The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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Pascal Contest

(Grade 9)

Tuesday, February 27, 2018

(in North America and South America)

Wednesday, February 28, 2018

(outside of North America and South America)

Time: 60 minutes

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Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.
10. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. Which of the following is the smallest number?

   (A) 1.4 **(B)** 1.2 **(C)** 2.0 **(D)** 1.5 **(E)** 2.1

2. The value of $\dfrac{2018 - 18 + 20}{2}$ is

   (A) 1010 **(B)** 2020 **(C)** 1008 **(D)** 2017 **(E)** 1011

3. July 3, 2030 is a Wednesday. What day of the week is July 14, 2030?

   (A) Wednesday **(B)** Saturday **(C)** Sunday
   (D) Monday **(E)** Tuesday

4. An electric car is charged 3 times per week for 52 weeks. The cost to charge the car each time is \$0.78. What is the total cost to charge the car over these 52 weeks?

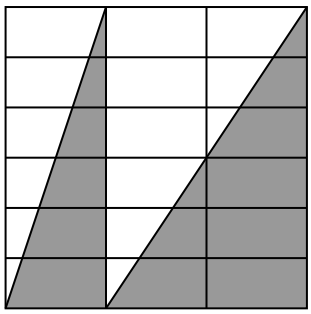
   (A) \$104.00 **(B)** \$81.12 **(C)** \$202.80 **(D)** \$162.24 **(E)** \$121.68

5. If $3 \times 3 \times 5 \times 5 \times 7 \times 9 = 3 \times 3 \times 7 \times n \times n$, what is a possible value of n?

   (A) 15 **(B)** 25 **(C)** 45 **(D)** 35 **(E)** 5

6. In the diagram, 18 identical 1×2 rectangles are put together to form a 6×6 square. Part of the square is shaded, as shown. What percentage of the area of the 6×6 square is shaded?

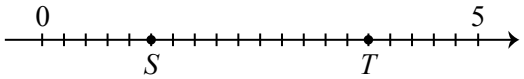
   (A) 50% **(B)** 67% **(C)** 75%
   (D) 33% **(E)** 25%



7. A box contains 5 black ties, 7 gold ties, and 8 pink ties. Stephen randomly chooses a tie from the box. Each tie is equally likely to be chosen. The probability that Stephen chooses a pink tie is equivalent to

   (A) $\frac{1}{4}$ **(B)** $\frac{7}{20}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{3}{4}$

8. In the diagram, the number line between 0 and 5 is divided into 20 equal parts. The numbers S and T are marked on the line. What is the value of $S + T$?



   (A) 5.25 **(B)** 5.5 **(C)** 4.5 **(D)** 4.75 **(E)** 5

9. The symbols $\heartsuit$ and ∇ represent different positive integers less than 20. If $\heartsuit \times \heartsuit \times \heartsuit = \nabla$, what is the value of $\nabla \times \nabla$?

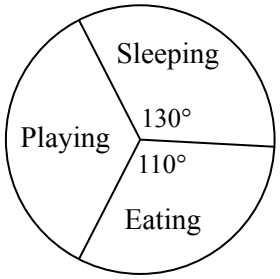
   (A) 12 **(B)** 16 **(C)** 36 **(D)** 64 **(E)** 81

10. Which of the following points lies on the line that passes through $(-2, 1)$ and $(2, 5)$?

(A) $(0, 0)$ **(B)** $(0, 2)$ **(C)** $(0, 3)$ **(D)** $(0, 4)$ **(E)** $(0, 5)$

Part B: Each correct answer is worth 6.

11. In the diagram, the circle graph shows how a baby polar bear spent 24 hours. How many hours did it spend playing?

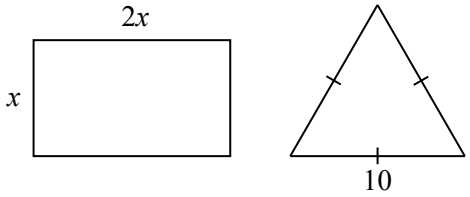


(A) 6 **(B)** 7 **(C)** 8
(D) 9 **(E)** 10

12. Glenda, Helga, Ioana, Julia, Karl, and Liu participated in the 2017 Canadian Team Mathematics Contest. On their team uniforms, each had a different number chosen from the list 11, 12, 13, 14, 15, 16. Helga's and Julia's numbers were even. Karl's and Liu's numbers were prime numbers. Glenda's number was a perfect square. What was Ioana's number?

(A) 11 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 12

13. A rectangle with height x and width $2x$ has the same perimeter as an equilateral triangle with side length 10. What is the area of the rectangle?



(A) 18 **(B)** 50 **(C)** 25 **(D)** 200 **(E)** 100

14. In the list $7, 9, 10, 11, 18$, which number is the average (mean) of the other four numbers?

(A) 9 **(B)** 18 **(C)** 7 **(D)** 11 **(E)** 10

15. A digital clock shows the time 4:56. How many minutes will pass until the clock next shows a time in which all of the digits are consecutive and are in increasing order?

(A) 458 **(B)** 587 **(C)** 376 **(D)** 315 **(E)** 518

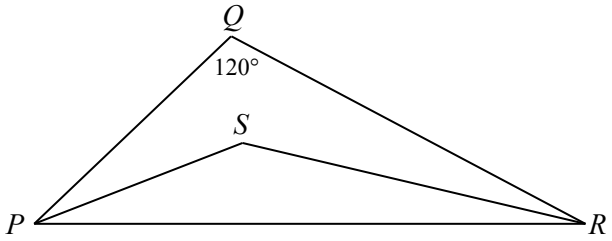
16. Reading from left to right, a sequence consists of 6 X's, followed by 24 Y's, followed by 96 X's. After the first n letters, reading from left to right, one letter has occurred twice as many times as the other letter. The sum of the four possible values of n is

(A) 72 **(B)** 54 **(C)** 135 **(D)** 81 **(E)** 111

17. Suppose that p and q are two different prime numbers and that $n = p^2q^2$. The number of possible values of n with $n < 1000$ is

(A) 5 **(B)** 6 **(C)** 4 **(D)** 8 **(E)** 7

18. In the diagram, $\triangle PQR$ has $\angle PQR = 120°$. Also, $\angle QPS = \angle RPS$ and $\angle QRS = \angle PRS$. (In other words, SP and SR bisect $\angle QPR$ and $\angle QRP$, respectively.) What is the measure of $\angle PSR$?

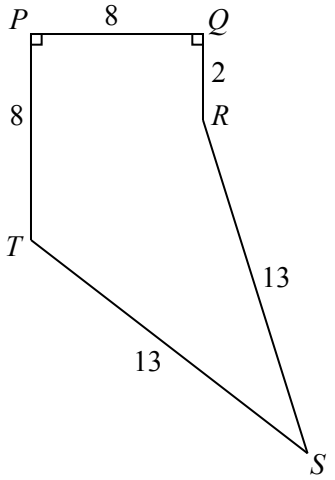


(A) 130° **(B)** 120° **(C)** 140° **(D)** 160° **(E)** 150°

19. On Monday, Mukesh travelled x km at a constant speed of 90 km/h. On Tuesday, he travelled on the same route at a constant speed of 120 km/h. His trip on Tuesday took 16 minutes less than his trip on Monday. The value of x is

(A) 90 **(B)** 112 **(C)** 100 **(D)** 96 **(E)** 92

20. In the diagram, $PQRST$ is a pentagon with $PQ = 8$, $QR = 2$, $RS = 13$, $ST = 13$, and $TP = 8$. Also, $\angle TPQ = \angle PQR = 90°$. What is the area of pentagon $PQRST$?

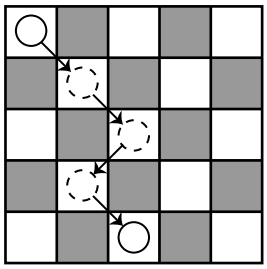


(A) 76 **(B)** 84 **(C)** 92

(D) 100 **(E)** 108

Part C: Each correct answer is worth 8.

21. A coin travels along a path that starts in an unshaded square in the top row of the figure, that uses only diagonal moves, and that ends in an unshaded square in the bottom row. A diagonal move takes the coin either one square down and one square left, or one square down and one square right. How many different paths from the top row to the bottom row are possible?

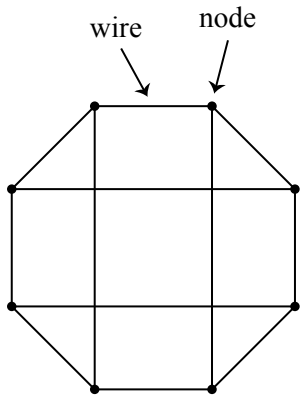


(A) 16 **(B)** 20 **(C)** 32

(D) 24 **(E)** 28

22. A Miniou circuit contains nodes and wires and obeys the following rules:

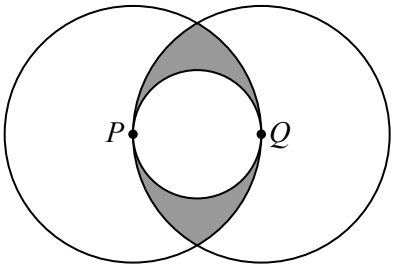
- Each wire connects two different nodes.
- There is at most one wire between each pair of nodes.
- Exactly three wires are connected to each node.



An example of a Miniou circuit is shown. If a Miniou circuit has 13 788 wires, how many nodes does it have?

(A) 9190 **(B)** 9192 **(C)** 9188 **(D)** 9186 **(E)** 9184

23. In the diagram, two larger circles with radius 1 have centres P and Q. Also, the smaller circle has diameter PQ. The region inside the two larger circles and outside the smaller circle is shaded.



The area of the shaded region is closest to

(A) 0.36 **(B)** 0.38 **(C)** 0.40 **(D)** 0.42 **(E)** 0.44

24. In Mrs. Warner's class, there are 30 students. Strangely, 15 of the students have a height of 1.60 m and 15 of the students have a height of 1.22 m. Mrs. Warner lines up n students so that the average height of any four consecutive students is greater than 1.50 m and the average height of any seven consecutive students is less than 1.50 m. What is the largest possible value of n?

(A) 8 **(B)** 12 **(C)** 11 **(D)** 9 **(E)** 10

25. P.J. starts with $m = 500$ and chooses a positive integer n with $1 \leq n \leq 499$. He applies the following algorithm to m and n:

- P.J. sets r equal to the remainder when m is divided by n.
- If $r = 0$, P.J. sets $s = 0$.
  If $r > 0$, P.J. sets s equal to the remainder when n is divided by r.
- If $s = 0$, P.J. sets $t = 0$.
  If $s > 0$, P.J. sets t equal to the remainder when r is divided by s.

For example, when $n = 8$, P.J. obtains $r = 4$, $s = 0$, and $t = 0$. For how many of the positive integers n with $1 \leq n \leq 499$ does P.J.'s algorithm give $1 \leq r \leq 15$ and $2 \leq s \leq 9$ and $t = 0$?

(A) 14 **(B)** 12 **(C)** 16 **(D)** 15 **(E)** 13

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For students...

Thank you for writing the 2018 Pascal Contest! Each year, more than 240 000 students from more than 75 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Fryer Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

- More information about the Fryer Contest
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- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

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