Next-Next-Gen Notes

Object-Oriented Maths

JP Guzman

February 8, 2018

1 Mathematical Logic

1.1 NaiveMaster

undefined terms: $set, tuple, element, nnumber, \in, \subseteq, =, \not, \subset, \cup, \cap, \emptyset, \{, \}, \langle, \rangle, |, \frown, \times, relation, property, binaryRelation,$ $domain, range, field,$

$$\tag{1}$$

$$\begin{gathered} element[x][] \in set[y][] \\ \text{\# x belongs to y} \end{gathered} \tag{2}$$

$$\begin{gathered} set[x][] \subseteq set[y][] \\ \text{\# x is included in y} \end{gathered} \tag{3}$$

$$\begin{gathered} set[x][] = set[y][] := (set[x][] \subseteq set[y][], set[y][] \subseteq set[x][]) \\ \text{\# x is the same set as y} \end{gathered} \tag{4}$$

$$\begin{gathered} set[x][] \subset set[y][]((= x \nsubseteq y)) := set[x][] \subseteq set[y][], set[x][] \neq set[y][] \\ \text{\# x is a proper subset of y} \end{gathered} \tag{5}$$

$$\begin{gathered} set[x][] \cup set[y][] \\ \text{\# all elements in x or y} \end{gathered} \tag{6}$$

$$\begin{gathered} set[x][] \cap set[y][] \\ \text{\# all elements in x and y} \end{gathered} \tag{7}$$

$$\begin{gathered} disjoint[x, y][] := set[x][] \cap set[y][] = \emptyset \\ \text{\# disjoint sets do not intersect} \end{gathered} \tag{8}$$

$$\begin{gathered} set[E][] = \{e_1, e_2, e_3, \cdots, e_n\} \\ \text{\# unordered set containing } e_1, e_2, e_3, \cdots, e_n \\ \{e_1, e_2, e_3\} = \{e_3, e_1, e_2\} \end{gathered} \tag{9}$$

$$\begin{gathered} tuple[E][] = \langle e_1, e_2, e_3, \cdots, e_n \rangle \\ \text{\# ordered tuple containing } e_1, e_2, e_3, \cdots, e_n \\ \langle e_1, e_2, e_3 \rangle \neq \langle e_2, e_3, e_1 \rangle \end{gathered} \tag{10}$$

$$\begin{gathered} set[X][] \frown nnumber[k][] \\ \text{\# set of all ordered k-tuples from the elements in X} \end{gathered}$$

$$X^\frown 1 = \{e_1, e_2, e_3, \cdots, e_n\}^\frown 1 = \{\langle e_1\rangle, \langle e_2\rangle, \langle e_3\rangle, \cdots, \langle e_n\rangle\} = \{e_1, e_2, e_3, \cdots, e_n\} = X \tag{11}$$

$$set[Y][] \times set[Z][]$$
$$\text{\# Cartesian product} \tag{12}$$

$$relation[R][S,k] \subseteq set[S][]^\frown nnumber[k][]$$
$$\text{\# k-tuple relation R on the set S takes only tuples that satisfy some relation} \tag{13}$$

$$property[P][S] = relation[P][S,1] \subseteq set[S][]^\frown 1 = S$$
$$\text{\# property P of the set S} \tag{14}$$

$$binaryRelation[B][S] = relation[B][S,2] \subseteq set[S][]^\frown 2$$
$$xBy = \langle x,y\rangle \in B \tag{15}$$

$$domain[X][B,S] = \{x \mid \langle x,y\rangle \in binaryRelation[B][S]\} \tag{16}$$

$$range[Y][B,S] = \{y \mid \langle x,y\rangle \in binaryRelation[B][S]\} \tag{17}$$

$$field[F][B,S] = domain[X][B,S] \cup range[Y][B,S] \tag{18}$$

$$inverseRelation[B^{-1}][B,S] := \{\langle y,x\rangle \mid \langle x,y\rangle \in binaryRelation[B][S]\} \tag{19}$$

$$reflexive[B][S] := x \in field[F][B,S], xBx define for all not in set, quantifiers, if and or etc. \tag{20}$$

$$symmetric[B][S] := \tag{21}$$

$$transitive[B][S] := \tag{22}$$

$$0000000000000000000000000000000000000000000000000000000000000000000000000000000$$
$$\langle a,b\rangle \in inverse(R^{-1}) : \langle b,a\rangle \in R$$
$$reflexive(R_X^2) : xR_X^2x$$
$$symmetric(R_X^2) : xR_X^2y = yR_X^2x$$
$$transitive(R_X^2) : xR_X^2y, yR_X^2z : xR_X^2z$$
$$equivalenceRelation(R_X^2) := reflexive(R_X^2), symmetric(R_X^2), transitive(R_X^2) \tag{23}$$

$$takethisintroshitmoresrsly \tag{24}$$

2 Logic and Set Theory

2.1 Logical Truths and Operators

$$\tag{25}$$

$$truth[t][] := t = \begin{cases} T \\ F \end{cases} \tag{26}$$

$$statement[s][] :=^{!} correctSyntaxSemantics[s][] \tag{27}$$

$$proposition[s,t][] := \big(statement[s][]\big), \big(truth[t][]\big). \tag{28}$$

$$operatorOR[\vee][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \vee y][] = \begin{cases} F & x=F, y=F \\ T & x=F, y=T \\ T & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{29}$$

$$operatorAND[\wedge][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \wedge y][] = \begin{cases} F & x=F, y=F \\ F & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{30}$$

$$operatorNOT[\neg][x] := \big(truth[x][]\big), \left(truth[\neg x][] = \begin{cases} T & x=F \\ F & x=T \end{cases} \right). \tag{31}$$

$$\begin{aligned} booleanAlgebra[\{T,F\}, \wedge, \vee, \neg][] :=^{POS-LCom} &\big((x \wedge y = y \wedge x), (x \vee y = y \vee x)\big) \ \#\ \text{Commutative}, \\ {}^{POS-LDis}\Big(\big(x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)\big), \big(x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)\big)\Big) &\ \#\ \text{Distributive}, \\ {}^{POS-LIdn}\big((x \wedge T = x), (x \vee F = x)\big) &\ \#\ \text{Identity}, \\ {}^{POS-LCmp}\big((x \wedge \neg x = F), (x \vee \neg x = T)\big) &\ \#\ \text{Complement}. \end{aligned} \tag{32}$$

$$operatorXOR[\veebar][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \veebar y][] = \begin{cases} F & x=F, y=F \\ T & x=F, y=T \\ T & x=T, y=F \\ F & x=T, y=T \end{cases} \right). \tag{33}$$

$$operatorIF[\Longrightarrow][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \Longrightarrow y][] = (\neg x) \vee y = \begin{cases} T & x=F, y=F \\ T & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{34}$$

$$\begin{aligned} &{}^{THM-LExp-1}_{POS-LCmp}(F = x \wedge \neg x) \Longrightarrow \\ &{}^{THM-LExp-2}_{THM-LExp-1}(x), \\ &{}^{THM-LExp-3}_{THM-LExp-1}(\neg x), \\ &{}^{THM-LExp-4}_{THM-LExp-2}(x \vee y), \\ &{}^{THM-LExp-5}_{\substack{THM-LExp-4 \\ THM-LExp-3}}(y). \\ &{}^{THM-LExp}_{\substack{THM-LExp-1 \\ THM-LExp-2 \\ THM-LExp-3 \\ THM-LExp-4 \\ THM-LExp-5}}(F \Longrightarrow y) \\ &\#\ \text{The Principle of Explosion, anything follows from a false } (F) \text{ premise} \end{aligned} \tag{35}$$

$$operatorOIF[\Longleftarrow][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \Longleftarrow y][] = (\neg y) \vee x = \begin{cases} T & x=F, y=F \\ F & x=F, y=T \\ T & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{36}$$

$$operatorIIF[\Longleftrightarrow][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \Longleftrightarrow y][] = (x \Longrightarrow y) \wedge (y \Longrightarrow x) = \begin{cases} T & x=F, y=F \\ F & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{37}$$