Math 105B Lab Report 3

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Purpose/Objective: In this lab I will first write a function that takes in three lists of x, y, and y prime, then outputs the Newton's divided differences. I will use the divided differences to compile a function handle for Hermite interpolating polynomial based on the nodes and the degree. Given some data, two specific Hermite polynomials will be computed in this lab. Last but not least, I will calculate the errors of said polynomials at a specific point and compare them to the error bounds respectively.

Introduction: This lab is much similar to lab 2, where we wrote a function for divided difference and used that function to compose the actual interpolating polynomial. The main difference is that we are making a new list Z where the x-coordinates double up. In addition, the first derivative values are involved, which makes Hermite polynomials a bit more complicated but more accurate.

Procedure(Algorithm method): When constructing the function for divided difference, I shifted every index in the pseudocode 1 unit to the right, since MATLAB starts with index 1, not 0 as described in the problem. For example, the first element z(1) in the list would denote "z_0" in the problem, the same applies to the matrix. So the output of the function would be a n +1 dimension matrix, where n is the degree. After calling the divided difference function, I manually created anonymous function handles for the corresponding Hermite interpolating polynomials. The rest of the coding were trivial, mostly evaluating the polynomials and finding the errors. Again, I didn't suppress some of the results so I could see what matrix Q looks like, what value H5_x(1.25) gives and etc.

Result:

```
Q =

    1.1052         0         0         0         0         0
    1.1052    0.2210         0         0         0         0
    1.4918    0.3867    0.1656         0         0         0
    1.4918    0.5967    0.2101    0.0445         0         0
    2.4596    0.9678    0.3710    0.0805    0.0180         0
    2.4596    1.4758    0.5080    0.1369    0.0282    0.0051


Z =

     1     1     2     2     3     3


H5 =

    1.1690


Q1 =

    1.1052         0         0         0
    1.1052    0.2210         0         0
    1.4918    0.3867    0.1656         0
    1.4918    0.5967    0.2101    0.0445


Z1 =

     1     1     2     2


H3 =

    1.1687


abs_error_H5 =

   1.0238e-04


abs_error_H3 =

   4.2166e-04


H5_error_bound =

   4.4263e-04


H3_error_bound =

   7.3775e-04
```

Conclusion: As expected, the absolute error of H5(x) is smaller than the absolute error of H3(x) from the real function value. For both Hermite polynomials, we can see that their absolute errors are contained in the error bounds that are calculated correspondingly.