Math 105B Lab Report 9

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Purpose/Objective: In this lab, we will code and test the accuracy of Chebyshev polynomial approximations. We will first write an algorithm that inputs the function, the endpoints and the desired degree, outputting the coefficients of Chebyshev polynomial. We will then write another algorithm that takes in the Chebyshev coefficients we obtained and yields the integral approximation of the chebyshev polynomial, with the help of composite simpson's rule. Lastly, we will test it on two different functions and explore what the degree N does to the accuracy of Chebyshev polynomial approximations.

Introduction: In the coefficients of Chebyshev polynomial, we are expected to use the transformation x=cos(theta) and then use composite Simpson's rule or any other approximation method to obtain the coefficients by formula given in the textbook. We are then asked to find a degree N that is large enough so that the error between the approximation of exp(x) and exact value is less than 1e-6 over the interval. Lastly, we are asked to compute Chebyshev approximations for a piecewise function with increasing N.

Algorithm: For writing the Chebyshev coefficients function, I transformed x to cos(theta) and followed the formula in the textbook. I did so by first simplifying both the numerator and denominator functions, and then plugging them into composite Simpson's rule to approximate the integrals. For the actual Chebyshev polynomial approximation, I used composite Simpson's rule as well. The rest of the coding is pretty trivial.

Result:

```
a_j =

    1.2661    1.1303    0.2715    0.0443    0.0055    0.0005    0.0000    0.0000    0.0000

We need N to be:

N =

     6


error =

   1.0e-15 *

    0.2220    0.2220    0.2220    0.2220
```

Conclusion: The degree needs to be 6 in order to achieve an absolute error of 1e-6 for the Chebyshev polynomial approximation of exp(x). The error for the piecewise function remains constant. One cannot make the error as small as one likes by increasing N.