Filter Design Coursework

Practical Engineering Design Solutions and Project Development (EEEE2046 UNUK)

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Introduction

The aim of this coursework is to design a unity gain Butterworth high pass filter with 100 Hz passband, 50 Hz stopband and 30 dB stopband attenuation, using Sallen-Key topology. In task 1, three types of filter: Chebyshev, Bessel and Butterworth, are analysed and compared in terms of advantages, disadvantages and applications. In task 2, a fifth-order high-pass filter was designed and its AC and transient response were simulated with exact calculated component values as per the required specifications. In task 3, these exact component values were taken place with E24 range values and new AC response simulation was analysed. The task response section details the associated methods, calculations and analysis in design.

Task Response

2.1 Task 1

The performance characteristics and applications of Chebyshev, Bessel and Butterworth filter are discussed in this section.

Performance characteristics:

Chebyshev filter has the steepest roll-off among the other two filter types. The advantage is that only small order number is needed to achieve sharp frequency cut-off. Hence fewer components are needed [1]. Steep roll-off also leads to high frequency selectivity. However, the cost of sharp roll-off is that the frequency response has plenty of ripples (ripples often occur in pass band for type I Chebyshev filter and stop band for type II) [2]. Unwanted ripple could cause plenty of ringing for the step response of the filter [3]. Another downside is high sensitivity to component tolerance [1]. In addition the phase response is often non-linear, leading to large phase shift, which could distort the pulse [1].

Bessel filter has the smoothest frequency response compared to the other two (no ripples). Contrary to Chebyshev filter, it has the slowest roll-off. Therefore, it has poor frequency range selectivity. To achieve sharp frequency cut-off, high order number is usually necessary, which could complicate the circuit with increasing order [1]. However, such slow roll-off often comes with more linear phase shift, as it has almost constant group delay, which can lead to less signal distortion [4]. Another advantage is that its slow roll-off can lead to low sensitivity to component tolerance. In addition, smooth frequency response can lead to gentle step response with minimal ringing [5].

Butterworth filter offers steep roll-off without causing ripple in passband [3]. Therefore, its frequency selectivity is greater than that of Bessel filter. It has greater component tolerance than Chebyshev filter. However, the downside is its non-linear phase response. The time delay is large and non-linear, which could cause the signal to be distorted [1]. In general, Butterworth filter can be viewed as a balance between Bessel filter and Chebyshev filter, as its frequency response has faster roll-off than Bessel filter but has less ripple compared to Chebyshev [3].

Summary of the advantages and disadvantages of these three filter types can be found in table 2.1.1:

Table 2.1.1: Summary of the Advantages and Disadvantages

Types	**Advantages**		**Disadvantages**	
	Time-domain performance	Frequency/Phase response	Time-domain performance	Frequency/Phase response
Butterworth	• Medium ringing [3]	• Slow Roll-off [3] • No ripples in the passband [1] • Flat passband response	• Some signal distortion [1] • Some ringing	• Poor phase linearity [1] • large phase shift
Chebyshev	• Need small order number [1]	• Steepest roll-off	• Plenty of ringing [3] • High sensitivity to component tolerance [1] • Signal distortion	• Ripples in passband or stop band. [2] • Non-linear phase shift
Bessel	• Minimal Ringing [5] • Low sensitivity to component tolerance [1]	• No ripple [1] • Smoothest curve • linear phase shift [4] • maximally constant group delay [4]	• Need rather large order number [1]	• Slowest roll-off [1]

The applications:

Chebyshev filter is often used in RF applications. Chebyshev filter's sharp roll-off can screen out unwanted frequency band easily. Its large ripples do not have a significant effect on the RF communication. Therefore, it fits for RF application [6].

Bessel filter is commonly found in analogue video signal processing as it offers close to linear phase shift due to constant group delay [7]. Video signal processing relies on high linearity phase shift and flap response of the circuit. It is also used in high speed, wideband data communication because it has high frequency selectivity [8].

Butterworth filter has flat frequency response in pass band and therefore has good anti-aliasing feature. It is often used in high quality audio circuits, where anti-aliasing is important [9]. It can also be used in motion analysis such as radar motion track detection because sharp roll-off and flat passband can filter low speed distraction [10].

2.2 Task 2

The order N of high pass Butterworth filter is determined by equation 2.2.1:

$$|H(j\omega_s)| = \frac{1}{\sqrt{1+\epsilon^2\left(\frac{\omega p}{\omega_s}\right)^{2N}}} \quad (2.2.1)$$

Where ϵ is the passband maximum gain, which in this case, is 1 (unity gain);
ω_p, ω_s are the passband and stopband frequency (in radian/second) respectively;
$|H(j\omega_s)|$ is the magnitude of stopband gain.

Convert passband and stopband frequency by equation 2.2.2 and equation 2.2.3 respectively:

$$\omega_p = 2\pi f_p = 2\pi \times 100\ Hz = 200\pi\ radian/s \quad (2.2.2)$$

$$\omega_s = 2\pi f_s = 2\pi \times 50\ Hz = 100\pi\ radian/s \quad (2.2.3)$$

Since stopband attenuation is 30 dB, the gain of stopband can be calculated by equation 2.2.5:

$$20log(|H(j\omega_s)|\ ) = -30\ dB \quad (2.2.4)$$

$$|H(j\omega_s)| = 10^{-\frac{30}{20}} \quad (2.2.5)$$

Therefore, the order of the filter can be calculated by equation 2.2.6 and 2.2.7:

$$10^{-\frac{30}{20}} = \frac{1}{\sqrt{1+\left(\frac{200\pi}{100\pi}\right)^{2N}}} \quad (2.2.6)$$

$$N = 4.98 \quad (2.2.7)$$

To meet the minimum requirement, the order *N* has to be 5.

Therefore, two second-order Butterworth filters and one first-order filter are to be cascaded to construct a fifth-order Butterworth filter.

Now determine the cut-off frequency ω_c at -3 dB amplitude by equation 2.2.8:

$$|H(j\omega_c)| = \frac{1}{\sqrt{1+\left(\frac{200\pi}{\omega_c}\right)^{2\times5}}} = 10^{\frac{-3}{20}} \quad (2.2.8)$$

Cut-off frequency ω_c was calculated to be 100.048 Hz ≈ 100 Hz or 200π radian/s.

First, analyse the transfer function of constituting component – second order high-pass filter:

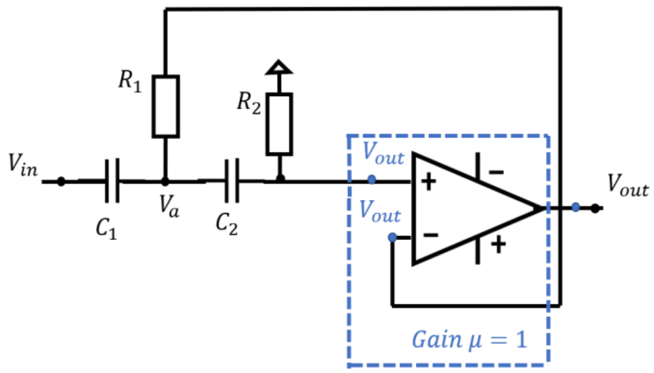

Figure 2.2.1: Second-order Unit Gain Filter Schematics for Calculation

Applying kirchhoff's law, transfer function can be obtained by equation 2.2.9 and 2.2.10:

$$\frac{V_{in}-V_a}{\frac{1}{sC_1}} + \frac{V_{out}-V_a}{R_1} + \frac{V_{out}-V_a}{\frac{1}{sC_2}} = 0 \tag{2.2.9}$$

$$\frac{V_a-V_{out}}{\frac{1}{sC_2}} + \frac{0-V_{out}}{R_2} = 0 \tag{2.2.10}$$

Therefore, TF for second-order high-pass filter is:

$$H(s) = \frac{s^2}{s^2+\frac{R_1C_1+R_1C_2}{C_1C_2R_1R_2}s+\frac{1}{C_1C_2R_1R_2}} \tag{2.2.11}$$

Comparing with general form:

$$H(s) = \frac{s^2}{s^2+\frac{\omega_c s}{Q}+\omega_c{}^2} = \frac{s^2}{s^2+\frac{200\pi s}{Q}+200\pi^2} \tag{2.2.12}$$

Where ω_c is the corner frequency;
Q is the quality factor.

Same analysis applies to the second second-order high-pass filter design.

Now analyse the transfer function of first order high-pass filter:

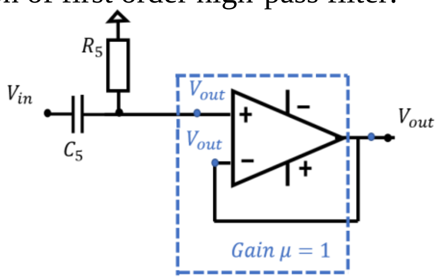

Figure 2.2.2: First-order Unit Gain Filter Schematics for Calculation

Apply potential divider rule, equation 2.2.13 can be obtained:

$$\frac{V_{out}}{V_{in}} = \frac{R_5C_5s}{1+R_5C_5s} = \frac{s}{\frac{1}{R_5C_5}+s} \tag{2.2.13}$$

Therefore, the transfer function for the final constructed fifth order filter is expressed in equation 2.2.14:

$$H(s) = \frac{s^2}{s^2 + \frac{R_1C_1+R_1C_2}{C_1C_2R_1R_2}s + \frac{1}{C_1C_2R_1R_2}} \times \frac{s^2}{s^2 + \frac{R_3C_3+R_3C_4}{C_3C_4R_3R_4}s + \frac{1}{C_3C_4R_3R_4}} \times \frac{s}{\frac{1}{R_5C_5}+s} \quad (2.2.14)$$

According to the Butterworth Polynomial table, denominator $(1+s)(1+0.618s+s^2)(1+1.618s+s^2)$ is to be used. Substituting s with $\frac{s}{\omega_c}$, the denominator would become:

$$\text{denominator} = \left(1+\frac{s}{\omega_c}\right)\left(1+0.618\frac{s}{\omega_c}+(\frac{s}{\omega_c})^2\right)\left(1+1.618\frac{s}{\omega_c}+(\frac{s}{\omega_c})^2\right) \quad (2.2.15)$$

Multiplying ${\omega_c}^5$ to both the numerator and denominator to obtain the standard form:

$$\text{denominator} = ({\omega_c}^2+0.618\omega_c s+s^2)({\omega_c}^2+1.618\omega_c s+s^2)(\omega_c+s) \quad (2.2.16)$$

Therefore, the following can be obtained:

$$\frac{R_1C_1+R_1C_2}{C_1C_2R_1R_2} = \frac{1}{Q}\omega_c = 0.618\omega_c = 0.618\times\ 200\pi = 388.3 \quad (2.2.17)$$

$$\frac{1}{C_1C_2R_1R_2} = {\omega_c}^2 = \ (200\pi)^2 = 3947.8 \quad (2.2.18)$$

$$\frac{R_3C_3+R_3C_4}{C_3C_4R_3R_4} = \frac{1}{Q}\omega_c = 1.618\omega_c = 1.618\times\ 200\pi = 1016.6 \quad (2.2.19)$$

$$\frac{1}{C_3C_4R_3R_4} = {\omega_c}^2 = \ (200\pi)^2 = 3947.8 \quad (2.2.20)$$

$$\frac{1}{R_5C_5} = \omega_c = 200\pi = 628.3 \quad (2.2.21)$$

Analysing equation 2.2.17 and 2.2.18, the following can be obtained:

$$C_1C_2 = \frac{1}{{\omega_c}^2R_1R_2}, C_1+C_2 = \frac{1}{Q\omega_cR_1}$$

Then equation 2.2.22 can be formulated (the roots are C_1 and C_2):

$$C^2 - \frac{1}{Q\omega_cR_1}C + \frac{1}{{\omega_c}^2R_1R_2} = 0 \quad (2.2.22)$$

In order to make equation 2.2.22 valid, the following must be followed:

$$\left(\frac{1}{Q\omega_cR_1}\right)^2 - 4\frac{1}{{\omega_c}^2R_1R_2} \geq 0$$

$$\frac{R_2}{R_1} \geq \frac{4}{\left(\frac{1}{Q}\right)^2}, \frac{1}{Q} = 0.618$$

Same applies for R_3 and R_4, therefore prerequisites: $\frac{R_2}{R_1} \geq 10.48$, $\frac{R_4}{R_3} \geq 1.53$ must be followed.

Choose $R_1 = 1K\Omega, R_2 = 15K\Omega$, which satisfises the minimum R_1, R_2 ratio. By equation 2.2.17 and 2.2.18, C_1, C_2 can be obtained:

$$C_1 = 0.761951\ uF,\ C_2 = 0.221627\ uF$$

Choose $R_3 = 1K\Omega, R_4 = 5K\Omega$. By equation 2.2.17 and 2.2.18, C_3, C_4 can be obtained:

$$C_3 = 2.360510\ uF,\ C_4 = 0.214617\ uF$$

Set $R_5 = 1\ K\Omega$, from equation 2.2.21, C_5 can be obtained:

$$C_5 = 1.591549\ uF$$

The chosen resistor values are all greater than 1 $K\Omega$ and capacitor values greater than 1 nF, having reasonable magnitude.

Figure 2.2.3 shows the LTspice schematic design:

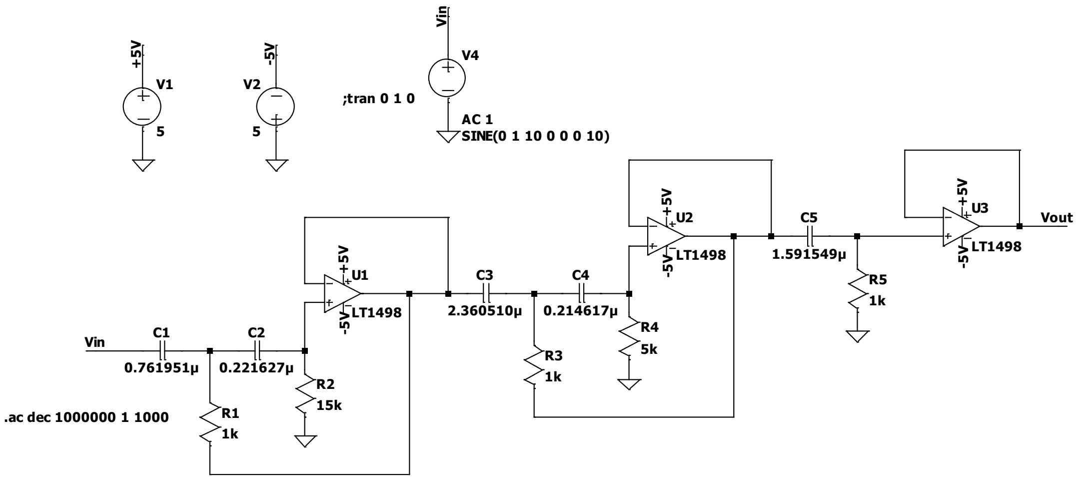

Figure 2.2.3: Sallen-Key Fifth-order High-pass Filter Schematics

AC Simulation:

The amplitude response of the circuit is shown in figure 2.2.4:

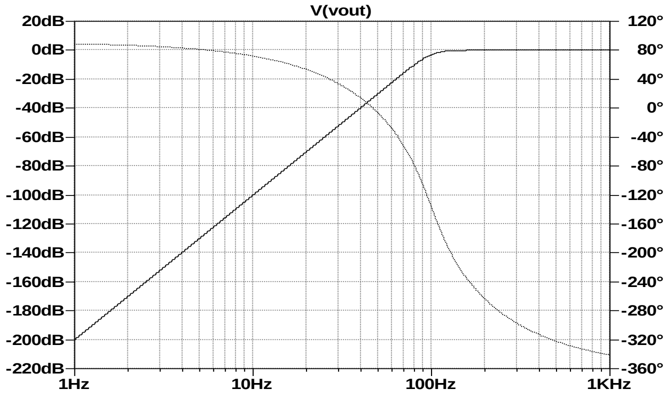

Figure 2.2.4: Amplitude Response of exact value components

Low cut off frequency can be obtained by moving cursor around the amplitude response plot.

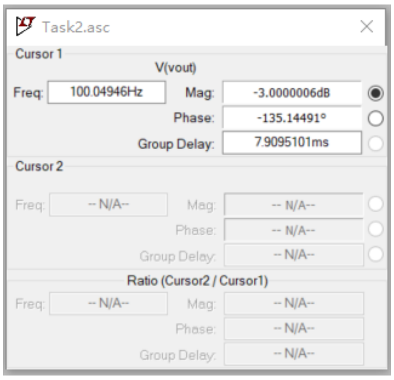

Figure 2.2.5: Screenshot Showing the -3dB Low Cut off Frequency Achieved

The -3dB low cut off frequency achieved is $100.04946\ Hz$, which is as expected.

Transient Response Simulation:

Figure 2.2.6 shows the transient response of the output and 500 Hz sinusoidal input waveform:

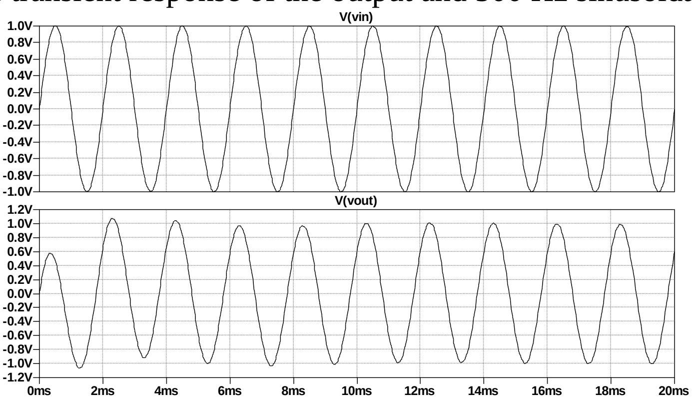

Figure 2.2.6: Transient Response of The Output and 500 Hz Sinusoidal Input Waveform

The steady state gain of the output voltage is 1, which indicates that the input 500 Hz signal is not attenuated.

Figure 2.2.7 shows the transient response of the output and 10 Hz sinusoidal input waveform

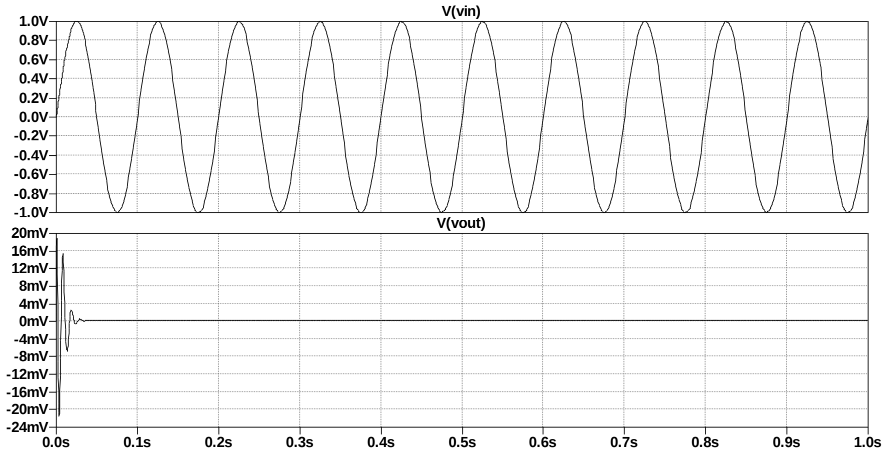

Figure 2.2.7: Transient Response of The Output and 10 Hz Sinusoidal Input Waveform

In steady state, the amplitude of the output signal is close to 0 mV, which indicates that signal of 10 Hz is attenuated to zero, as expected.

In high frequency ranges (up to MHz), AC frequency response is shown in figure 2.2.7

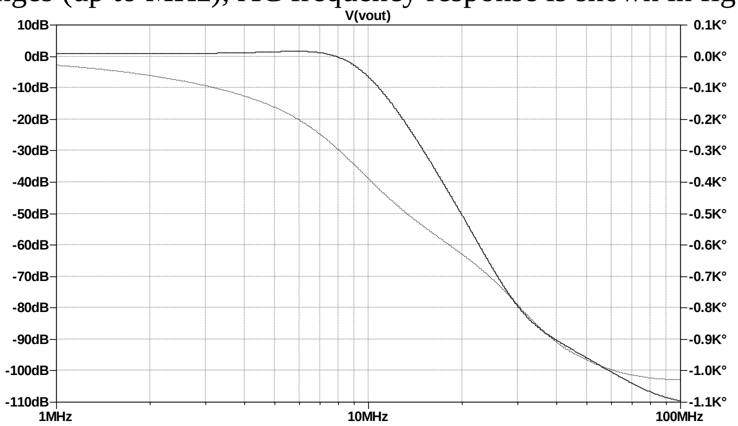

Figure 2.2.8: AC Amplitude Response in High Frequency Range

Moving around the cursor, high cut-off frequency can be obtained to be $9.0441946\ MHz$. This high cut-off frequency is caused by the upper limit of op-amp response. High frequency could cause phase difference between input and output signal through the op-amp, leading to oscillation in feedback circuits [11].

2.3 Task 3

New design schematics with E24 values is shown in figure 2.3.1:

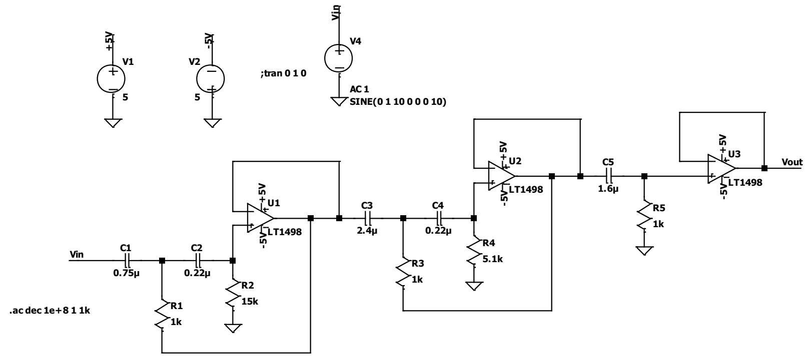

Figure 2.3.1: New LTspice Schematics with E24 values

AC amplitude response is shown in figure 2.3.2:

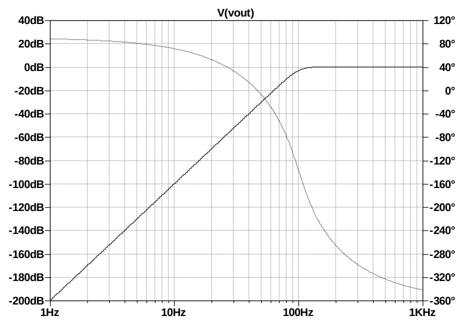

Figure 2.3.2: AC Amplitude Response for new LTspice Schematics

Moving the cursor around, the low cut-off frequency when amplitude is -3 dB is 98.704796 Hz as shown in figure 2.3.3:

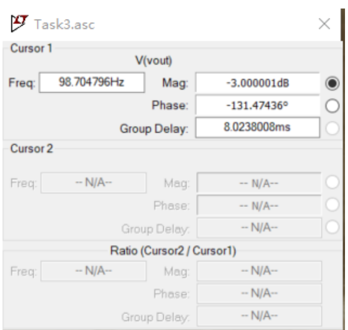

Figure 2.3.3: Screenshot Showing the -3dB Low Cut-off Frequency Achieved

This value does not differ significantly from the value shown in Figure 2.2.5, which indicates changes to E-value components have only minor effect to cut-off frequency.

Second order Sallen Key filter usually has low sensitivity to component tolerance. Cut-off frequency ω_c is affected by the value of resistors and capacitors because $\omega_c = \frac{1}{\sqrt{C_1 C_2 R_1 R_2}}$.
In extreme case, ω_c would have overall rating from $\frac{1}{\sqrt{110\% \times 110\% \times 105\% \times 105\%}} - 1 = -13.42\%$ to $\frac{1}{\sqrt{90\% \times 90\% \times 95\% \times 95\%}} - 1 = 16.95\%$. This effect could escalate as the order of the filter increases. The higher the order of the circuit, the more sensitive it is to component tolerance.

For a second order Butterworth filter, changes can be made as following to include amplification:

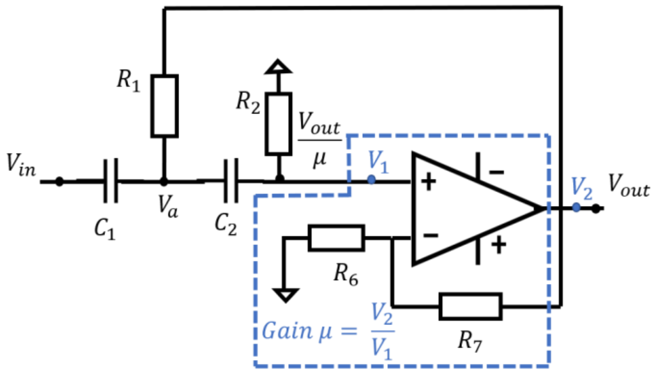

Figure 2.3.4: Second-order Filter with Amplification Gain Schematics

The circuit enclosed in dash lines is the changes made to provide gain. The gain μ can be described by equation 2.3.1:

$$\mu = 1 + \frac{R_6}{R_7} \tag{2.3.1}$$

For first order filter with gain, changes are shown in figure 2.3.5:

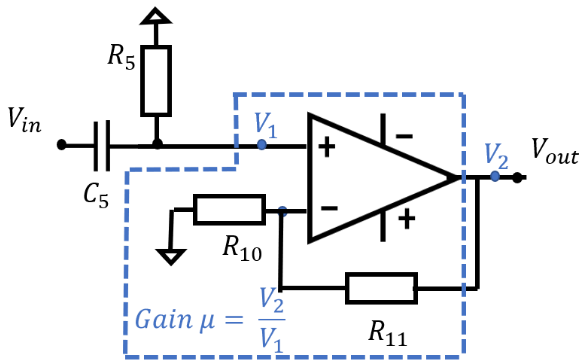

Figure 2.3.5: First-order Filter with Amplification Gain Schematics

Same as before, the gain is:

$$\mu = 1 + \frac{R_{11}}{R_{10}} \tag{2.3.2}$$

Therefore, the overall fifth-order filter amplification circuit is shown in figure 2.3.6:

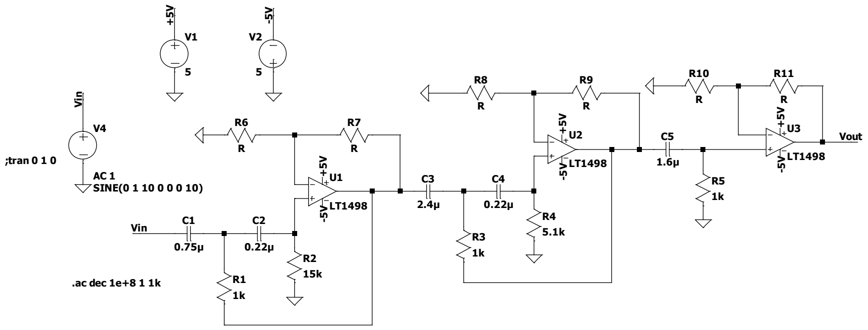

Figure 2.3.6: Fifth-order Filter with Amplification Gain Schematics

The overall gain can be expressed by equation 2.3.3:

$$\mu = (1 + \frac{R_7}{R_6})(1 + \frac{R_9}{R_8})(1 + \frac{R_{11}}{R_{10}}) \tag{2.3.3}$$

Conclusion

In conclusion, a fifth-order high pass Butterworth filter has been designed as per the required specifications. In task 1, performance characteristics of three types of filter is analysed. Butterworth filter tends to be a compromise between Bessel and Chebyshev filter, having more reliable output for this design. In task 2, the order of the filter was determined, followed by choosing suitable resistor and capacitor values, to meet design specifications. AC response and transient response show the design requirement is met and its high cut-off frequency is found by applying high input frequency range to the filter. In task 3, the component values in original design were taken place with E24 range values. The AC response shows that the changes to E24 range component values do not have a significant impact in filter response. Finally, a new circuit diagram was presented to include amplification gain in the passband.

(Word count: 1915)

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