Pascal Contest

(Grade 9)

Thursday, February 24, 2011

UNIVERSITY OF
WATERLOO

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Time: 60 minutes   ©2010 Centre for Education in Mathematics and Computing
Calculators are permitted
Instructions

1. Do not open the Contest booklet until you are told to do so.

2. You may use rulers, compasses and paper for rough work.

3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.

4. On your response form, print your school name, city/town, and province in the box in the upper left corner.

5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**

6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.

7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

8. Diagrams are *not* drawn to scale. They are intended as aids only.

9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Part A: Each correct answer is worth 5.

1. What is the value of $6 \times (5 - 2) + 4$?

    (A) 18   **(B)** 22   **(C)** 24   **(D)** 32   **(E)** 42

2. Nine hundred forty-three minus eighty-seven equals

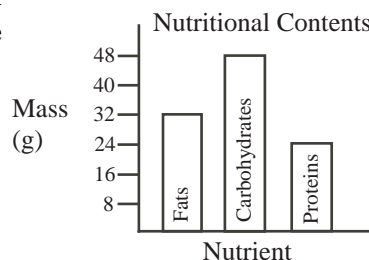   **(A)** -1030 **(B)** -856 **(C)** 770   **(D)** 1030   **(E)** 856

3. Which list of numbers is written in increasing order?

    (A) $2011, \sqrt{2011}, 2011^2$
    (B) $2011, 2011^2, \sqrt{2011}$
    (C) $\sqrt{2011}, 2011, 2011^2$
    (D) $\sqrt{2011}, 2011^2, 2011$
    (E) $2011^2, \sqrt{2011}, 2011$

4. The graph shows the nutritional contents of a Pascal Burger. Which ratio compares the mass of fats to the mass of carbohydrates?

    (A) $3 : 2$ **(B)** $2 : 3$ **(C)** $2 : 1$

    (D) $4 : 3$ **(E)** $3 : 4$

5. When $x = -2$, the value of $(x + 1)^3$ is

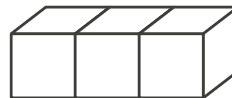   **(A)** -1 **(B)** -8 **(C)** -5 **(D)** 1   **(E)** -3

6. Peyton puts 30 L of oil and 15 L of vinegar into a large empty can. He then adds 15 L of oil to create a new mixture. What percentage of the new mixture is oil?

    (A) 75   **(B)** 25   **(C)** 45   **(D)** 50   **(E)** 60

7. Three 1 by 1 by 1 cubes are joined side by side, as shown. What is the surface area of the resulting prism?

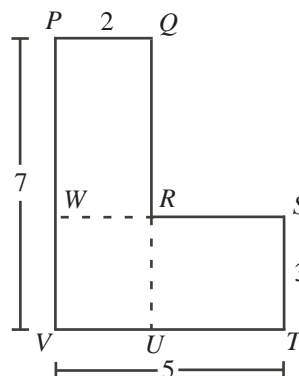   **(A)** 13   **(B)** 14   **(C)** 15

    (D) 16   **(E)** 17

8. The 17th day of a month is Saturday. The first day of that month was

    (A) Monday   **(B)** Tuesday   **(C)** Wednesday **(D)** Thursday   **(E)** Friday

9. Two rectangles $PQUV$ and $WSTV$ overlap as shown. What is the area of $PQRSTV$?


(A) 35   (B) 24   (C) 25

(D) 17   (E) 23

10. John lists the integers from 1 to 20 in increasing order. He then erases the first half of the integers in the list and rewrites them in order at the end of the second half of the list. Which integer in the new list has exactly 12 integers to its left?

(A) 1   (B) 2   (C) 3   (D) 12   (E) 13

Part B: Each correct answer is worth 6.

11. Which of the following numbers is closest to 1?

(A) $\frac{11}{10}$ (B) $\frac{111}{100}$ (C) 1.101   (D) $\frac{1111}{1000}$ (E) 1.011

12. The number of odd integers between $\frac{17}{4}$ and $\frac{35}{2}$ is

(A) 4   (B) 5   (C) 6   (D) 7   (E) 8

13. The first four terms of a sequence are 1, 4, 2, and 3. Beginning with the fifth term in the sequence, each term is the sum of the previous four terms. Therefore, the fifth term is 10. What is the eighth term?

(A) 66   (B) 65   (C) 69   (D) 134   (E) 129

14. In the diagram, a garden is enclosed by six straight fences. If the area of the garden is 97 m^2, what is the length of the fence around the garden?

(A) 48 m   (B) 47 m   (C) 40 m

(D) 38 m   (E) 37 m
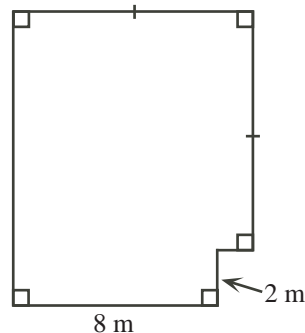
15. Six friends ate at a restaurant and agreed to share the bill equally. Because Luxmi forgot her money, each of her five friends paid an extra $3 to cover her portion of the total bill. What was the total bill?

(A) $90   (B) $84   (C) $75   (D) $108   (E) $60

16. The set $S = \{1, 2, 3, \ldots, 49, 50\}$ contains the first 50 positive integers. After the multiples of 2 and the multiples of 3 are removed, how many integers remain in the set S?
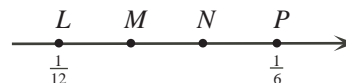
(A) 8   (B) 9   (C) 16   (D) 17   (E) 18

17. In the subtraction shown, K, L, M, and N are digits. What is the value of $K + L + M + N$?

$$\begin{array}{r} 6\ K\ 0\ L \\ -\ M\ 9\ N\ 4 \\ \hline 2\ 0\ 1\ 1 \end{array}$$

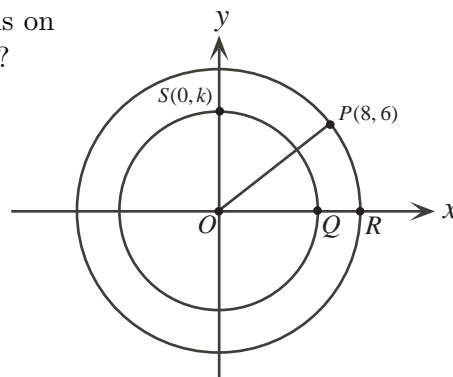(A) 17   (B) 18   (C) 19

(D) 23   (E) 27

18. On the number line, points M and N divide LP into three equal parts. What is the value at M?

(A) $\frac{1}{7}$ (B) $\frac{1}{8}$ (C) $\frac{1}{9}$

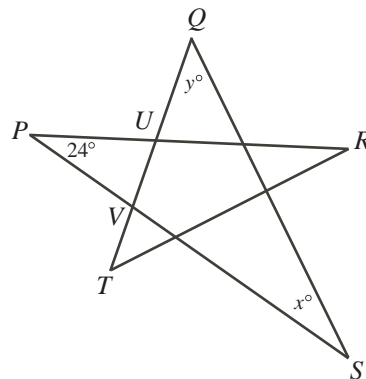(D) $\frac{1}{10}$ (E) $\frac{1}{11}$

19. Two circles are centred at the origin, as shown. The point $P(8,6)$ is on the larger circle and the point $S(0,k)$ is on the smaller circle. If $QR = 3$, what is the value of k?

(A) 3.5   (B) 4   (C) 6

(D) 6.5   (E) 7

20. In the diagram, PR, PS, QS, QT, and RT are straight line segments. QT intersects PR and PS at U and V, respectively. If $PU = PV$, $\angle UPV = 24°$, $\angle PSQ = x°$, and $\angle TQS = y°$, what is the value of $x + y$?

(A) 48   (B) 66   (C) 72

(D) 78   (E) 156


Part C: Each correct answer is worth 8.

21. In the diagram, there are 26 levels, labelled $A, B, C, \ldots, Z$. There is one dot on level A. Each of levels $B, D, F, H, J, \ldots$, and Z contains twice as many dots as the level immediately above. Each of levels $C, E, G, I, K, \ldots$, and Y contains the same number of dots as the level immediately above. How many dots does level Z contain?

(A) 1024   (B) 2048   (C) 4096

(D) 8192   (E) 16 384
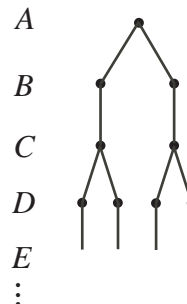
22. Each of the integers 1 to 7 is to be written, one in each circle in the diagram. The sum of the three integers in any straight line is to be the same. In how many different ways can the centre circle be filled?

   (A) 1   (B) 2   (C) 3

   (D) 4   (E) 5
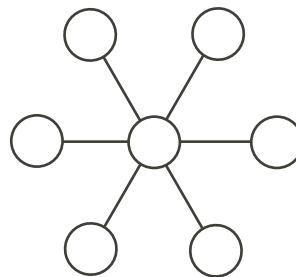
23. An ordered list of four numbers is called a *quadruple*.
   A quadruple (p, q, r, s) of integers with $p, q, r, s \geq 0$ is chosen at random such that

$$2p + q + r + s = 4$$

   What is the probability that $p + q + r + s = 3$?

   (A) $\frac{3}{22}$ (B) $\frac{3}{11}$ (C) $\frac{3}{19}$ (D) $\frac{6}{19}$ (E) $\frac{2}{7}$

24. Let n be the largest integer for which $14n$ has exactly 100 digits. Counting from right to left, what is the 68th digit of n?

   (A) 1   (B) 2   (C) 4   (D) 5   (E) 8

25. Dolly, Molly and Polly each can walk at 6 km/h. Their one motorcycle, which travels at 90 km/h, can accommodate at most two of them at once (and cannot drive by itself!). Let t hours be the time taken for all three of them to reach a point 135 km away. Ignoring the time required to start, stop or change directions, what is true about the smallest possible value of t?

   (A) $t < 3.9$ (B) $3.9 \leq t < 4.1$ (C) $4.1 \leq t < 4.3$
   (D) $4.3 \leq t < 4.5$ (E) $t \geq 4.5$

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