The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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Pascal Contest

(Grade 9)

Tuesday, February 23, 2021
(in North America and South America)

Wednesday, February 24, 2021
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. Point Q is on a line segment between P and R, as shown. If $PR = 12$ and $PQ = 3$, what is the length of QR?

(A) 6 **(B)** 10 **(C)** 8 **(D)** 9 **(E)** 4

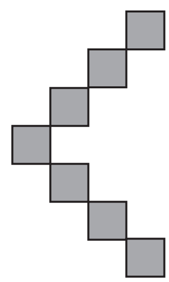
2. What number should be placed in the $\square$ to make the equation $\frac{1}{2} = \frac{\square}{8}$ true?

(A) 1 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 7

3. Elena earns \$13.25 per hour working at a store. How much does Elena earn in 4 hours?

(A) \$54.00 **(B)** \$56.25 **(C)** \$52.25 **(D)** \$51.00 **(E)** \$53.00

4. In the diagram, squares of side length 1 meet each other at their vertices. The perimeter of the figure is



(A) 14 **(B)** 20 **(C)** 24 **(D)** 28 **(E)** 32

5. Wesley is a professional runner. He ran five laps around a track. His times for the five laps were 63 seconds, 1 minute, 1.5 minutes, 68 seconds, and 57 seconds. What is the median of these times?

(A) 63 seconds **(B)** 1 minute **(C)** 1.5 minutes **(D)** 68 seconds **(E)** 57 seconds

6. A rectangle has length 13 and width 10. The length and the width of the rectangle are each increased by 2. By how much does the area of the rectangle increase?

(A) 50 **(B)** 20 **(C)** 38 **(D)** 35 **(E)** 26

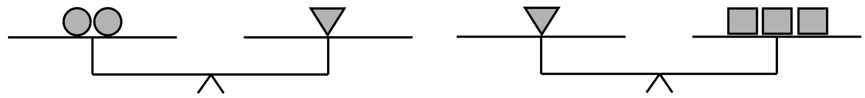
7. Which of the following is equal to 110% of 500?

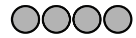
(A) 610 **(B)** 510 **(C)** 650 **(D)** 505 **(E)** 550

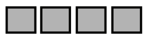
8. An integer n is decreased by 2 and then multiplied by 5. If the result is 85, the value of n is

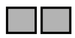
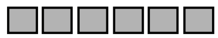
(A) 17 **(B)** 19 **(C)** 21 **(D)** 23 **(E)** 25

9. The two equal-arm scales shown are balanced.



Which of the following has the same mass as  ?

(A)  **(B)**  **(C)** 

(D)  **(E)** 

10. How many integers between 100 and 300 are multiples of both 5 and 7, but are not multiples of 10?

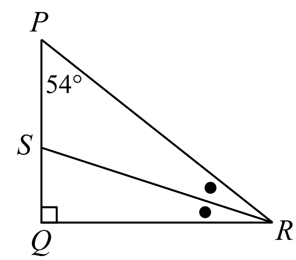
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

Part B: Each correct answer is worth 6.

11. If a and b are positive integers, the operation ∇ is defined by $a \nabla b = a^b \times b^a$. What is the value of $2 \nabla 3$?

(A) 36 **(B)** 72 **(C)** 3125 **(D)** 7776 **(E)** 46656

12. In the diagram, $\triangle PQR$ is right-angled at Q and has $\angle QPR = 54^\circ$. Also, point S lies on PQ such that $\angle PRS = \angle QRS$. What is the measure of $\angle RSQ$?



(A) 36° **(B)** 54° **(C)** 108°

(D) 18° **(E)** 72°

13. If $m + 1 = \dfrac{n-2}{3}$, what is the value of $3m - n$?

(A) -1 **(B)** -5 **(C)** -3 **(D)** -9 **(E)** -7

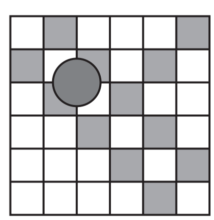
14. A robot is placed on the grid shown. The robot starts on square 25, initially facing square 32. The robot (i) moves 2 squares forward in the direction that it is facing, (ii) rotates clockwise 90°, and (iii) moves 1 square forward in the new direction. Thus, the robot moves to square 39, then turns to face square 38, then moves to square 38. The robot repeats the sequence of moves (i), (ii), (iii) two more times. Given that the robot never leaves the grid, on which square does it finish?

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

(A) 16 **(B)** 20 **(C)** 29

(D) 24 **(E)** 25

15. Nate has a grid made of shaded and unshaded 2 cm by 2 cm squares, as shown. He randomly places a circle with a diameter of 3 cm on the board so that the centre of the circle is at the meeting point of four squares. What is the probability that he places the disk so that it is touching an equal number of shaded and unshaded squares?



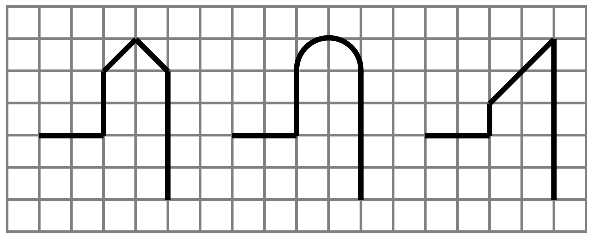
(A) $\frac{13}{25}$ **(B)** $\frac{17}{25}$ **(C)** $\frac{11}{25}$

(D) $\frac{21}{25}$ **(E)** $\frac{3}{5}$

16. The integer m is a *perfect cube* exactly when it is equal to n^3 for some integer n. For example, 1000 is a perfect cube since $1000 = 10^3$. What is the smallest positive integer k for which the integer $2^4 \times 3^2 \times 5^5 \times k$ is a perfect cube?

(A) 12 **(B)** 30 **(C)** 60 **(D)** 480 **(E)** 1620

17. In the diagram, Paths 1, 2 and 3 are drawn on a grid.



Paths 1 and 3 consist entirely of straight line segments. Path 2 consists of straight line segments and a semi-circle. If the length of Path 1 is x, the length of Path 2 is y, and the length of Path 3 is z, which of the following is true?

(A) $x < y$ and $y < z$ **(B)** $x < z$ and $z < y$ **(C)** $x = z$ and $z < y$
(D) $z < x$ and $x < y$ **(E)** $y < z$ and $z = x$

18. Trains arrive at Pascal Station every x minutes, where x is a positive integer. Trains arrive at Pascal Station at many different times, including at 10:10 a.m., 10:55 a.m., and 11:58 a.m. Which of the following is a possible value of x?

(A) 9 **(B)** 7 **(C)** 10 **(D)** 5 **(E)** 11

19. A group of friends are sharing a bag of candy.
On the first day, they eat $\frac{1}{2}$ of the candies in the bag.
On the second day, they eat $\frac{2}{3}$ of the remaining candies.
On the third day, they eat $\frac{3}{4}$ of the remaining candies.
On the fourth day, they eat $\frac{4}{5}$ of the remaining candies.
On the fifth day, they eat $\frac{5}{6}$ of the remaining candies.
At the end of the fifth day, there is 1 candy remaining in the bag.
How many candies were in the bag before the first day?

(A) 512 **(B)** 720 **(C)** 1024 **(D)** 1440 **(E)** 2048

20. Suppose that R, S and T are digits and that N is the four-digit positive integer $8RST$. That is, N has thousands digit 8, hundreds digit R, tens digits S, and ones (units) digit T, which means that $N = 8000 + 100R + 10S + T$. Suppose that the following conditions are all true:

- The two-digit integer $8R$ is divisible by 3.
- The three-digit integer $8RS$ is divisible by 4.
- The four-digit integer $8RST$ is divisible by 5.
- The digits of N are not necessarily all different.

The number of possible values for the integer N is

(A) 8 **(B)** 16 **(C)** 12 **(D)** 10 **(E)** 14

Part C: Each correct answer is worth 8.

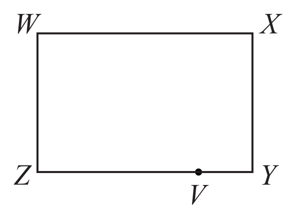
21. Three cubes have edge lengths 3 cm, 12 cm, and x cm. The average volume of the three cubes is 700 cm^3. The value of x, rounded to the nearest integer, is

(A) 6 **(B)** 10 **(C)** 8 **(D)** 9 **(E)** 7

22. Azmi has four blocks, each in the shape of a rectangular prism and each with dimensions $2 \times 3 \times 6$. She carefully stacks these four blocks on a flat table to form a tower that is four blocks high. The number of possible heights for this tower is

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

23. Rectangle $WXYZ$ has $WX = 4$, $WZ = 3$, and $ZV = 3$. The rectangle is curled without overlapping into a cylinder so that sides WZ and XY touch each other. In other words, W touches X and Z touches Y. The shortest distance from W to V through the inside of the cylinder can be written in the form $\sqrt{\dfrac{a+b\pi^2}{c\pi^2}}$ where a, b and c are positive integers. The smallest possible value of $a+b+c$ is



(A) 12 **(B)** 26 **(C)** 18
(D) 19 **(E)** 36

24. Suppose that $k \geq 2$ is a positive integer. An *in-shuffle* is performed on a list with $2k$ items to produce a new list of $2k$ items in the following way:

- The first k items from the original are placed in the odd positions of the new list in the same order as they appeared in the original list.
- The remaining k items from the original are placed in the even positions of the new list, in the same order as they appeared in the original list.

For example, an in-shuffle performed on the list $P\ Q\ R\ S\ T\ U$ gives the new list $P\ S\ Q\ T\ R\ U$. A second in-shuffle now gives the list $P\ T\ S\ R\ Q\ U$. Ping has a list of the 66 integers from 1 to 66, arranged in increasing order. He performs 1000 in-shuffles on this list, recording the new list each time. In how many of these 1001 lists is the number 47 in the 24th position?

(A) 90 **(B)** 71 **(C)** 83 **(D)** 72 **(E)** 84

25. Yann writes down the first n consecutive positive integers, $1, 2, 3, 4, \ldots, n-1, n$. He removes four different integers p, q, r, s from the list. At least three of p, q, r, s are consecutive and $100 < p < q < r < s$. The average of the integers remaining in the list is 89.5625. The number of possible values of s is

(A) 25 **(B)** 23 **(C)** 21 **(D)** 20 **(E)** 22

(The original version of this problem was missing the correct answer.)

2021 Pascal Contest (English)

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For students...

Thank you for writing the 2021 Pascal Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Fryer Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

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- Find your school's contest results