Next-Next-Gen Notes
Object-Oriented Maths

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Format: $characteristic((subjects), (dependencies)) \iff (conditions(dependencies)) \wedge (conditions(subjects))$
TODO should really define union intersection complement etc TESTMEEEEEEEEEEEEEEEEEEEEEEEE

1 Mathematical Analysis

1.0.1 Formal Logic

$$statement(s, ()) \iff well\text{-}formedString(s, ()) \tag{1}$$

$$\begin{aligned} proposition((p,t),()) \iff & (statement(p,()))\wedge \\ & (t = eval(p))\wedge \\ & (t = true \veebar t = false) \end{aligned} \tag{2}$$

$$operator(o, ((p)_{n\in\mathbb{N}})) \iff proposition(o((p)_{n\in\mathbb{N}}), ()) \tag{3}$$

$$\begin{aligned} operator(\neg, (p_1)) \iff & (proposition((p_1, true), ()) \implies ((\neg p_1, false), ()))\wedge \\ & (proposition((p_1, false), ()) \implies ((\neg p_1, true), ())) \\ & \text{\# an operator takes in propositions and returns a proposition} \end{aligned} \tag{4}$$

$$\begin{aligned} & operator(\neg) \iff \textbf{NOT} \text{ ; } operator(\vee) \iff \textbf{OR} \text{ ; } operator(\wedge) \iff \textbf{AND} \text{ ; } operator(\veebar) \iff \textbf{XOR} \\ & operator(\implies) \iff \textbf{IF} \text{ ; } operator(\impliedby) \iff \textbf{OIF} \text{ ; } operator(\iff) \iff \textbf{IFF} \end{aligned} \tag{5}$$

$$\begin{aligned} & proposition((false \implies true), true, ()) \wedge proposition((false \implies false), true, ()) \\ & \text{\# truths based on a false premise is not false (ex falso quodlibet)} \end{aligned} \tag{6}$$

$$(a \implies b \implies c) \iff (a \implies (b \implies c)) \iff ((a \wedge b) \implies c) \tag{7}$$

$$predicate(P, (V)) \iff \forall_{v\in V}(proposition((P(v), t), ())) \tag{8}$$

$$\begin{aligned} 0thOrderLogic(P, ()) \iff & proposition((P, t), ()) \\ & \text{\# individual proposition} \end{aligned} \tag{9}$$

$$\begin{aligned} 1stOrderLogic(P, (V)) \iff & (\forall_{v\in V}(0thOrderLogic(v, ())))\wedge \\ & (\forall_{v\in V}(proposition((P(v), t), ()))) \\ & \text{\# propositions defined over a set of (1-1=0)th-order logical statements} \end{aligned} \tag{10}$$

$$\begin{aligned} quantifier(q, (p, V)) \iff & (predicate(p, (V)))\wedge \\ & (proposition((q(p), t), ())) \\ & \text{\# a quantifier takes in a predicate and returns a proposition} \end{aligned} \tag{11}$$

$$quantifier(\forall,(p,V)) \iff proposition((\wedge_{v\in V}(p(v)),t),()) \quad \text{\# universal quantifier} \tag{12}$$

$$quantifier(\exists,(p,V)) \iff proposition((\vee_{v\in V}(p(v)),t),()) \quad \text{\# existential quantifier} \tag{13}$$

$$quantifier(\exists!,(p,V)) \iff \exists_{x\in V}(P(x) \wedge \neg(\exists_{y\in V\setminus\{x\}}(P(y)))) \quad \text{\# uniqueness quantifier} \tag{14}$$

$$\forall_x p(x) \iff \neg\exists_x \neg p(x) \quad \text{\# De Morgan's law} \tag{15}$$

$$\forall_x\exists_y p(x,y) = \forall_x\neg\forall_y\neg p(x,y) \neq \exists_y\forall_x p(x,y) = \neg\forall_y\neg(\forall_x p(x,y)) = \neg\forall_y\exists_x\neg p(x,y) \quad \text{\# different quantifiers are not interchangeable} \tag{16}$$

★★★ (17)

proof = truths derived from a finite number of axioms and deductions (tautologies) (18)

elementary arithmetics = system with substitutions, and some notion of addition, multiplication, and prime nuumbers for encoding metamathematics (19)

Gödel theorem $\implies$ **axiomatic systems equivalent in power to elementary mathematics are either incomplete (has unprovables) or inconsistent (has contradictions)** (20)

★★★ (21)

1.1 Axiomatic Set Theory

★★★ (22)

Zermelo-Fraenkel-Choice (ZFC) set theory = standard form of axiomatic set theory (23)

ZFC axioms = Existence (EE), Construction (PURP), ECAdv (IC), Nonexistent (F) (24)

$$A \subseteq B = \forall_x x \in A \implies x \in B \tag{25}$$

$$(A = B) = A \subseteq B \wedge B \subseteq A \tag{26}$$

$$\in \textbf{basis} \implies \{x,y\} = \{y,x\} \wedge \{x\} = \{x,x\} \tag{27}$$

$\in$**-relation and sets ({. . . }) works as follows (9 ZFC axioms):** (28)

$$\forall_x\forall_y(x \in y \veebar \neg(x \in y)) \quad \text{\# E: } \in \text{ is only a proposition on sets} \tag{29}$$

$$\exists_\emptyset \forall_y \neg y \in \emptyset \text{ \# E: existence of empty set} \tag{30}$$

$$\forall_x \forall_y \exists_m \forall_u u \in m \iff u = x \vee u = y \text{ \# C: pair set construction} \tag{31}$$

$$\forall_s \exists_u \forall_x \forall_y (x \in s \wedge y \in x \implies y \in u) \text{ \# C: union set construction} \tag{32}$$

$$x = \{\{a\}, \{b\}\} \text{ \# from the pair set axiom} \tag{33}$$

$$u = \cup x = \cup\{\{a\}, \{b\}\} = \{a, b\} \tag{34}$$

$$\forall_x \exists!_y R(x, y) \text{ \# functional relation } R \tag{35}$$

$$\begin{aligned} \exists_i \forall_x \exists!_y R(x, y) \implies y \in i \text{ \# C: image } i \text{ of set } m \text{ under a relation } R \text{ is assumed to be a set} \\ \implies \{y \in m | P(y)\} \text{ \# Restricted Comprehension } \not\Longrightarrow \{y | P(y)\} \text{ \# Universal Comprehension} \end{aligned} \tag{36}$$

$$\forall_{x \in m} P(x) = \forall_x (x \in m \implies P(x)) \text{ \# ignores out of scope} \neq \forall_x (x \in m \wedge P(x)) \text{ \# restricts entirety} \tag{37}$$

$$\forall_m \forall_n \exists_{\mathcal{P}(m)} (n \subseteq m \implies n \subseteq \mathcal{P}(m)) \text{ \# C: existence of power set} \tag{38}$$

$$\exists_I (\emptyset \in I \wedge \forall_{x \in I} (\{x\} \in I)) \text{ \# I: axiom of infinity ; } I = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \ldots\}; I \cong \mathbb{N} \implies \mathbb{N} \textbf{ is a set} \tag{39}$$

$$\forall_x ((\emptyset \notin x \wedge x \cap x' = \emptyset) \implies \exists_y (\textbf{set of each e} \in x)) \text{ \# C: axiom of choice LOLOL} \tag{40}$$

$$\forall_x x \neq \emptyset \implies x \notin x \text{ \# F: axiom of foundation covers further paradoxes} \tag{41}$$

$$\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star\star \tag{42}$$

1.2 Classification of sets

$$\begin{aligned} \mathit{space}((\mathit{set}, \mathit{structure}), ()) \iff \mathit{structure}(\mathit{set}) \\ \text{\# a space a set equipped with some structure} \\ \text{\# various spaces can be studied through structure preserving maps between those spaces} \end{aligned} \tag{43}$$

$$\begin{aligned} \mathit{map}(\phi, (A, B)) \iff (\forall_{a \in A} \exists!_{b \in B} (\phi(a, b))) \vee \\ (\forall_{a \in A} \exists!_{b \in B} (b = \phi(a))) \\ \text{\# maps elements of a set to elements of another set} \end{aligned} \tag{44}$$

$$\mathit{domain}(A, (\phi, A, B)) \iff (\mathit{map}(\phi, (A, B))) \tag{45}$$

$$\mathit{codomain}(B, (\phi, A, B)) \iff (\mathit{map}(\phi, (A, B))) \tag{46}$$

$$\begin{aligned} \mathit{image}(B, (A, q, M, N)) \iff (\mathit{map}(q, (M, N)) \wedge A \subseteq M) \wedge \\ (B = \{n \in N | \exists_{a \in A} (q(a) = n)\}) \end{aligned} \tag{47}$$

$$preimage(A,(B,q,M,N)) \iff (map(q,(M,N)) \land B \subseteq N) \land \\ (A = \{m \in M | \exists_{b \in B}(b = q(m))\}) \tag{48}$$

$$injection(q,(M,N)) \iff (map(q,(M,N))) \land \\ \forall_{u,v \in M}(q(u) = q(v) \implies u = v) \\ \#\text{ every } m \text{ has at most 1 image} \tag{49}$$

$$surjection(q,(M,N)) \iff (map(q,(M,N))) \land \\ \forall_{n \in N} \exists_{m \in M}(n = q(m)) \\ \#\text{ every } n \text{ has at least 1 preimage} \tag{50}$$

$$bijection(q,(M,N)) \iff (injection(q,(M,N))) \land \\ (surjection(q,(M,N))) \\ \#\text{ every unique } m \text{ corresponds to a unique } n \tag{51}$$

$$isomorphicSets((A,B),()) \iff \exists_{\phi}(bijection(\phi,(A,B))) \tag{52}$$

$$infiniteSet(S,()) \iff \exists_{T \subset S}(isomorphicSets((T,S),())) \tag{53}$$

$$finiteSet(S,()) \iff (\neg infiniteSet(S,())) \lor (|S| \in \mathbb{N}) \tag{54}$$

$$countablyInfinite(S,()) \iff (infiniteSet(S,())) \land (isomorphicSets((S,\mathbb{N}),())) \tag{55}$$

$$uncountablyInfinite(S,()) \iff (infiniteSet(S,())) \land (\neg isomorphicSets((S,\mathbb{N}),())) \tag{56}$$

$$inverseMap(q^{-1},(q,M,N)) \iff (bijection(q,(M,N))) \land \\ (map(q^{-1},(N,M))) \land \\ (\forall_{n \in N} \exists!_{m \in M}(q(m) = n \implies q^{-1}(n) = m)) \tag{57}$$

$$mapComposition(\phi \circ \psi,(\phi,\psi,A,B,C)) \iff map(\psi,(A,B)) \land map(\phi,(B,C)) \land \\ \forall_{a \in A}(\phi \circ \psi(a) = \phi(\psi(a))) \tag{58}$$

$$equivalenceRelation(\sim,(M)) \iff (\forall_{m \in M}(m \sim m)) \land \\ (\forall_{m,n \in M}(m \sim n \implies n \sim m)) \land \\ (\forall_{m,n,p \in M}(m \sim n \land n \sim p \implies m \sim p)) \\ \#\text{ behaves like equivalences} \tag{59}$$

$$equivalenceClass([m],(m,M,\sim)) \iff [m] = \{n \in M | n \sim m\} \\ \#\text{ set of elements satisfying the equivalence relation with } m \tag{60}$$

$$a \in [m] \implies [a] = [m]\ ;\ [m] = [n] \veebar [m] \cap [n] = \emptyset \\ \#\text{ equivalence class properties} \tag{61}$$

$$quotientSet(M/\sim,(M,\sim)) \iff M/\sim = \{[m] \in \mathcal{P}(M) | m \in M\} \\ \#\text{ set of all equivalence classes} \tag{62}$$

$$\textbf{axiom of choice} \implies \forall_{[m]\in M/\sim}\exists_r(r\in[m])$$
$$\#\text{ well-defined maps may be defined in terms of chosen representative elements } r \quad (63)$$

1.3 Construction of number sets

$$\bigstar \quad (64)$$

$$\textbf{axiom of infinity} \implies \{\emptyset,\{\emptyset\},\{\{\emptyset\}\},\ldots\}\cong\mathbb{N} \quad (65)$$

$$\mathbb{N}^\star=\mathbb{N}\backslash\{0\} \quad (66)$$

$$\textbf{addition}=\textbf{successor map: } \mathbb{N}\to\mathbb{N}=S(n)=\{n\}\ \#\text{ adds a layer of brackets} \quad (67)$$

$$\textbf{subtraction}=\textbf{predecessor map: } \mathbb{N}^\star\to\mathbb{N}=P(n)=m|m\in n\ \#\text{ removes a layer of brackets} \quad (68)$$

$$S^0=id\ ;\ n\in\mathbb{N}^\star \implies S^n=S\circ S^{P(n)} \quad (69)$$

$$\textbf{addition}=+:\mathbb{N}\times\mathbb{N}\to\mathbb{N}=+(m,n)=m+n=S^n(m) \quad (70)$$

$$S^x=id=S^0 \implies x=\textbf{additive identity}=0 \quad (71)$$

$$S^n(x)=0 \implies x=\textbf{additive inverse}\notin\mathbb{N}\ \#\text{ git gud smh -\_-} \quad (72)$$

$$\mathbb{Z}=\mathbb{N}\times\mathbb{N}/\sim\textbf{, s.t.:}$$
$$(m,n)\sim(p,q) \iff m+q=p+n\ \#\text{ span } \mathbb{Z} \text{ using differences then group equal differences} \quad (73)$$

$$\mathbb{N}\hookrightarrow\mathbb{Z}:\forall_{n\in\mathbb{N}}n\to[(n,0)]\ \#\ \mathbb{N}\text{ embedded in }\mathbb{Z} \quad (74)$$

$$+_{\mathbb{Z}}=[(m+_{\mathbb{N}}p,n+_{\mathbb{N}}q)]\ \#\text{ well-defined and consistent} \quad (75)$$

$$\textbf{multiplication}\ldots M^x=id \implies x=\textbf{multiplicative identity}=1\ldots\textbf{multiplicative inverse}\notin\mathbb{N} \quad (76)$$

$$\mathbb{Q}=(\mathbb{Z}\times\mathbb{Z}^\star)/\sim\textbf{, s.t.: }(x,y)\sim(u,v) \iff x\cdot v=u\cdot y \quad (77)$$

$$\mathbb{Z}\hookrightarrow\mathbb{Q}\forall_{q\in\mathbb{Q}}q\to[(q,1)]\ ;\ \ldots\{x|x^2=2\}\notin\mathbb{Q} \quad (78)$$

$$\mathbb{R}=\textbf{almost homomorphisms on }\mathbb{Z}/\sim\ \#\ \text{http://blog.sigfpe.com/2006/05/defining-reals.html} \quad (79)$$

$$\bigstar \quad (80)$$

1.4 Topology and induced topology

$$\begin{aligned} topology(\mathcal{O},(M)) \iff (\mathcal{O} \subseteq \mathcal{P}(M))\wedge \\ (\emptyset, M \in \mathcal{O})\wedge \\ ((F \in \mathcal{O} \wedge |F| < |\mathbb{N}|) \implies \cap F \in \mathcal{O})\wedge \\ (C \subseteq \mathcal{O} \implies \cup C \in \mathcal{O}) \end{aligned}$$

\# topology is defined by a set of open sets which provide the characteristics needed to define continuity, etc.
\# arbitrary unions of open sets always result in an open set
\# open sets do not contain their boundaries and infinite intersections of open sets may approach and
\# induce boundaries resulting in a closed set (81)

$$topologicalSpace((M,\mathcal{O}),()) \iff topology(\mathcal{O},(M)) \tag{82}$$

$$\begin{aligned} open(S,(M,\mathcal{O})) \iff (topologicalSpace((M,\mathcal{O}),()))\wedge \\ (S \subseteq M) \wedge (S \in \mathcal{O}) \end{aligned}$$

\# an open set do not contains its own boundaries (83)

$$\begin{aligned} closed(S,(M,\mathcal{O})) \iff (topologicalSpace((M,\mathcal{O}),()))\wedge \\ (S \subseteq M) \wedge (S \in \mathcal{P}(M) \setminus \mathcal{O}) \end{aligned}$$

\# a closed set contains the boundaries an open set (84)

$$clopen(S,(M,\mathcal{O})) \iff (closed(S,(M,\mathcal{O}))) \wedge (open(S,(M,\mathcal{O}))) \tag{85}$$

$$neighborhood(U,(a,\mathcal{O})) \iff (a \in U \in \mathcal{O}) \tag{86}$$

$$\begin{aligned} M = \{a,b,c,d\} \wedge \mathcal{O} = \{\emptyset, \{c\}, \{a,b\}, \{c,d\}, \{a,b,c\}, M\} \implies \\ (open(X,(M,\mathcal{O})) \iff X = \{\emptyset, \{c\}, \{a,b\}, \{c,d\}, \{a,b,c\}, M\})\wedge \\ (closed(Y,(M,\mathcal{O})) \iff Y = \{\emptyset, \{a,b,d\}, \{c,d\}, \{a,b\}, \{d\}, M\})\wedge \\ (clopen(Z,(M,\mathcal{O})) \iff Z = \{\emptyset, \{a,b\}, \{c,d\}, M\}) \end{aligned} \tag{87}$$

$$\mathcal{O}_{chaotic} = \{0, M\} \; ; \; \mathcal{O}_{discrete} = \mathcal{P}(M) \tag{88}$$

$$\begin{aligned} distance(d,(M)) \iff (x,y,z \in M)\wedge \\ (d(x,y) \in \mathbb{R}^+)\wedge \\ (d(x,y) = 0 \iff x = y)\wedge \\ (d(x,y) = d(y,x))\wedge \\ (d(x,z) \leq d(x,y) + d(y,z)) \end{aligned} \tag{89}$$

$$TEST\mathcal{O}_{chaotic} \tag{90}$$

$$\begin{aligned} openBall(B,(r,p,M,d)) \iff (r \in \mathbb{R}^+, p \in M)\wedge \\ (B = \{q \in M | d(p,q) < r\}) \end{aligned} \tag{91}$$

$$metricTopology(\mathcal{O},(M,d)) \iff \mathcal{O} = \{U \subseteq M | \forall_{p \in U} \exists_{r \in \mathbb{R}^+} (B(r,p,M,d) \subseteq U)\} \tag{92}$$

$$\begin{aligned} limitPoint(p,(S,M,\mathcal{O},d)) \iff (S \subseteq M)\wedge \\ \forall_{r \in \mathbb{R}^+}(openBall \cap S \neq \emptyset) \end{aligned}$$

$$\# \text{ every open ball contains some intersection} \tag{93}$$

$$\begin{aligned} interiorPoint(p,(S,M,\mathcal{O},d)) \iff (S \subseteq M) \\ \exists_{r\in\mathbb{R}^+}(openBall \subseteq S) \\ \# \text{ there is an open ball that is fully enclosed} \end{aligned} \tag{94}$$

$$n \in \mathcal{O} \iff interiorPoint(n) \tag{95}$$

$$closure(\bar{S},(S,M,\mathcal{O},d)) \iff \bar{S} = S \cup limitPoints(S) \tag{96}$$

$$\begin{aligned} dense(S,(M,\mathcal{O},d)) \iff (S \subseteq M)\wedge \\ \forall_{p\in M}(p \in closure(S)) \\ \# \text{ every of point in } X \text{ is a point or a limit point of } S \end{aligned} \tag{97}$$

$$\begin{aligned} eucD(d,(\mathbb{R}^n)) \iff (x_i \in \mathbb{R})\wedge \\ (d = \sqrt{\sum_{i=1}^{n} {x_i}^2}) \end{aligned} \tag{98}$$

$$\begin{aligned} \mathcal{O}_{standard} = \mathcal{O}(\mathbb{R}^n, eucD) \\ \textbf{L1:)}\ \forall_{p\in U=\emptyset}(\ldots) \implies \forall_p((p\in\emptyset) \implies \ldots) \implies \forall_p((\textbf{False}) \implies \ldots) \implies \emptyset \in \mathcal{O}_{standard} \\ \textbf{L2:)}\ \forall_{p\in\mathbb{R}^n} B(r,p,\mathbb{R}^n,eucD) \subseteq \mathbb{R}^n \implies M \in \mathcal{O}_{standard} \\ \textbf{L3:}\ U,V \in \mathcal{O}_{standard} \implies p \in U\cap V \implies p\in U \wedge p \in V \implies \\ \exists_{r\in\mathbb{R}^+}B(r,p,\mathbb{R}^n,eucD) \wedge \exists_{s\in\mathbb{R}^+}B(s,p,\mathbb{R}^n,eucD) \implies \\ B(min(r,s),p,\mathbb{R}^n,eucD) \subseteq U \wedge B(min(r,s),q,\mathbb{R}^n,eucD) \subseteq V \implies \\ B(min(r,s),p,\mathbb{R}^n,eucD) \in U\cap V \implies U \cap V \in \mathcal{O}_{standard} \\ \# \text{ could fail on infinite sets since } min \text{ could approach } 0 \\ \textbf{L4:}\ C \subseteq \mathcal{O}_{standard} \implies \forall_{U\in C}\forall_{p\in U}\exists_{r\in\mathbb{R}^+}(B_r(p) \subseteq U \subseteq \cup C) \implies \cup C \in \mathcal{O}_{standard} \end{aligned} \tag{99}$$

$$\begin{aligned} subsetTopology(\mathcal{O}|_N,(M,\mathcal{O},N)) \iff (N \subseteq M)\wedge \\ (\mathcal{O}|_N = \{U\cap N | U \in \mathcal{O}\}) \end{aligned} \tag{100}$$

$$\begin{aligned} topology(\mathcal{O}|_N(M,\mathcal{O},N),(N)) \impliedby \\ \textbf{L1:}\ \emptyset\in\mathcal{O} \implies U=\emptyset \implies \emptyset\cap N=\emptyset \implies \emptyset \in \mathcal{O}|_N \\ \textbf{L2:}\ M\in\mathcal{O} \implies U=M \implies M\cap N = N \implies N \in \mathcal{O}|_N \\ \textbf{L3:}\ S,T\in\mathcal{O}|_N \implies \exists_{U\in\mathcal{O}}(S=U\cap N)\wedge\exists_{V\in\mathcal{O}}(T=V\cap N) \implies S\cap T = (U\cap N)\cap(V\cap N) \\ =(U\cap V)\cap N \wedge U\cap V\in\mathcal{O} \implies S\cap T\in\mathcal{O}|_N \\ \textbf{L4:}\ TODO: EXERCISE \end{aligned} \tag{101}$$

$$(\mathbb{R},\mathcal{O}_s)\ ;\ N=\{x\in\mathbb{R}|-1\le x\le 1\}\ ;\ (0,1]\notin\mathcal{O}_s \tag{102}$$

$$(0,1] = (0,2)\cap N \wedge (0,2) \in (O)_s \implies (0,1] \in \mathcal{O}_s|_N \ \# \text{ openness depends on topology} \tag{103}$$

$$productTopology(\mathcal{O}_{A\times B},((A,\mathcal{O}_A),(B,\mathcal{O}_B)))$$

$$\iff \mathcal{O}_{A\times B} = \{(a,b) \in A \times B | \exists_S (a \in S \in \mathcal{O}_A) \exists_T (b \in T \in \mathcal{O}_B)\} \text{ \# open in cross iff open in each} \tag{104}$$

1.5 Convergence

$$sequence(q,(M)) \iff q : \mathbb{N} \to M \tag{105}$$

$$\begin{aligned} convergeAgainst((q,a),(M,\mathcal{O})) \iff (sequence(q,(M)))\wedge \\ (\forall_{U|a\in U\in\mathcal{O}} \exists_{N\in\mathbb{N}} \forall_{n>N} (q(n) \in U)) \\ \text{\# each neighborhood of } a \text{ has a tail-end sequence that does not map to outside points} \end{aligned} \tag{106}$$

$$\forall_a \forall_q (convergeAgainst((q,a),(M,\mathcal{O}_{chaotic}))) \impliedby \forall n(q(n) \in M) \tag{107}$$

$$\begin{aligned} convergeAgainst((q,a),(M,\mathcal{P}(M))) \iff \exists_{N\in\mathbb{N}} \forall_{k>N} (q(N) = q(k)) \\ \text{\# single element neighborhood can only converge if } q \text{ is almost constant} \end{aligned} \tag{108}$$

$$\begin{aligned} \textbf{convergence generalizes to: the sequence } q : \mathbb{N} \to \mathbb{R}^d \textbf{ converges against } a \in \mathbb{R}^d \textbf{ if:} \\ \forall_{r>0} \exists_{N\in\mathbb{N}} \forall_{n>N} (||q(n) - a|| < \epsilon) \text{ \# distance based convergence} \end{aligned} \tag{109}$$

$$\begin{aligned} q(n) = 1 - \frac{1}{n+1} \implies \\ q \textbf{ is not almost constant } \implies q \textbf{ does not converge in } (\mathbb{R}, \mathcal{P}(\mathbb{R}))\ ; \\ q \textbf{ satisfies distance based convergence } \implies q \textbf{ does converge in } (\mathbb{R}, \mathcal{O}_s) \end{aligned} \tag{110}$$

1.6 Continuity

$$\begin{aligned} continuous(\phi,((M,\mathcal{O}_M),(N,\mathcal{O}_N))) \iff (\phi : M \to N)\wedge \\ (\forall_{V\in\mathcal{O}_N} (preimage(V,\phi) \in \mathcal{O}_M)) \text{ \# preimage of open sets are open} \end{aligned} \tag{111}$$

$$\begin{aligned} homeomorphism(\phi,((M,\mathcal{O}_M),(N,\mathcal{O}_N))) \iff (bijection(\phi,(M,N)))\wedge \\ (continuous(\phi,((M,\mathcal{O}_M),(N,\mathcal{O}_N))))\wedge \\ (continuous(\phi^{-1},((N,\mathcal{O}_N),(M,\mathcal{O}_M)))) \\ \text{\# structure preserving maps in topology, one-to-one pairing of open sets} \\ \text{\# homeomorphic spaces share topological properties} \end{aligned} \tag{112}$$

$$isomorphic(\cong,((M,\mathcal{O}_M),(N,\mathcal{O}_N))) \iff \exists_\phi (homeomorphism(\phi,((M,\mathcal{O}_M),(N,\mathcal{O}_N)))) \tag{113}$$

$$(M,\mathcal{O}_M) \cong (N,\mathcal{O}_N) \implies \exists_\phi (bijection(\phi,(M,N))) \implies M \cong N \tag{114}$$

1.7 Separation

$$\begin{aligned} T0Separate((M,\mathcal{O}),()) \iff \forall_{x,y\in M\wedge x\neq y} \exists_{U\in\mathcal{O}} ((x \in U \wedge y \notin U) \vee (y \in U \wedge x \notin U)) \\ \text{\# each pair of points has a neighborhood s.t. one is inside and the other is outside} \end{aligned} \tag{115}$$

$$T1Separate((M,\mathcal{O}),()) \iff \forall_{x,y\in M \wedge x\neq y}\exists_{U,V\in\mathcal{O}\wedge U\neq V}((x\in U \wedge y\notin U)\wedge(y\in V \wedge x\notin V))$$
$$\text{\# every point has a neighborhood that does not contain another point} \quad (116)$$

$$T2Separate((M,\mathcal{O}),()) \iff \forall_{x,y\in M \wedge x\neq y}\exists_{U,V\in\mathcal{O}\wedge U\neq V}(U\cap V=\emptyset)$$
$$\text{\# every point has a neighborhood that does not intersect with a neighborhood of another point}$$
$$\text{\# also known as Hausdorff space} \quad (117)$$

$$T2Separate \implies T1Separate \implies T0Separate \quad (118)$$

1.8 Compactness and Paracompactness

$$openCover(C,(M,\mathcal{O})) \iff (C\subseteq\mathcal{O})\wedge$$
$$(\cup C = M) \text{ \# collection of open sets whose elements cover the entire space} \quad (119)$$

$$finiteSubcover(\widetilde{C},(C,M,\mathcal{O})) \iff (\widetilde{C}\subseteq C)\wedge$$
$$(openCover(C,(M,\mathcal{O})))\wedge$$
$$(|\widetilde{C}|<|\mathbb{N}|)\wedge$$
$$(openCover(\widetilde{C},(M,\mathcal{O})))$$
$$\text{\# finite subset of a cover that is also a cover} \quad (120)$$

$$compact((M,\mathcal{O}),()) \iff \forall_{C\subseteq\mathcal{O}\wedge openCover(C,(M,\mathcal{O}))}\exists_{\widetilde{C}\subseteq C}(finiteSubcover(\widetilde{C},(C,M,\mathcal{O})))$$
$$\text{\# every possible cover has a finite representation}$$
$$\text{\# "the entire space can be surveyed by a finite number of guards patrolling neighborhoods"} \quad (121)$$

$$compact(N,(M,\mathcal{O})) \iff (N\subseteq M)\wedge$$
$$(compact((N,\mathcal{O}|_N),())) \quad (122)$$

$$bounded(N,(M,d)) \iff (\exists_{p\in M}\exists_{r\in\mathbb{R}^+}(N\subseteq openBall(B,(r,p,M,d))))\vee$$
$$(\forall_{p,q\in n}\exists_{r\in\mathbb{R}^+}(d(p,q)<r)) \quad (123)$$

$$HeineBorel(S,(M,metricTopology(\mathcal{O}_d,(M,d)))) \implies$$
$$\forall_{S\in\mathcal{P}(M)}((closed(S,(M,\mathcal{O}_d))\wedge bounded(S,(M,\mathcal{O}_d))) \iff compact(S,(M,\mathcal{O}_d)))$$
$$\text{\# in some situations, compactness is equivalent to being closed and bounded} \quad (124)$$

$$compact((M,\mathcal{O}_M),())\wedge compact((N,\mathcal{O}_N),()) \implies compact(\mathcal{O}_{A\times B}((A,\mathcal{O}_A),(B,\mathcal{O}_B)),()) \quad (125)$$

$$openRefinement(\widetilde{C},(C,M,\mathcal{O})) \iff (openCover(C,(M,\mathcal{O})))\wedge$$
$$(openCover(\widetilde{C},(M,\mathcal{O})))\wedge$$
$$(\forall_{U\in C}\exists_{\widetilde{U}\in\widetilde{C}}(\widetilde{U}\subseteq U))$$
$$\text{\# open sets in the open refinement only needs to be a subset of some in the open cover}$$
$$\text{\# one could refine the cover by removing the excess open set elements that lie outside the space} \quad (126)$$

$$finiteSubcover \implies openRefinement \quad (127)$$

$$locallyFinite(C,(M,\mathcal{O})) \iff (openCover(C,(M,\mathcal{O})))\wedge$$
$$\forall_{p\in M}\exists_{U\in\mathcal{O}|p\in U}(finiteSet(\{U_c \in C|U\cap U_c \neq \emptyset\},()))$$
$$\text{\# each point has a neighborhood that intersects with only finitely many sets in the cover} \quad (128)$$

$$paracompact((M,\mathcal{O}),()) \iff$$
$$\forall_C(openCover(C,(M,\mathcal{O})) \implies \exists_{\widetilde{C}}(locallyFinite(openRefinement(\widetilde{C},(C,M,\mathcal{O})),(M,\mathcal{O}))))$$
$$\text{\# every open cover has a locally finite open refinement}$$
$$\text{\# each point has a neighborhood that is in contact with only finitely many open refinement elements} \quad (129)$$

thm: every metrizable space is paracompact (130)

thm: product of a paracompact and finitely many compact topologies is paracompact (131)

$$partitionOfUnitySOTCover(\mathcal{F},(C,M,\mathcal{O})) \iff (openCover(C,(M,\mathcal{O})))\wedge$$
$$(locallyFinite(C,M,\mathcal{O}))\wedge$$
$$(f\in\mathcal{F})\wedge$$
$$(continuous(f,((M,\mathcal{O}),([0,1],\mathcal{O}_{standard}|_{[0,1]}))))\wedge$$
$$(\exists_{U_C\in C}\forall_{p\in M}(f(p)\neq 0 \implies p\in U_C))\wedge$$
$$(\forall_{p\in M}\exists_{U\in\mathcal{O}|p\in U}((f_U)_n = \{f\in\mathcal{F}|p\in M \wedge f(p)\neq 0\}))\wedge$$
$$(locallyFinite(C,M,\mathcal{O}) \implies finiteSet((f_U)_n,()))\wedge$$
$$(\forall_{p\in M}\exists_{U\in\mathcal{O}|p\in U}\left(\sum_{i=1}^{|(f_U)_n|}(f_U)_i(p)=1\right))$$
$$\text{\# useful for defining integrals between overlapping neighborhoods} \quad (132)$$

$$T2Separate((M,\mathcal{O}),()) \implies (paracompact((M,\mathcal{O}),()) \iff$$
$$\forall_C(openCover(C,(M,\mathcal{O})) \implies partitionOfUnitySOTCover(\mathcal{F},(C,M,\mathcal{O})))) \quad (133)$$

1.9 Connectedness and path-connectedness

$$connected((M,\mathcal{O}),()) \iff \neg(\exists_{A,B\in\mathcal{O}\setminus\emptyset}(A\cap B\neq\emptyset \wedge A\cup B = M)) \quad (134)$$

$$\neg connected((\mathbb{R}\setminus\{0\},\mathcal{O}_{standard}|_{\mathbb{R}\setminus\{0\}}),()) \impliedby$$
$$(A=(-\infty,0)\in\mathcal{O}_{standard}|_{\mathbb{R}\setminus\{0\}})\wedge$$
$$(B=(0,\infty)\in\mathcal{O}_{standard}|_{\mathbb{R}\setminus\{0\}})\wedge$$
$$(A\cap B=\emptyset)\wedge$$
$$(A\cup B=\mathbb{R}\setminus\{0\}) \quad (135)$$

$$connected((M,\mathcal{O}),()) \iff \forall_{S\in\mathcal{O}}((S=\emptyset\vee S=M) \implies clopen(S,(M,\mathcal{O}))) \quad (136)$$

$$pathConnected((M,\mathcal{O}),()) \iff$$
$$\forall_{p,q\in M}\exists_\gamma(continuous(\gamma,(([0,1],\mathcal{O}_{standard}|_{[0,1]}),(M,\mathcal{O})))\wedge\gamma(0)=p\wedge\gamma(1)=q) \quad (137)$$

$$pathConnected \implies connected \quad (138)$$

1.10 Homotopic curve and the fundamental group

$$homotopic(\sim,(\gamma,\delta,M,\mathcal{O})) \iff (map(\gamma,([0,1],M)) \wedge map(\delta,([0,1],M)))\wedge$$
$$(\gamma(0)=\delta(0)\wedge\gamma(1)=\delta(1))\wedge$$
$$(\exists_H\forall_{\lambda\in[0,1]}(continuous(H,(([0,1]\times[0,1],\mathcal{O}_{standard^2}|_{[0,1]\times[0,1]}),(M,\mathcal{O}))\wedge H(0,\lambda)=\gamma(\lambda)\wedge H(1,\lambda)=\delta(\lambda))))$$
$$\#\ H \text{ is a continuous deformation of one curve into another} \quad (139)$$

$$homotopic(\sim) \implies equivalenceRelation(\sim) \quad (140)$$

$$loopSpace(\mathcal{L}_p,(p,M,\mathcal{O})) \iff \mathcal{L}_p=\{map(\gamma,([0,1],M))|continuous(\gamma)\wedge\gamma(0)=\gamma(1)\} \quad (141)$$

$$concatination(\star,(p,\gamma,\delta)) \iff (\gamma,\delta\in loopSpace(\mathcal{L}_p))\wedge$$
$$(\forall_{\lambda\in[0,1]}((\gamma\star\delta)(\lambda)=\begin{cases}\gamma(2\lambda) & 0\leq\lambda<0.5\\ \delta(2\lambda-1) & 0.5\leq\lambda\leq 1\end{cases})) \quad (142)$$

$$group((G,\bullet),()) \iff (map(\bullet,(G\times G,G)))\wedge$$
$$(\forall_{a,b\in G}(a\bullet b\in G))$$
$$(\forall_{a,b,c\in G}((a\bullet b)\bullet C=a\bullet(b\bullet c)))$$
$$(\exists_{\boldsymbol{e}}\forall_{a\in G}(\boldsymbol{e}\bullet a=a=a\bullet\boldsymbol{e}))\wedge$$
$$(\forall_{a\in G}\exists_{a^{-1}}(a\bullet a^{-1}=\boldsymbol{e}=a^{-1}\bullet a))$$
$$\#\ \text{characterizes symmetry of a set structure} \quad (143)$$

$$isomorphic(\cong,(X,\odot),(Y,\ominus))) \iff \exists_f\forall_{a,b\in X}(bijection(f,(X,Y))\wedge f(a\odot b)=f(a)\ominus f(b)) \quad (144)$$

$$fundamentalGroup((\pi_{1,p},\bullet),(p,M,\mathcal{O})) \iff (\pi_{1,p}=\mathcal{L}_p/\sim)\wedge$$
$$(map(\bullet,(\pi_{1,p}\times\pi_{1,p},\pi_{1,p})))\wedge$$
$$(\forall_{A,B\in\pi_{1,p}}([A]\bullet[B]=[A\star B]))\wedge$$
$$(group((\pi_{1,p},\bullet),()))$$
$$\#\ \text{an equivalence class of all loops induced from the homotopic equivalence relation} \quad (145)$$

$$fundamentalGroup_1\ncong fundamentalGroup_2 \implies topologicalSpace_1\ncong topologicalSpace_2 \quad (146)$$

---

there exists no known list of topological properties that can imply homeomorphisms (147)

MISSING SOME IFF SETUP CONDITIONS CHANGE QUANTIFIER LAND TO S.T. $\odot$ $\oplus$ $\otimes$ $\ominus$ $\oslash$

1.11 Lecture 6 manifolds

$$manifold((M,\mathcal{O}),()) \iff (paracompact\wedge T2separable)\wedge$$
$$(\exists_{d\in\mathbb{N}^+}\forall_{p\in M}\exists_{U\in\mathcal{O}|p\in U}\exists_{F\in\mathbb{R}^d}((U,\mathcal{O}|_U)\cong(F,\mathcal{O}_{standard^d})))\ \#\ \text{topology that is locally flat} \quad (148)$$

$$0 \quad (149)$$

2 Statistics

2.1 Overview

$$randomExperiment(X, (\Omega)) \iff \forall_{\omega \in \Omega}(outcome(\omega, (X))) \tag{150}$$

$$sampleSpace(\Omega, (X)) \iff \Omega = \{\omega | outcome(\omega, (X))\} \tag{151}$$

$$event(A, (\Omega)) \implies A \subseteq \Omega \text{ \# that is of interest} \tag{152}$$

$$eventOccured(A, (\omega, \Omega)) \iff \omega \in A, \Omega \wedge event(A, (\Omega)) \tag{153}$$

$$\begin{aligned} algebra(\mathcal{F}_0, (\Omega)) \iff & (\mathcal{F}_0 \subseteq \mathcal{P}(\Omega)) \wedge \\ & (\Omega \in \mathcal{F}_0) \wedge \\ & (\forall_{A \in \mathcal{F}_0}(A^C \in \mathcal{F}_0)) \wedge \\ & (\forall_{A,B \in \mathcal{F}_0}(A \cup B \in \mathcal{F}_0)) \\ & \text{\# but this is unable to capture some countable events} \end{aligned} \tag{154}$$

$$\begin{aligned} \sigma\text{-}algebra(\mathcal{F}, (\Omega)) \iff & (\mathcal{F}_0 \subseteq \mathcal{P}(\Omega)) \wedge \\ & (\Omega \in \mathcal{F}) \wedge \\ & (\forall_{A \in \mathcal{F}}(A^C \in \mathcal{F})) \wedge \\ & (\forall_{F \subseteq \mathcal{F}}(\neg uncountablyInfinite(F, ()) \implies \cup F \in \mathcal{F})) \end{aligned} \tag{155}$$

$$NONINDIANSHIT \tag{156}$$

$$\begin{aligned} \sigma\text{-}algebra(\sigma, (M)) \iff & (\sigma \subseteq \mathcal{P}(M)) \wedge \\ & (M \in \sigma) \wedge \\ & (\forall_{A \in \sigma}(M \setminus A \in \sigma)) \wedge \\ & ((A)_{\mathbb{N}} \subseteq \sigma \implies \cup((A)_{\mathbb{N}}) \in \sigma) \end{aligned} \tag{157}$$

$$measurableSpace((M, \sigma), ()) \iff \sigma\text{-}algebra(\sigma, (M) \tag{158}$$

$$measurableSet(A, (M, \sigma)) \iff A \in \sigma) \tag{159}$$

$$\begin{aligned} measure(\mu, (M, \sigma)) \iff & (map(\mu, (\sigma, \overline{\mathbb{R}}_0^+))) \wedge \\ & (\mu(\emptyset) = 0) \wedge \\ ((A)_{\mathbb{N}} \subseteq \sigma \wedge \forall_{i \in \mathbb{N}} \forall_{j \in \mathbb{N} \setminus \{i\}}(A_i \cap A_j = \emptyset) \implies & \mu(\cup_{i \in \mathbb{N}}(A_i)) = \sum_{i \in \mathbb{N}}(\mu(A_i))) \end{aligned} \tag{160}$$

$$measureSpace((M, \sigma, \mu), ()) \tag{161}$$

$$\begin{aligned} & measure \implies \\ & \forall_{A,B \in \sigma}(A \subseteq B \implies \mu(A) \leq \mu(B)) \\ & (A)_{\mathbb{N}} \subseteq \sigma \implies \mu \cup \leq \sum \mu \\ A_1 \subseteq A_2 ... = A \implies & \lim_{n \to \infty}(\mu(A_n)) = \mu(\cup A_n) = \mu(A) \\ & ...A_2 \subseteq A_1 = A \end{aligned} \tag{162}$$

maybe tupleify algebra like topology
generalized form

3 Statistical Learning Theory

3.1 Overview

$$ \tag{163}$$

$$curve - fitting/explaining \neq prediction \tag{164}$$

$$ill - definedproblem + solutionspaceconstraints \implies well - definedproblem \tag{165}$$

$$x \text{ \# input ; } y \text{ \# output} \tag{166}$$

$$S_n = \{(x_1, y_1), \dots, (x_n, y_n)\} \text{ \# training set} \tag{167}$$

$$f_S(x) \sim y \text{ \# solution} \tag{168}$$

$$each(x, y) \in p(x, y) \text{ \# training data } x, y \text{ is a sample from an unknown distribution } p \tag{169}$$

$$V(f(x), y) = d(f(x), y) \text{ \# loss function} \tag{170}$$

$$I[f] = \int_{X \times Y} V(f(x), y)p(x, y)dxdy \text{ \# expected error} \tag{171}$$

$$I_n[f] = \frac{1}{n} \sum_{i=1}^{n} V(f(x_i), y_i) \text{ \# empirical error} \tag{172}$$

$$probabilisticConvergence(X, ()) \iff \forall_{\epsilon > 0} \lim_{n \to \infty} Pxn - x \leq \epsilon = 0 \tag{173}$$

$$I - Ingeneralizationerror \tag{174}$$

$$well - posed := exists, unique, stable; elseill - posed \tag{175}$$

3.2 Background maths

$$\begin{aligned} vectorSpace(V, (+, *)) \iff (u, v, w \in V), (c, d \in \mathbb{R} \in F) \wedge \\ (u + v, c * u = c(u) = cu \in V) \wedge \\ (u + v = v + u) \wedge \\ ((u + v) + w = u + (v + w)) \wedge \\ (\exists_{\boldsymbol{0}}(u + \boldsymbol{0} = u)) \wedge \\ (\exists_{-u}(u + (-u) = \boldsymbol{0})) \wedge \\ ((1)u = u) \\ ((cd)u = c(du)) \wedge \\ ((c + d)u = cu + du) \wedge \text{ \# linearity} \end{aligned}$$

$$(c(u+v) = cu + cv) \wedge \ \# \text{ linearity}$$
$$\# \text{ behaves similar to vectors} \tag{176}$$

$$innerProduct(\langle \cdot, \cdot \rangle, (V)) \iff (u, v, w \in V), (c \in \mathbb{R} \in F) \wedge$$
$$(\langle v, w \rangle = \langle w, v \rangle) \wedge$$
$$(\langle cv, w \rangle = c\langle v, w \rangle) \wedge$$
$$(\langle u + v, w \rangle = \langle u, w \rangle + \langle v, w \rangle) \wedge \ \# \text{ linearity}$$
$$(\langle u, u \rangle \geq 0 \in \mathbb{R}_0^+) \wedge \ \# \text{ metric inducing}$$
$$(\langle u, u \rangle = 0 \iff u = \boldsymbol{0}) \tag{177}$$

$$innerProductNorm(|| \cdot ||, (V)) \iff (v, w \in V), (r \in R) \wedge$$
$$(||v|| = \sqrt{\langle v, v \rangle} \in \mathbb{R}_0^+) \wedge$$
$$(||v|| = 0 \iff v = \boldsymbol{0}) \wedge$$
$$(||rv|| = |r|||v||) \wedge$$
$$(||v + w|| \leq ||v|| + ||w||) \ \# \text{ triangle inequality} \tag{178}$$

$$normConvergences(v, (V, (v_n)_{n \in \mathbb{N}})) \iff (\{v\} \cup (v_n)_{n \in \mathbb{N}} \subseteq V) \wedge$$
$$(\lim_{n \to \infty} ||v - v_n|| = 0) \tag{179}$$

$$cauchySequence((v_n)_{n \in \mathbb{N}}, (V)) \iff$$
$$(\forall_{\epsilon > 0} \exists_{n \in \mathbb{N}} \forall_{x, y > n} (||v_x - v_y|| < \epsilon)) \tag{180}$$

$$normConvergences \implies cauchySequence \ \# \text{ there might be holes in the space} \tag{181}$$

$$completeSpace(V, (innerProductNorm)) \iff (cauchySequence \iff normConvergences) \tag{182}$$

$$completion(R, (Q)) \iff R = QUcauchyUs = Qbar \tag{183}$$

$$hilbertSpace(H, (+, *, \langle \cdot, \cdot \rangle)) \iff (vectorSpace(H, (+, *))) \wedge$$
$$(innerProduct(\langle \cdot, \cdot \rangle, (H))) \wedge$$
$$completeSpace(H, (innerProductNorm)) \tag{184}$$

$$separable(H, ()) \iff \exists_{S \subseteq V}(countable(S, ()) \wedge Sbar = V) \ \# \text{ has a countable basis} \tag{185}$$

$$hilbertSpace \wedge seperable \iff \exists countableortho(gonal)normalbasisforspace, allnorm = 1, IP = 0 \tag{186}$$

$$x = \sum < x, v > v \ \# \text{ countable projection times v} \tag{187}$$

$$0000000000 \tag{188}$$

$$linearOperator(L, (V)) \iff (u, v \in V), (c, d \in \mathbb{R}) \wedge$$
$$(L(cu + dv) = cL(u) + dL(v)) \tag{189}$$

$$adjoint(L^\dagger, (L, V)) \iff (\forall_{u, v \in V} < L(u), v >=< u, L^\dagger(v) >_\dagger) \tag{190}$$

$$selfAdjoint(L, ()) \iff L = L^\dagger \tag{191}$$

$$eigenvector(V) \iff Lv = kv \tag{192}$$

$$30mins \tag{193}$$

4 Machine Learning

4.0.1 Overview

$$X \text{ \# input ; } Y \text{ \# output ; } S(X, Y) \text{ \# dataset} \tag{194}$$

$$\textbf{learned parameters} = \textbf{parameters to be fixed by training with the dataset} \tag{195}$$

$$\textbf{hyperparameters} = \textbf{parameters that depends on a dataset} \tag{196}$$

$$\begin{gathered}\textbf{validation} = \textbf{partitions dataset into training and testing partitions, then evaluates the} \\ \textbf{accuracy of the parameters learned from the training partition in predicting the} \\ \textbf{outputs of the testing partition} \text{ \# useful for fixing hyperparameters}\end{gathered} \tag{197}$$

$$\textbf{cross-validation} = \textbf{average accuracy of validation for different choices of testing partition} \tag{198}$$

$$\textbf{L1} = \textbf{scales linearly} \text{ ; } \textbf{L2} = \textbf{scales quadratically} \tag{199}$$

$$d = \textbf{distance} = \textbf{quantifies the the similarity between data points} \tag{200}$$

$$d_{L1}(A, B) = \sum_p |A_p - B_p| \text{ \# Manhattan distance} \tag{201}$$

$$d_{L2}(A, B) = \sqrt{\sum_p (A_p - B_p)^2} \text{ \# Euclidean distance} \tag{202}$$

$$\textbf{kNN classifier} = \textbf{classifier based on } k \textbf{ nearest data points} \tag{203}$$

$$s = \textbf{class score} = \textbf{quantifies bias towards a particular class} \tag{204}$$

$$s_{linear} = f_{c\times 1}(x_{n\times 1}, W_{c\times n}, b_{c\times 1}) = W_{c\times n}x_{n\times 1} + b_{c\times 1} \text{ \# linear score function} \tag{205}$$

$$l = \textbf{loss} = \textbf{quantifies the errors by the learned parameters} \tag{206}$$

$$l = \frac{1}{|c_i|}\sum_{c_i} l_i \text{ \# average loss for all classes} \tag{207}$$

$$l_{SVM_i} = \sum_{y_i \neq c_i} \max(0, s_{y_i} - s_{c_i} + 1) \text{ \# SVM hinge class loss function:}$$

$$\text{\# ignores incorrect classes with lower scores including a non-zero margin} \tag{208}$$

$$l_{MLR_i} = -\log\left(\frac{e^{s_{c_i}}}{\sum_{y_i} e^{y_i}}\right) \text{ \# Softmax class loss function}$$
$$\text{\# lower scores correspond to lower exponentiated-normalized probabilities} \tag{209}$$

$$R = \textbf{regularization} = \textbf{optimizes the choice of learned parameters to minimize test error} \tag{210}$$

$$\lambda \text{ \# regularization strength hyperparameter} \tag{211}$$

$$R_{L1}(W) = \sum_{W_i} |W_i| \text{ \# L1 regularization} \tag{212}$$

$$R_{L2}(W) = \sum_{W_i} {W_i}^2 \text{ \# L2 regularization} \tag{213}$$

$$L' = L + \lambda R(W) \text{ \# weight regularization} \tag{214}$$

$$\nabla_W L = \overrightarrow{\frac{\partial}{\partial W_i}} L = \textbf{loss gradient w.r.t. weights} \tag{215}$$

$$\frac{\partial L_E}{\partial W_I} = \frac{\partial L_L}{\partial W_I}\frac{\partial L_E}{\partial L_L} \text{ \# loss gradient w.r.t. input weight in terms of external and local gradients} \tag{216}$$

$$s = \textbf{forward API} \text{ ; } \frac{\partial L_L}{\partial W_I} = \textbf{backward API} \tag{217}$$

$$W_{t+1} = W_t - \nabla_{W_t} L \text{ \# weight update loss minimization} \tag{218}$$

$$\textbf{TODO:Research on Activation functions, Weight Initialization, Batch Normalization} \tag{219}$$

$$review5meanvardiscussion/hyperparameteroptimization/babysittinglearning \tag{220}$$

TODO loss L or l ??