Next-Next-Gen Notes

Object-Oriented Maths

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Format: $characteristic((subjects),(dependencies)) \iff (conditions(dependencies)) \wedge (conditions(subjects))$
Note: All weaker objects automatically induces notions inherited from stronger objects.
TODO assign free variables as parameters
TODO define || abs cross-product and other missing refs
TODO distinguish new condition vs implied proposition - separate propositions into new line thms
TODO silent link expressions! - e.g. $backslashsilentPLPL_X$

1 Logic and Set Theory

1.1 Logic SHENANNIGANS

$$truth[t][] := t = \begin{cases} T \\ F \end{cases} \tag{1}$$

$$statement[s][] := correctSyntaxSemantics[s][] \tag{2}$$

$$proposition[s,t][] := (statement[s][]),(truth[t][]) \tag{3}$$

$$operatorNOT[\neg][x] := (truth[x][]),(truth[\neg x][] = \begin{cases} T & x = F \\ F & x = T \end{cases}) \tag{4}$$

$$operatorAND[\wedge][x,y] := (truth[x][]),(truth[y][]),(truth[x \wedge y][] = \begin{cases} F & x = F, y = F \\ F & x = F, y = T \\ F & x = T, y = F \\ T & x = T, y = T \end{cases}) \tag{5}$$

$$operatorOR[\vee][x,y] := (truth[x][]),(truth[y][]),(truth[x \vee y][] = \begin{cases} F & x = F, y = F \\ T & x = F, y = T \\ T & x = T, y = F \\ T & x = T, y = T \end{cases}) \tag{6}$$

$$operatorXOR[\veebar][x,y] := (truth[x][]),(truth[y][]),(truth[x \veebar y][] = \begin{cases} F & x = F, y = F \\ T & x = F, y = T \\ T & x = T, y = F \\ F & x = T, y = T \end{cases}) \tag{7}$$

$$^{POS-LIdn}(x \wedge T = x),(x \vee F = x)$$
$$\#\ \text{Identity} \tag{8}$$

$$^{POS-LCmp}(x\wedge\neg x=F),(x\vee\neg x=T)$$
$$\#\ \text{Complement} \tag{9}$$

$$^{POS-LCom}(x\wedge y=y\wedge x),(x\vee y=y\vee x)$$
$$\#\ \text{Commutative} \tag{10}$$

$$^{POS-LDis}(x\wedge(y\vee z)=(x\wedge y)\vee(x\wedge z)),(x\vee(y\wedge z)=(x\vee y)\wedge(x\vee z))$$
$$\#\ \text{Distributive} \tag{11}$$

$$operatorIF[\Longrightarrow][x,y]:=(truth[x][]),(truth[y][]),(truth[x\Longrightarrow y][]=\begin{cases}T & x=F,y=F\\ T & x=F,y=T\\ F & x=T,y=F\\ T & x=T,y=T\end{cases}) \tag{12}$$

$$^{THM-LExp-1}_{POS-LCmp}(F=x\wedge\neg x)\Longrightarrow$$
$$^{THM-LExp-2}_{THM-LExp-1}(x),$$
$$^{THM-LExp-3}_{THM-LExp-1}(\neg x),$$
$$^{THM-LExp-4}_{THM-LExp-2}(x\vee y),$$
$$^{THM-LExp-5}_{\substack{THM-LExp-4\\THM-LExp-3}}(y).$$
$$^{THM-LExp}_{\substack{THM-LExp-1\\THM-LExp-2\\THM-LExp-3\\THM-LExp-4\\THM-LExp-5}}(F\Longrightarrow y)$$
$$\#\ \text{The Principle of Explosion, anything follows from a false } (F) \text{ premise} \tag{13}$$

$$operatorOIF[\Longleftarrow][x,y]:=(truth[x][]),(truth[y][]),(truth[x\Longleftarrow y][]=\begin{cases}T & x=F,y=F\\ F & x=F,y=T\\ T & x=T,y=F\\ T & x=T,y=T\end{cases}) \tag{14}$$

$$operatorIIF[\Longleftrightarrow][x,y]:=(truth[x][]),(truth[y][]),(truth[x\Longleftrightarrow y][]=\begin{cases}T & x=F,y=F\\ F & x=F,y=T\\ F & x=T,y=F\\ T & x=T,y=T\end{cases}) \tag{15}$$

$$^{THM-LUNt-1}((x\vee y=T=x\vee z)\wedge(x\wedge y=F=x\wedge z))\Longrightarrow$$
$$^{THM-LUNt-2}_{POS-LIdn}(y=y\wedge T),$$
$$^{THM-LUNt-3}_{THM-LUNt-1}(y\wedge T=y\wedge(x\vee z)),$$
$$^{THM-LUNt-4}_{POS-LDis}(y\wedge(x\vee z)=(y\wedge x)\vee(y\wedge z)),$$
$$^{THM-LUNt-5}_{\substack{POS-LCom\\THM-LUNt-4}}((y\wedge x)\vee(y\wedge z)=(x\wedge z)\vee(y\wedge z)),$$
$$^{THM-LUNt-6}_{\substack{POS-LCom\\POS-LDis}}((x\wedge z)\vee(y\wedge z)=z\wedge(x\vee y)),$$
$$^{THM-LUNt-7}_{THM-LUNt-1}(z\wedge(x\vee y)=z\wedge T),$$
$$^{THM-LUNt-8}_{POS-LIdn}(z\wedge T=z).$$
$$(((x\vee y=T=x\vee z)\wedge(x\wedge y=F=x\wedge z))\Longrightarrow(y=z))$$
$$\#\ \text{Uniqueness of Complements} \tag{16}$$

$$ {}^{THM-Dual}(\textbf{swapping AND/OR and T/F is valid}) \;\#\; \text{TODO} \tag{17}$$

$$\begin{gathered}
{}^{THM-LDom-1}_{POS-LIdn}(x \vee T = (x \vee T) \wedge T), \\
{}^{THM-LDom-2}_{POS-LCmp}((x \vee T) \wedge T = (x \vee T) \wedge (x \vee \neg x)), \\
{}^{THM-LDom-3}_{POS-LDis}((x \vee T) \wedge (x \vee \neg x) = x \vee (T \wedge \neg x)), \\
{}^{THM-LDom-4}_{POS-LIdn}(x \vee (T \wedge \neg x) = x \vee \neg x), \\
{}^{THM-LDom-5}_{POS-LCmp}(x \vee \neg x = T). \\
{}^{THM-LDom-6}_{\substack{THM-LDom-1\\THM-LDom-2\\THM-LDom-3\\THM-LDom-4\\THM-LDom-5}}(x \vee T = T), \\
{}^{THM-LDom}_{\substack{THM-LDom-6\\THM-Dual}}(x \vee T = T), (x \wedge F = F). \\
\#\ \text{Domination}
\end{gathered} \tag{18}$$

$$\begin{gathered}
{}^{THM-LIdm-1}_{POS-LIdn}(x \vee x = (x \vee x) \wedge T), \\
{}^{THM-LIdm-2}_{POS-LCmp}((x \vee x) \wedge T = (x \vee x) \wedge (x \vee \neg x)), \\
{}^{THM-LIdm-3}_{POS-LDis}((x \vee x) \wedge (x \vee \neg x) = x \wedge (x \vee \neg x)), \\
{}^{THM-LIdm-4}_{POS-LCmp}(x \wedge (x \vee \neg x) = x \wedge T), \\
{}^{THM-LIdm-5}_{POS-LIdn}(x \wedge T = x), \\
{}^{THM-LIdm-6}_{\substack{THM-LIdm-1\\THM-LIdm-2\\THM-LIdm-3\\THM-LIdm-4\\THM-LIdm-5}}(x \vee x = x), \\
{}^{THM-LIdm}_{\substack{THM-LIdm-6\\THM-Dual}}(x \vee x = x), (x \wedge x = x). \\
\#\ \text{Idempotent}
\end{gathered} \tag{19}$$

$$\begin{gathered}
{}^{THM-LInv-1}_{POS-LIdn}(\neg\neg x = \neg\neg x \vee F), \\
{}^{THM-LInv-2}_{POS-LCmp}(\neg\neg x \vee F = \neg\neg x \vee (x \wedge \neg x)), \\
{}^{THM-LInv-3}_{POS-LDis}(\neg\neg x \vee (x \wedge \neg x) = (\neg\neg x \vee x) \wedge (\neg\neg x \vee \neg x)), \\
{}^{THM-LInv-4}_{POS-LCmp}((\neg\neg x \vee x) \wedge (\neg\neg x \vee \neg x) = (\neg\neg x \vee x) \wedge T), \\
{}^{THM-LInv-5}_{POS-LCmp}((\neg\neg x \vee x) \wedge T = (\neg\neg x \vee x) \wedge (x \vee \neg x)), \\
{}^{THM-LInv-6}_{POS-LDis}((\neg\neg x \vee x) \wedge (x \vee \neg x) = x \vee (\neg\neg x \wedge \neg x)), \\
{}^{THM-LInv-7}_{POS-LCmp}(x \vee (\neg\neg x \wedge \neg x) = x \vee F), \\
{}^{THM-LInv-8}_{POS-LIdn}(x \vee F = x), \\
{}^{THM-LInv}_{\substack{THM-LInv-1\\THM-LInv-2\\THM-LInv-3\\THM-LInv-4\\THM-LInv-5\\THM-LInv-6\\THM-LInv-7\\THM-LInv-8}}(\neg\neg x = x). \\
\#\ \text{Involution}
\end{gathered} \tag{20}$$

$$0 \tag{21}$$

1.2 Predicate shennanigans

$$0 \tag{22}$$

$$0 \tag{23}$$

2 Glossary