

Math 105B Lab Report 4

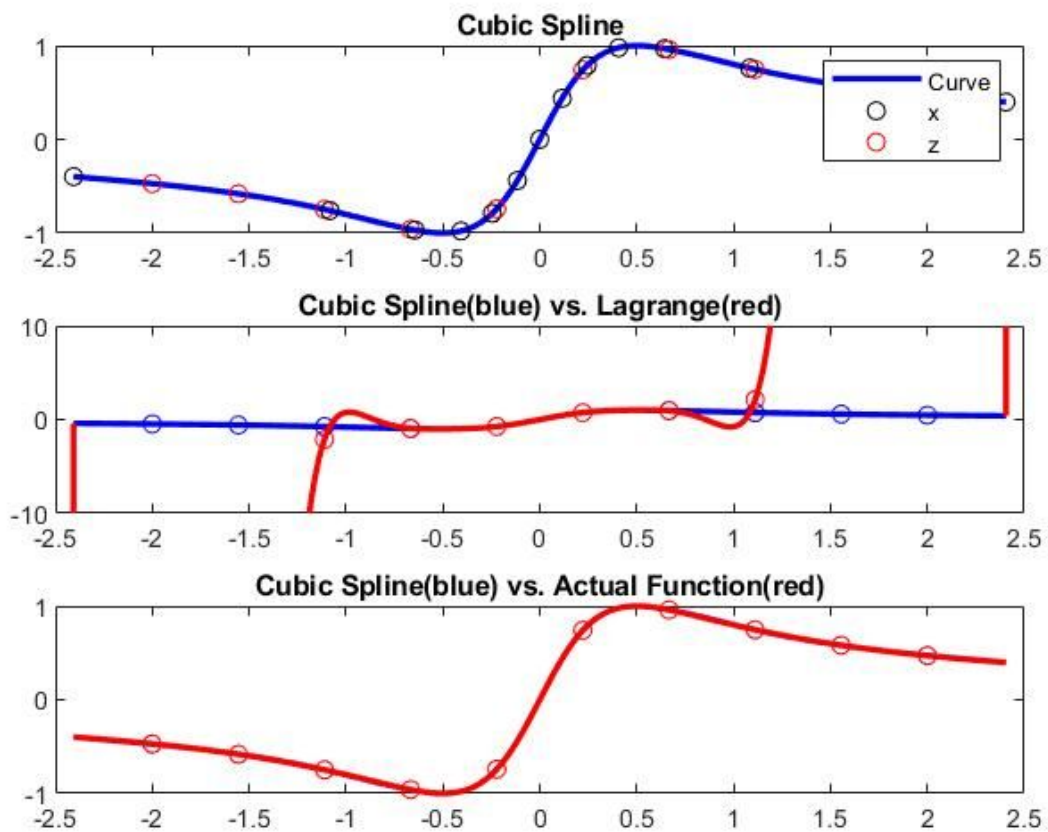
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Purpose/Objective: In this lab, I will write a function that computes the coefficients of a cubic spline interpolating function from the given X and Y list. I will graph the cubic spline interpolant using the given table and plot the 13 nodes and 10 equally spaced points. Next, I will compare it with the graph of Lagrange polynomial interpolant, and with the graph of the actual function. Last but not least, I will calculate the maximum error and the theoretical error in this problem.

Introduction: In this lab, we will compute the cubic spline interpolation and measure how accurate it is versus Lagrange interpolant and the actual function. The cubic spline interpolation requires us to break the whole value into n intervals, where n is 12 in this case. The first step is to solve the tridiagonal linear system to find the coefficients of each interval, iteratively. Based on the coefficients, we construct the polynomial formulaically and plot each interval onto the graph.

Algorithm: For finding the coefficients, I just followed the pseudocode given in the assignment, and shifted the index when it was needed like previous computer assignments. Then I generated 3 plots corresponding to each part. For the first plot, I looped through the intervals and plotted the functions based on their own coefficients. For every interval, I used 100 equally spaced points between the endpoints as the x values, where the y values will be computed and stored in a list. I also plotted the table points and the z points in the loop process. For part 2, it was almost the same process. The only difference is when I generated 1200 equally spaced points as the x values of the Lagrange polynomial. Part 3 is essentially the same, where I defined the function f and used 1200 points to compute the function values. The most important thing about plotting was the label choices, like changing the color and changing the axis etc. Lastly, I calculated the error at the 10 points and stored them in a list, and then found the maximum among them.

Graph:



```
maximum_error =  
  
    0.0023  
  
theoretical_error =  
  
    24.8905
```

Conclusion: The cubic spline interpolating polynomial we constructed here is extremely accurate, as it almost perfectly matched with the actual function. The maximum error is 0.0023, which is very low compared to the theoretical error.