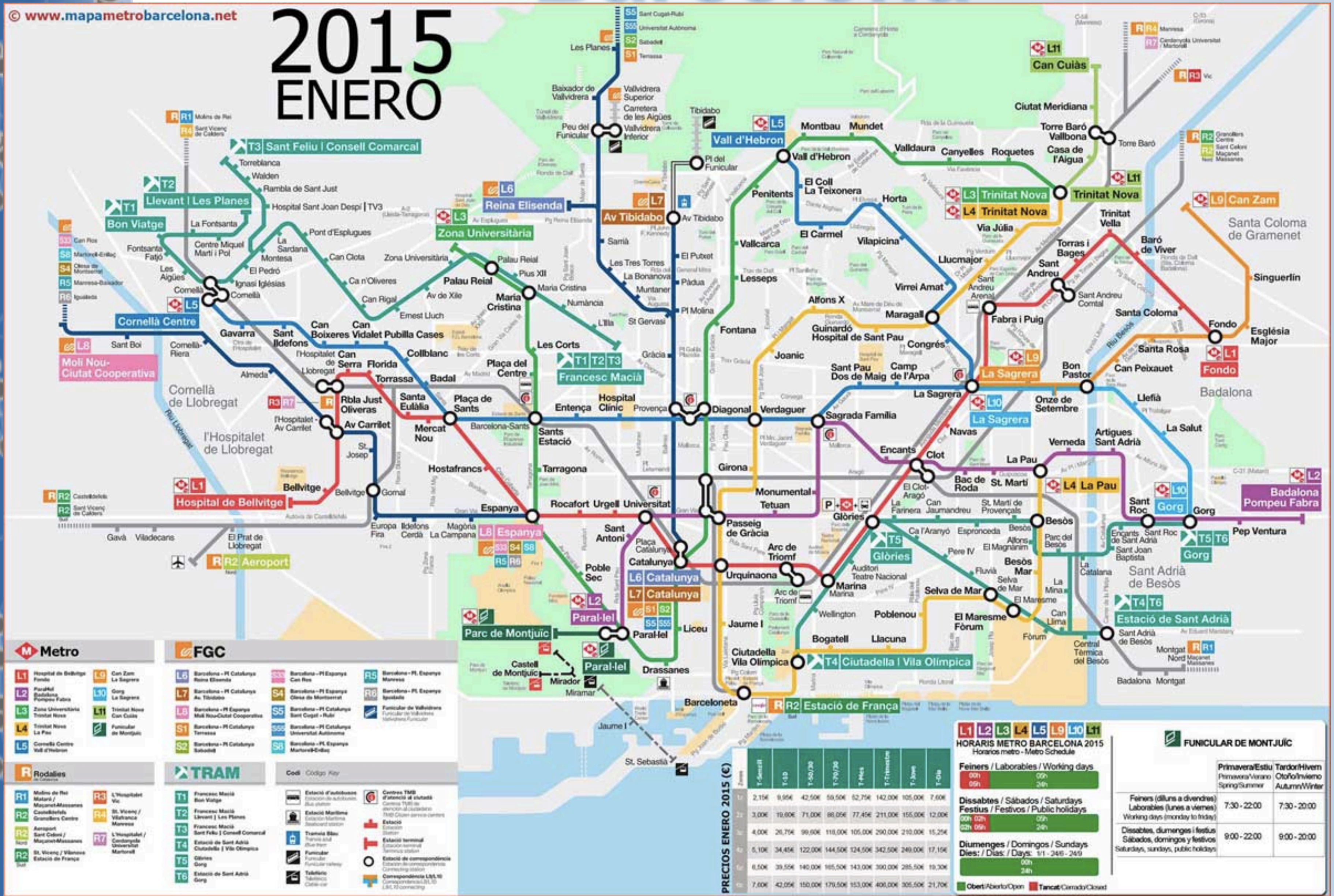
Network Data

Current concept of a network (with physical substrate)

- Structure (complex) in which something flows:
  - Energy
  - Information
  - Passengers
  - Capital
  - Viruses
- There are other networks where the substrate is some sort of virtual relation
  - WWW vs. Internet

Barcelona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2015
ENERO
Metro
FGC
Rodalies
TRAM
T3 Sant Feliu | Consell Comarcal
T2 Llevant | Les Planes
T1 Bon Viatge
L5 Cornellà Centre
L8 Molí Nou-Ciutat Cooperativa
L1 Hospital de Bellvitge
R2 Aeroport
L6 Reina Elisenda
L3 Zona Universitària
L7 Av Tibidabo
L5 Vall d'Hebron
L11 Can Cuiàs
L3 Trinitat Nova
L4 Trinitat Nova
L9 Can Zam
L1 Fondo
L9 La Sagrera
L10 La Sagrera
L4 La Pau
L2 Badalona Pompeu Fabra
L10 Gorg
T5 T6 Gorg
T5 Glòries
T4 T6 Estació de Sant Adrià
T4 Ciutadella | Vila Olímpica
R2 Estació de França
T1 T2 T3 Francesc Macià
L8 Espanya
L6 Catalunya
L7 Catalunya
L2 Paral·lel
Parc de Montjuïc
Paral·lel
Santa Coloma de Gramenet
Badalona
Sant Adrià de Besòs
Cornellà de Llobregat
l'Hospitalet de Llobregat
HORARIS METRO BARCELONA 2015
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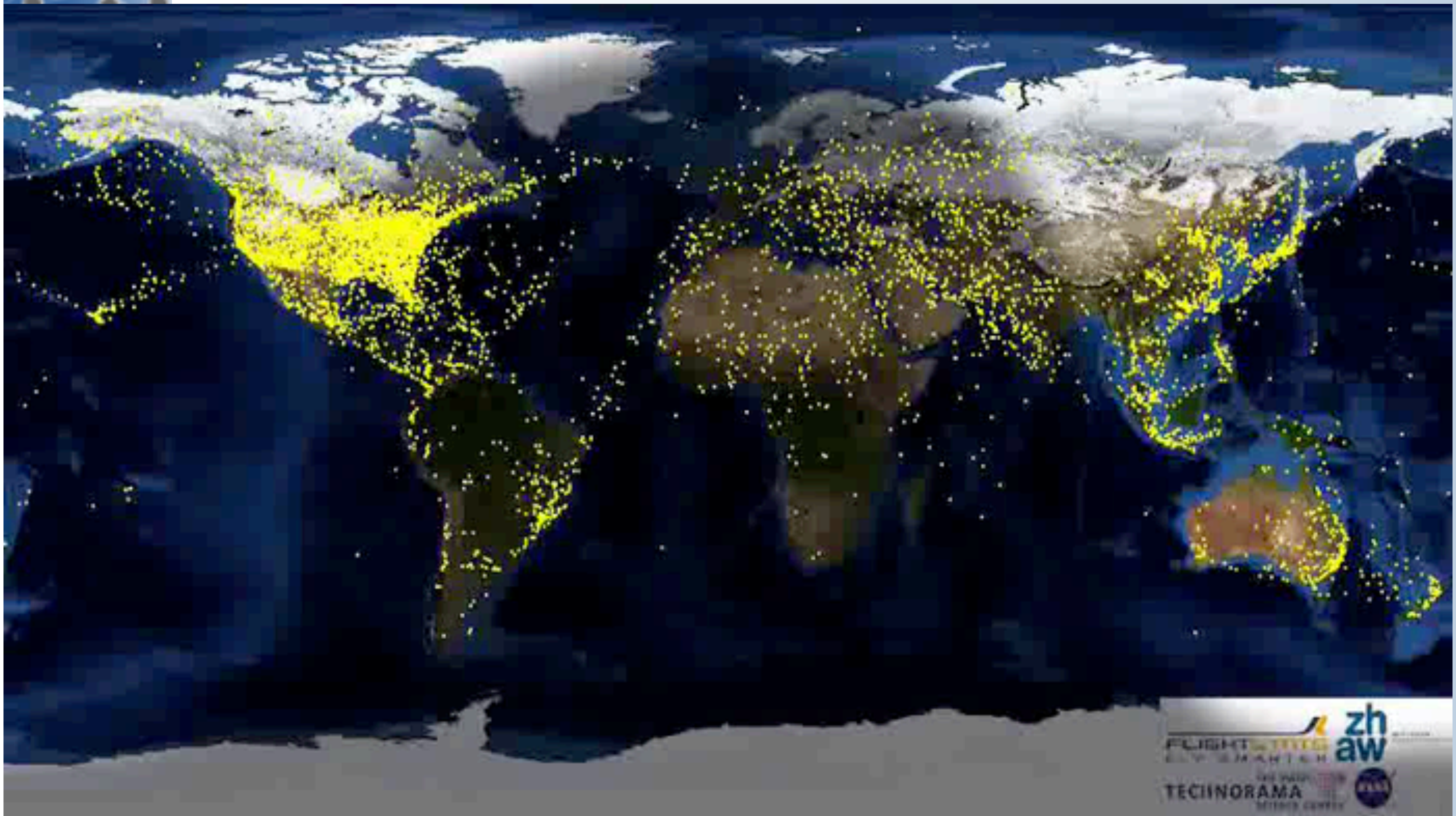
PRECIOS ENERO 2015 (€)

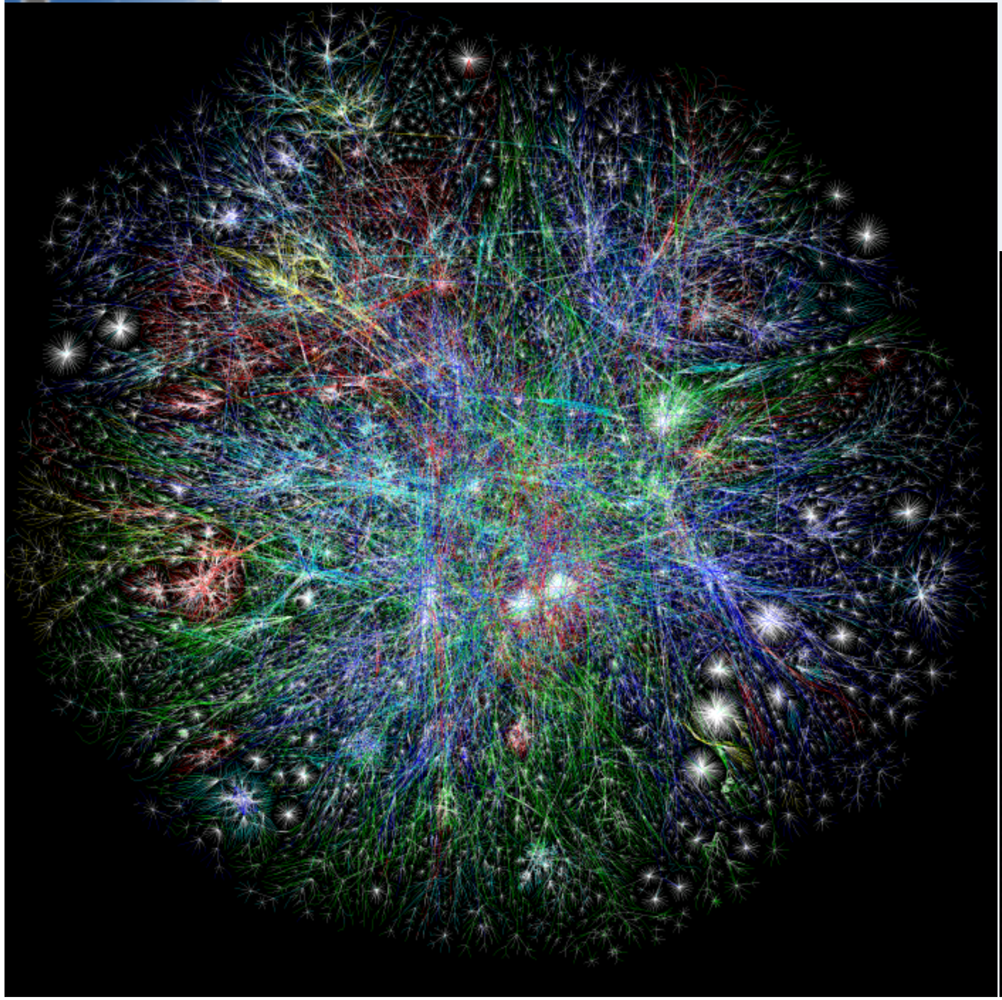
Power-grid

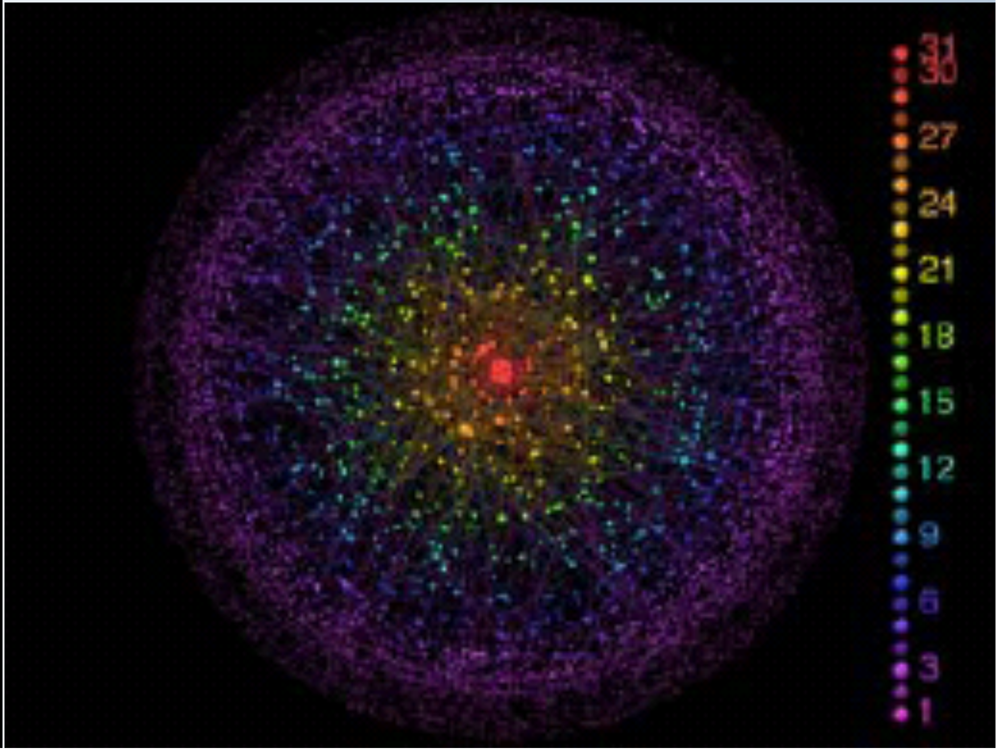
Aerial transportation networks



https://vimeo.com/147174695

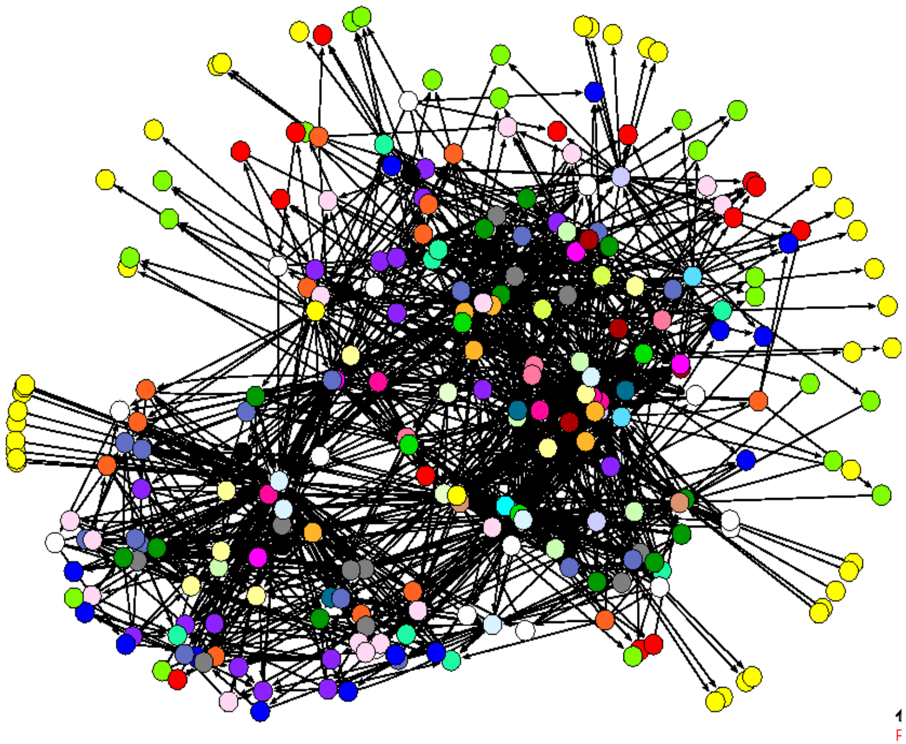
Internet





Neural network of C. Elegans

- Nodes are neurons
- Links are synapses



Networks

- Nodes: subjects
- Links: relations between agents
- Transportation networks:
  - Nodes: cities, computers, airports, …
  - Link: cables, tracks, ….

Social networks

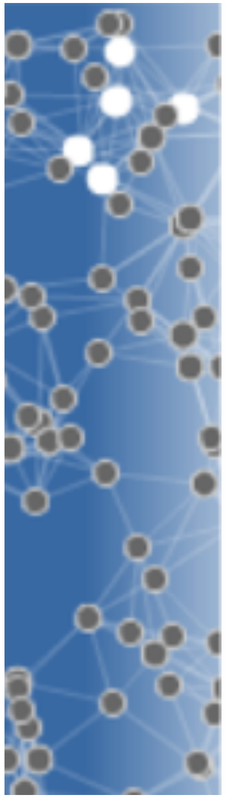
- Nodes: individuals or groups
- Links: ??????
    - Friendship
    - Business
    - Family
    - Articles co-authorship
    - Co-starring in movies (actors/actresses)
    - Board of directors
    - Sexual relations
    - Cybercommunities: TWitter, blogs, Facebook, Instagram, ….
    - Phone calls
    - ....

How do we stablish te links in a social network?

- Questionaires
- Biased

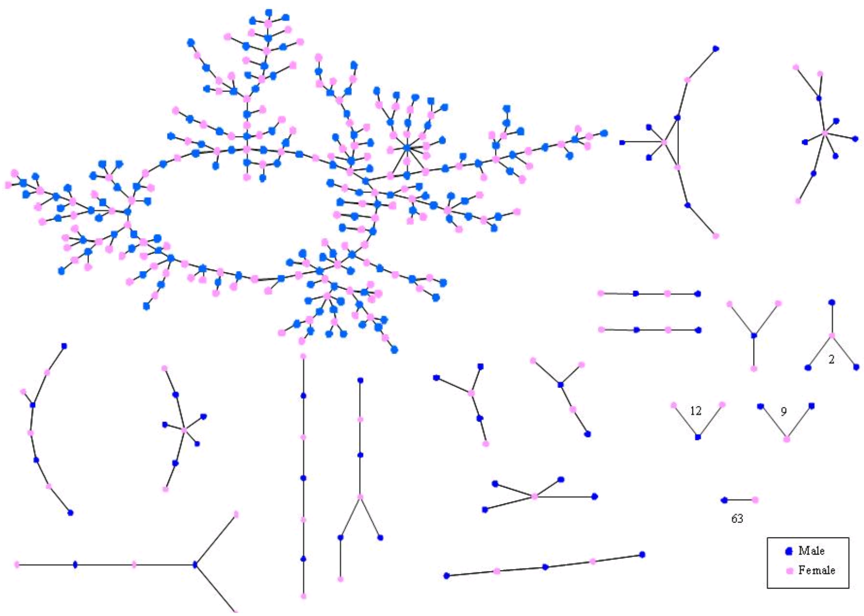
- Acces to large databases:
  - Mobile phone users for some European operator
  - Microsoft Messenger users
  - Twitter followers / retweets / likes

- More powerful computers to extract the relevant relationships between the data



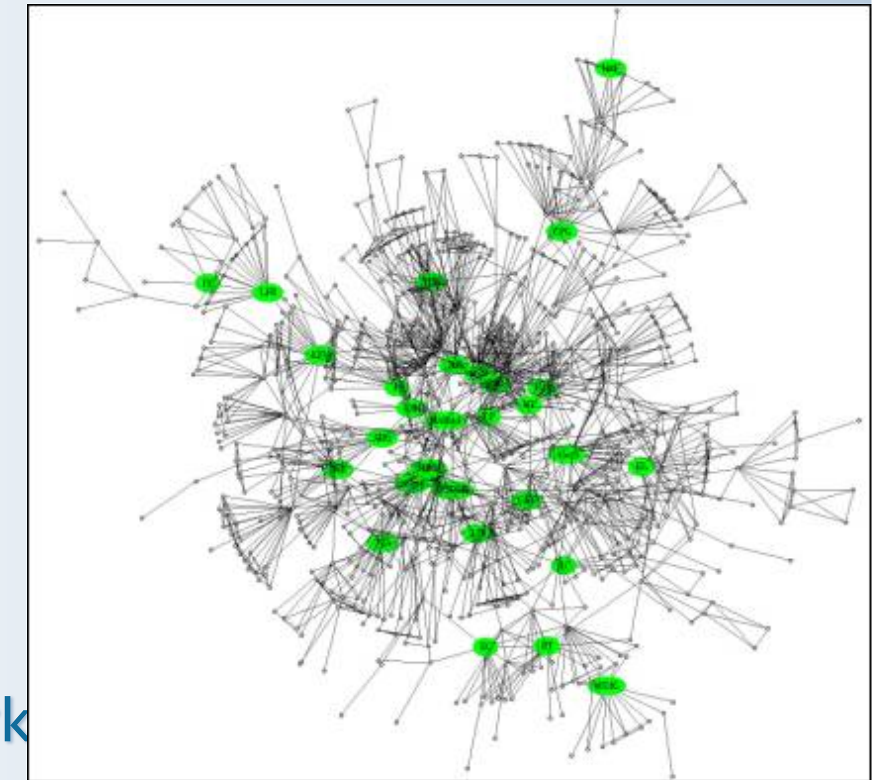
Small social network

- Role of the nodes
- Which is the most important?



Social I: Networks of collaboration

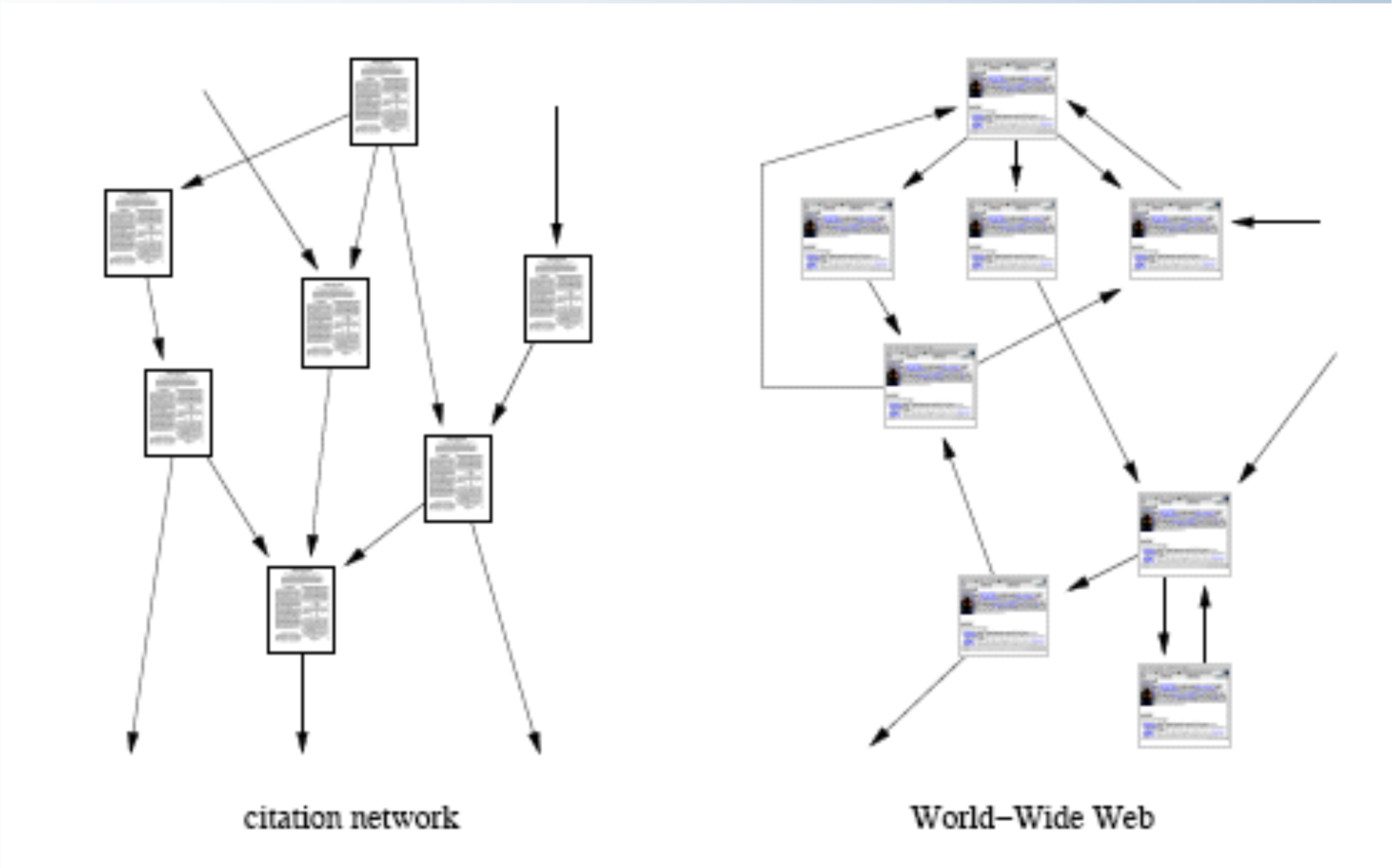
- Through collaboration acts
- Examples:
  - movie actor
  - board of directors
  - musicians
  - scientific collaboration network (MEDLINE, Mathematical, neuroscience, e-archives,..)



$\Rightarrow$Erdös number

Information networks

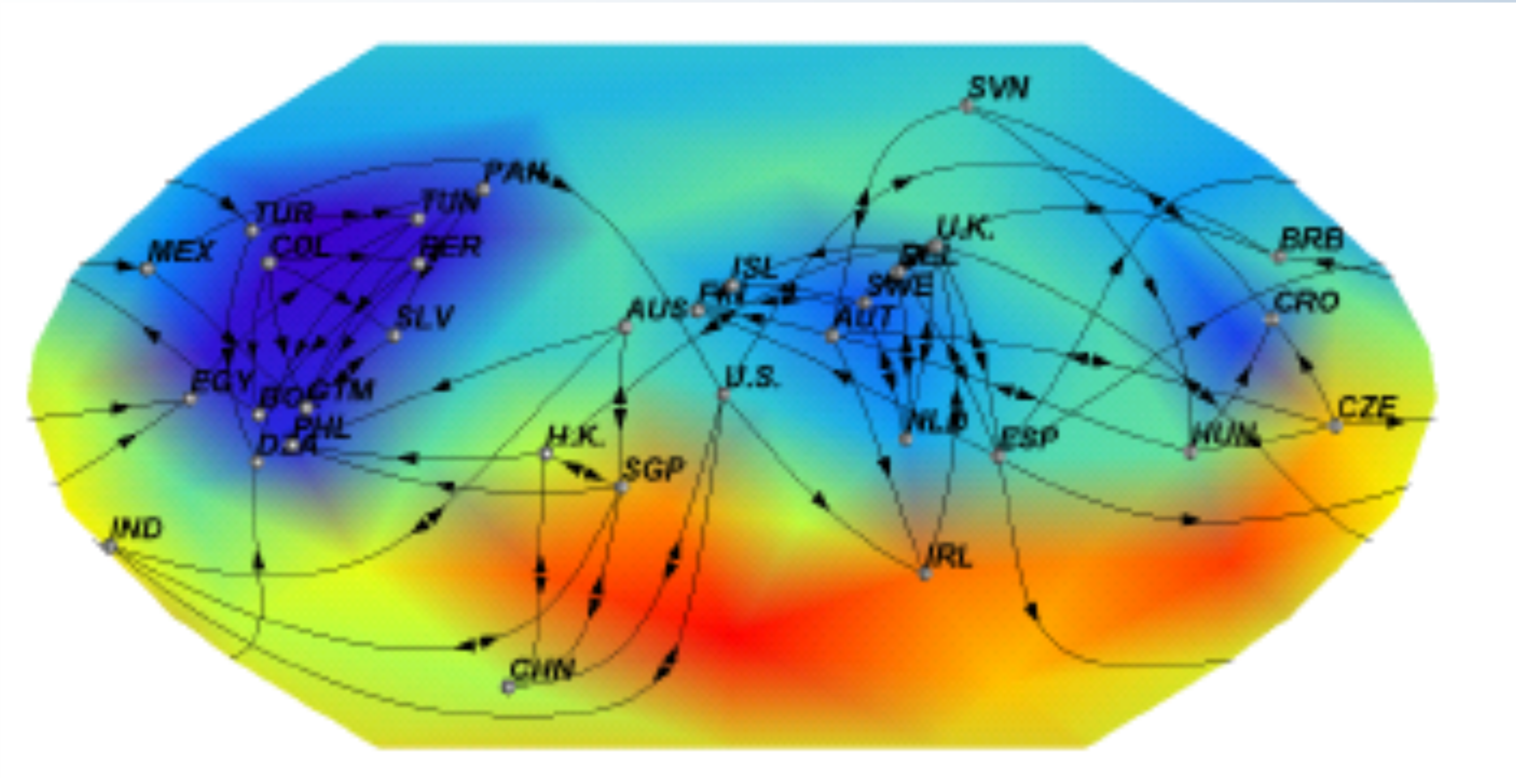
- The World-Wide Web
- Citation networks



Economical systems

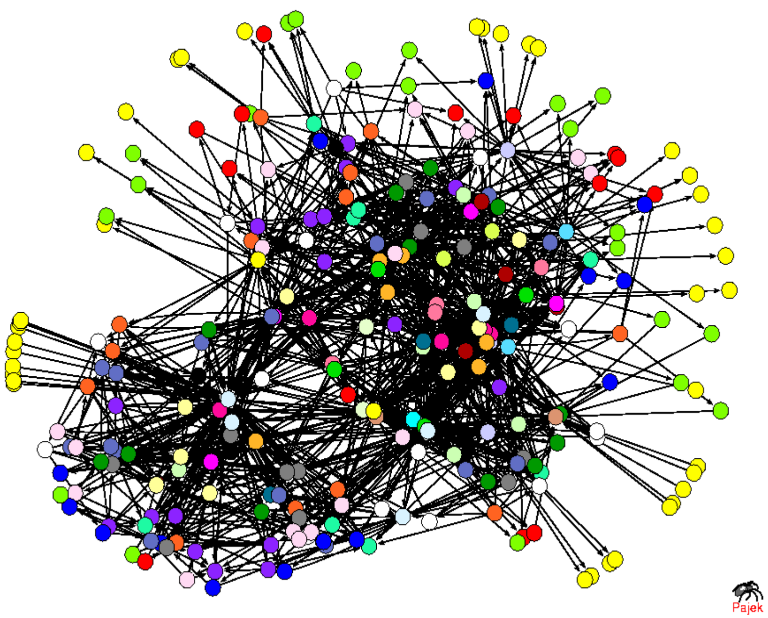
- Relations between stocks
- Relations between economic agents: banks, customers, …
- World Trade Web

Directly measurable connectivities



Biological networks

- Neural networks: neurons – synapses
- Metabolic reactions: compounds – reactions
- Protein networks: protein-protein interaction
- Food-webs: predator-prey (directed)
- Genetic regulatory networks



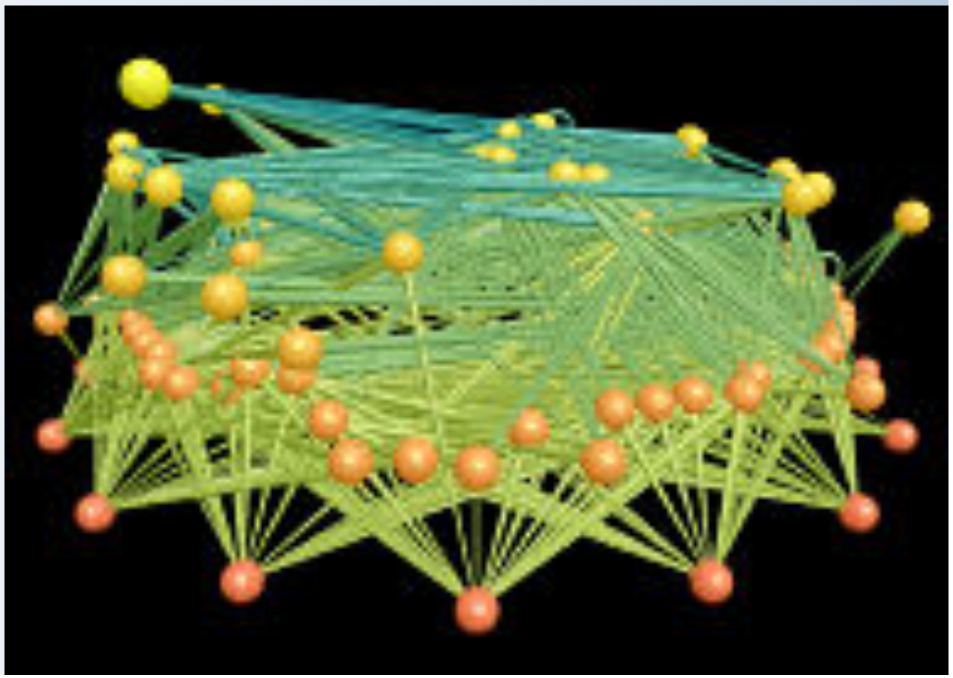


Table 2.1 **Canonical Network Maps**

The basic characteristics of ten networks used throughout this book to illustrate the tools of network science. The table lists the nature of their nodes and links, indicating if links are directed or undirected, the number of nodes (N) and links (L), and the average degree for each network. For directed networks the average degree shown is the average number of in- or out-degrees $\langle k \rangle = \langle k_{in} \rangle = \langle k_{out} \rangle$ (see equation **(2.5)**).

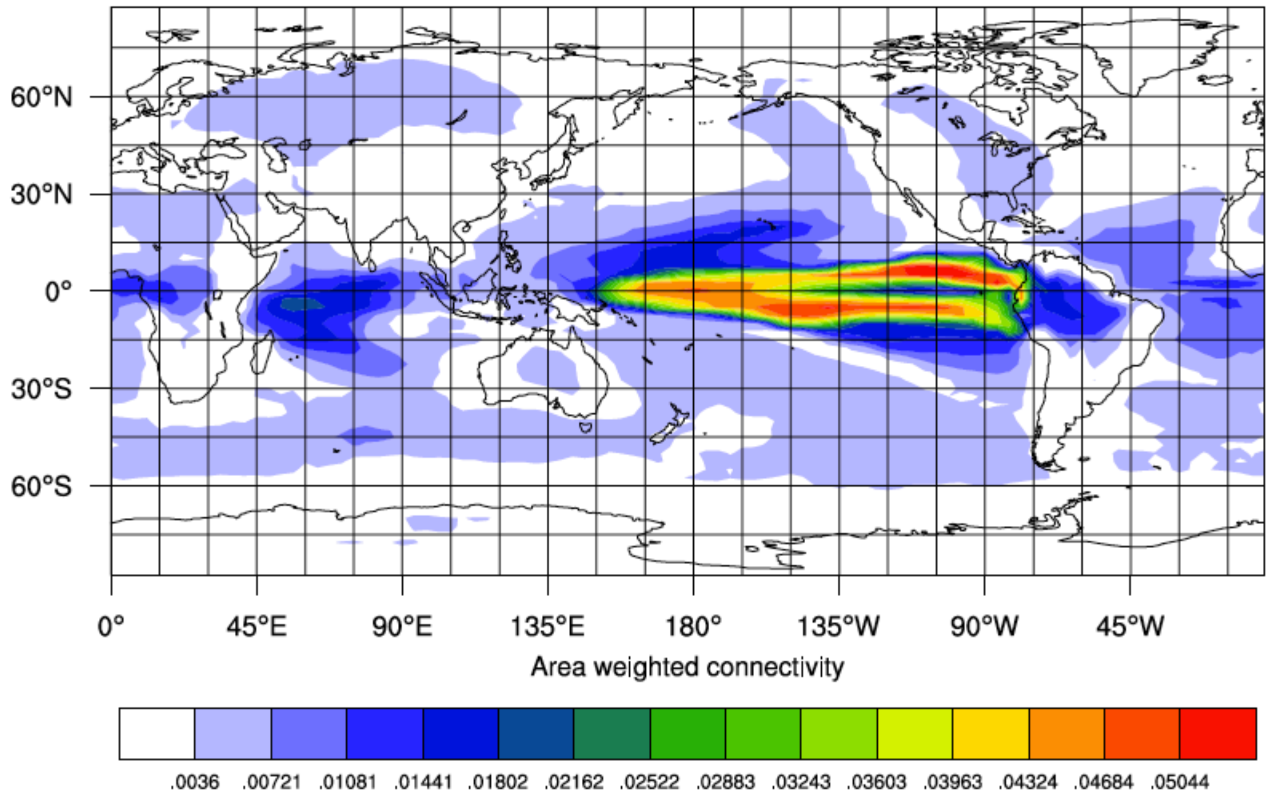
Network	Nodes	Links	Directed or undirected	N	L	$\langle k \rangle$
Internet	Routers	Internet connections	Undirected	192,244	609,066	6.34
WWW	Webpages	Links	Directed	325,729	1,497,134	4.60
Power grid	Power plants, transformers	Cables	Undirected	4,941	6,594	2.67
Mobile phone calls	Subscribers	Calls	Directed	36,595	91,826	2.51
Email	Email addresses	Emails	Directed	57,194	103,731	1.81
Science collaboration	Scientists	Co-authorship	Undirected	23,133	93,439	8.08
Actor network	Actors	Co-acting	Undirected	702,388	29,397,908	83.71
Citation network	Papers	Citations	Directed	449,673	4,689,479	10.43
***E. coli* metabolism**	Metabolites	Chemical reactions	Directed	1,039	5,802	5.58
Protein interactions	Proteins	Binding interactions	Undirected	2,018	2,930	2.90

Networks in climatology

Complex networks reveal global pattern of extreme-rainfall teleconnections

Niklas Boers, Bedartha Goswami, Aljoscha Rheinwalt, Bodo Bookhagen, Brian Hoskins & Jürgen Kurths

Nature **566**, 373–377 (2019) | Download Citation

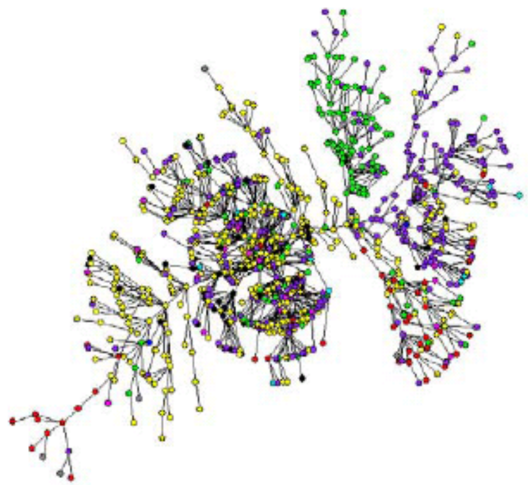


Indirect measures

- Correlations between dynamical properties: returns, GDP,...

$$\rho_{ij}(\Delta t) = \frac{\langle p_i p_j \rangle - \langle p_i \rangle \langle p_j \rangle}{\sqrt{(\langle p_i^2 \rangle - \langle p_i \rangle^2)(\langle p_j^2 \rangle - \langle p_j \rangle}}$$

$$d_{ij} = \sqrt{2(1 - \rho_{ij})}$$



Networks where there are neither nodes nor links????? Time series converted to networks

PRL **96,** 238701 (2006) PHYSICAL REVIEW LETTERS week ending 16 JUNE 2006

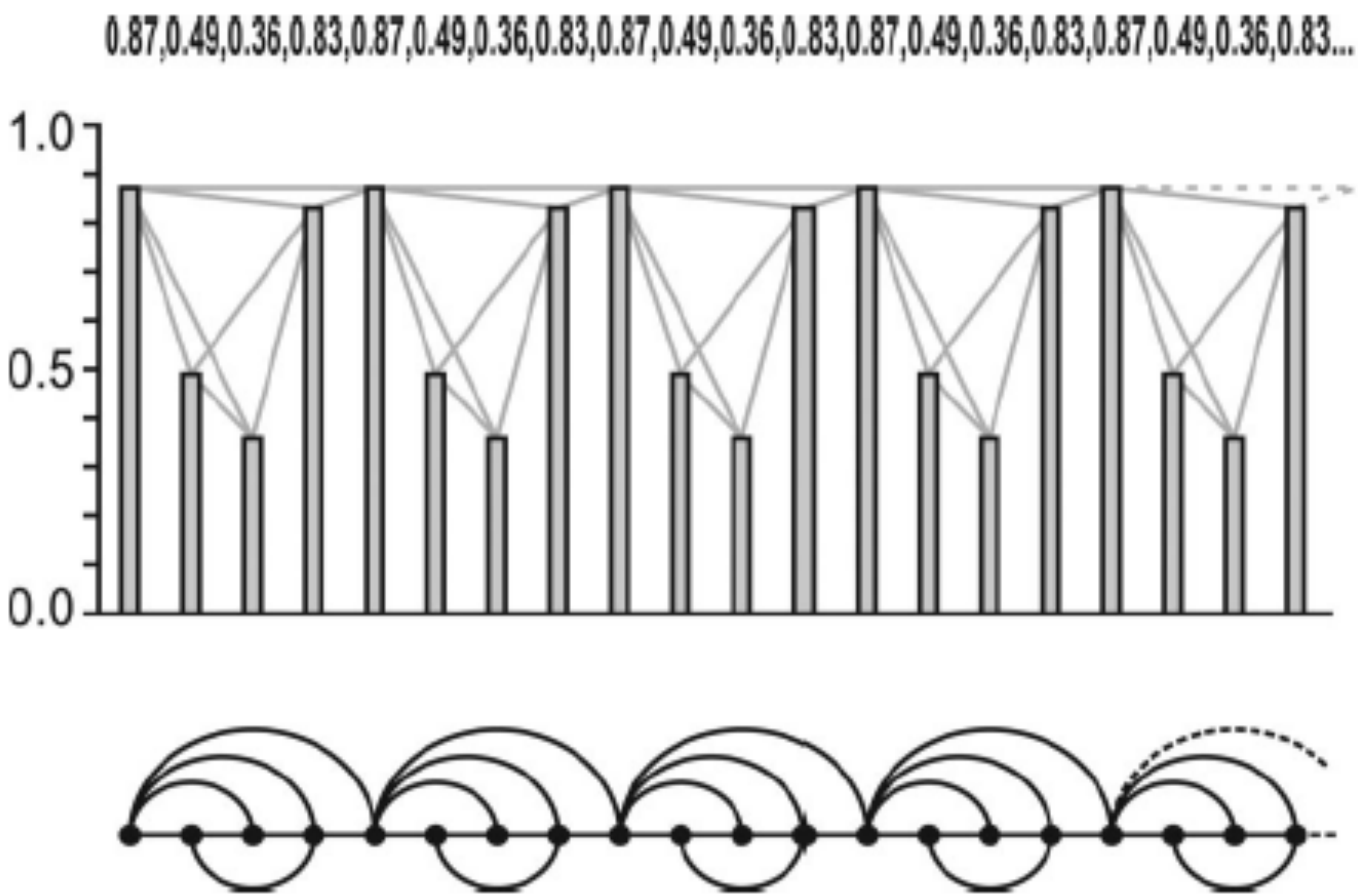
Complex Network from Pseudoperiodic Time Series: Topology versus Dynamics

J. Zhang and M. Small

We construct complex networks from pseudoperiodic time series, with each cycle represented by a single node in the network. We investigate the statistical properties of these networks for various time series and find that time series with different dynamics exhibit distinct topological structures. Specifically,

From time series to complex networks: The visibility graph

Lucas Lacasa*†, Bartolo Luque*, Fernando Ballesteros‡, Jordi Luque§, and Juan Carlos Nuño¶



Duality between time series and networks

Campanharo ASLO, Sirer MI, Malmgren RD, Ramos FM, Amaral LAN.

PLoS ONE 6, e23378 (2011)

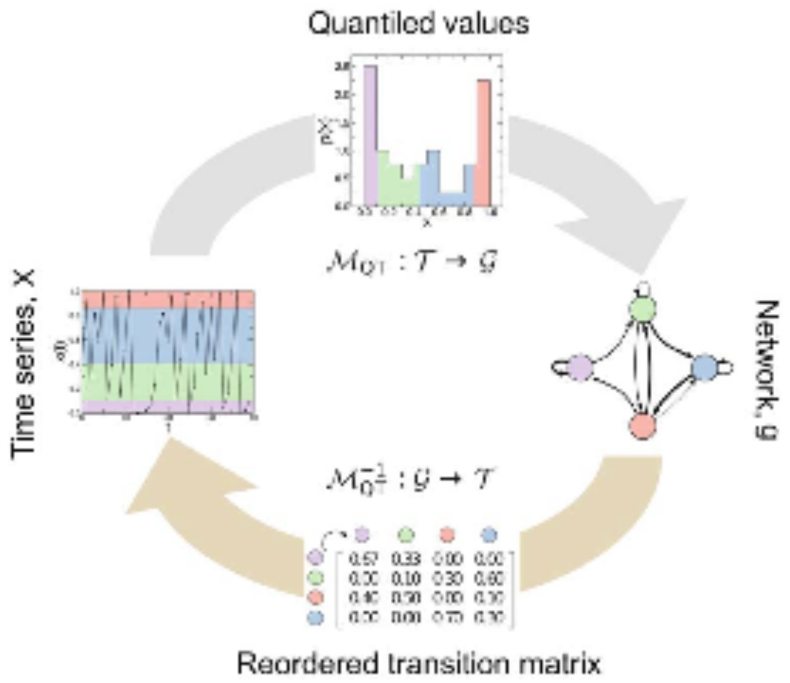


Figure 1. Illustration of the proposed map. *Forward map:* A time series X is split into $Q=4$ quantiles (colored shading) and each quantile q_i is assigned to a node $n_i \in \mathcal{N}$ in the corresponding network g. Two nodes n_i and n_j are then connected in the network with a weighted arc $(n_i, n_j, w_{ij}) \in \mathcal{A}$ where the weight w_{ij} of the arc is given by the probability that a point in quantile q_i is followed by a point in quantile q_j. Repeated transitions between quantiles results in arcs in the network with larger weights (represented by thicker lines). *Inverse map:* The weighted adjacency matrix W of network g is first normalized such that it is a Markov transition matrix with $\sum_j w_{ij} = 1$. The association between nodes and quantiles is obtained by reordering W to have large w_{ij} near to the diagonal such that the resulting time series is as continuously smooth as possible [29]. The time series is constructed by repeatedly moving from node n_i to node n_j with probability w_{ij} and choosing a random number from the corresponding quantile q_j until we have obtained a time series of length T.

Networks that help to understand

Figura 2 Problema d'escacs: intercanvieu les posicions dels dos cavalls blancs i els dos cavalls negres.

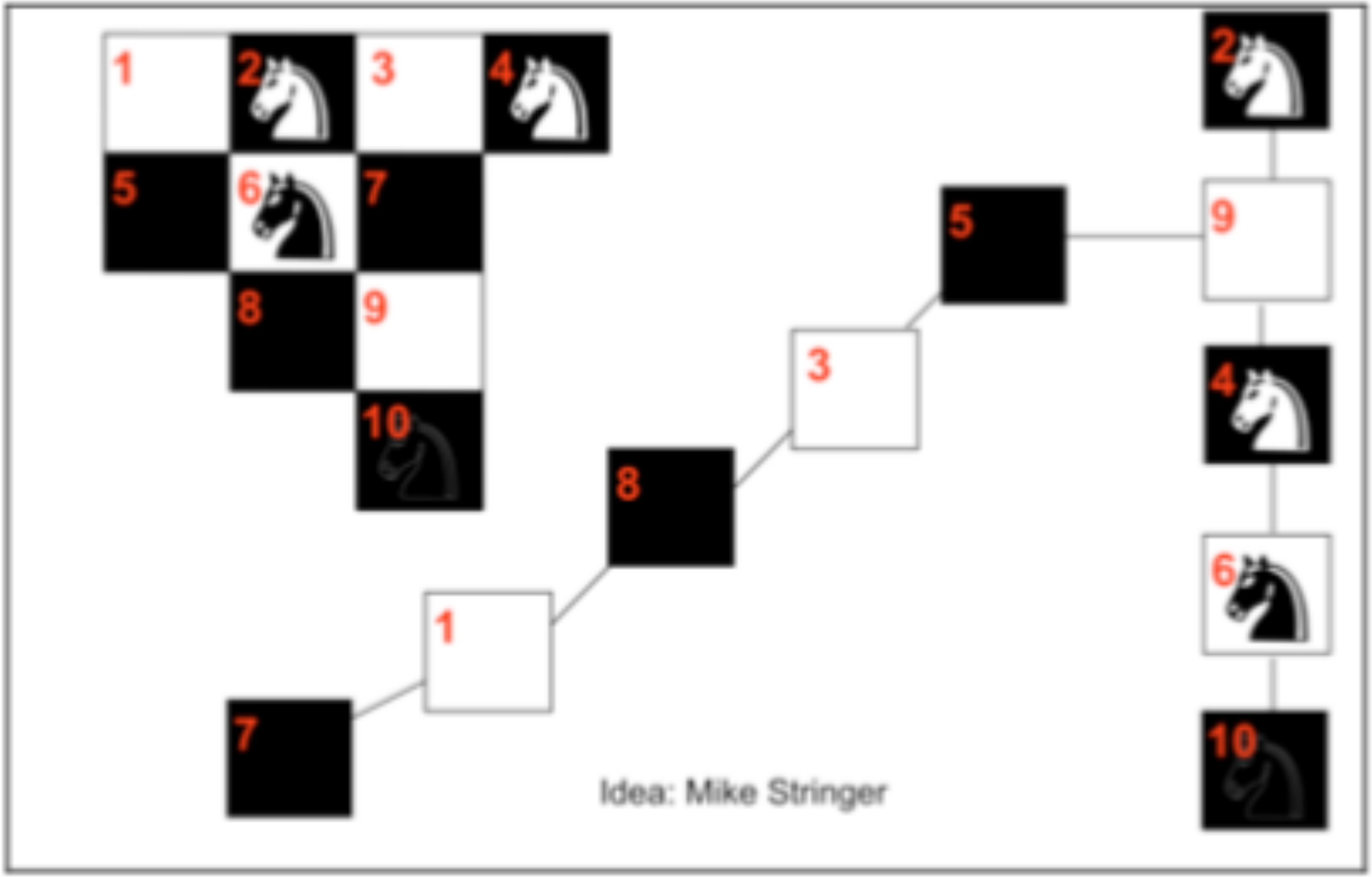
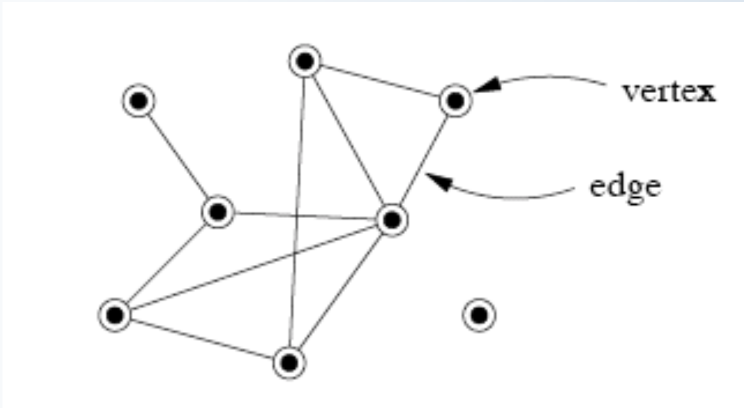


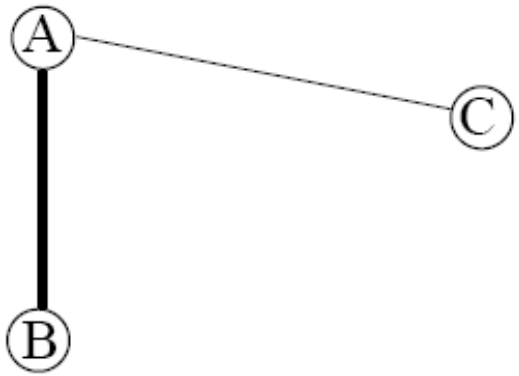
Figura 3 Problema d'escacs representat mitjançant una xarxa, on els nodes són les caselles i els enllaços els possibles moviments del cavall entre elles.

Representations

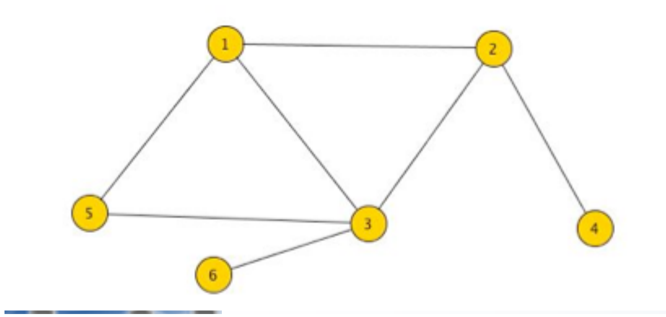
- Network (graph): nodes (vertices) + links (edges)



- Network (matrix): adjancency matrix



$$A=\begin{pmatrix} 0 & \mathbf{1} & 1 \\ \mathbf{1} & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$



1	2	2	1	1	2	1
1	3	3	1	1	3	2
1	5	3	2	1	5	1
2	3	5	1	2	3	1
2	4	4	2	2	4	2
3	5	3	5	3	5	2
3	6	3	6	3	6	1
		5	3			
		6	3			
		1	5			

links

1 :	2, 3, 5	1 :	2, 3, 5
2 :	1, 3, 4	2 :	3, 4
3 :	1, 2, 5, 6	3 :	5, 6
4 :	2	4 :	
5 :	1, 3	5 :	1, 3
6 :	3	6 :	3

neighbors

2.1 Formal Definition of Networks

We have seen that networks can appear in a variety of guises but they can always be thought of as a collection of items and the connections between them. In order to analyse networks, we need to turn this loose statement in formal mathematical language. To do this, we first introduce some basic set notation.

Let V be a finite set and let $E \subseteq V \otimes V$, whose elements are not necessarily all distinct. E is **reflexive** if $(v, v) \in E$ for all v in V, it is **anti-reflexive** if $(v, v) \notin E$ for all v in V and it is symmetric if $(v_1, v_2) \in E \iff (v_2, v_1) \in E$.

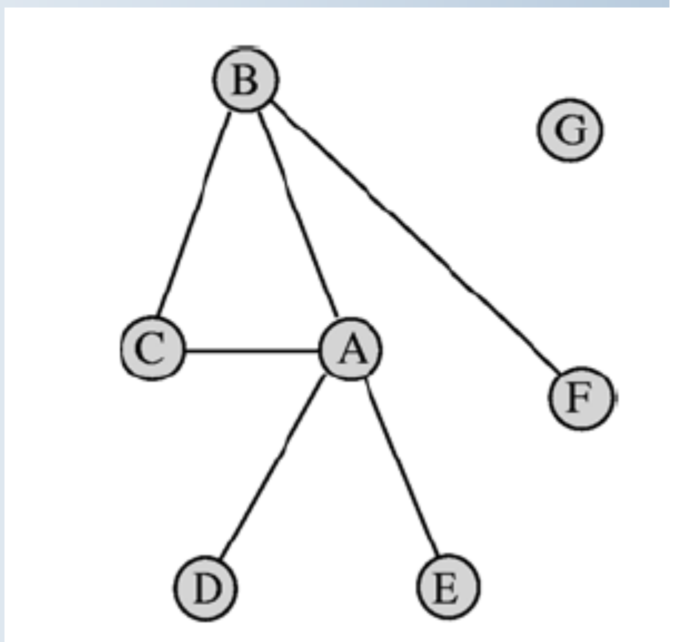
Definition 2.1.1 *A* **network**, *G, is a pair* (V, E). *Networks are also known as* **graphs**. *V is called the* **vertex set** *of G, its elements are the* **vertices** *of G (also known as* **nodes**).

- *If E is symmetric then G is an* **undirected network**.
- *If E is symmetric and anti-reflexive and contains no duplicate edges G is a* **simple network**.
- *If E is nonsymmetric then G is a* **directed network** *(or* **digraph**).

Basic concepts

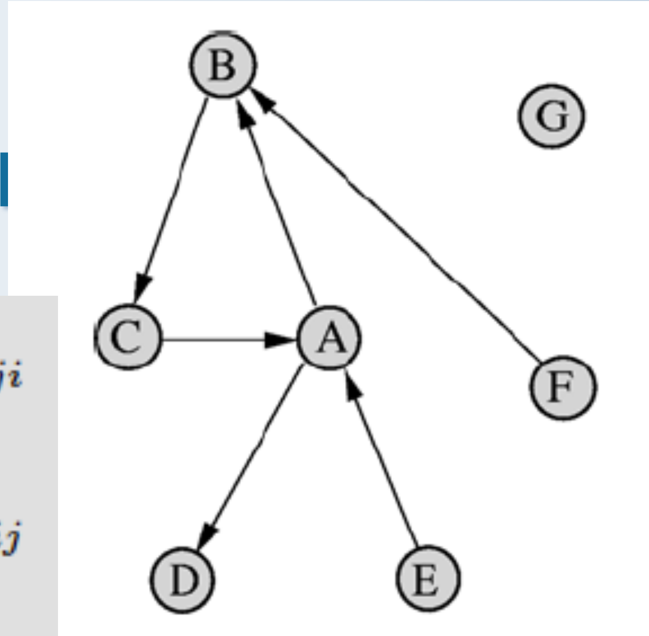
- Graph G(7,6)
- Order or size 7
- Number of links 6
- Degree A=4

$$k_i = \sum_{j=1,n} a_{ij}.$$



No binary networks

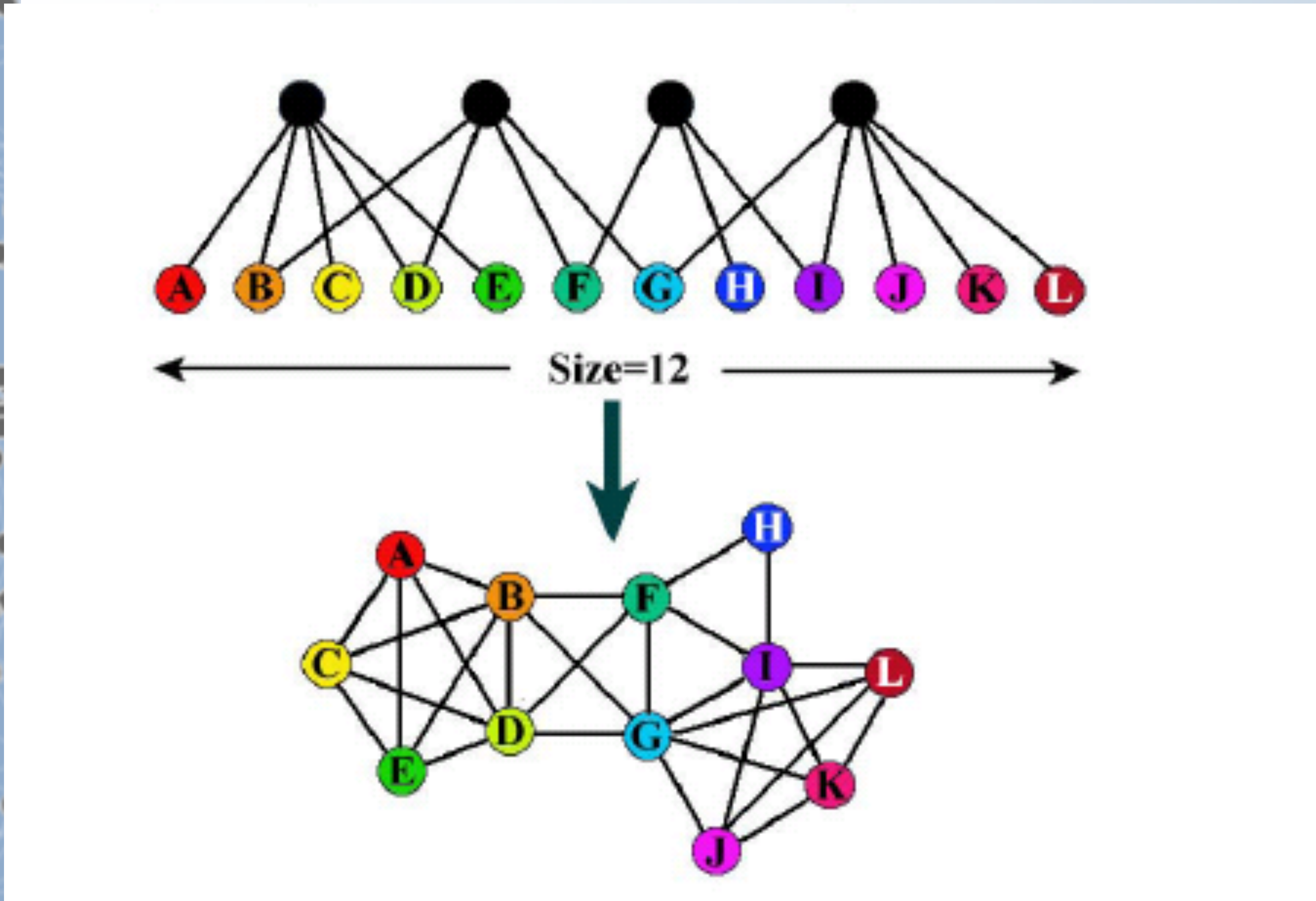
- Directed networ
  - Indegree $k_i^{in} = \sum_{j=1,n} a_{ji}$
  - outdegree $k_i^{out} = \sum_{j=1,n} a_{ij}$



- Weighted networks (flow)

$$s_i = \sum_{j=1,N} w_{ij}$$

Bipartite networks



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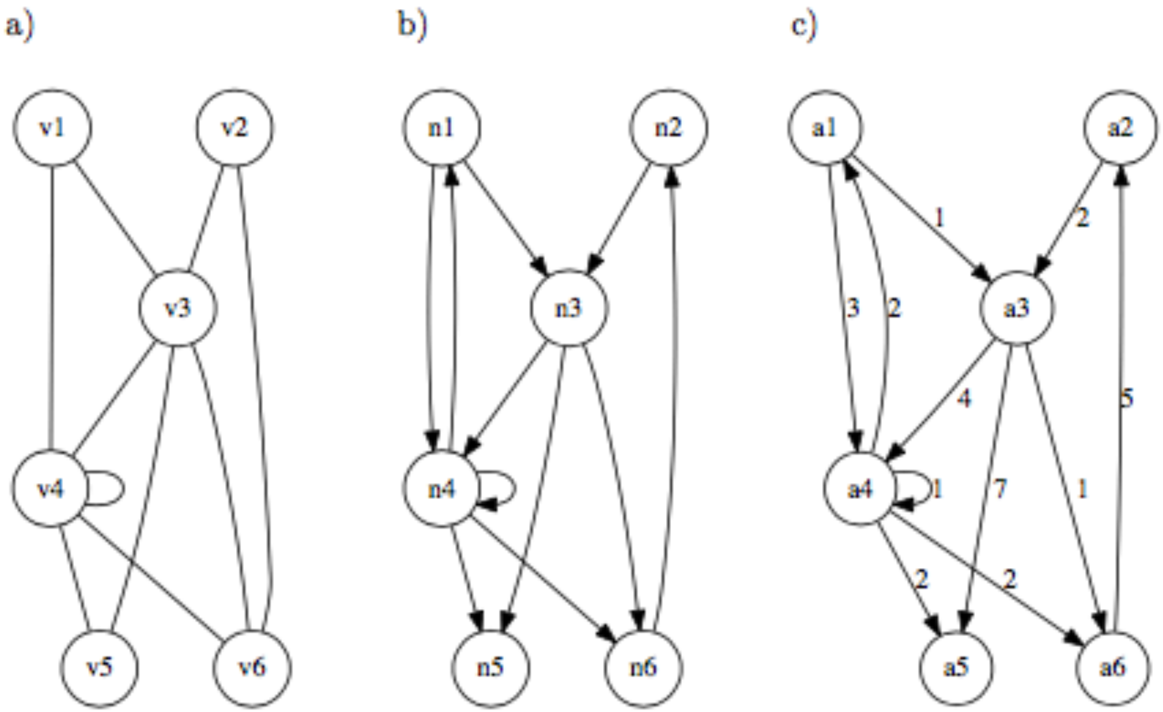

Figure 2.6 Different kinds of networks: a) undirected unweighted; b) directed unweighted; c) directed weighted.

a)

0	0	1	1	0	0
0	0	1	0	0	1
1	1	0	1	1	1
1	0	1	1	1	1
0	0	1	1	0	0
0	1	1	1	0	0

b)

0	0	1	1	0	0
0	0	1	0	0	0
0	0	0	1	1	1
1	0	0	1	1	1
0	0	0	0	0	0
0	1	0	0	0	0

c)

0	0	1	3	0	0
0	0	2	0	0	0
0	0	0	4	7	1
2	0	0	1	2	2
0	0	0	0	0	0
0	5	0	0	0	0

Figure 2.7 Adjacency (a, b) and weights (a, b, c) matrices of the respective networks in Fig. 2.6.

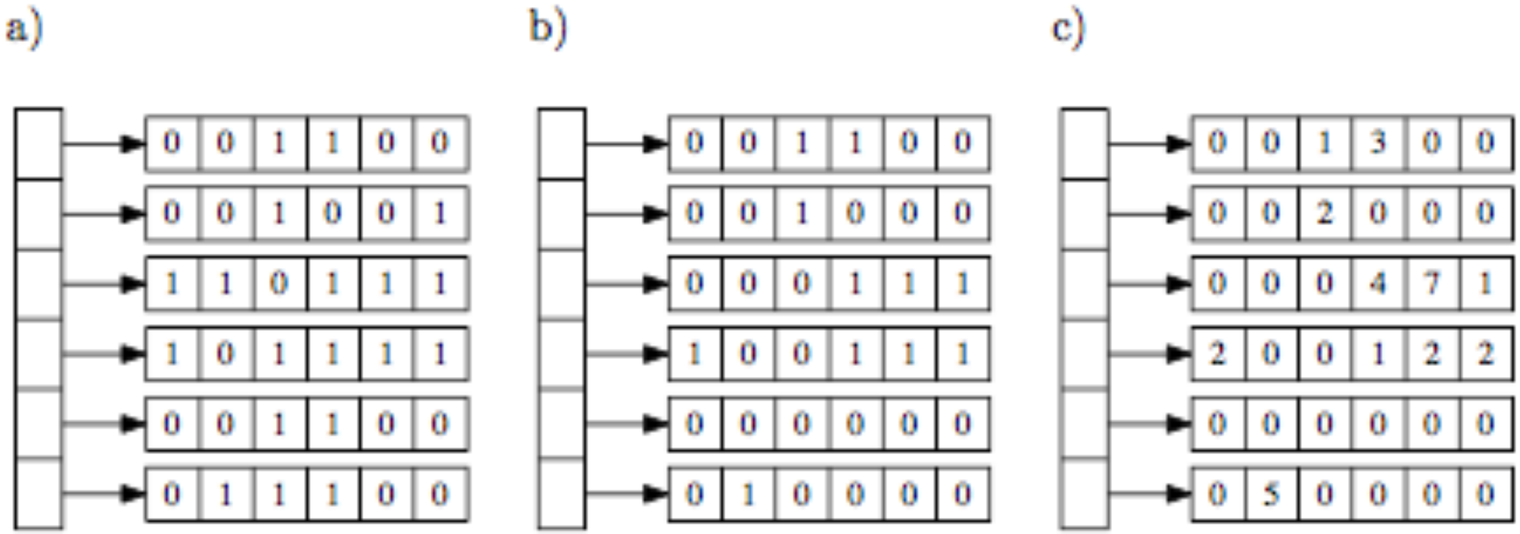

Figure 2.8 Weights matrices, as arrays of arrays, for the respective networks in Fig. 2.6.

Table 2.1 *Size in memory of networks with mean degree equal to 40.*

N	Matrix representation	Adjacency list representation
100	40 kB	50 kB
1000	4 MB	500 kB
10000	400 MB	5 MB
100000	40 GB	50 MB
1000000	4 TB	500 MB

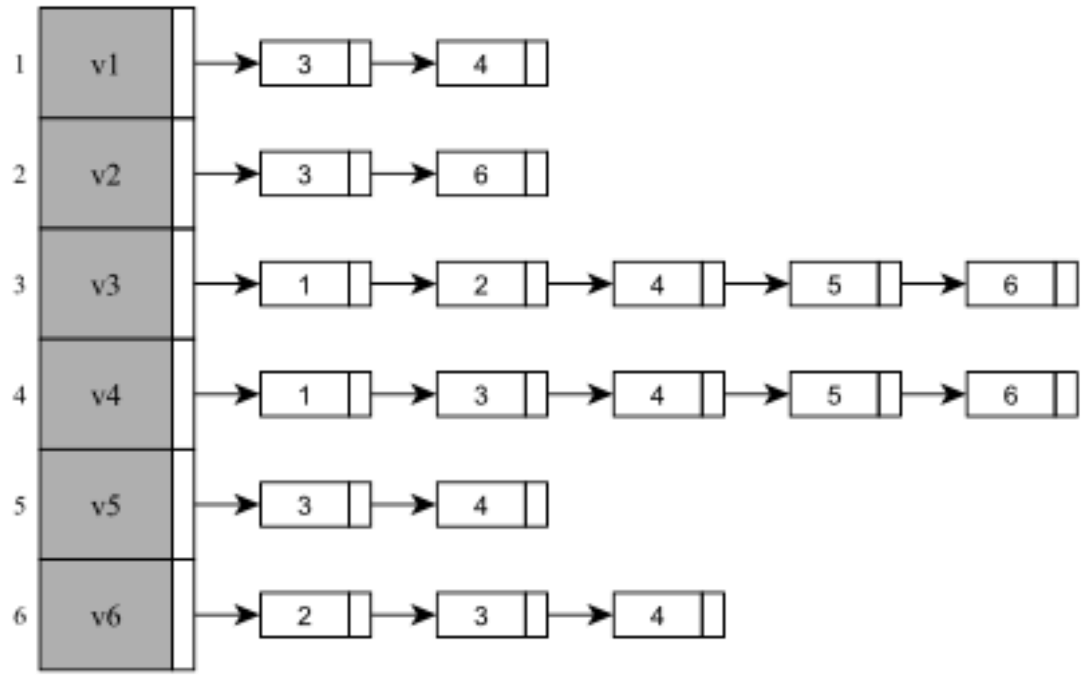

Figure 2.9 Adjacency list representation of the undirected unweighted network in Fig. 2.6a.