Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Pascal Contest (Grade 9)

Wednesday, February 18, 1998

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 credits in Part A, 6 credits in Part B, and 8 credits in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

Part A: Each question is worth 5 credits.

1. The value of $\frac{1+3+5}{10+6+2}$ is

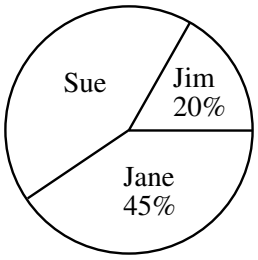
   (**A**) $\frac{1}{6}$ (**B**) 2 (**C**) $\frac{1}{2}$ (**D**) $1\frac{1}{2}$ (**E**) $3\frac{1}{10}$

2. If $3(x-5)=3(18-5)$, then x is

   (**A**) $\frac{44}{3}$ (**B**) $\frac{32}{3}$ (**C**) 9 (**D**) 18 (**E**) 81

3. The pie chart shows a percentage breakdown of 1000 votes in a student election. How many votes did Sue receive?

   (**A**) 550 (**B**) 350 (**C**) 330
   (**D**) 450 (**E**) 935



4. The value of $\left(\sqrt{169}-\sqrt{25}\right)^2$ is

   (**A**) 64 (**B**) 8 (**C**) 16 (**D**) 144 (**E**) 12

5. The value of $\frac{5^6\times5^9\times5}{5^3}$ is

   (**A**) 5^{18} (**B**) 25^{18} (**C**) 5^{13} (**D**) 25^{13} (**E**) 5^{51}

6. If $x=3$, which of the following expressions is an even number?

   (**A**) $9x$ (**B**) x^3 (**C**) $2\left(x^2+9\right)$ (**D**) $2x^2+9$ (**E**) $3x^2$

7. The value of $490-491+492-493+494-495+\ldots-509+510$ is

   (**A**) 500 (**B**) -10 (**C**) -11 (**D**) 499 (**E**) 510

8. The average (mean) of a list of 10 numbers is 0. If 72 and -12 are added to the list, the new average will be

   (**A**) 30 (**B**) 6 (**C**) 0 (**D**) 60 (**E**) 5

9. What is one-half of 1.2×10^{30}?

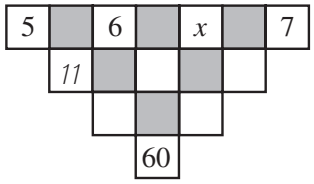
   (**A**) 6.0×10^{30} (**B**) 6.0×10^{29} (**C**) 0.6×5^{30} (**D**) 1.2×10^{15} (**E**) 1.2×5^{30}

10. If $x+y+z=25$ and $y+z=14$, then x is

    (**A**) 8 (**B**) 11 (**C**) 6 (**D**) -6 (**E**) 31

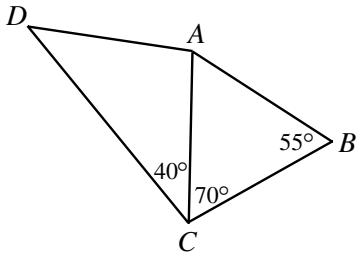
Part B: Each question is worth 6 credits.

11. The number in an unshaded square is obtained by adding the numbers connected to it from the row above. (The '*11*' is one such number.) The value of x is



(**A**) 4 (**B**) 6 (**C**) 9
(**D**) 15 (**E**) 10

12. In the diagram, $DA = CB$. What is the measure of $\angle DAC$?



(**A**) 70° (**B**) 100° (**C**) 95°
(**D**) 125° (**E**) 110°

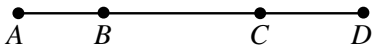
13. A three-wheeled vehicle travels 100 km. Two spare wheels are available. Each of the five wheels is used for the same distance during the trip. For how many kilometres is each wheel used?

(**A**) 20 (**B**) 25 (**C**) $33\frac{1}{3}$ (**D**) 50 (**E**) 60

14. The sum of the digits of a five-digit positive integer is 2. (A five-digit integer cannot start with zero.) The number of such integers is

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

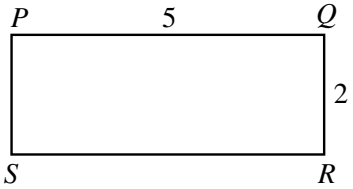
15. Four points are on a line segment, as shown.



If $AB : BC = 1 : 2$ and $BC : CD = 8 : 5$, then $AB : BD$ equals

(**A**) 4 : 13 (**B**) 1 : 13 (**C**) 1 : 7
(**D**) 3 : 13 (**E**) 4 : 17

16. On a rectangular table 5 units long and 2 units wide, a ball is rolled from point P at an angle of 45° to PQ and bounces off SR. The ball continues to bounce off the sides at 45° until it reaches S. How many bounces of the ball are required?

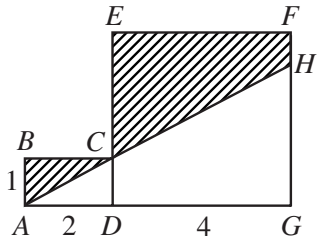


(**A**) 9 (**B**) 8 (**C**) 7
(**D**) 5 (**E**) 4

17. If $1998 = p^s q^t r^u$, where p, q and r are prime numbers, what is the value of $p + q + r$?

(**A**) 222 (**B**) 48 (**C**) 42 (**D**) 66 (**E**) 122

18. In the diagram, $DEFG$ is a square and $ABCD$ is a rectangle. A straight line is drawn from A, passes through C and meets FG at H. The area of the shaded region is



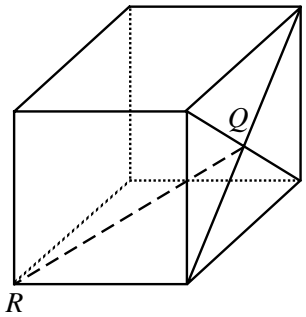
(**A**) 8 (**B**) 8.5 (**C**) 10
(**D**) 9 (**E**) 10.5

19. Using only digits 1, 2, 3, 4, and 5, a sequence is created as follows: one 1, two 2's, three 3's, four 4's, five 5's, six 1's, seven 2's, and so on.
The sequence appears as: 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 1, 1, 1, 1, 1, 1, 2, 2, ...
The 100th digit in the sequence is
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

20. Driving between two towns at 110 km/h instead of 100 km/h saves 9 minutes. The distance in kilometres between the two towns is
(**A**) 210 (**B**) 99 (**C**) 165 (**D**) 9900 (**E**) 150

Part C: Each question is worth 8 credits.

21. Q is the point of intersection of the diagonals of one face of a cube whose edges have length 2 units. The length of QR is
(**A**) 2 (**B**) $\sqrt{8}$ (**C**) $\sqrt{5}$
(**D**) $\sqrt{12}$ (**E**) $\sqrt{6}$



22. A deck of 100 cards is numbered from 1 to 100. Each card has the same number printed on both sides. One side of each card is red and the other side is yellow. Barsby places all the cards, red side up, on a table. He first turns over every card that has a number divisible by 2. He then examines all the cards, and turns over every card that has a number divisible by 3. How many cards have the red side up when Barsby is finished?
(**A**) 83 (**B**) 17 (**C**) 66 (**D**) 50 (**E**) 49

23. The numbers 123 456 789 and 999 999 999 are multiplied. How many of the digits in the final result are 9's?
(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 17

24. Three rugs have a combined area of 200 m^2. By overlapping the rugs to cover a floor area of 140 m^2, the area which is covered by exactly two layers of rug is 24 m^2. What area of floor is covered by three layers of rug?
(**A**) 12 m^2 (**B**) 18 m^2 (**C**) 24 m^2 (**D**) 36 m^2 (**E**) 42 m^2

25. One way to pack a 100 by 100 square with 10 000 circles, each of diameter 1, is to put them in 100 rows with 100 circles in each row. If the circles are repacked so that the centres of any three tangent circles form an equilateral triangle, what is the maximum number of additional circles that can be packed?
(**A**) 647 (**B**) 1442 (**C**) 1343 (**D**) 1443 (**E**) 1344