The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Pascal Contest

(Grade 9)

Wednesday, February 23, 2022
(in North America and South America)

Thursday, February 24, 2022
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes   ©2021 University of Waterloo

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.

2. You may use rulers, compasses and paper for rough work.

3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.

4. On your response form, print your school name and city/town in the box in the upper right corner.

5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**

6. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.

7. The correct answer to each question in Part C is an integer from 0 to 99, inclusive. After deciding on your answer, fill in the appropriate two circles on the response form. A one-digit answer (such as "7") must be coded with a leading zero ("07").

8. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

9. Diagrams are *not* drawn to scale. They are intended as aids only.

10. When your supervisor tells you to begin, you will have 60 minutes of working time.

11. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Part A: Each correct answer is worth 5.

1. The expression $\dfrac{20 + 22}{2}$ is equal to

    (A) 1   **(B)** 4   **(C)** 20   **(D)** 21   **(E)** 22

2. The graph to the right shows the amount of money that each of four students donated to a school fundraiser. The total amount of money that they donated was

    (A) \$20   **(B)** \$18   **(C)** \$16
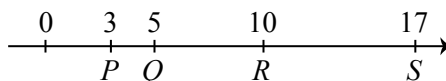
    (D) \$14   **(E)** \$22

    Donations to school fundraiser
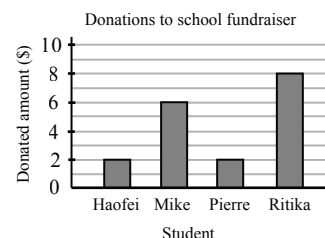
3. The value of $\dfrac{1}{2} + \dfrac{2}{4} + \dfrac{4}{8} + \dfrac{8}{16}$ is

    (A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** 0   **(D)** 4   **(E)** 2

4. Which of the following numbers is closest to -3.4?

    (A) -4 **(B)** -3 **(C)** 0   **(D)** 3   **(E)** 4

5. Points P, Q, R, and S are on a number line, as shown.
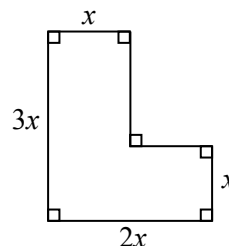
    The ratio of lengths $PR : QS$ is

    (A) $7 : 12$ **(B)** $10 : 17$ **(C)** $1 : 1$ **(D)** $5 : 12$ **(E)** $7 : 17$

6. Robyn has 4 tasks to do and Sasha has 14 tasks to do. In order for Robyn and Sasha to do the same number of tasks, how many of Sasha's tasks should Robyn do?

    (A) 6   **(B)** 4   **(C)** 5   **(D)** 7   **(E)** 2

7. In the diagram, the lengths of four of the sides of the figure are shown in terms of x. Assuming that $x \neq 0$, the perimeter of the figure is

   **(A)** $8x$ **(B)** $9x$ **(C)** $12x$

    (D) $11x$ **(E)** $10x$

8. A circular spinner is divided into 4 sections, as shown. The angles at the centre of the circle in the sections labelled Green and Blue each measure 90°. An arrow is attached to the centre of the spinner. The arrow is spun once. What is the probability that the arrow lands on either Red or Yellow?
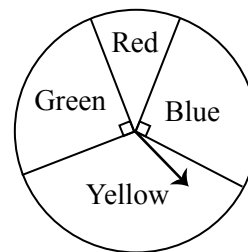
(A) $\dfrac{1}{8}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{3}{8}$

(D) $\dfrac{1}{2}$ (E) $\dfrac{3}{4}$

9. The line with equation $y = 2x + b$ passes through the point $(-4, 0)$. The value of b is

(A) $\dfrac{1}{2}$ (B) 1 (C) 2 (D) 4 (E) 8

10. On the map shown, there are a number of routes from Mathville to Algebratown.
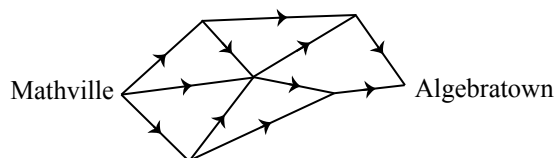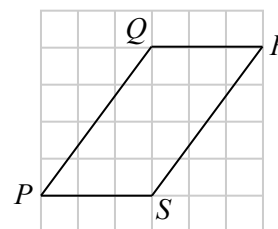
Each route must travel along the roads in the direction marked by the arrows. The total number of routes from Mathville to Algebratown is

(A) 3 (B) 4 (C) 8 (D) 6 (E) 10

Part B: Each correct answer is worth 6.

11. In the diagram, points P, Q, R, and S are at intersections of gridlines in a 6×6 grid. What is the perimeter of parallelogram $PQRS$?

(A) 14 (B) 15 (C) 16

(D) 17 (E) 18

12. How many of the integers from 1 to 100, inclusive, have at least one digit equal to 6?

(A) 17 (B) 18 (C) 19 (D) 21 (E) 22

13. Mayar and Rosie are 90 metres apart. Starting at the same time, they run towards each other. Mayar runs twice as fast as Rosie. How far has Mayar run when they meet?

(A) 15 m (B) 30 m (C) 45 m (D) 60 m (E) 75 m

14. Dhruv is older than Bev. Bev is older than Elcim. Elcim is younger than Andy. Andy is younger than Bev. Bev is younger than Cao. Who is the third oldest?

(A) Andy (B) Bev (C) Cao (D) Dhruv (E) Elcim

15. How many of the integers 19, 21, 23, 25, 27 can be expressed as the sum of two prime numbers?

(A) 3 (B) 4 (C) 1 (D) 2 (E) 5

16. Alvin, Bingyi and Cheska play a two-player game that never ends in a tie. In a recent tournament between the three players, a total of 60 games were played and each pair of players played the same number of games.

  • When Alvin and Bingyi played, Alvin won 20% of the games.
  • When Bingyi and Cheska played, Bingyi won 60% of the games.
  • When Cheska and Alvin played, Cheska won 40% of the games.

How many games did Bingyi win?

(A) 12   (B) 24   (C) 28   (D) 30   (E) 36

17. The integers a, b and c satisfy the equations $a + 5 = b$ and $5 + b = c$ and $b + c = a$. The value of b is

(A) -30 (B) -20 (C) -10 (D) 0   (E) 5

18. Five balls, numbered 1 to 5, are placed in order on a table. A sequence of steps is performed on the balls. In step 1, the rightmost ball is picked up and put in the middle of the four remaining balls. (The remaining balls are shifted to make room for the inserted ball.) Then in step 2, the leftmost ball is picked up and put in the middle of the four remaining balls. These steps repeat, with the rightmost and leftmost balls alternately picked up and put in the middle of the four remaining balls. Immediately after step N, the balls are in the reverse of their original order. Which of the following is a possible value of N?

Start:
① ② ③ ④ ⑤

After step 1:
① ② ⑤ ③ ④

After step 2:
② ⑤ ① ③ ④

(A) 2020   (B) 2028   (C) 2031

(D) 2027   (E) 2025

19. Miyuki texted a six-digit integer to Greer. Two of the digits of the six-digit integer were 3s. Unfortunately, the two 3s that Miyuki texted did not appear and Greer instead received the four-digit integer 2022. The number of possible six-digit integers that Miyuki could have texted is

(A) 20   (B) 10   (C) 5   (D) 25   (E) 15

20. A pizza is cut into 10 pieces. Two of the pieces are each $\frac{1}{24}$ of the whole pizza, four are each $\frac{1}{12}$, two are each $\frac{1}{8}$, and two are each $\frac{1}{6}$. A group of n friends share the pizza by distributing all of these pieces. They do not cut any of these pieces. Each of the n friends receives, in total, an equal fraction of the whole pizza. The sum of the values of n with $2 \leq n \leq 10$ for which this is *not* possible is

(A) 31   (B) 35   (C) 40   (D) 39   (E) 36

Part C: Each correct answer is worth 8.
Each correct answer is an integer from 0 to 99, inclusive.
**A one-digit answer (such as "7") must be coded with a leading zero ("07").**
Note: The integer formed by the rightmost two digits of 12 345 is 45.
The integer formed by the rightmost two digits of 6307 is 7, coded 07.

21. A 5 cm by 5 cm pegboard and a 10 cm by 10 cm pegboard each have holes at the intersection of invisible horizontal and vertical lines that occur in 1 cm intervals from each edge. Pegs are placed into the holes on the two main diagonals of both pegboards. The 5 cm by 5 cm pegboard is shown; it has 16 holes. The 8 shaded holes have pegs, and the 8 unshaded holes do not. How many empty holes does the 10 cm by 10 cm pegboard have?
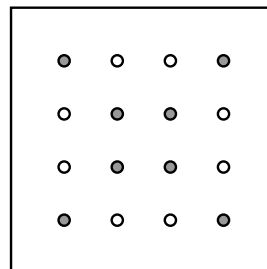
22. What is the integer formed by the rightmost two digits of the integer equal to $4^{127} + 5^{129} + 7^{131}$?

23. In the diagram, two circles are centred at O. The smaller circle has a radius of 1 and the larger circle has a radius of 3. Points P and Q are placed on the larger circle so that the areas of the two shaded regions are equal. If $\angle POQ = x°$, what is the value of x?
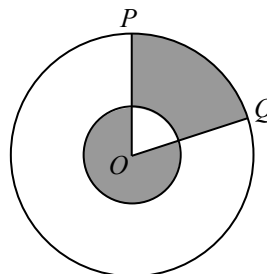
24. A *Pretti number* is a seven-digit positive integer with the following properties:

    - The integer formed by its leftmost three digits is a perfect square.
    - The integer formed by its rightmost four digits is a perfect cube.
    - Its ten thousands digit and ones (units) digit are equal.
    - Its thousands digit is not zero.

    How many Pretti numbers are there?

25. A hexagonal prism has a height of 165 cm. Its two hexagonal faces are regular hexagons with sides of length 30 cm. Its other six faces are rectangles. A fly and an ant start at point X on the bottom face and travel to point Y on the top face. The fly flies directly along the shortest route through the prism. The ant crawls around the outside of the prism along a path of constant slope so that it winds around the prism exactly $n + \frac{1}{2}$ times, for some positive integer n. The distance crawled by the ant is more than 20 times the distance flown by the fly. What is the smallest possible value of n?
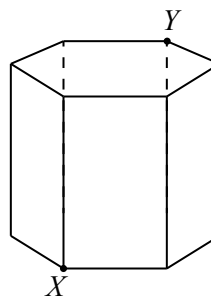
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For students...

Thank you for writing the 2022 Pascal Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Fryer Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

- More information about the Fryer Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
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- Find your school's contest results