



Canadian Mathematics Competition

An activity of the Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Pascal Contest (Grade 9)

Wednesday, February 23, 2005

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Time: 60 minutes

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Calculators are permitted.

Instructions

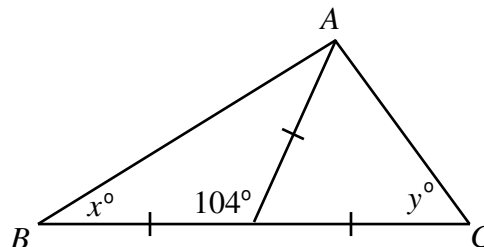
1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.

Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

- What is the value of $\frac{200 + 10}{20 + 10}$?
(A) 2 (B) 10 (C) 1 (D) 11 (E) 7
- The expression $6a - 5a + 4a - 3a + 2a - a$ is equal to
(A) $3a$ (B) $3a^6$ (C) 3 (D) $-21a$ (E) $-21a^6$
- When $x = 3$, the value of $x(x - 1)(x - 2)(x - 3)(x - 4)$ is
(A) 6 (B) -6 (C) 0 (D) 24 (E) -24
- Six balls, numbered 2, 3, 4, 5, 6, 7, are placed in a hat. Each ball is equally likely to be chosen. If one ball is chosen, what is the probability that the number on the selected ball is a prime number?
(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{5}{6}$
- The value of $\sqrt{36 \times \sqrt{16}}$ is
(A) 12 (B) 144 (C) 24 (D) 26 (E) 96
- A glass filled with water has a mass of 1000 g. When half the water is removed from the glass, the mass of the glass and the remaining water is 700 g. What is the mass of the empty glass?
(A) 600 g (B) 500 g (C) 350 g (D) 400 g (E) 300 g
- If $\frac{1}{3}x = 12$, then $\frac{1}{4}x$ equals
(A) 1 (B) 16 (C) 9 (D) 144 (E) 64
- Which of the numbers -5 , $\frac{3}{2}$, 2 , $\frac{3}{5}$, 8 is larger than its square?
(A) -5 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{3}{5}$ (E) 8
- In triangle ABC , the value of $x + y$ is
(A) 104 (B) 76 (C) 180
(D) 90 (E) 166



- In the sequence 32, 8, —, —, x , each term after the second is the average of the two terms immediately before it. The value of x is
(A) 17 (B) 20 (C) 44 (D) 24 (E) 14

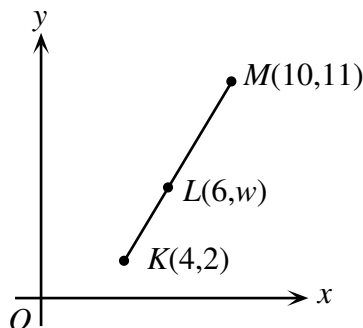
Part B: Each correct answer is worth 6.

11. If a , b and c are positive integers with $a \times b = 13$, $b \times c = 52$, and $c \times a = 4$, the value of $a \times b \times c$ is

(A) 2704 (B) 104 (C) 676 (D) 208 (E) 52

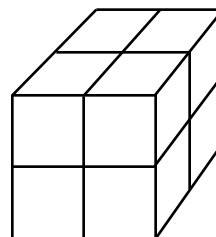
12. Point L lies on line segment KM , as shown.
The value of w is

(A) 4 (B) 5 (C) 6
(D) 7 (E) 8



13. Eight unit cubes are used to form a larger 2 by 2 by 2 cube. The six faces of this larger cube are then painted red. When the paint is dry, the larger cube is taken apart. What fraction of the total surface area of the unit cubes is red?

(A) $\frac{1}{6}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$
(D) $\frac{1}{4}$ (E) $\frac{1}{3}$



14. A positive integer whose digits are the same when read forwards or backwards is called a palindrome. For example, 4664 is a palindrome. How many integers between 2005 and 3000 are palindromes?

(A) 0 (B) 8 (C) 9 (D) 10 (E) more than 10

15. When 14 is divided by 5, the remainder is 4. When 14 is divided by a positive integer n , the remainder is 2. For how many different values of n is this possible?

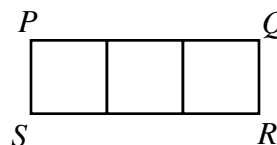
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

16. The digits 1, 2, 5, 6, and 9 are all used to form five-digit *even* numbers, in which no digit is repeated. The difference between the largest and smallest of these numbers is

(A) 83 916 (B) 79 524 (C) 83 952 (D) 79 236 (E) 83 016

17. In the diagram, rectangle $PQRS$ is divided into three identical squares. If $PQRS$ has perimeter 120 cm, what is its area, in cm^2 ?

(A) 225 (B) 675 (C) 360
(D) 432 (E) 144



18. When the expression $2005^2 + 2005^0 + 2005^0 + 2005^5$ is evaluated, the final two digits are

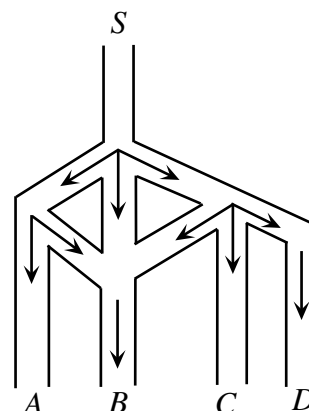
(A) 52 (B) 25 (C) 20 (D) 50 (E) 05

19. A whole number is called *decreasing* if each digit of the number is less than the digit to its left. For example, 8540 is a decreasing four-digit number. How many decreasing numbers are there between 100 and 500?

(A) 11 (B) 10 (C) 9 (D) 8 (E) 7

20. Harry the Hamster is put in a maze, and he starts at point S . The paths are such that Harry can move forward only in the direction of the arrows. At any junction, he is equally likely to choose any of the forward paths. What is the probability that Harry ends up at B ?

(A) $\frac{2}{3}$ (B) $\frac{13}{18}$ (C) $\frac{11}{18}$
(D) $\frac{1}{3}$ (E) $\frac{1}{4}$



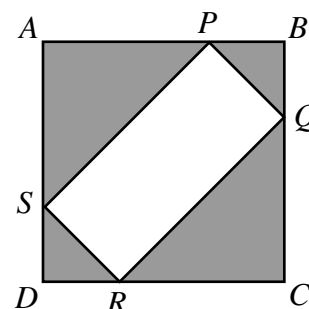
Part C: Each correct answer is worth 8.

21. Integers m and n are each greater than 100. If $m + n = 300$, then $m : n$ could be equal to

(A) 9 : 1 (B) 17 : 8 (C) 5 : 3 (D) 4 : 1 (E) 3 : 2

22. In the diagram, two pairs of identical isosceles triangles are cut off of square $ABCD$, leaving rectangle $PQRS$. The total area cut off is 200 m^2 . The length of PR , in metres, is

(A) $\sqrt{200}$ (B) 20 (C) $\sqrt{800}$
(D) 25 (E) 15



23. Starting with the 2, the number 2005 can be formed by moving either horizontally, vertically, or diagonally from square to square in the grid. How many different paths can be followed to form 2005?

(A) 96 (B) 72 (C) 80
(D) 64 (E) 88

5	5	5	5	5
5	0	0	0	5
5	0	2	0	5
5	0	0	0	5
5	5	5	5	5

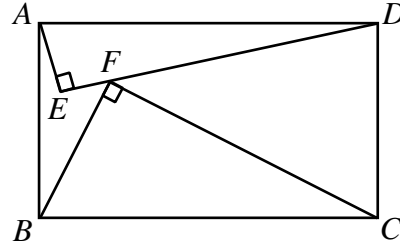
24. A positive integer is called a *perfect power* if it can be written in the form a^b , where a and b are positive integers with $b \geq 2$. For example, 32 and 125 are perfect powers because $32 = 2^5$ and $125 = 5^3$.

The increasing sequence

$$2, 3, 5, 6, 7, 10, \dots$$

consists of all positive integers which are not perfect powers. The sum of the squares of the digits of the 1000th number in this sequence is

- (A) 42 (B) 26 (C) 33 (D) 18 (E) 21
25. In the diagram, right-angled triangles AED and BFC are constructed inside rectangle $ABCD$ so that F lies on DE . If $AE = 21$, $ED = 72$ and $BF = 45$, what is the length of AB ?
- (A) 50 (B) 48 (C) 52
(D) 54 (E) 56





Canadian Mathematics Competition



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Thank you for writing the 2005 Pascal Contest!

In 2004, more than 83 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

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