Math 105B Lab Report 6

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Purpose/Objective: In this lab, we will test the accuracy of the trapezoid and Simpson's rules over the given interval, as well as their composite formulas, for computing definite integrals numerically. By comparing the error and the error estimate of each formula, we will be able to determine which numerical quadrature is more accurate. We will also explore the relationship between n, the number of subintervals, and the accuracy of composite formulas as we produce one plot showing the value as a function of n, and the other plot showing the error as a function of n.

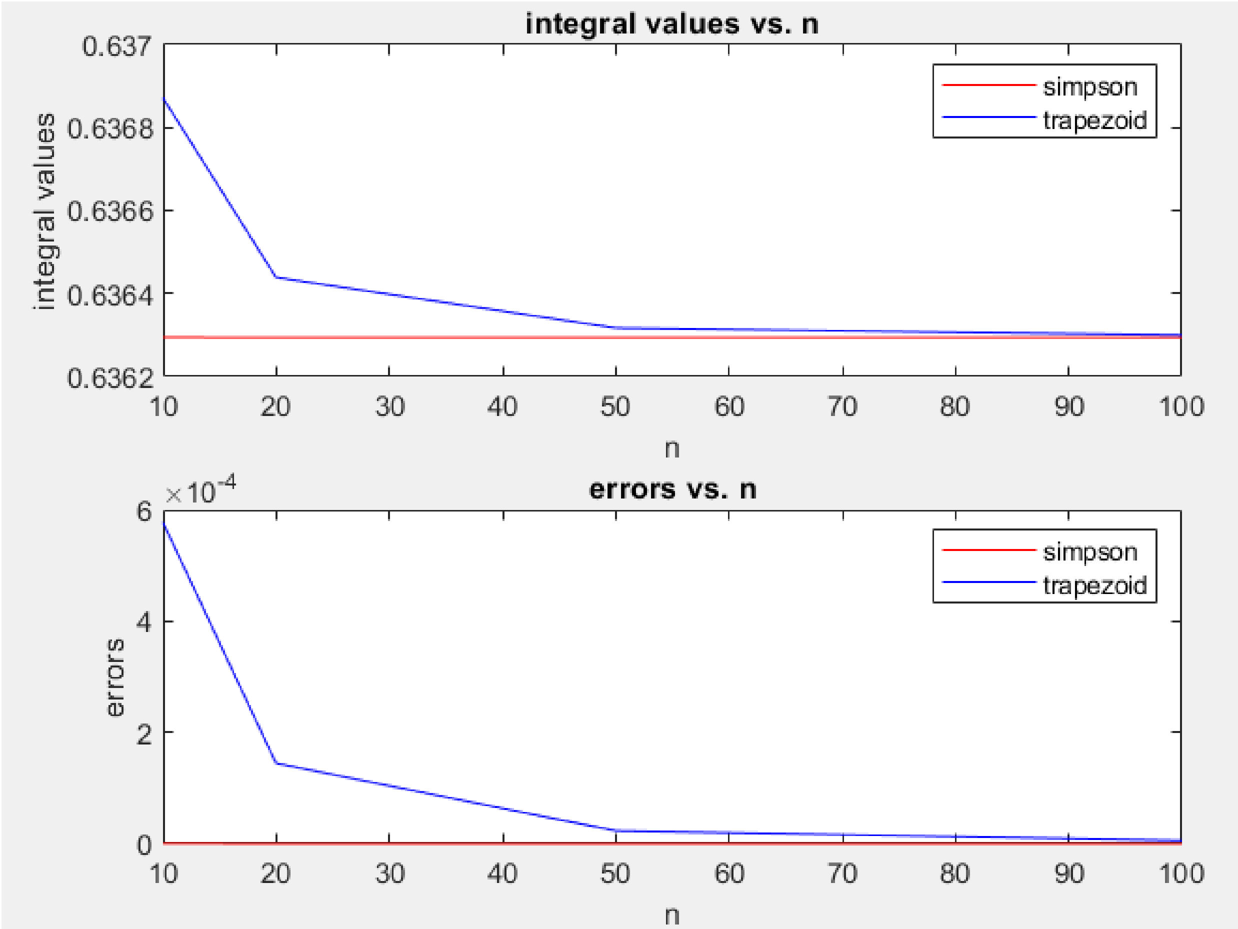
Introduction: We are asked to integrate the function xln(x) over the interval [1,2] to find its exact value. We are then expected to use simpson's rule and trapezoidal rule separately and find their errors and error estimates. We will then develop algorithms for their composite formulas in two separate functions, outputting the interval approximation with inputs of endpoints and the number of subintervals. Lastly, we will use those functions to approximate the integral using different values of n and finding the errors as usual.

Algorithm: When coding the composite formulas into functions, I made some slight changes. I figured that we didn't really have to make I subscript n into a matrix, so I only constructed it as a list. In order for the given pseudo code to work, we needed to define function f in it. Other than that, the main assignment followed closely with the structure of previous assignments. The important features include making the n value list and looping through it, creating empty lists and adding desired values to them in each iteration, and creating labels/legends for each plot.

Results:

```
Exact_value =

    0.6363

Part 2 answers:

trapezoid =

    0.6931


trapezoid_error =

    0.0569


error_estimate1 =

    0.0833


simpson =

    0.6365


simpson_error =

   2.1981e-04
```

```
   6.9444e-04

Part 5 answers:

simpson_error1 =

   1.0e-06 *

    0.4130    0.0260    0.0007    0.0000


simpson_error_estimate =

   1.0e-05 *

    0.1111    0.0069    0.0002    0.0000


trapezoid_error1 =

   1.0e-03 *

    0.5775    0.1444    0.0231    0.0058


trapezoid_error_estimate =

   1.0e-03 *

    0.8333    0.2083    0.0333    0.0083
```



Conclusion: We can clearly see that Simpson's rule is more accurate than trapezoidal rule, which matches with our expectation because Simpson's rule uses one more point in each interval. From the graphs above, we can obviously tell that both numerical quadrature approximations converge to the exact value as n increases, in other words, their errors approaches zero.