The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Pascal Contest
(Grade 9)

Wednesday, February 28, 2024
(in North America and South America)

Thursday, February 29, 2024
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.

2. You may use rulers, compasses and paper for rough work.

3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.

4. On your response form, print your school name and city/town in the box in the upper right corner.

5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**

6. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.

7. The correct answer to each question in Part C is an integer from 0 to 99, inclusive. After deciding on your answer, fill in the appropriate two circles on the response form. A one-digit answer (such as "7") must be coded with a leading zero ("07").

8. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

9. Diagrams are *not* drawn to scale. They are intended as aids only.

10. When your supervisor tells you to begin, you will have 60 minutes of working time.

11. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Part A: Each correct answer is worth 5.

1. The value of $2 - 0 + 2 - 4$ is
    (A) -2 **(B)** 0 **(C)** 2 **(D)** 4 **(E)** 6

2. The integers -5 and 6 are shown on a number line. The distance between them is
    (A) -9 **(B)** -1 **(C)** 1
    (D) 11 **(E)** 10
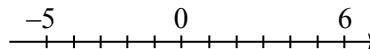
3. In the diagram, the word PASCAL is written inside a circle. When the diagram is rotated $180°$ in the clockwise direction, the resulting figure is
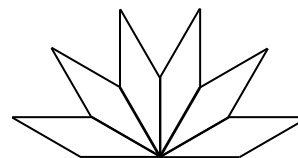
    (A) PASCAL   **(B)**   **(C)**

    (D)   **(E)**

4. In a certain year, July 1 was a Wednesday. In that year, July 17 was a
    (A) Thursday   **(B)** Saturday   **(C)** Tuesday   **(D)** Friday   **(E)** Wednesday

5. Six rhombi of side length 1 are arranged as shown. What is the perimeter of this figure?
    (A) 6 **(B)** 14 **(C)** 10
    (D) 16 **(E)** 18

6. Narsa buys a package of 45 cookies on Monday morning. The bar graph shows the number of cookies that Narsa eats each day from Monday to Friday.

    Cookies eaten by Narsa
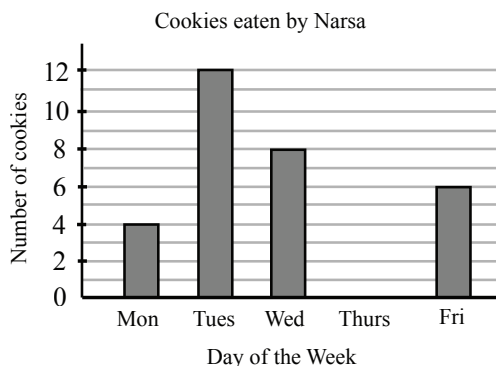
    How many cookies are left in the package after Friday?
    (A) 45 **(B)** 25 **(C)** 20 **(D)** 15 **(E)** 12

7. Shuxin begins with 10 red candies, 7 yellow candies, and 3 blue candies. After eating some of the candies, there are equal numbers of red, yellow, and blue candies remaining. What is the smallest possible number of candies that Shuxin ate?

    (A) 17   (B) 7   (C) 11   (D) 20   (E) 14

8. There are 20 students in a class. In total, 10 of them have black hair, 5 of them wear glasses, and 3 of them both have black hair and wear glasses. How many of the students have black hair but do not wear glasses?

    (A) 7   (B) 12   (C) 2   (D) 8   (E) 15

9. A hiker is exploring a trail. The trail has three sections: the first 25% of the trail is along a river, the next $\frac{5}{8}$ of the trail is through a forest, and the remaining 3 km of the trail is up a hill. How long is the trail?

    (A) 24 km   (B) $3\frac{3}{7}$ km   (C) 12 km   (D) 15 km   (E) $10\frac{3}{8}$ km

10. The operation ∇ is defined by $a\nabla b = 4a + b$. The value of $(5\nabla 2)\nabla 2$ is

    (A) 21   (B) 22   (C) 30   (D) 88   (E) 90

Part B: Each correct answer is worth 6.

11. Lauren plays basketball with her friends. She makes 10 baskets. Each of these baskets is worth either 2 or 3 points. Lauren scores a total of 26 points. How many 3 point baskets did she make?

    (A) 5   (B) 6   (C) 7   (D) 8   (E) 9

12. Glen, Hao, Ioana, Julia, Karla, and Levi participated in the 2023 Canadian Team Mathematics Contest. On their team uniforms, each had a different number chosen from the list 11, 12, 13, 14, 15, 16. Hao's and Julia's numbers were even. Karla's and Levi's numbers were prime numbers. Glen's number was a perfect square. What was Ioana's number?

    (A) 11   (B) 15   (C) 14   (D) 13   (E) 12

13. Figure 1 shows an arrangement of 3 lines with 1 intersection point, and Figure 2 shows an arrangement of 3 lines with 3 intersection points.
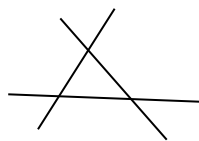

    Figure 1   Figure 2

    What is the maximum number of intersection points that can appear in an arrangement of 4 lines?

    (A) 5   (B) 4   (C) 8   (D) 6   (E) 7

14. The average (mean) of a list of 10 numbers is 17. When one number is removed from the list, the new average is 16. What number was removed?

    (A) 26   (B) 36   (C) 17   (D) 16   (E) 20

15. In $\triangle ABC$, points D and E lie on AB, as shown. If $AD = DE = EB = CD = CE$, the measure of $\angle ABC$ is

(A) $30°$ (B) $45°$ (C) $60°$

(D) $15°$ (E) $40°$
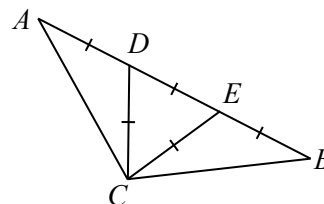
16. The value of $\dfrac{x}{2}$ is less than the value of x^2.

The value of x^2 is less than the value of x.

Which of the following could be a value of x?

(A) 2 (B) $\frac{1}{3}$ (C) $\frac{3}{4}$ (D) -1 (E) $-\frac{3}{2}$

17. The first two hours of Melanie's trip was spent travelling at 100 km/h. The remaining 200 km of Melanie's trip was spent travelling at 80 km/h. Melanie's average speed during this trip is closest to

(A) 80 km/h   (B) 89 km/h   (C) 90 km/h   (D) 94 km/h   (E) 100 km/h

18. A numerical value is assigned to each letter of the alphabet. The value of a word is determined by adding up the numerical values of each of its letters. The value of SET is 2, the value of HAT is 7, the value of TASTE is 3, and the value of MAT is 4. What is the value of the word MATH?

(A) 6   (B) 7   (C) 8   (D) 9   (E) 10

19. In the diagram, $\triangle ABC$ has $AB = BC = 3x + 4$ and $AC = 2x$ and rectangle $DEFG$ has $EF = 2x - 2$ and $FG = 3x - 1$.
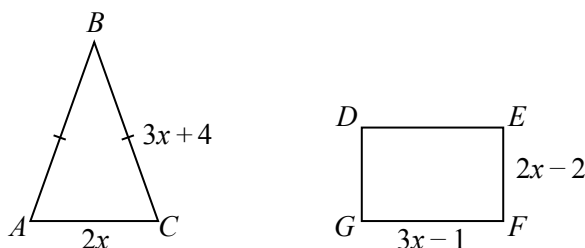
The perimeter of $\triangle ABC$ is equal to the perimeter of rectangle $DEFG$. What is the area of $\triangle ABC$?

(A) 84   (B) 87.5   (C) 168   (D) 175   (E) 336

20. If N is a positive integer between $1\,000\,000$ and $10\,000\,000$, inclusive, what is the maximum possible value for the sum of the digits of $25 \times N$?

(A) 63   (B) 65   (C) 67   (D) 69   (E) 66

Part C: Each correct answer is worth 8.
Each correct answer is an integer from 0 to 99, inclusive.
A one-digit answer (such as "7") must be coded with a leading zero ("07").
Note: The integer formed by the rightmost two digits of 12 345 is 45.
       The integer formed by the rightmost two digits of 6307 is 7, coded 07.

21. A 3×3 table starts with every entry equal to 0 and is modified using the following steps:

    (i) adding 1 to all three numbers in any row;

    (ii) adding 2 to all three numbers in any column.

    After step (i) has been used a total of a times and step (ii) has been used a total of b times, the table appears as shown. What is the value of $a + b$?

7	1	5
9	3	7
8	2	6

22. Pablo has 27 solid $1 \times 1 \times 1$ that he assembles in a larger $3 \times 3 \times 3$ cube. If 10 of the smaller cubes are red, 9 are blue, and 8 are yellow, what is the smallest possible surface area of the larger cube that is red?

23. A lock code is made up of four digits that satisfy the following rules:

    • At least one digit is a 4, but neither the second digit nor the fourth digit is a 4.

    • Exactly one digit is a 2, but the first digit is not 2.

    • Exactly one digit is a 7.

    • The code includes a 1, or the code includes a 6, or the code includes two 4s.

    How many codes are possible?

24. In the diagram, $CD = CE = 30$ and F is the midpoint of CE. Two quarter circles are drawn: one with centre C and passing through D and E, and the other with centre F and passing through E. Let x be the area of the region that is inside rectangle $GDCF$ and outside the larger quarter circle. Let y be the area that is inside the larger quarter circle, outside the smaller quarter circle, and outside rectangle $GDCF$. Let d be the positive difference between x and y. What is the integer closest to d?
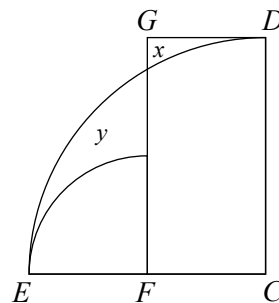
25. Each of a, b and c is equal to a number from the list 3^1, 3^2, 3^3, 3^4, 3^5, 3^6, 3^7, 3^8. There are N triples (a, b, c) with $a \le b \le c$ for which each of $\dfrac{ab}{c}$, $\dfrac{ac}{b}$ and $\dfrac{bc}{a}$ is equal to an integer. What is the value of N?

2024
Pascal
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(English)
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For students...

Thank you for writing the 2024 Pascal Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Fryer Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

- More information about the Fryer Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Look at our free online courseware
- Use our free Problem Set Generator to create problem sets for curriculum support and enrichment
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
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- Find your school's contest results