UNIVERSITAT DE BARCELONA

Complex Networks

Albert Díaz Guilera

http://diaz-guilera.net

@anduviera

complexity lab barcelona

COMPLEXITAT

Matrices (and their spectra)

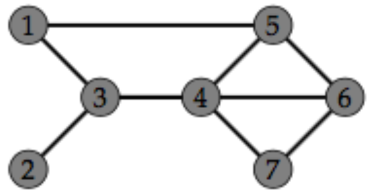
- Adjacency
- Laplacian

Adjacency

- Undirected network → Symmetric
- Eigenvalues are real $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n.$
- Certain structures have well defined spectra

Eigenvectors

- Can be used to highlight the importance of a given node
- Comparing components

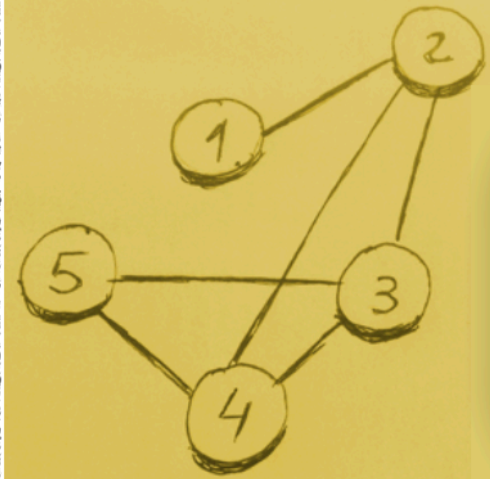


The principal eigenvector of the adjacency matrix is

$$\begin{bmatrix} 0.106 & 0.044 & 0.128 & 0.219 & 0.177 & 0.186 & 0.140 \end{bmatrix}^T.$$

The largest element (the fourth) is **associated** with the node at the hub of the network and that the smallest element is associated with the most peripheral.

Laplacian matrix



Adjacency Matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Network Laplacian

$$\mathcal{L} = \begin{pmatrix} k_1 & 0 & . & 0 & 0 \\ 0 & k_2 & . & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & . & k_{N-1} & 0 \\ 0 & 0 & . & 0 & k_N \end{pmatrix} - A = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & -1 & 0 \\ 0 & -1 & 3 & -1 & -1 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & -1 & 2 \end{pmatrix}$$

Discrete versus continuous

- Diffusion equation. Laplacian operator

$$\frac{dn(x,t)}{dt} = \nabla^2 n(x,t)$$

- Discrete Laplacian operator (1d)

$$\frac{dn_i(t)}{dt} = (n_{i+1}(t) - n_i(t)) - (n_i(t) - n_{i-1}(t))$$

$$\frac{dn_i(t)}{dt} = n_{i+1}(t) + n_{i-1}(t) - 2 \cdot n_i(t)$$

- Discrete Laplacian operator (2d)

$$\frac{dn_{i,j}(t)}{dt} = n_{i+1,j}(t)+n_{i-1,j}(t)+n_{i,j+1}(t)+n_{i,j-1}(t)-4\cdot n_{i,j}(t)$$

- In general

$$\frac{dn_i(t)}{dt} = \sum_{i}^{N} a_{i,j}n_j(t) - k_i \cdot n_i(t) = -L_{ij}n_j(t)$$

- Where we have introduced the Laplacian matrix

$$L_{ij} = k_i\delta_{ij} - a_{ij}$$

Laplacian eigenvalues

- There is always a 0 eigenvalue
- Number of 0 eigenvalues is equal to the number of (dis) connected components
- Smallest nonzero and largest eigenvalues

$$0 = \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_N$$