The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Pascal Contest
(Grade 9)

Wednesday, February 22, 2023
(in North America and South America)

Thursday, February 23, 2023
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.

2. You may use rulers, compasses and paper for rough work.

3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.

4. On your response form, print your school name and city/town in the box in the upper right corner.

5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**

6. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.

7. The correct answer to each question in Part C is an integer from 0 to 99, inclusive. After deciding on your answer, fill in the appropriate two circles on the response form. A one-digit answer (such as "7") must be coded with a leading zero ("07").

8. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

9. Diagrams are *not* drawn to scale. They are intended as aids only.

10. When your supervisor tells you to begin, you will have 60 minutes of working time.

11. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.
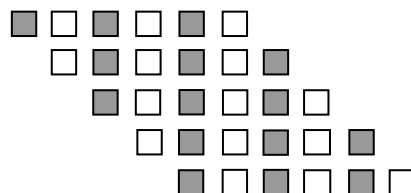
Part A: Each correct answer is worth 5.

1. Which of the following integers has the greatest value?

   (A) $100\,004$ **(B)** $110\,003$ **(C)** $102\,002$ **(D)** $100\,301$ **(E)** $100\,041$

2. In the diagram, 30 identical small squares are shown.
   How many of these 30 squares are shaded?

   (A) 10   **(B)** 12   **(C)** 15

   (D) 18   **(E)** 20

3. The value of $2^3 - 2 + 3$ is

   (A) 2   **(B)** 7   **(C)** 9   **(D)** 10   **(E)** 12

4. If $3 + \triangle = 5$ and $\triangle + \square = 7$, the value of $\triangle + \triangle + \triangle + \square + \square$ is

   (A) 2   **(B)** 5   **(C)** 12   **(D)** 19   **(E)** 16

5. The expression $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000}$ is equal to

   (A) 0.333   **(B)** 0.9   **(C)** 0.963   **(D)** 0.369   **(E)** 0.30303

6. If $\frac{1}{3}$ of x is equal to 4, then $\frac{1}{6}$ of x is equal to

   (A) 3   **(B)** 4   **(C)** 2   **(D)** $\frac{7}{6}$ **(E)** $\frac{4}{3}$

7. Jurgen is travelling to Waterloo by bus. He packs for 25 minutes. He then walks to the bus station, which takes 35 minutes. He arrives 60 minutes before his bus leaves. His bus leaves at 6:45 p.m. At what time did he start packing?

   (A) 4:45 p.m.   **(B)** 4:40 p.m.   **(C)** 4:35 p.m.   **(D)** 4:55 p.m.   **(E)** 4:50 p.m.

8. A sign has 31 spaces on a single line.
   The word RHOMBUS is written from left to right in 7 consecutive spaces.
   There is an equal number of empty spaces on each side of the word.
   Counting from the left, in what space number should the letter R be put?

   (A) 12   **(B)** 13   **(C)** 14   **(D)** 15   **(E)** 16

9. The decimal representation of $\frac{1}{11}$ is 0.09090909....
   Another way to write this decimal representation is $0.\overline{09}$.
   Similarly, $0.\overline{125}$ represents the number $0.125125125\ldots$.
   The decimal representation of $\frac{1}{7}$ is $0.\overline{142857}$.
   In the decimal representation of $\frac{1}{7}$, the 100th digit to the right of the decimal is

   (A) 1   **(B)** 4   **(C)** 2   **(D)** 8   **(E)** 5

10. In the diagram, points A, B and C are plotted on a 7×10 grid. Line segments join A, B and C. An ant walks directly from A to B to C to A along these line segments. The distance that the ant walks is equal to

    (A) $40 + \sqrt{13}$ (B) $13 + \sqrt{39}$ (C) $15 + \sqrt{149}$
   (D) $13 + \sqrt{89}$ (E) 26
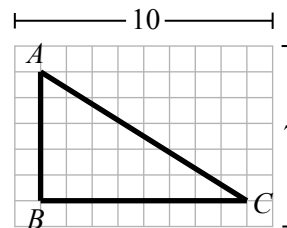
Part B: Each correct answer is worth 6.

11. A rectangular prism has a volume of 12 cm^3. A new prism is formed by doubling the length, doubling the width, and tripling the height of the original prism. The volume of this new prism is

    (A) 24 cm^3 (B) 36 cm^3 (C) 72 cm^3 (D) 96 cm^3 (E) 144 cm^3

12. Morgan uses a spreadsheet to create a table of values. In the first column, she lists the positive integers from 1 to 400. She then puts integers in the second column in the following way: if the integer in the first column of a given row is n, the number in the second column of that row is $3n + 1$. Which of the following integers does not appear in the second column?

    (A) 31   (B) 94   (C) 131   (D) 331   (E) 907

13. On February 1, it was 16.2°C outside Jacinta's house at 3:00 p.m. On February 2, it was -3.6°C outside Jacinta's house at 2:00 a.m. If the temperature changed at a constant rate between these times, the rate at which the temperature decreased was

    (A) 1.1°C/h   (B) 1.8°C/h   (C) 2.2°C/h   (D) 3.6°C/h   (E) 4.4°C/h

14. Each of four doors is randomly either open or closed. What is the probability that exactly two of the four doors are open?

    (A) $\frac{3}{8}$ (B) $\frac{5}{16}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) $\frac{3}{16}$

15. Nasim buys trading cards in packages of 5 cards and in packages of 8 cards. He can purchase exactly 18 cards by buying two 5-packs and one 8-pack, but he cannot purchase exactly 12 cards with any combination of packages. For how many of the integers $n = 24, 25, 26, 27, 28, 29$ can he buy exactly n cards?

    (A) 5   (B) 3   (C) 2   (D) 4   (E) 6

16. At the start of this month, Mathilde and Salah each had 100 coins. For Mathilde, this was 25% more coins than she had at the start of last month. For Salah, this was 20% fewer coins than he had at the start of last month. The total number of coins that they had at the start of last month was

    (A) 180   (B) 185   (C) 190   (D) 200   (E) 205

17. In a survey, 100 students were asked if they like lentils and were also asked if they like chickpeas. A total of 68 students like lentils. A total of 53 like chickpeas. A total of 6 like neither lentils nor chickpeas. How many of the 100 students like both lentils and chickpeas?

    (A) 32   (B) 27   (C) 26   (D) 21   (E) 15

18. In the diagram, A, B, D, F, and G lie on a vertical line, $\triangle BCD$ is right-angled at C, and $\triangle DEF$ is right-angled at E. Also, $\angle ABC = x°$, $\angle CDE = 80°$, and $\angle EFG = y°$. What is the value of $x + y$?

   (A) 250   (B) 260   (C) 270

    (D) 280   (E) 290
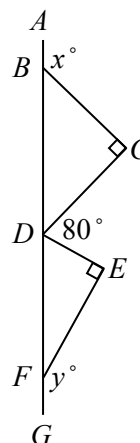
19. Ellie's drawer of hair clips contains 4 red clips, 5 blue clips, and 7 green clips. Each morning, she randomly chooses one hair clip to wear for the day. She returns this clip to the drawer each evening. One morning, Kyne removes k hair clips before Ellie can make her daily selection. As a result, the probability that Ellie chooses a red clip is doubled. Which of the following is a possible value of k?

    (A) 6   (B) 9   (C) 12   (D) 4   (E) 13

20. Four larger circles with radius 5 are arranged so that their centres are the vertices of a square. Each of the larger circles is tangent to (that is, just touches) two of the other circles, as shown. A smaller circle with radius r is drawn in the region between the four larger circles. The smaller circle is tangent to each of the larger circles. The value of r is closest to

    (A) 1.9   (B) 2.0   (C) 2.1

    (D) 2.2   (E) 2.3
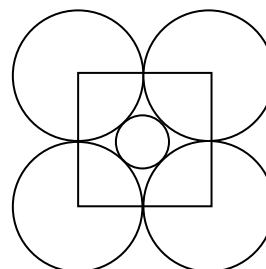
Part C: Each correct answer is worth 8.
Each correct answer is an integer from 0 to 99, inclusive.
A one-digit answer (such as "7") must be coded with a leading zero ("07").
Note: The integer formed by the rightmost two digits of 12 345 is 45.
**   The integer formed by the rightmost two digits of 6307 is 7, coded 07.**

21. Starting with a positive integer m, Alicia creates a sequence by applying the following algorithm:

   - Step 1: Alicia writes down the number m as the first term of the sequence.
   - Step 2: If m is even, Alicia sets $n = \frac{1}{2}m$. If m is odd, Alicia sets $n = m + 1$.
   - Step 3: Alicia writes down the number $m+n+1$ as the next term of the sequence.
   - Step 4: Alicia sets m equal to the value of the term that she just wrote down in Step 3.
   - Step 5: Alicia repeats Steps 2, 3, 4 until she has five terms, at which point she stops.

   For example, starting with $m = 1$, Alicia's sequence would be 1, 4, 7, 16, 25.
   Alicia starts a sequence with $m = 3$. What is the fifth term of her sequence?

22. The integers 1, 2, 4, 5, 6, 9, 10, 11, 13 are to be placed in the circles and squares below with one number in each shape.

   $$\boxed{x}\ \bigcirc\ \square\ \bigcirc\ \square\ \bigcirc\ \square\ \bigcirc\ \boxed{y}$$

   Each integer must be used exactly once and the integer in each circle must be equal to the sum of the integers in the two neighbouring squares. If the integer x is placed in the leftmost square and the integer y is placed in the rightmost square, what is the largest possible value of $x + y$?

23. Dewa writes down a list of four integers. He calculates the average of each group of three of the four integers. These averages are 32, 39, 40, 44. What is the largest of the four integers?

24. Cube $ABCDEFGH$ has edge length 100. Point P is on AB, point Q is on AD, and point R is on AF, as shown, so that $AP = x$, $AQ = x + 1$ and $AR = \dfrac{x+1}{2x}$ for some integer x. For how many integers x is the volume of triangular-based pyramid $APQR$ between 0.04% and 0.08% of the volume of cube $ABCDEFGH$? (The volume of a pyramid is equal to one-third of the area of its base times its height.)
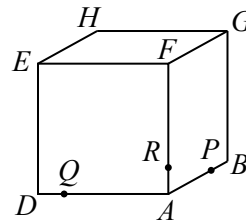
25. Consider positive integers $a \le b \le c \le d \le e$. There are N lists a, b, c, d, e with a mean of 2023 and a median of 2023, in which the integer 2023 appears more than once, and in which no other integer appears more than once. What is the sum of the digits of N?

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For students...

Thank you for writing the 2023 Pascal Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Fryer Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

- More information about the Fryer Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Look at our free online courseware
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
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- Find your school's contest results