Canadian
Mathematics
Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Pascal Contest (Grade 9)

Wednesday, February 23, 2000

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

Part A: Each correct answer is worth 5.

1. The value of $5^2 + 2(5-2)$ is

   (**A**) 16 (**B**) 19 (**C**) 31 (**D**) 36 (**E**) 81

2. The sum of $29 + 12 + 23$ is

   (**A**) 32^2 (**B**) 2^6 (**C**) 3^4 (**D**) 1^{64} (**E**) 64^0

3. If $x = 4$ and $y = -3$, then the value of $\dfrac{x-2y}{x+y}$ is

   (**A**) $-\frac{1}{2}$ (**B**) -2 (**C**) $\frac{10}{7}$ (**D**) $-\frac{2}{7}$ (**E**) 10

4. If the following sequence of five arrows repeats itself continuously, what arrow would be in the 48th position?

   (**A**) (**B**)  (**C**)  (**D**) (**E**) 

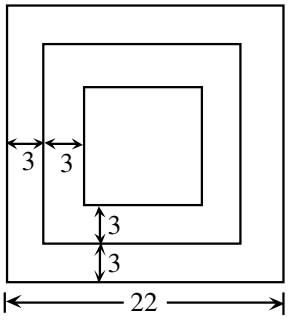
5. If $y = 6 + \dfrac{1}{6}$, then $\dfrac{1}{y}$ is

   (**A**) $\frac{6}{37}$ (**B**) $\frac{37}{6}$ (**C**) $\frac{6}{7}$ (**D**) $\frac{7}{6}$ (**E**) 1

6. If $\frac{2}{3}$, $\frac{23}{30}$, $\frac{9}{10}$, $\frac{11}{15}$, and $\frac{4}{5}$ are written from smallest to largest then the middle fraction will be

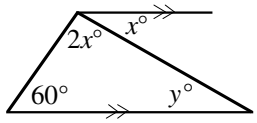
   (**A**) $\frac{23}{30}$ (**B**) $\frac{4}{5}$ (**C**) $\frac{2}{3}$ (**D**) $\frac{9}{10}$ (**E**) $\frac{11}{15}$

7. Three squares with the same centre and corresponding parallel sides are drawn. The distance between the sides of successive squares is 3 and the side length of the largest square is 22, as shown. What is the perimeter of the smallest square?



   (**A**) 40 (**B**) 100 (**C**) 10
   (**D**) 64 (**E**) 20

8. In the diagram, the value of y is



(**A**) 30 (**B**) 20 (**C**) 80
(**D**) 60 (**E**) 40

9. The ages of three contestants in the Pascal Contest are 14 years, 9 months; 15 years, 1 month; and 14 years, 8 months. Their average (mean) age is

(**A**) 14 years, 8 months (**B**) 14 years, 9 months (**C**) 14 years, 10 months
(**D**) 14 years, 11 months (**E**) 15 years

10. The number of integers between $-\sqrt{8}$ and $\sqrt{32}$ is

(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 19

Part B: Each correct answer is worth 6.

11. A store had a sale on T-shirts. For every two T-shirts purchased at the regular price, a third T-shirt was bought for \$1.00. Twelve T-shirts were bought for \$120.00. What was the regular price for one T-shirt?

(**A**) \$10.00 (**B**) \$13.50 (**C**) \$14.00 (**D**) \$14.50 (**E**) \$15.00

12. In the diagram, every number beginning at 30 equals twice the sum of the two numbers to its immediate left. The value of c is

10	a	30	b	c

(**A**) 50 (**B**) 70 (**C**) 80
(**D**) 100 (**E**) 200

13. In the expression $\frac{a}{b}+\frac{c}{d}+\frac{e}{f}$ each letter is replaced by a different digit from 1, 2, 3, 4, 5, and 6. What is the largest possible value of this expression?

(**A**) $8\frac{2}{3}$ (**B**) $9\frac{5}{6}$ (**C**) $9\frac{1}{3}$ (**D**) $9\frac{2}{3}$ (**E**) $10\frac{1}{3}$

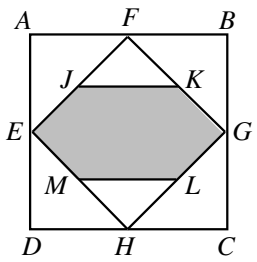
14. The numbers 6, 14, x, 17, 9, y, 10 have a mean of 13. What is the value of $x+y$?

(**A**) 20 (**B**) 21 (**C**) 23 (**D**) 25 (**E**) 35

15. The digits 1, 1, 2, 2, 3, and 3 are arranged to form an odd six digit integer. The 1's are separated by one digit, the 2's by two digits, and the 3's by three digits. What are the last three digits of this integer?

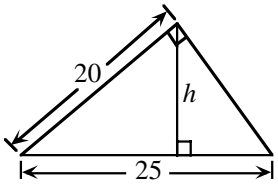
(**A**) 3 1 2 (**B**) 1 2 3 (**C**) 1 3 1 (**D**) 1 2 1 (**E**) 2 1 3

16. The area of square $ABCD$ is 64. The midpoints of its sides are joined to form the square $EFGH$. The midpoints of its sides are J, K, L, and M. The area of the shaded region is



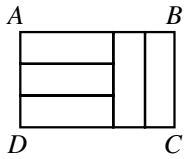
(**A**) 32 (**B**) 24 (**C**) 20
(**D**) 28 (**E**) 16

17. In the diagram, the value of the height h is



(**A**) 6 (**B**) 9 (**C**) 10
(**D**) 12 (**E**) 15

18. In the diagram the five smaller rectangles are identical in size and shape. The ratio of $AB:BC$ is



(**A**) 3:2 (**B**) 2:1 (**C**) 5:2
(**D**) 5:3 (**E**) 4:3

19. The year 2000 is a leap year. The year 2100 is not a leap year. The following are the complete rules for determining a leap year:
(i) Year Y is not a leap year if Y is not divisible by 4.
(ii) Year Y is a leap year if Y is divisible by 4 but not by 100.
(iii) Year Y is not a leap year if Y is divisible by 100 but not by 400.
(iv) Year Y is a leap year if Y is divisible by 400.
How many leap years will there be from the years 2000 to 3000 inclusive?

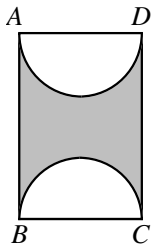
(**A**) 240 (**B**) 242 (**C**) 243 (**D**) 244 (**E**) 251

20. A straight line is drawn across an 8 by 8 checkerboard. What is the greatest number of 1 by 1 squares through which this line could pass?

(**A**) 12 (**B**) 14 (**C**) 16 (**D**) 11 (**E**) 15

Part C: Each correct answer is worth 8.

21. $ABCD$ is a rectangle with $AD = 10$. If the shaded area is 100, then the shortest distance between the semicircles is



(**A**) 2.5π (**B**) 5π (**C**) π
(**D**) $2.5\pi + 5$ (**E**) $2.5\pi - 2.5$

22. A wooden rectangular prism has dimensions 4 by 5 by 6. This solid is painted green and then cut into 1 by 1 by 1 cubes. The ratio of the number of cubes with exactly two green faces to the number of cubes with three green faces is

(**A**) 9:2 (**B**) 9:4 (**C**) 6:1 (**D**) 3:1 (**E**) 5:2

23. The left most digit of an integer of length 2000 digits is 3. In this integer, any two consecutive digits must be divisible by 17 or 23. The 2000th digit may be either 'a' or 'b'. What is the value of $a+b$?

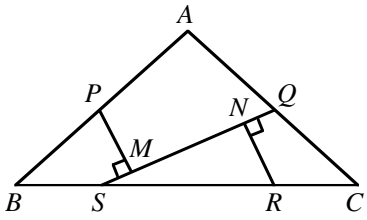
(**A**) 3 (**B**) 7 (**C**) 4 (**D**) 10 (**E**) 17

24. There are seven points on a piece of paper. Exactly four of these points are on a straight line. No other line contains more than two of these points. Three of these seven points are selected to form the vertices of a triangle. How many triangles are possible?

(**A**) 18 (**B**) 28 (**C**) 30 (**D**) 31 (**E**) 33

continued ...

25. ΔABC is an isosceles triangle in which $AB = AC = 10$ and $BC = 12$. The points S and R are on BC such that $BS:SR:RC = 1:2:1$. The midpoints of AB and AC are P and Q respectively. Perpendiculars are drawn from P and R to SQ meeting at M and N respectively. The length of MN is



(A) $\frac{9}{\sqrt{13}}$ **(B)** $\frac{10}{\sqrt{13}}$ **(C)** $\frac{11}{\sqrt{13}}$

(D) $\frac{12}{\sqrt{13}}$ **(E)** $\frac{5}{2}$