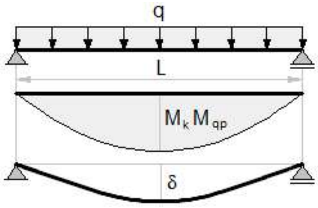
SLS Design of RC Beam with Tee Section

According to **Eurocode**: EN 1992-1-1

Static scheme



Simply supported beam with uniformly distributed load

Beam length - L = 5.75 m

Exposure class:

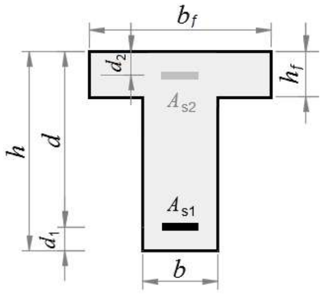
X0 or XC1

Bending moments

Characteristic combination ($g + q$) - M_k = 200 kNm

Quasi-permanent combination ($g + \psi_2 q$) - M_{qp} = 157 kNm

Cross section dimensions



Stem: b = 250 mm, h = 550 mm

Flange: b_f = 2400 mm, h_f = 140 mm

Concrete cover - c = 30 mm (to bars surface)

Tension reinforcement

Bar count - n_1 = 4 with diameter - Φ_1 = 20 mm

Reinforcement area - $A_{s1} = \dfrac{n_1 \cdot \pi \cdot \Phi_1{}^2}{4} = \dfrac{4 \cdot 3.14 \cdot 20^2}{4} = 1256.64$ mm²

Concrete cover to the center of reinforcement - d_1 = 45 mm

Effective cross section depth - $d = h - d_1 = 550 - 45 = 505$ mm

Spacing between bar centers - s = 50 mm

Compression reinforcement

Bar count - n_2 = 2 with diameter - Φ_2 = 8 mm

Reinforcement area - $A_{s2} = \dfrac{n_2 \cdot \pi \cdot \Phi_2{}^2}{4} = \dfrac{2 \cdot 3.14 \cdot 8^2}{4} = 100.53$ mm²

Concrete cover to the center of reinforcement - d_2 = 45 mm

Material properties

Concrete [EN 1992-1-1, Table 3.1]

Characteristic compressive cylinder strength - f_{ck} = 25 MPa

Mean value of cylinder compressive strength - $f_{cm} = f_{ck} + 8 = 25 + 8 = 33$ MPa

Mean value of axial tensile strength - $f_{ctm} = 0.3 \cdot f_{ck}^{\frac{2}{3}} = 0.3 \cdot 25^{\frac{2}{3}} = 2.56$ MPa

Secant modulus of elasticity - $E_{cm} = 22 \cdot \left(\dfrac{f_{cm}}{10}\right)^{0.3} = 22 \cdot \left(\dfrac{33}{10}\right)^{0.3} = 31.48$ GPa

Steel

Characteristic yield strength - $f_{yk} = 500$ MPa

Modulus of elasticity - $E_s = 200$ GPa

Ratio of steel to concrete moduli of elasticity - $\alpha = \frac{E_s}{E_{cm}} = \frac{200}{31.48} = 6.35$

Concrete creep and shrinkage ▲

Creep [EN 1992-1-1 B.1(1)]

Relative humidity of the environment - $RH = 60$ %

Perimeter of cross section in contact with the atmosphere

$u = 2 \cdot (b + h) = 2 \cdot (250 + 550) = 1600$ mm

Cross section area - $A_c = b \cdot h = 250 \cdot 550 = 137500$ mm²

Notional size of the cross section

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{2 \cdot 137500}{1600} = 171.88 \text{ mm}$$ (Formula B.6)

Relative humidity factor

$$\varphi_{RH} = 1 + \frac{1 - \frac{RH}{100}}{0.1 \cdot h_0^{0.333}} = 1 + \frac{1 - \frac{60}{100}}{0.1 \cdot 171.88^{0.333}} = 1.72$$ (Formula B.3a)

Concrete strength factor

$$\beta_{f\_cm} = \frac{16.8}{\sqrt{f_{cm}}} = \frac{16.8}{\sqrt{33}} = 2.92$$ (Formula B.4)

Concrete age at the moment of loading - $t_0 = 28$ дни

Concrete age factor

$$\beta_{t0} = \frac{1}{0.1 + t_0^{0.2}} = \frac{1}{0.1 + 28^{0.2}} = 0.488$$ (Formula B.5)

Notional creep coefficient

$$\varphi_{8\_t0} = \varphi_{RH} \cdot \beta_{f\_cm} \cdot \beta_{t0} = 1.72 \cdot 2.92 \cdot 0.488 = 2.46$$ (Formula B.2)

Effective concrete modulus of elasticity

$$E_{c\_eff} = \frac{E_{cm}}{1 + \varphi_{8\_t0}} = \frac{31.48}{1 + 2.46} = 9.1 \text{ GPa}$$ [EN 1992-1-1 §7.4.3(5)]

Effective ratio of modules of elasticity

$$\alpha_e = \frac{E_s}{E_{c\_eff}} = \frac{200}{9.1} = 21.97$$ [EN 1992-1-1 §7.4.3(6)]

Shrinkage [EN 1992-1-1 B.2(1)]

Coefficient depending on the notional size

$k_h = 1 - 0.0015 \cdot (h_0 - 100) = 1 - 0.0015 \cdot (171.88 - 100) = 0.892$

$\alpha_{ds1} = 4$ - for cement Class N

$\alpha_{ds2} = 0.12$ - for cement Class N

$\beta_{RH} = 1.55 \cdot \left(1 - \left(\frac{RH}{100}\right)^3\right) = 1.55 \cdot \left(1 - \left(\frac{60}{100}\right)^3\right) = 1.22$ (Formula B.12)

Basic drying shrinkage strain

$\varepsilon_{cd0} = 0.85 \cdot (220 + 110 \cdot \alpha_{ds1}) \cdot e^{\frac{-\alpha_{ds2} \cdot f_{cm}}{10}} \cdot 10^{-6} \cdot \beta_{RH} = 0.85 \cdot (220 + 110 \cdot 4) \cdot 2.72^{\frac{-0.12 \cdot 33}{10}} \cdot 10^{-6} \cdot 1.22 = 0.000459$ (Formula B.11)

Drying shrinkage strain in time [EN 1992-1-1 §3.1.4(6)]

$\varepsilon_{cd} = k_h \cdot \varepsilon_{cd0} = 0.892 \cdot 0.000459 = 0.000409$ (Formula 3.9)

Autogenous shrinkage strain

$\varepsilon_{ca} = 2.5 \cdot (f_{ck} - 10) \cdot 10^{-6} = 2.5 \cdot (25 - 10) \cdot 10^{-6} = 3.75 \times 10^{-5}$ (Formula 3.11)

Total shrinkage strain

$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca} = 0.000409 + 3.75 \times 10^{-5} = 0.000447$ (Formula 3.8)

Cross section properties ▲

Total reinforcement area - $A_s = A_{s1} + A_{s2} = 1256.64 + 100.53 = 1357.17\ \text{mm}^2$

Flange area - $A_f = (b_f - b) \cdot h_f = (2400 - 250) \cdot 140 = 301000\ \text{mm}^2$

T-section area - $A_c = b \cdot h + A_f = 250 \cdot 550 + 301000 = 438500\ \text{mm}^2$

Effective section area

$A_{red} = A_c + \alpha_e \cdot A_s = 438500 + 21.97 \cdot 1357.17 = 468320\ \text{mm}^2$

Section modulus about the bottom edge

$S_c = \frac{b \cdot h^2}{2} + A_f \cdot \left(h - \frac{h_f}{2}\right) = \frac{250 \cdot 550^2}{2} + 301000 \cdot \left(550 - \frac{140}{2}\right) = 182292500\ \text{mm}^3$

Effective section modulus about the bottom edge

$S_{red} = S_c + \alpha_e \cdot (A_{s1} \cdot d_1 + A_{s2} \cdot (h - d_2)) = 182292500 + 21.97 \cdot (1256.64 \cdot 45 + 100.53 \cdot (550 - 45)) = 184650460\ \text{mm}^3$

Effective depth of cross section center

$z_c = \frac{S_{red}}{A_{red}} = \frac{184650460}{468320} = 394.28\ \text{mm}$

Second moment of area of the concrete section

$I_c = \frac{b \cdot h^3 + (b_f - b) \cdot h_f^3}{12} + b \cdot h \cdot \left(z_c - \frac{h}{2}\right)^2 + A_f \cdot \left(h - z_c - \frac{h_f}{2}\right)^2 = \frac{250 \cdot 550^3 + (2400 - 250) \cdot 140^3}{12} + 250 \cdot 550 \cdot \left(394.28 - \frac{550}{2}\right)^2 + 301000 \cdot \left(550 - 394.28 - \frac{140}{2}\right)^2 = 8125757925\ \text{mm}^4$

Effective second moment of area for uncracked section

$I_{red} = I_c + \alpha_e \cdot (A_{s1} \cdot (z_c - d_1)^2 + A_{s2} \cdot (h - z_c - d_2)^2) = 8125757925 + 21.97 \cdot (1256.64 \cdot (394.28 - 45)^2 + 100.53 \cdot (550 - 394.28 - 45)^2) = 11521310457\ \text{mm}^4$

Uncracked section properties for SLS

Equivalent bending moment and axial force due to concrete stress about the neutral axis

$N_c(x) = \frac{b_f \cdot x^2}{2} - \frac{(x - h_f)^2 \cdot (b_f - b)}{2} \cdot (x > h_f)$

Equivalent axial force due to internal stress

$N_{bal}(x) = N_c(x) + \alpha_e \cdot (A_{s2} \cdot (x - d_2) - A_{s1} \cdot (d - x))$

Depth of neutral axis

$x = \$Root\{N_{bal}(x) = 0; x \in [0; h]\} = 96.46$ mm

Second moment of area for cracked section

$I_{red\_II} = \dfrac{b_f \cdot x^3}{3} + \alpha_e \cdot (A_{s1} \cdot (d - x)^2 + A_{s2} \cdot (x - d_2)^2) = \dfrac{2400 \cdot 96.46^3}{3} + 21.97 \cdot (1256.64 \cdot (505 - 96.46)^2 + 100.53 \cdot (96.46 - 45)^2) = 5332235820\ \text{mm}^4$

Control of concrete stress ▲

Characteristic combination

Stress at the bottom edge of the section

$\sigma_{ct} = \dfrac{M_k \cdot 10^6}{I_{red}} \cdot z_c = \dfrac{200 \cdot 10^6}{11521310457} \cdot 394.28 = 6.84$ MPa

Check for crack opening

$\sigma_{ct} = 6.84$ MPa $> f_{ctm} = 2.56$ MPa - section is cracked

Concrete stress (cracked section)

$\sigma_c = \dfrac{M_k \cdot 10^6}{I_{red\_II}} \cdot x = \dfrac{200 \cdot 10^6}{5332235820} \cdot 96.46 = 3.62$ MPa

Reinforcement stress (cracked section)

$\sigma_{s1} = \dfrac{\alpha_e \cdot M_k \cdot 10^6}{I_{red\_II}} \cdot (d - x) = \dfrac{21.97 \cdot 200 \cdot 10^6}{5332235820} \cdot (505 - 96.46) = 336.68$ MPa

$\sigma_{s2} = \dfrac{-\alpha_e \cdot M_k \cdot 10^6}{I_{red\_II}} \cdot (x - d_2) = \dfrac{-21.97 \cdot 200 \cdot 10^6}{5332235820} \cdot (96.46 - 45) = -42.41$ MPa

Stress limitation

Concrete stress, $k_1 = 0.6$ [EN 1992-1-1 §7.2(2)]

$\sigma_c = 3.62$ MPa $\le k_1 \cdot f_{ck} = 0.6 \cdot 25 = 15$ MPa - The condition is satisfied!

Reinforcement stress, $k_3 = 0.8$ [EN 1992-1-1 §7.2(5)]

$\sigma_{s1} = 336.68$ MPa $\le k_3 \cdot f_{yk} = 0.8 \cdot 500 = 400$ MPa - The condition is satisfied

Quasi-permanent combination

Concrete stress for cracked section

$\sigma_c = \dfrac{M_{qp} \cdot 10^6}{I_{red\_II}} \cdot x = \dfrac{157 \cdot 10^6}{5332235820} \cdot 96.46 = 2.84$ MPa

Stress limitation, $k_2 = 0.45$ [EN 1992-1-1 §7.2(3)]

$\sigma_c = 2.84$ MPa $\le k_2 \cdot f_{ck} = 0.45 \cdot 25 = 11.25$ MPa - The condition is satisfied!

Non-linear creep is NOT calculated!

Control of cracks ▲

Minimum reinforcement area [EN 1992-1-1 §7.3.2(2)]

Allowable reinforcement stress - $\sigma_s = f_{yk} = 500$ MPa

Approximate value of axial tensile strength - $f_{ct\_eff} = f_{ctm} = 2.56$ MPa

Area of tensile zone before opening of first crack

$A_{ct} = b \cdot z_c = 250 \cdot 394.28 = 98570.7\ \text{mm}^2$

$k = 1 - \dfrac{0.35 \cdot (h - 300)}{500} = 1 - \dfrac{0.35 \cdot (550 - 300)}{500} = 0.825$ - for 300 < h < 800

$k_c = 0.4$

$h_1 = h = 550$ - for h ≤ 1000

Minimum reinforcement

$A_{s\_min} = \dfrac{k_c \cdot k \cdot f_{ct\_eff} \cdot A_{ct}}{\sigma_s} = \dfrac{0.4 \cdot 0.825 \cdot 2.56 \cdot 98570.7}{500} = 166.87\ \text{mm}^2$

$A_{s1} = 1256.64\ \text{mm}^2 \geq A_{s\_min} = 166.87\ \text{mm}^2$ - The condition is satisfied!

Calculation of crack widths

Depth of the effective area [EN 1992-1-1 §7.3.2(3)]

$h_{c\_eff} = \min\left(2.5 \cdot (h - d); \min\left(h - \dfrac{x}{3}; \dfrac{h}{2}\right)\right) = \min\left(2.5 \cdot (550 - 505); \min\left(550 - \dfrac{96.46}{3}; \dfrac{550}{2}\right)\right) =$ 112.5 mm

Concrete effective area within tensile zone

$A_{c\_eff} = b \cdot h_{c\_eff} = 250 \cdot 112.5 = 28125\ \text{mm}^2$

$\rho_{p\_eff} = \dfrac{A_{s1}}{A_{c\_eff}} = \dfrac{1256.64}{28125} = 0.0447$

For $s = 50$ mm $\leq 5 \cdot \left(c + \dfrac{\Phi_1}{2}\right) = 5 \cdot \left(30 + \dfrac{20}{2}\right) = 200$ mm: [EN 1992-1-1 §7.3.4(3)]

$k_1 = 0.8$ - for high bond bars

$k_2 = 0.5$ - for bending

$k_3 = 3.4$, $k_4 = 0.425$

Maximum final crack spacing

$s_{r\_max} = k_3 \cdot c + \dfrac{k_1 \cdot k_2 \cdot k_4 \cdot \Phi_1}{\rho_{p\_eff}} = 3.4 \cdot 30 + \dfrac{0.8 \cdot 0.5 \cdot 0.425 \cdot 20}{0.0447} = 178.1$ mm

The check is performed for quasi-permanent combination

Reinforcement stress

$\sigma_s = \dfrac{\alpha_e \cdot M_{qp} \cdot 10^6}{I_{red\_II}} \cdot (d - x) = \dfrac{21.97 \cdot 157 \cdot 10^6}{5332235820} \cdot (505 - 96.46) = 264.3$ MPa

Long-term load factor - $k_t = 0.4$

Difference between mean values of concrete and reinforcement strains [EN 1992-1-1 §7.3.4(2)]

$$\varepsilon_{sm} - \varepsilon_{cm} = \Delta\varepsilon = \frac{\max\left(\sigma_s - \frac{k_t \cdot f_{ct\_eff}}{\rho_{p\_eff}} \cdot (1 + \alpha \cdot \rho_{p\_eff}); 0.6 \cdot \sigma_s\right)}{E_s \cdot 10^3} = \frac{\max\left(264.3 - \frac{0.4 \cdot 2.56}{0.0447} \cdot (1 + 6.35 \cdot 0.0447); 0.6 \cdot 264.3\right)}{200 \cdot 10^3} = 0.00117$$

Crack widths [EN 1992-1-1 §7.3.4(1)]

$w_k = s_{r\_max} \cdot \Delta\varepsilon = 178.1 \cdot 0.00117 = 0.209$ mm

Limiting crack width value [EN 1992-1-1 §7.3.1(5)]

$w_{max} = 0.4$ mm - for exposure classes X0, XC1 [EN 1992-1-1, Table NA.4]

Crack width limitation

$w_k = 0.209$ mm ≤ $w_{max} = 0.4$ mm - The condition is satisfied!

Control of deflections ▲

Crack opening moment

$$M_{cr} = \frac{f_{ctm} \cdot I_{red}}{z_c} \cdot 10^{-6} = \frac{2.56 \cdot 11521310457}{394.28} \cdot 10^{-6} = 74.95 \text{ kNm}$$

Coefficient for duration of loads

$\beta = 0.5$ - for long-term loads

Distribution coefficient [EN 1992-1-1 §7.4.3(3)]

$$\zeta = 1 - \beta \cdot \left(\frac{M_{cr}}{M_{qp}}\right)^2 = 1 - 0.5 \cdot \left(\frac{74.95}{157}\right)^2 = 0.886$$ - for cracked section

Calculation of curvature

First moment of reinforcement area about centroid of uncracked section

$S = A_{s1} \cdot (z_c - d_1) - A_{s2} \cdot (h - z_c - d_2) = 1256.64 \cdot (394.28 - 45) - 100.53 \cdot (550 - 394.28 - 45) = 427791$ mm^3

Curvature due to shrinkage of uncracked section [EN 1992-1-1 §7.4.3(6)]

$$\theta_{cs\_I} = \frac{\varepsilon_{cs} \cdot \alpha_e \cdot S}{I_{red}} = \frac{0.000447 \cdot 21.97 \cdot 427791}{11521310457} = 3.65 \times 10^{-7}$$

Curvature of uncracked section

$$\theta_I = \frac{M_{qp}}{E_{c\_eff} \cdot I_{red}} \cdot 10^3 + \theta_{cs\_I} = \frac{157}{9.1 \cdot 11521310457} \cdot 10^3 + 3.65 \times 10^{-7} = 1.86 \times 10^{-6}$$

First moment of reinforcement area about centroid of cracked section

$S = A_{s1} \cdot (d - x) - A_{s2} \cdot (x - d_2) = 1256.64 \cdot (505 - 96.46) - 100.53 \cdot (96.46 - 45) = 508208$ mm^3

Curvature due to shrinkage of cracked section [EN 1992-1-1 §7.4.3(6)]

$$\theta_{cs\_II} = \frac{\varepsilon_{cs} \cdot \alpha_e \cdot S}{I_{red\_II}} = \frac{0.000447 \cdot 21.97 \cdot 508208}{5332235820} = 9.36 \times 10^{-7}$$

Curvature of cracked section

$$\theta_{II} = \frac{M_{qp}}{E_{c\_eff} \cdot I_{red\_II}} \cdot 10^3 + \theta_{cs\_II} = \frac{157}{9.1 \cdot 5332235820} \cdot 10^3 + 9.36 \times 10^{-7} = 4.17 \times 10^{-6}$$

Total curvature

$\theta = \zeta \cdot \theta_{II} + (1 - \zeta) \cdot \theta_{I} = 0.886 \cdot 4.17\times10^{-6} + (1 - 0.886) \cdot 1.86\times10^{-6} = 3.91\times10^{-6}$ [EN 1992-1-1 §7.4.3(3)]

Factor considering the static scheme

$k = \frac{5}{48} = 0.104$ - for simply supported beam with uniformly distributed load

Deflection due to quasi-permanent combination - $\delta = k \cdot \theta \cdot L^2 \cdot 10^6 = 0.104 \cdot 3.91\times10^{-6} \cdot 5.75^2 \cdot 10^6 = 13.46$ mm

Maximum deflection - $\delta_{max} = \frac{L \cdot 10^3}{250} = \frac{5.75 \cdot 10^3}{250} = 23$ mm [EN 1992-1-1 §7.4.1(4)]

Deflection check:

$\delta = 13.46$ mm $\le \delta_{max} = 23$ mm - The condition is satisfied!