Next-Next-Gen Notes

Object-Oriented Maths

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February 7, 2018

1 Mathematical Logic

1.1 NaiveMaster

$$ \tag{1} $$

$$set, element, \in \tag{2}$$

$$element[x] \in set[y] := \text{x belongs to y} \tag{3}$$

$$x \subseteq y := \text{x is included in y} \tag{4}$$

$$\begin{aligned} x = y &:= \text{x is the same thing as y} \\ &:= x \subseteq y, y \subseteq x \end{aligned} \tag{5}$$

$$\begin{aligned} x \subset y, x \nsubseteq y &:= \text{proper subset} \\ &:= x \neq y, x \subseteq y \end{aligned} \tag{6}$$

$$x \cup y := \text{all elements in x or y} \tag{7}$$

$$x \cap y := \text{all elements in x and y} \tag{8}$$

$$\begin{aligned} disjoint(x, y) &:= \text{disjoint sets} \\ &:= x \cap y = \emptyset \end{aligned} \tag{9}$$

$$\begin{aligned} \{e_1, e_2, e_3, \cdots, e_n\} &:= \text{unordered set containing } e_1, e_2, e_3, \cdots, e_n \\ \{e_1, e_2, e_3\} &= \{e_3, e_1, e_2\} \end{aligned} \tag{10}$$

$$\begin{aligned} \langle e_1, e_2, e_3, \cdots, e_n \rangle &:= \text{ordered tuple containing } e_1, e_2, e_3, \cdots, e_n \\ \langle e_1, e_2, e_3 \rangle &\neq \langle e_2, e_3, e_1 \rangle \end{aligned} \tag{11}$$

$$\begin{aligned} X^k = \{e_1, e_2, e_3, \cdots, e_n\}^k &:= \text{set of all ordered k-tuples from the elements of } e_1, e_2, e_3, \cdots, e_n \\ X^1 = \{e_1, e_2, e_3, \cdots, e_n\}^1 = \{\langle e_1 \rangle, \langle e_2 \rangle, \langle e_3 \rangle, \cdots, \langle e_n \rangle\} &= \{e_1, e_2, e_3, \cdots, e_n\} = X \end{aligned} \tag{12}$$

$$\begin{aligned} Y \times Z = \{y_1, y_2, y_3, \cdots, y_i\} \times \{z_1, z_2, z_3, \cdots, z_j\} &:= \text{Cartesian product} \\ &:= \bigcup_{a \leq i, b \leq j} (\{\langle y_a, z_b \rangle\}) \end{aligned} \tag{13}$$

$$R_Y^k \subseteq Y^k := \text{k-tuple relation R on the set Y takes only tuples that satisfy some relation}$$
$$P_Y \subseteq Y := \text{property P of the set Y} \tag{14}$$

$$\langle y,z\rangle \in binaryRelation(R_X^2) = yR_X^2 z$$
$$domain(Y), range(Z)$$
$$field(R) = Y \cup Z$$
$$\langle a,b\rangle \in inverse(R^{-1}) : \langle b,a\rangle \in R$$
$$reflexive(R_X^2) : xR_X^2 x$$
$$symmetric(R_X^2) : xR_X^2 y = yR_X^2 x$$
$$transitive(R_X^2) : xR_X^2 y, yR_X^2 z : xR_X^2 z$$
$$equivalenceRelation(R_X^2) := reflexive(R_X^2), symmetric(R_X^2), transitive(R_X^2) \tag{15}$$

$$takethisintroshitmoresrsly \tag{16}$$

2 Logic and Set Theory

2.1 Logical Truths and Operators

$$\tag{17}$$

$$truth[t][] := t = \begin{cases} T \\ F \end{cases} \tag{18}$$

$$statement[s][] := correctSyntaxSemantics^{!}[s][] \tag{19}$$

$$proposition[s,t][] := \big(statement[s][]\big), \big(truth[t][]\big). \tag{20}$$

$$operatorOR[\vee][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \vee y][] = \begin{cases} F & x=F, y=F \\ T & x=F, y=T \\ T & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{21}$$

$$operatorAND[\wedge][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \wedge y][] = \begin{cases} F & x=F, y=F \\ F & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{22}$$

$$operatorNOT[\neg][x] := \big(truth[x][]\big), \left(truth[\neg x][] = \begin{cases} T & x=F \\ F & x=T \end{cases} \right). \tag{23}$$

$$booleanAlgebra[\{T,F\}, \wedge, \vee, \neg][] :=^{POS-LCom} \big((x \wedge y = y \wedge x), (x \vee y = y \vee x)\big) \text{ \# Commutative,}$$
$$^{POS-LDis}\Big(\big(x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)\big), \big(x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)\big)\Big) \text{ \# Distributive,}$$
$$^{POS-LIdn}\big((x \wedge T = x), (x \vee F = x)\big) \text{ \# Identity,}$$

$$^{POS-LCmp}\big((x \wedge \neg x = F), (x \vee \neg x = T)\big) \text{ \# Complement.} \tag{24}$$

$$operatorXOR[\veebar][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \veebar y][] = \begin{cases} F & x=F, y=F \\ T & x=F, y=T \\ T & x=T, y=F \\ F & x=T, y=T \end{cases} \right). \tag{25}$$

$$operatorIF[\Longrightarrow][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \Longrightarrow y][] = (\neg x) \vee y = \begin{cases} T & x=F, y=F \\ T & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{26}$$

$$\begin{aligned} &{}^{THM-LExp-1}_{POS-LCmp}(F = x \wedge \neg x) \Longrightarrow \\ &{}^{THM-LExp-2}_{THM-LExp-1}(x), \\ &{}^{THM-LExp-3}_{THM-LExp-1}(\neg x), \\ &{}^{THM-LExp-4}_{THM-LExp-2}(x \vee y), \\ &{}^{THM-LExp-5}_{\substack{THM-LExp-4 \\ THM-LExp-3}}(y). \\ &{}^{THM-LExp}_{\substack{THM-LExp-1 \\ THM-LExp-2 \\ THM-LExp-3 \\ THM-LExp-4 \\ THM-LExp-5}}(F \Longrightarrow y) \\ &\text{\# The Principle of Explosion, anything follows from a false } (F) \text{ premise} \end{aligned} \tag{27}$$

$$operatorOIF[\Longleftarrow][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \Longleftarrow y][] = (\neg y) \vee x = \begin{cases} T & x=F, y=F \\ F & x=F, y=T \\ T & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{28}$$

$$\begin{aligned} &operatorIIF[\Longleftrightarrow][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \\ &\left(truth[x \Longleftrightarrow y][] = (x \Longrightarrow y) \wedge (y \Longrightarrow x) = \begin{cases} T & x=F, y=F \\ F & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \end{aligned} \tag{29}$$