Next-Next-Gen Notes

Object-Oriented Maths

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January 13, 2018

Format: $characteristic\big((subjects),(dependencies)\big) \iff \big(conditions(dependencies)\big) \wedge \big(conditions(subjects)\big)$
Note: All weaker objects automatically induces notions inherited from stronger objects.
TODO assign free variables as parameters
TODO define || abs cross-product and other missing refs
TODO distinguish new condition vs implied proposition - separate propositions into new line thms
TODO silent link expressions! - e.g. $backslashsilentPLPL_X$

1 Logic and Set Theory

1.1 Logical Truths and Operators

$$truth[t][] := t = \begin{cases} T \\ F \end{cases} \tag{1}$$

$$statement[s][] := correctSyntaxSemantics[s][] \tag{2}$$

$$proposition[s,t][] := \big(statement[s][]\big), \big(truth[t][]\big). \tag{3}$$

$$operatorOR[\vee][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \vee y][] = \begin{cases} F & x=F, y=F \\ T & x=F, y=T \\ T & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{4}$$

$$operatorAND[\wedge][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \wedge y][] = \begin{cases} F & x=F, y=F \\ F & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{5}$$

$$operatorNOT[\neg][x] := \big(truth[x][]\big), \left(truth[\neg x][] = \begin{cases} T & x=F \\ F & x=T \end{cases} \right). \tag{6}$$

$$\begin{aligned} booleanAlgebra[\{T,F\},\wedge,\vee,\neg][] := {}^{POS-LCom}\big((x \wedge y = y \wedge x), (x \vee y = y \vee x)\big) \ \# \text{ Commutative,} \\ {}^{POS-LDis}\Big(\big(x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)\big), \big(x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)\big)\Big) \ \# \text{ Distributive,} \\ {}^{POS-LIdn}\big((x \wedge T = x), (x \vee F = x)\big) \ \# \text{ Identity,} \\ {}^{POS-LCmp}\big((x \wedge \neg x = F), (x \vee \neg x = T)\big) \ \# \text{ Complement.} \end{aligned} \tag{7}$$

$$\mathit{operatorXOR}[\veebar][x,y] := \big(\mathit{truth}[x][]\big), \big(\mathit{truth}[y][]\big), \left(\mathit{truth}[x \veebar y][] = \begin{cases} F & x=F, y=F \\ T & x=F, y=T \\ T & x=T, y=F \\ F & x=T, y=T \end{cases} \right). \tag{8}$$

$$\mathit{operatorIF}[\implies][x,y] := \big(\mathit{truth}[x][]\big), \big(\mathit{truth}[y][]\big), \left(\mathit{truth}[x \implies y][] = (\neg x) \vee y = \begin{cases} T & x=F, y=F \\ T & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{9}$$

$$\begin{aligned} & {}^{THM-LExp-1}_{POS-LCmp}(F = x \wedge \neg x) \implies \\ & {}^{THM-LExp-2}_{THM-LExp-1}(x), \\ & {}^{THM-LExp-3}_{THM-LExp-1}(\neg x), \\ & {}^{THM-LExp-4}_{THM-LExp-2}(x \vee y), \\ & {}^{THM-LExp-5}_{\substack{THM-LExp-4 \\ THM-LExp-3}}(y). \\ & {}^{THM-LExp}_{\substack{THM-LExp-1 \\ THM-LExp-2 \\ THM-LExp-3 \\ THM-LExp-4 \\ THM-LExp-5}}(F \implies y) \\ & \text{\# The Principle of Explosion, anything follows from a false } (F) \text{ premise} \end{aligned} \tag{10}$$

$$\mathit{operatorOIF}[\impliedby][x,y] := \big(\mathit{truth}[x][]\big), \big(\mathit{truth}[y][]\big), \left(\mathit{truth}[x \impliedby y][] = \begin{cases} T & x=F, y=F \\ F & x=F, y=T \\ T & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{11}$$

$$\mathit{operatorIIF}[\iff][x,y] := \big(\mathit{truth}[x][]\big), \big(\mathit{truth}[y][]\big), \left(\mathit{truth}[x \iff y][] = \begin{cases} T & x=F, y=F \\ F & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{12}$$

1.2 Boolean Algebra Properties

$$\begin{aligned} & {}^{THM-Dual-1}_{POS-LCom}\Bigg(\mathit{booleanAlgebra}[\{T,F\}, \wedge, \vee, \neg][] \iff \\ & \big((x \vee y = y \vee x), (x \wedge y = y \wedge x)\big) \text{ \# Reordered Commutative,} \\ & \Big(\big(x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)\big), \big(x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)\big)\Big) \text{ \# Reordered Distributive,} \\ & \big((x \vee F = x), (x \wedge T = x)\big) \text{ \# Reordered Identity,} \\ & \big((x \vee \neg x = T), (x \wedge \neg x = F)\big) \text{ \# Reordered Complement.} \iff \\ & \mathit{booleanAlgebra}[\{F,T\}, \vee, \wedge, \neg][] \Bigg) \\ & {}^{THM-Dual}_{THM-Dual-1}\big(\mathit{booleanAlgebra}[\{T,F\}, \wedge, \vee, \neg][] \iff \mathit{booleanAlgebra}[\{F,T\}, \vee, \wedge, \neg][]\big) \\ & \text{\# Boolean Algebra Duality follows from the swap symmetry of } (\wedge, T) \text{ and } (\vee, F) \text{ within the axioms} \end{aligned} \tag{13}$$

$$\substack{THM-LUNt-1}\big((x\vee y=T=x\vee z)\wedge(x\wedge y=F=x\wedge z)\big)\Longrightarrow$$

$$\substack{THM-LUNt-2\\ POS-LIdn}\big(y=y\wedge T\big),$$

$$\substack{THM-LUNt-3\\ THM-LUNt-1}\big(y\wedge T=y\wedge(x\vee z)\big),$$

$$\substack{THM-LUNt-4\\ POS-LDis}\big(y\wedge(x\vee z)=(y\wedge x)\vee(y\wedge z)\big),$$

$$\substack{THM-LUNt-5\\ POS-LCom\\ THM-LUNt-4}\big((y\wedge x)\vee(y\wedge z)=(x\wedge z)\vee(y\wedge z)\big),$$

$$\substack{THM-LUNt-6\\ POS-LCom\\ POS-LDis}\big((x\wedge z)\vee(y\wedge z)=z\wedge(x\vee y)\big),$$

$$\substack{THM-LUNt-7\\ THM-LUNt-1}\big(z\wedge(x\vee y)=z\wedge T\big),$$

$$\substack{THM-LUNt-8\\ POS-LIdn}\big(z\wedge T=z\big).$$

$$\substack{THM-LUNt\\ THM-LUNt-1\\ THM-LUNt-2\\ THM-LUNt-3\\ THM-LUNt-4\\ THM-LUNt-5\\ THM-LUNt-6\\ THM-LUNt-7\\ THM-LUNt-8}\Big(\big((x\vee y=T=x\vee z)\wedge(x\wedge y=F=x\wedge z)\big)\Longrightarrow(y=z)\Big)$$

\# Uniqueness of Complements (14)

$$\substack{THM-LDom-1\\ POS-LIdn}\big(x\vee T=(x\vee T)\wedge T\big),$$

$$\substack{THM-LDom-2\\ POS-LCmp}\big((x\vee T)\wedge T=(x\vee T)\wedge(x\vee\neg x)\big),$$

$$\substack{THM-LDom-3\\ POS-LDis}\big((x\vee T)\wedge(x\vee\neg x)=x\vee(T\wedge\neg x)\big),$$

$$\substack{THM-LDom-4\\ POS-LIdn}\big(x\vee(T\wedge\neg x)=x\vee\neg x\big),$$

$$\substack{THM-LDom-5\\ POS-LCmp}\big(x\vee\neg x=T\big).$$

$$\substack{THM-LDom-6\\ THM-LDom-1\\ THM-LDom-2\\ THM-LDom-3\\ THM-LDom-4\\ THM-LDom-5}\big(x\vee T=T\big),$$

$$\substack{THM-LDom\\ THM-LDom-6\\ THM-Dual}\big((x\vee T=T),(x\wedge F=F)\big).$$

\# Domination (15)

$$\substack{THM-LIdm-1\\ POS-LIdn}\big(x\vee x=(x\vee x)\wedge T\big),$$

$$\substack{THM-LIdm-2\\ POS-LCmp}\big((x\vee x)\wedge T=(x\vee x)\wedge(x\vee\neg x)\big),$$

$$\substack{THM-LIdm-3\\ POS-LDis}\big((x\vee x)\wedge(x\vee\neg x)=x\wedge(x\vee\neg x)\big),$$

$$\substack{THM-LIdm-4\\ POS-LCmp}\big(x\wedge(x\vee\neg x)=x\wedge T\big),$$

$$\substack{THM-LIdm-5\\ POS-LIdn}\big(x\wedge T=x\big),$$

$$\substack{THM-LIdm-6\\ THM-LIdm-1\\ THM-LIdm-2\\ THM-LIdm-3\\ THM-LIdm-4\\ THM-LIdm-5}\big(x\vee x=x\big),$$

$$\substack{THM-LIdm\\ THM-LIdm-6\\ THM-Dual}\big((x\vee x=x),(x\wedge x=x)\big).$$

\# Idempotent (16)

$$\substack{THM-LInv-1\\ POS-LIdn}\big(\neg\neg x=\neg\neg x\vee F\big),$$

$$\substack{THM-LInv-2\\ POS-LCmp}\big(\neg\neg x\vee F=\neg\neg x\vee(x\wedge\neg x)\big),$$

$$\substack{THM-LInv-3\\ POS-LDis}\big(\neg\neg x\vee(x\wedge\neg x)=(\neg\neg x\vee x)\wedge(\neg\neg x\vee\neg x)\big),$$

$$\substack{THM-LInv-4\\ POS-LCmp}\big((\neg\neg x\vee x)\wedge(\neg\neg x\vee\neg x)=(\neg\neg x\vee x)\wedge T\big),$$

$$\substack{THM-LInv-5\\ POS-LCmp}\big((\neg\neg x\vee x)\wedge T=(\neg\neg x\vee x)\wedge(x\vee\neg x)\big),$$

$$\substack{THM-LInv-6\\ POS-LDis}\big((\neg\neg x\vee x)\wedge(x\vee\neg x)=x\vee(\neg\neg x\wedge\neg x)\big),$$

$$\overset{THM-LInv-7}{\underset{POS-LCmp}{}}(x \lor (\neg\neg x \land \neg x) = x \lor F),$$
$$\overset{THM-LInv-8}{\underset{POS-LIdn}{}}(x \lor F = x),$$
$$\overset{THM-LInv}{\underset{\substack{THM-LInv-1\\THM-LInv-2\\THM-LInv-3\\THM-LInv-4\\THM-LInv-5\\THM-LInv-6\\THM-LInv-7\\THM-LInv-8}}{}}(\neg\neg x = x).$$
$$\#\ \text{Involution} \tag{17}$$

$$\overset{THM-LAbs-1}{\underset{POS-LIdn}{}}\big(x \lor (x \land y) = (x \land T) \lor (x \land y)\big),$$
$$\overset{THM-LAbs-2}{\underset{POS-LDis}{}}\big((x \land T) \lor (x \land y) = x \land (T \lor y)\big),$$
$$\overset{THM-LAbs-3}{\underset{THM-LDom}{}}\big(x \land (T \lor y) = x \land T\big),$$
$$\overset{THM-LAbs-4}{\underset{THM-LIdn}{}}(x \land T = x),$$
$$\overset{THM-LAbs-5}{\underset{\substack{THM-LAbs-1\\THM-LAbs-2\\THM-LAbs-3\\THM-LAbs-4}}{}}(x \lor (x \land y) = x),$$
$$\overset{THM-LAbs}{\underset{\substack{THM-LAbs-5\\THM-Dual}}{}}\Big((x \lor (x \land y) = x), (x \land (x \lor y) = x)\Big).$$
$$\#\ \text{Absorption} \tag{18}$$

$$\overset{THM-LAsc-1}{}((A = x \lor (y \lor z)), (B = (x \lor y) \lor z)) \Longrightarrow$$
$$\overset{THM-LAsc-2}{\underset{THM-LAsc-1}{}}(x \land A = x \land (x \lor (y \lor z))),,$$
$$\overset{THM-LAsc-3}{\underset{THM-LAbs}{}}(x \land (x \lor (y \lor z)) = x),,$$
$$\overset{THM-LAsc-4}{\underset{THM-LAsc-1}{}}(x \land B = x \land ((x \lor y) \lor z)),,$$
$$\overset{THM-LAsc-5}{\underset{POS-LDis}{}}(x \land ((x \lor y) \lor z) = (x \land (x \lor y)) \lor (x \land z)),,$$
$$\overset{THM-LAsc-6}{\underset{THM-LAbs}{}}((x \land (x \lor y)) \lor (x \land z) = x \lor (x \land z)),,$$
$$\overset{THM-LAsc-7}{\underset{THM-LAbs}{}}(x \lor (x \land z) = x),,$$
$$\overset{THM-LAsc-8}{\underset{THM-LAbs}{}}((x \land (x \lor y)) \lor (x \land z) = x \lor (x \land z)),,$$
$$\overset{THM-LAsc-9}{\underset{\substack{THM-LAsc-2\\THM-LAsc-3\\THM-LAsc-4\\THM-LAsc-5\\THM-LAsc-6\\THM-LAsc-7\\THM-LAsc-8}}{}}(x \land A = x = x \land B).,$$
$$\overset{THM-LAsc-10}{\underset{THM-LAsc-1}{}}(\neg x \land A = \neg x \land (x \lor (y \lor z))),,$$
$$\overset{THM-LAsc-11}{\underset{POS-LDis}{}}(\neg x \land (x \lor (y \lor z)) = (\neg x \land x) \lor (\neg x \land (y + z))),,$$
$$\overset{THM-LAsc-12}{\underset{POS-LCom}{}}((\neg x \land x) \lor (\neg x \land (y \lor z)) = F \lor (\neg x \land (y \lor z))),,$$
$$\overset{THM-LAsc-13}{\underset{POS-LIdn}{}}(F \lor (\neg x \land (y + z)) = \neg x \land (y \lor z)),,$$
$$\overset{THM-LAsc-14}{\underset{THM-LAsc-1}{}}(\neg x \land B = \ldots),,$$
$$\#\ \text{Associative} \tag{19}$$

$$\overset{000}{\underset{000}{}}()$$
$$\#\ \text{Boolean De Morgan's Laws} \tag{20}$$

$$000 TODOIFPROPERTIES \tag{21}$$