* 1. **Indices**

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* 1. **Parameters**
     1. **Cost Parameters**

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* + 1. **Capacity-Related Parameters**

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* + 1. **Inventory-Related Parameters**

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* + 1. **Time-Related Parameters**

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* + 1. **Bundle Composition and Pricing Parameters**

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* 1. **Variables**
     1. **Decision Variables**

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* + 1. **System Variables**

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* + 1. **Binary Variables**

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* 1. **Objective Function**

The objective of the model is to maximize the profit of the upstream firm. The profit equation revolves around the revenue generated from selling products/bundles to the retailers and the different cost components namely the production cost, inventory costs in the factories and in the warehouses, transportation costs to warehouses and to retailers, backorder costs, introductory costs for new bundles, setup costs for setting up bundling activities, and the bundle creation cost. The whole objective function is shown in equation 1.

Equation 1 Objective Function

* + 1. **Total Sales**

The total sales of the firm or the total revenue generated is basically the number of units ordered by the retailers multiplied by the selling price of each unit for all products and bundles across all time periods. However, if the order is “Lost” i.e. not catered, it will not generate any revenue. The sales expression in the objective function is shown below.

* + 1. **Total Production Cost**

The total production cost is based on the unit cost of producing each individual product in each factory multiplied by the total number of units produced across all periods. This unit cost technically has two components – the variable unit cost that covers the direct labor and materials used in the production and is constant, and the fixed expenses incurred by the factory for production per time period (such as utilities and rent) expressed on a per unit basis hence would change depending on the number of units produced. These two components are expressed as one under the production cost per unit, PCift. This production cost per unit, therefore, also changes based on the number of units produced. With VCif as the variable unit cost and FCf as the fixed expenses in factory f per time period, the total production cost per unit can be computed as: PCift = VCif + (FCf/∑Pift). The fixed expenses are expressed on a per unit basis by dividing the amount by the total number of products produced in the factory per time period. The production cost per unit becomes higher as less products are produced therefore accounting for the cost of not fully utilizing the production capacity. The total production cost expression in the objective function is shown below.

* + 1. **Total Inventory Cost**

Inventory costs are incurred by ending inventories in both the factory and the warehouse. This is based on the unit inventory cost per product/bundle in the factory or in the warehouse. This unit cost includes labor and materials for carrying, storage costs, and other inventory-related costs. The total inventory cost is basically the ending inventory per period in the factories and in the warehouses multiplied by the corresponding unit inventory cost across all periods. This expression in the objective function is shown below.

* + 1. **Total Transportation Cost**

Transportation costs are incurred in transporting units to warehouses and to retailers. This is based on the unit transportation cost per product to and from each entity (factory, warehouse, or retailer). The unit cost includes the labor and equipment for carrying, truck storage, and other transportation-related costs. The total transportation cost is basically the total number of units transported from entity to entity (factory to warehouse, factory to retailer, and warehouse to retailer) multiplied by the corresponding unit transportation cost from point to point across all periods. This expression in the objective function is shown below.

* + 1. **Total Backorder Cost**

Backorder costs are incurred when orders are not met fully at a given due date. This is based on the unit backorder cost per retailer group. The unit backorder cost is constant for all products/bundles and varies only per retailer group since this backorder cost represents the penalty imposed by the retailers for late deliveries. The total backorder cost is basically the number of backordered units per period multiplied by the backorder cost per unit for each retailer across all periods and for all products/bundles.

* + 1. **Total Bundle Introductory Cost**

The introductory costs for new bundles are incurred when a bundle is produced for the first time in a factory or in a warehouse. This is based on the fixed introductory cost for each option in each location. The fixed introductory cost includes necessary equipment, supplies, utilities, registration or listing fees for the new “product”, and other fixed costs required for the initial setup. The total bundle introductory cost is basically the fixed introductory cost per bundle in each entity multiplied by the binary variable indicating that the bundle was produced for the first time in that entity. This expression in the objective function is shown below.

* + 1. **Total Bundling Activity Setup Cost**

The setup costs for the bundling activity are incurred when new setups will be made at the start of the period. This is based on the fixed setup cost per bundle in each location. This fixed cost includes the labor and the materials required for setting up the activity at the start of the period after coming from a “no setup” period. The total bundling activity setup cost is basically the fixed cost of setting up the activity for the bundle in the location multiplied by the binary variable indicating that a new setup was made for the day for that bundle. This expression in the objective function is shown below.

* + 1. **Total Bundle Creation Cost**

Finally, the bundle creation cost is based on the unit cost of forming each bundle in each location. The unit cost includes the labor, materials, and other direct costs for processing. The total bundle creation cost is basically the total number of units of the bundles created in the factories and in the warehouses multiplied by the corresponding unit bundle creation cost across all periods. This expression in the objective function is shown below.

* 1. **Constraints and Functions**
     1. **Beginning Inventory of Individual Product i in Factory f**

This constraint sets the amount of inventory of each individual product in each factory available at the start of the period. The beginning inventory of each individual product would basically take the amount of inventory left at the end of the previous period. The beginning inventory for the very first time period is basically equal to the given initial inventory of the product in the factory.

Equation 2 Beginning Inventory of Product in Factory

* + 1. **Beginning Inventory of Individual Product i in Warehouse w**

This constraint sets the amount of inventory of each individual product in each warehouse available at the start of the period. The beginning inventory of each individual product would basically take the amount of inventory left at the end of the previous period plus the amount of units set to arrive at that period from the different factories. The beginning inventory for the very first time period is basically equal to the given initial inventory of the product in the warehouse.

Note that all ’s when are equal to zero. This is because no deliveries can be made at time 0, -1, -2, and so on as these time periods are inexistent.

In notation form, .

Equation 3 Beginning Inventory of Product in Warehouse

* + 1. **Beginning Inventory of Bundle k in Factory f**

This constraint sets the amount of inventory of each bundle in each factory available at the start of the period. The beginning inventory of each bundle would basically take the amount of inventory left at the end of the previous period.

Equation 4 Beginning Inventory of Bundle in Factory

* + 1. **Beginning Inventory of Bundle k in Warehouse w**

This constraint sets the amount of inventory of each bundle in each warehouse available at the start of the period. The beginning inventory of each bundle would basically take the amount of inventory left at the end of the previous period plus the amount of units set to arrive at that period from the different factories. Note that all ’s when are equal to zero. This is because no deliveries can be made at time 0, -1, -2, and so on as these time periods are inexistent.

In notation form,

Equation 5 Beginning Inventory of Bundle in Warehouse

* + 1. **Production Capacity**

This constraint assures that the number of units of a product to be produced in a factory per period will not exceed the production capacity.

Equation 6 Production Capacity

* + 1. **Inventory Capacity in Factory**

This constraint assures that the total amount of inventory to be held in each factory at any given time will not exceed the inventory capacity of the factory or the storage space available in the factory. Since the inventory capacity is in terms of standard units or STDU, the total amount of inventory (at the start of the period) will be converted into STDU units and such amount will be the basis of the storage occupied.

Equation 7 Inventory Capacity in Factory

* + 1. **Inventory Capacity in Warehouse**

This constraint assures that the total amount of inventory to be held in each warehouse at any given time will not exceed the inventory capacity of the warehouse or the storage space available in the warehouse. Since the inventory capacity is in terms of standard units or STDU, the total amount of inventory (at the start of the period) will be converted into STDU units and such amount will be the basis of the storage occupied.

Equation 8 Inventory Capacity in Warehouse

* + 1. **Product Dispatch Supply Constraint for Factory**

This constraint assures that the number of units of a product/bundle a factory can dispatch will not exceed the supply of products in the factory. That is, the number of units of a product/bundle a factory can transport both to the warehouses and to the retailers should not exceed the amount of inventory available in the factory at the start of the period.

Equation 9 Product Dispatch Supply for Factory

* + 1. **Product Dispatch Supply Constraint for Warehouse**

This constraint assures that the number of units of a product/bundle a warehouse can dispatch will not exceed the supply of products in the warehouse. That is, the number of units of a product/bundle a warehouse can transport to the retailers should not exceed the amount of inventory available in the warehouse at the start of the period.

Equation 10 Product Dispatch Supply for Warehouse

* + 1. **Bundle Creation Supply Constraint for Factory**

This constraint assures that the number of units of a bundle that can be created from individual products will not exceed the supply of the individual products available in the factory after all dispatches have been made. That is, the number of bundled units that can be formed should not exceed the remaining inventory of the individual products the bundle comprises after the dispatches.

Equation 11 Bundle Creation Supply in Factory

* + 1. **Bundle Creation Supply Constraint for Warehouse**

This constraint assures that the number of units of a bundle that can be created from individual products will not exceed the supply of the individual products available in the warehouse after all dispatches have been made. That is, the number of bundled units that can be formed in the warehouse should not exceed the remaining inventory of the individual products the bundle comprises after the dispatches.

Equation 12 Bundle Creation Supply in Warehouse

* + 1. **Bundle Formation Capacity in Factory**

This constraint assures that the number of units of a bundle formed in a factory will not exceed the maximum amount that can be formed per period. That is, the amount of bundles created per period in a factory should not exceed the capacity or the amount that can be created in the location per period. This capacity is based on the labor and the nature of the bundle formation activity (i.e. how long it takes to bundle).

Equation 13 Bundle Formation Capacity in Factory

* + 1. **Bundle Formation Capacity in Warehouse**

This constraint assures that the number of units of a bundle formed in a warehouse will not exceed the maximum amount that can be formed per period. That is, the amount of bundles created per period in a warehouse should not exceed the capacity or the amount that can be created in the warehouse per period. This capacity is based on the labor and the nature of the bundle formation activity (i.e. how long it takes to bundle).

Equation 14 Bundle Formation Capacity in Warehouse

* + 1. **Bundling Activity in Factory**

This constraint sets the variable, BSUFkft, to 1 if a bundling activity (for a specific bundle) is being performed in the factory at the current period. That is, if the number of bundled units is positive, then an activity is being executed hence the said variable takes a value of 1.

Equation 15 Bundling Activity in Factory

* + 1. **Bundling Activity in Warehouse**

This constraint sets the variable, BSUWkwt, to 1 if a bundling activity (for a specific bundle) is being performed in the warehouse at the current period. That is, if the number of bundled units is positive, then an activity is being executed hence the said variable takes a value of 1.

Equation 16 Bundling Activity in Warehouse

* + 1. **Bundle Introduction in Factory**

This constraint sets the variable, INTROBF­kft, to 1 if a bundle is produced for the first time in the factory. That is, as the bundle is introduced for the first time in the location, the said variable will take a value of one. This idea was represented through the summation of the bundling activity variable up until the current period (equations a to e). If the summation would exceed one, this denotes that the bundling activity has been performed for more than once already, then the introductory variable will have no value. Likewise, if the sum is zero, meaning the bundling activity hasn’t been performed at all, then no value will be given to the introductory variable as well. Only when the summation is equal to one will the variable have a value. The purpose of this introduction variable is to represent when introductory costs will be incurred. Introductory costs include equipment purchases and other necessary tools for the bundle formation, legality costs associated with introducing new “products”, and other initiation costs required.

Equation 17 Bundle Introduction in Factory

* + 1. **Bundle Introduction in Warehouse**

This constraint sets the variable, INTROBW­kwt, to 1 if a bundle is produced for the first time in the warehouse. That is, as the bundle is introduced for the first time in the location, the said variable will take a value of one. This idea was represented through the summation of the bundling activity variable up until the current period (equations a to e). If the summation would exceed one, this denotes that the bundling activity has been performed for more than once already, then the introductory variable will have no value. Likewise, if the sum is zero, meaning the bundling activity hasn’t been performed at all, then no value will be given to the introductory variable as well. Only when the summation is equal to one will the variable have a value. The purpose of this introduction variable is to represent when introductory costs will be incurred. Introductory costs include equipment purchases and other necessary tools for the bundle formation, legality costs associated with introducing new “products”, and other initiation costs required.

Equation 18 Bundle Introduction in Warehouse

* + 1. **Introductory Variables Incurring Cost in Factory**

This constraint assures that once a bundle activity (for a specific bundle) is introduced and established in the factory, it cannot be introduced again hence the introductory variable will only be counted once. That is, the cost that can be incurred with the introduction can only be incurred once or none at all (if bundle was not introduced).

Equation 19 Introductory Variables Incurring Cost in Factory

* + 1. **Introductory Variables Incurring Cost in Warehouse**

This constraint assures that once a bundle activity (for a specific bundle) is introduced and established in the warehouse, it cannot be introduced again hence the introductory variable will only be counted once. That is, the cost that can be incurred with the introduction can only be incurred once or none at all (if bundle was not introduced).

Equation 20 Introductory Variables Incurring Cost in Warehouse

* + 1. **Bundling Setup in Factory**

This constraint sets the variable, SETUPFkft, to 1 if a bundling activity will be setup in the factory at the start of the current period. That is, if the activity was not previously performed at the last period then performed at the current period (i.e. if no bundles were formed during the previous period then formation will be done at the current period), there is a need to setup the bundling activity hence the variable takes a value of 1. Setting up includes the labor to work for the setup. With additional labor for setting up, costs are incurred but the capacity of the bundle formation will not be jeopardized.

Equation 21 Bundling Setup in Factory

* + 1. **Bundling Setup in Warehouse**

This constraint sets the variable, SETUPWkwt, to 1 if a bundling activity will be setup in the warehouse at the start of the current period. That is, if the activity was not previously performed at the last period then performed at the current period (i.e. if no bundles were formed during the previous period then formation will be done at the current period), there is a need to setup the bundling activity hence the variable takes a value of 1. Setting up includes the labor to work for the setup. With additional labor for setting up, costs are incurred but the capacity of the bundle formation will not be jeopardized.

Equation 22 Bundling Setup in Warehouse

* + 1. **Ending Inventory of Individual Product in Factory**

This constraint sets the amount of inventory of the individual products left in the factory at the end of the current period. The amount of individual products left would be equal to the beginning inventory and the newly produced products less all dispatches done going to the warehouses and the retailers and, less all individual products used to create the bundles formed.

Equation 23 Ending Inventory of Product in Factory

* + 1. **Ending Inventory of Individual Product in Warehouse**

This constraint sets the amount of inventory of the individual products left in the warehouse at the end of the current period. The amount of individual products left would be equal to the beginning inventory less all dispatches done going to the the retailers and, less all individual products used to create the bundles formed.

Equation 24 Ending Inventory of Product in Warehouse

* + 1. **Ending Inventory of Bundle in Factory**

This constraint sets the amount of inventory of the bundles left in the factory at the end of the current period. The amount of bundles in-store at the end of the period would be equal to the inventory at the start of the period plus all bundles created during the period less all dispatches done going to the warehouses and the retailers.

Equation 25 Ending Inventory of Bundle in Factory

* + 1. **Ending Inventory of Bundle in Warehouse**

This constraint sets the amount of inventory of the bundles left in the warehouse at the end of the current period. The amount of bundles in-store at the end of the period would be equal to the inventory at the start of the period plus all bundles created during the period less all dispatches done going to the retailers.

Equation 26 Ending Inventory of Bundle in Warehouse

* + 1. **Inventory Allocation of Individual Product on Each Option**

This constraint sets the total amount of inventory of each individual product allocated to each option. The allocation amount would be based on the beginning inventory of the options in every period.

Equation 27 Inventory Allocation

* + 1. **Individual Product or Bundle Offering**

This constraint sets the variable, OFFERjt, to 1 if the option is offered during the current period. That is, the variable will have a value of 1 if the inventory of the option at the start of the period is positive. If there is positive inventory for an option, then that option is offered.

Equation 28 Product or Bundle Offering

* + 1. **Lost Orders Based on Product/Bundle Offering**

This constraint assures that an order cannot be catered if the ordered option is not available at the time the order was made. That is, if a retailer expects units of products/bundles to be delivered to her at the current period, that order will be lost if the products/bundles were not offered at the time the order was made (i.e. t – xr time periods from the current t period). If an order was made and the option was not offered at that time, the order will be lost hence the variable LOSTjrt will have a value of 1.

Note that all ’s,’s, and ’s when are equal to zero. This means that, given a retailer wants her order delivered *x*r periods after the order was made, no order has been made yet at the time *t* and all previous periods wherein . The retailer wants her order to arrive at the start of time *t*, therefore the order was made at . If , then the order was made at time 0 or at time -1, -2, and so on, which are inexistent hence the value zero assigned to it. In other words, no order can be made at time 0, -1, and so on.

In notation form,

Equation 29 Lost Orders

* + 1. **Order Fulfillment**

This constraint assures that the number of units to be transported to the retailers from both the warehouses and the factory will not exceed the amount the firm is supposed to deliver to the retailers at the current period. That is, the units to be delivered to the retailers should not be greater than the amount ordered by the retailers (expected to arrive) at the start of the period plus the amount of the previous orders the firm was not able to meet (backorder). The amount expected to be delivered can either have a value or not as influenced by the firm’s ability to cater the order (LOSTjrt variable from previous constraint).

Note that all ’s when and ’s when are equal to zero. This is because no deliveries can be made at time 0, -1, -2, and so on as these time periods are inexistent.

In notation form, or

Equation 30 Order Fulfillment

* + 1. **Backorder Occurrence**

This constraint assures that the amount of order that will not be fulfilled on time will be considered as a backorder. That is, if the firm fails to deliver completely the order of the retailer, the remaining unfulfilled amount shall be delivered as backorders on the next period. In expression form, the amount of the order (given that it will be catered or it is not list) less all the amounts fulfilled and delivered will give the backorder amount. Backorder incurs additional costs representing a penalty imposed by retailers for late deliveries. Backorder is allowed since the retailer itself stocks its own inventory of the products.

Equation 31 Backorder Occurrence

* + 1. **Retailer Order Amount**

This constraint serves as the basis for how much the retailer would want to order per period (i.e. how much of a product the retailer wants to be delivered to her per period). This sets the order amount of the retailer. Such amount is dependent on the demand (or the desired purchases) of the end customers. This demand is influenced by the end customers’ reservation prices and the selling price of the products/bundles set by the retailer. *The next subsection presents the stochastic model for the derivation of this demand based on the previously discussed purchasing behavior of the customers*. The amount the retailer orders for delivery for a certain time period is equal to the demand for that period less the backordered units to arrive at that period. When the upstream firm will not be able to deliver on time i.e. backorder is present, the retailer loses sales for that period. In other words, when a customer arrives in the retailer store at a specific time period and her preferred option is not available, she leaves without a purchase hence a stockout on the retailer’s part. Therefore, that portion of the demand is already foregone. The retailer allows backorders because she can use those orders for the next periods. Hence, what the retailer does with the backordered units from previous periods is that, knowing the demand for the current period based on consumer purchasing decisions, she orders a lesser amount instead. This amount would be equal to the supposed demand less the backordered units from the previous period. She orders a lesser amount for the current period but the demand can still be met because the total number of units to arrive for that period includes not only the new order but also the backorder all equating to the demand.

Equation 32 Retailer Order Amount

* + 1. **Upper Bound Bundle Price**

This constraint assures that the selling price to be given by the firm to a bundle will not exceed the sum of the selling prices of its components. That is, the upper bound of the price that can be set for a bundle is the sum of the prices of the products it comprises. Such constraint is necessary since one of the major ideas of bundling is to combine products together and sell them at a relatively discounted price. If the price of the bundle will exceed the aggregated price of its components, then the discount essence will be gone.

Equation 33 Upper Bound Bundle Price

* + 1. **Retailer Selling Price Declaration**

This constraint sets the retail price of the products and the bundles based on the selling price of the upstream firm and a constant percentage markup set by the retailer. The retailer selling price is the same across all retailers and the markup percentage is constant for all products/bundles.

Equation 34 Retailer Selling Price

* + 1. **Production Cost per Unit Declaration**

This constraint computes for the total production cost per unit (for each product in each time period) that will be used in the objective function calculation. As discussed in the objective function section, the production cost per unit has two components – the variable cost per unit and the fixed expenses incurred by the factory for production per time period expressed on a per unit basis. The total production cost per unit can be computed using the equation below.

Equation 35 Production Cost per Unit

* + 1. **Sales Monitoring Variable Declaration**

This constraint allows the model to monitor the cumulative amount of orders (for delivery) made for a certain product/bundle as of any time period. Since one of the sub-problems to be addressed is to investigate the influence of low sales on the bundling decisions, this variable that will keep track of the cumulative orders for a product is necessary. By analyzing this variable, the researcher will be able to determine if the sales of the product is low relative to that of the other products. And, the researcher will be able to analyze the decisions made alongside i.e. how production decisions, bundling decisions, or pricing decisions changed.

Equation 36 Sales Monitoring Variable

* 1. **Demand Derivation**

The end customers in the supply chain arrive at retail stores (r) with a fixed arrival rate of customers per period in each period t . Customers, in this system, are already those who are intending to buy the products of the upstream firm (other products from other firms are no longer part of the study). The purchasing behavior of the customers follow the maximum surplus rule wherein the customer chooses to purchase the option giving her the highest surplus. The customer’s choice is limited to one option hence the highest surplus option. The customer may also leave without a purchase if all options yield a negative surplus. Meaning, if the prices the customer is willing to pay for all options are lower than the actual prices, then she would not be willing to buy anything. The surplus, as discussed in the previous chapters, is the difference between the customer’s reservation price and the selling price of the product/bundle.

The reservation prices of customers for the individual products are considered as random variables following a multiivariate normal distribution with mean, , standard deviation, , and correlation coefficient for all pairings of the individual products. The joint probability density function of the reservation prices, R1, R2, …, Rh, is . The reservation price of the bundle is first assumed to be the sum of the reservation prices of its individual components, i.e. R123 = R1 + R2 + R3. This assumption will be relaxed by allowing subadditivity and superadditivity of the reservation prices for bundles. This relaxation will be presented in the “Superadditivity and subadditivity of reservation prices” subsection. For reservation price estimates, the researcher suggests that the readers refer to Jedidi et al (2003) and adopt the model that they developed for capturing heterogeneity in joint distributions of product reservation prices. In their study, they also provided examples wherein they conducted experiments and used the model to estimate reservation price distributions.

An arriving customer can take several possible actions based on the maximum surplus rule. The customer may leave without a purchase, purchase product 1, purchase product 2, purchase product 3, purchase any individual product, purchase bundle 12, purchase bundle 13, purchase bundle 23, purchase bundle 123, or any other possible bundle combination. Each action has its own probability of occurrence , respectively (based on *h* individual products and *k* bundles). The purchasing probability and the demand rate derivation was based on the approach of Bulut et al (2009) but was extended for multiple individual products.

* + 1. **Preliminary Functions**

The above notations must first be defined before the discussion of the purchasing probability derivation. The reservation prices are assumed to follow a multivariate normal distribution adopting the approach of Bulut et al (2009). With this distribution, the joint probability density function of the reservation prices, R1, R2, …, Rh, is as follows.

where *h* is the number of individual products

*r* is a 1xh matrix for the reservation price notations of each individual product

is a 1xh matrix for the mean reservation price of each individual product

is an hxh matrix for the covariance matrix of the components (the diagonal elements of this matrix contain the variances of each variable or individual product while the off-diagonal elements contain the covariances [e.g. ] between variables)

is the determinant of matrix

In notation form:

* + 1. **Purchasing Probabilities**

In this section, the derivation of the purchasing probabilities will be presented similar to the discussion of Bulut et al (2009). For simplicity purposes and to clearly demonstrate the logic, only three individual products will be considered. The purchasing probability equations differ as the number of individual products changes. A general procedure and function will be developed for when there are *h* number of individual products to aid on how to come up with the final integral equation. Note that the subscript *t* refers to the time period as a multi-period system is studied and changes in the selling prices are allowed.

* + 1. *No Purchase Option*

The probability of a customer to purchase nothing is equal to the probability that all surpluses that can be obtained from the options are negative. That is, a customer will purchase nothing when her reservation price for the products and the bundles are all lower than the option’s respective selling prices.

* + 1. *Purchase of Product 1*

The probability of a customer to purchase an option *j* is equal to the probability that the surplus she can get from that option is positive and is larger than the surplus she can get from all other options. That is, a customer will only purchase a product if the difference between the reservation price and the selling price is positive, and if this difference is higher than the differences for all other options. In other words, a customer will purchase an option if that option gives her the highest positive surplus.

The probability of purchasing individual product 1 is shown below.

* + 1. *Purchase of Product 2*

The probability of purchasing individual product 2 is shown below.

* + 1. *Purchase of Product 3*

The probability of purchasing individual product 3 is shown below.

* + 1. *Purchase of Bundle 12*

The probability of purchasing bundle 12 is shown below.

* + 1. *Purchase of Bundle 13*

The probability of purchasing bundle 13 is shown below.

* + 1. *Purchase of Bundle 23*

The probability of purchasing bundle 23 is shown below.

* + 1. *Purchase of Bundle 123*

The probability of purchasing bundle 123 is shown below.

* + - 1. **General Purchasing Probability Derivation Function and Procedure**

The previous discussion considered only three individual products for simplification and better understanding of the logic and the procedure. However, the number of individual products may increase indefinitely. This general function and procedure can be used for when there are *h* number of individual products and *k* number of bundles.

1. *No Purchase Option*

The general function for a customer not purchasing anything is shown below. Note that there are *h* individual products and *k* bundles. The notation *x* denotes the last component of bundle k. For the no purchase action of the customer, the reservation price for all options should be lower than the selling price. This general idea is reflected on the first equation line. For an action involving a purchase, the general idea is that the surplus from purchasing the option should be positive and larger than the surplus from purchasing all other options. The next step to derive the purchasing probability is to translate the reservation price notations in terms of individual product reservation price notations only. That is, bundle reservation price notations (e.g. R­12) should be expressed in terms of individual product RP notations only (e.g. R1 + R2, assuming additive reservation prices). The output of this step is shown in the line 2 of the function. After expressing all RPs in terms of individual RP notations, a function should be obtained for each individual RP notation (e.g. R1 < f(SPR1t), R2 < f(R1, SPR1…k(t)). The first individual RP should be expressed as a function of the selling prices only, the second individual RP should be expressed as a function of the selling prices and the first individual RP, the third individual RP should be expressed as a function of the selling prices and the first and second individual RPs, and so on. The output of this step is shown in the line 3 of the function. This accumulation of expressions is necessary to trim down the variables in each step. Lastly, the probability expression must be translated into integral form, making use of the probability density function of the variables. In this integral form, continuous integration will be made from the last individual RP to the first individual RP. Limits of each integration depend on the function created in line 3. The output of this step is shown in line 4 of the function.

*ii. With Purchase Option*

The general function for when a purchase will be made by the customer is shown below (treating the option *1* as the option purchased). The logic is the same across all options that can be purchased so there is no need to elaborate on each. The same general procedure applies as aforementioned: Identify the conditions for when the action will be done and express in terms of notations of reservation prices and selling prices (line 1), Translate all reservation price notations into individual product RP notations only (line 2), obtain a function for each individual product RP notation in a hierarchical manner, i.e. first function is a function of SPRs only, second function is a function of SPRs and the first individual product RP notation, third function is a function of SPRs and the first and second individual product RP notation, and so on (line 3 and line 4), and last is to translate the whole probability function into integral form integrating from last RP notation to first RP notation with limits based on the functions from line 3 (line 5).

* + - 1. **Purchasing Probability Derivation with Superadditivity and Subadditivity in Bundle Reservation Prices**

The previously discussed purchasing probability computation assumed that the reservation price of a customer for a bundle is the sum of her reservation prices for the bundle’s individual components. However, when the bundle comprises products that are complements or substitutes, this assumption will no longer be applicable. According to Venkatesh and Kamakura (2003), the reservation price of customers for bundles may not always be strictly additive. The bundle reservation price may be superadditive (e.g. ) or subadditive (e.g. ) of the reservation prices of its components. This superadditivity or subadditivity can be measured through the degree of contingency, , or the degree of complementarity or substitutability. This measure can be obtained as follows wherein *k* denotes the bundle and denotes the set whose elements are components of the bundle. Venkatesh and Kamakura (2003) noted that “the degree of contingency captures the perceived value enhancement or reduction of a bundle within each customer”.

Using this concept, the new bundle reservation price can be expressed as , e.g. . The calculation of the purchasing probabilities can be done similarly as previously discussed. However, the second step involving the translation of RP notations in terms of individual product RP notations only will change as the translation will be different this time, involving the degree of contingency. Nevertheless, the general procedure still applies. Shown below is an example of the calculation of the purchasing probability for the no purchase action with 3 individual products given the superadditive/subadditive reservation prices of the bundles.

* + 1. **Demand Rate Function**

With the known arrival rates for each retailer in each time period and the calculated purchasing probabilities, the demand for each product in each time period can now be computed with rates . The demand Djrt then takes these values.