Next-Next-Gen Notes
Object-Oriented Maths

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Format: $characteristic\big((subjects),(dependencies)\big) \iff \big(conditions(dependencies)\big) \wedge \big(conditions(subjects)\big)$
Note: All weaker objects automatically induces notions inherited from stronger objects.
TODO assign free variables as parameters
TODO define || abs cross-product and other missing refs
TODO distinguish new condition vs implied proposition - separate propositions into new line thms
TODO silent link expressions! - e.g. $backslashsilentPLPL_X$

1 Mathematical Logic

1.1 NaiveMaster

$$x \in y := \text{x belongs to y} \tag{1}$$

$$x \subseteq y := \text{x is included in y} \tag{2}$$

$$\begin{aligned} x = y &:= \text{x is the same thing as y} \\ &:= x \subseteq y, y \subseteq x \end{aligned} \tag{3}$$

$$\begin{aligned} x \subset y, x \not\subseteq y &:= \text{proper subset} \\ &:= x \neq y, x \subseteq y \end{aligned} \tag{4}$$

$$x \cup y := \text{all elements in x or y} \tag{5}$$

$$x \cap y := \text{all elements in x and y} \tag{6}$$

$$\begin{aligned} disjoint(x,y) &:= \text{disjoint sets} \\ &:= x \cap y = \emptyset \end{aligned} \tag{7}$$

$$\begin{aligned} \{e_1, e_2, e_3, \cdots, e_n\} &:= \text{unordered set containing } e_1, e_2, e_3, \cdots, e_n \\ \{e_1, e_2, e_3\} &= \{e_3, e_1, e_2\} \end{aligned} \tag{8}$$

$$\begin{aligned} \langle e_1, e_2, e_3, \cdots, e_n \rangle &:= \text{ordered tuple containing } e_1, e_2, e_3, \cdots, e_n \\ \langle e_1, e_2, e_3 \rangle &\neq \langle e_2, e_3, e_1 \rangle \end{aligned} \tag{9}$$

$$\begin{aligned} X^k = \{e_1, e_2, e_3, \cdots, e_n\}^k &:= \text{set of all ordered k-tuples from the elements of } e_1, e_2, e_3, \cdots, e_n \\ X^1 = \{e_1, e_2, e_3, \cdots, e_n\}^1 = \{\langle e_1 \rangle, \langle e_2 \rangle, \langle e_3 \rangle, \cdots, \langle e_n \rangle\} &= \{e_1, e_2, e_3, \cdots, e_n\} = X \end{aligned} \tag{10}$$

$$\begin{aligned} Y \times Z = \{y_1, y_2, y_3, \cdots, y_i\} \times \{z_1, z_2, z_3, \cdots, z_j\} &:= \text{Cartesian product} \\ &:= \bigcup_{a \leq i, b \leq j} (\{\langle y_a, z_b \rangle\}) \end{aligned} \tag{11}$$

$$R^k_Y \subseteq Y^k := \text{k-tuple relation R on the set Y takes only tuples that satisfy some relation}$$
$$P_Y \subseteq Y := \text{property P of the set Y} \tag{12}$$

$$\langle y,z\rangle \in binaryRelation(R^2_X) = yR^2_X z$$
$$domain(Y), range(Z)$$
$$field(R) = Y \cup Z$$
$$\langle a,b\rangle \in inverse(R^{-1}) : \langle b,a\rangle \in R$$
$$reflexive(R^2_X) : xR^2_X x$$
$$symmetric(R^2_X) : xR^2_X y = yR^2_X x$$
$$transitive(R^2_X) : xR^2_X y, yR^2_X z : xR^2_X z$$
$$equivalenceRelation(R^2_X) := reflexive(R^2_X), symmetric(R^2_X), transitive(R^2_X) \tag{13}$$

$$takethisintroshitmoresrsly \tag{14}$$

2 Logic and Set Theory

2.1 Logical Truths and Operators

$$truth[t][] := t = \begin{cases} T \\ F \end{cases} \tag{15}$$

$$statement[s][] := correctSyntaxSemantics[s][] \tag{16}$$

$$proposition[s,t][] := \big(statement[s][]\big), \big(truth[t][]\big). \tag{17}$$

$$operatorOR[\vee][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \vee y][] = \begin{cases} F & x=F, y=F \\ T & x=F, y=T \\ T & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{18}$$

$$operatorAND[\wedge][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \wedge y][] = \begin{cases} F & x=F, y=F \\ F & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{19}$$

$$operatorNOT[\neg][x] := \big(truth[x][]\big), \left(truth[\neg x][] = \begin{cases} T & x=F \\ F & x=T \end{cases} \right). \tag{20}$$

$$booleanAlgebra[\{T,F\}, \wedge, \vee, \neg][] :=^{POS-LCom} \big((x \wedge y = y \wedge x), (x \vee y = y \vee x)\big) \ \# \text{ Commutative,}$$
$$^{POS-LDis}\Big(\big(x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)\big), \big(x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)\big)\Big) \ \# \text{ Distributive,}$$
$$^{POS-LIdn}\big((x \wedge T = x), (x \vee F = x)\big) \ \# \text{ Identity,}$$
$$^{POS-LCmp}\big((x \wedge \neg x = F), (x \vee \neg x = T)\big) \ \# \text{ Complement.} \tag{21}$$

$$operatorXOR[\veebar][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \veebar y][] = \begin{cases} F & x=F, y=F \\ T & x=F, y=T \\ T & x=T, y=F \\ F & x=T, y=T \end{cases} \right). \tag{22}$$

$$operatorIF[\Longrightarrow][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \Longrightarrow y][] = (\neg x) \vee y = \begin{cases} T & x=F, y=F \\ T & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{23}$$

$$\begin{aligned} {}^{THM-LExp-1}_{POS-LCmp}(F = x \wedge \neg x) \Longrightarrow& \\ {}^{THM-LExp-2}_{THM-LExp-1}(x),& \\ {}^{THM-LExp-3}_{THM-LExp-1}(\neg x),& \\ {}^{THM-LExp-4}_{THM-LExp-2}(x \vee y),& \\ {}^{THM-LExp-5}_{\substack{THM-LExp-4\\THM-LExp-3}}(y).& \\ {}^{THM-LExp}_{\substack{THM-LExp-1\\THM-LExp-2\\THM-LExp-3\\THM-LExp-4\\THM-LExp-5}}(F \Longrightarrow y)& \\ \text{\# The Principle of Explosion, anything follows from a false } (F) \text{ premise}& \end{aligned} \tag{24}$$

$$operatorOIF[\Longleftarrow][x,y] := \big(truth[x][]\big), \big(truth[y][]\big), \left(truth[x \Longleftarrow y][] = (\neg y) \vee x = \begin{cases} T & x=F, y=F \\ F & x=F, y=T \\ T & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{25}$$

$$\begin{aligned} operatorIIF[\Longleftrightarrow][x,y] := \big(truth[x][]\big), \big(truth[y][]\big),& \\ \left(truth[x \Longleftrightarrow y][] = (x \Longrightarrow y) \wedge (y \Longrightarrow x) = \begin{cases} T & x=F, y=F \\ F & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right).& \end{aligned} \tag{26}$$

2.2 Boolean Algebra Properties

$$\begin{aligned} {}^{THM-Dual-1}_{POS-LCom}\Big(booleanAlgebra[\{T,F\}, \wedge, \vee, \neg][] \Longleftrightarrow& \\ \big((x \vee y = y \vee x), (x \wedge y = y \wedge x)\big) \text{ \# Reordered Commutative},& \\ \Big(\big(x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)\big), \big(x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)\big)\Big) \text{ \# Reordered Distributive},& \\ \big((x \vee F = x), (x \wedge T = x)\big) \text{ \# Reordered Identity},& \\ \big((x \vee \neg x = T), (x \wedge \neg x = F)\big) \text{ \# Reordered Complement.} \Longleftrightarrow& \\ booleanAlgebra[\{F,T\}, \vee, \wedge, \neg][] \Big)& \end{aligned}$$

$$\substack{\mathit{THM{-}Dual}\\ \mathit{THM{-}Dual{-}1}}\Big(\mathit{booleanAlgebra}[\{T,F\},\wedge,\vee,\neg][] \iff \mathit{booleanAlgebra}[\{F,T\},\vee,\wedge,\neg][]\Big)$$

Boolean Algebra Duality follows from the swap symmetry of $(\wedge,T)$ and $(\vee,F)$ within the axioms (27)

$$\substack{\mathit{THM{-}LUNt{-}1}}\Big((x\vee y=T=x\vee z)\wedge(x\wedge y=F=x\wedge z)\Big)\implies$$

$$\substack{\mathit{THM{-}LUNt{-}2}\\ \mathit{POS{-}LIdn}}\big(y=y\wedge T\big),$$

$$\substack{\mathit{THM{-}LUNt{-}3}\\ \mathit{THM{-}LUNt{-}1}}\big(y\wedge T=y\wedge(x\vee z)\big),$$

$$\substack{\mathit{THM{-}LUNt{-}4}\\ \mathit{POS{-}LDis}}\big(y\wedge(x\vee z)=(y\wedge x)\vee(y\wedge z)\big),$$

$$\substack{\mathit{THM{-}LUNt{-}5}\\ \mathit{POS{-}LCom}\\ \mathit{THM{-}LUNt{-}4}}\big((y\wedge x)\vee(y\wedge z)=(x\wedge z)\vee(y\wedge z)\big),$$

$$\substack{\mathit{THM{-}LUNt{-}6}\\ \mathit{POS{-}LCom}\\ \mathit{POS{-}LDis}}\big((x\wedge z)\vee(y\wedge z)=z\wedge(x\vee y)\big),$$

$$\substack{\mathit{THM{-}LUNt{-}7}\\ \mathit{THM{-}LUNt{-}1}}\big(z\wedge(x\vee y)=z\wedge T\big),$$

$$\substack{\mathit{THM{-}LUNt{-}8}\\ \mathit{POS{-}LIdn}}\big(z\wedge T=z\big).$$

$$\substack{\mathit{THM{-}LUNt}\\ \mathit{THM{-}LUNt{-}1}\\ \mathit{THM{-}LUNt{-}2}\\ \mathit{THM{-}LUNt{-}3}\\ \mathit{THM{-}LUNt{-}4}\\ \mathit{THM{-}LUNt{-}5}\\ \mathit{THM{-}LUNt{-}6}\\ \mathit{THM{-}LUNt{-}7}\\ \mathit{THM{-}LUNt{-}8}}\Big(\big((x\vee y=T=x\vee z)\wedge(x\wedge y=F=x\wedge z)\big)\implies(y=z)\Big)$$

Uniqueness of Complements (28)

$$\substack{\mathit{THM{-}LDom{-}1}\\ \mathit{POS{-}LIdn}}\big(x\vee T=(x\vee T)\wedge T\big),$$

$$\substack{\mathit{THM{-}LDom{-}2}\\ \mathit{POS{-}LCmp}}\big((x\vee T)\wedge T=(x\vee T)\wedge(x\vee\neg x)\big),$$

$$\substack{\mathit{THM{-}LDom{-}3}\\ \mathit{POS{-}LDis}}\big((x\vee T)\wedge(x\vee\neg x)=x\vee(T\wedge\neg x)\big),$$

$$\substack{\mathit{THM{-}LDom{-}4}\\ \mathit{POS{-}LIdn}}\big(x\vee(T\wedge\neg x)=x\vee\neg x\big),$$

$$\substack{\mathit{THM{-}LDom{-}5}\\ \mathit{POS{-}LCmp}}\big(x\vee\neg x=T\big).$$

$$\substack{\mathit{THM{-}LDom{-}6}\\ \mathit{THM{-}LDom{-}1}\\ \mathit{THM{-}LDom{-}2}\\ \mathit{THM{-}LDom{-}3}\\ \mathit{THM{-}LDom{-}4}\\ \mathit{THM{-}LDom{-}5}}\big(x\vee T=T\big),$$

$$\substack{\mathit{THM{-}LDom}\\ \mathit{THM{-}LDom{-}6}\\ \mathit{THM{-}Dual}}\big((x\vee T=T),(x\wedge F=F)\big).$$

Domination (29)

$$\substack{\mathit{THM{-}LIdm{-}1}\\ \mathit{POS{-}LIdn}}\big(x\vee x=(x\vee x)\wedge T\big),$$

$$\substack{\mathit{THM{-}LIdm{-}2}\\ \mathit{POS{-}LCmp}}\big((x\vee x)\wedge T=(x\vee x)\wedge(x\vee\neg x)\big),$$

$$\substack{\mathit{THM{-}LIdm{-}3}\\ \mathit{POS{-}LDis}}\big((x\vee x)\wedge(x\vee\neg x)=x\wedge(x\vee\neg x)\big),$$

$$\substack{\mathit{THM{-}LIdm{-}4}\\ \mathit{POS{-}LCmp}}\big(x\wedge(x\vee\neg x)=x\wedge T\big),$$

$$\substack{\mathit{THM{-}LIdm{-}5}\\ \mathit{POS{-}LIdn}}\big(x\wedge T=x\big),$$

$$\substack{\mathit{THM{-}LIdm{-}6}\\ \mathit{THM{-}LIdm{-}1}\\ \mathit{THM{-}LIdm{-}2}\\ \mathit{THM{-}LIdm{-}3}\\ \mathit{THM{-}LIdm{-}4}\\ \mathit{THM{-}LIdm{-}5}}\big(x\vee x=x\big),$$

$$\substack{\mathit{THM{-}LIdm}\\ \mathit{THM{-}LIdm{-}6}\\ \mathit{THM{-}Dual}}\big((x\vee x=x),(x\wedge x=x)\big).$$

Idempotent (30)

$$\substack{\mathit{THM{-}LInv{-}1}\\ \mathit{POS{-}LIdn}}\big(\neg\neg x=\neg\neg x\vee F\big),$$

$$\substack{\mathit{THM{-}LInv{-}2}\\ \mathit{POS{-}LCmp}}\big(\neg\neg x\vee F=\neg\neg x\vee(x\wedge\neg x)\big),$$

$$\substack{\mathit{THM{-}LInv{-}3}\\ \mathit{POS{-}LDis}}\big(\neg\neg x\vee(x\wedge\neg x)=(\neg\neg x\vee x)\wedge(\neg\neg x\vee\neg x)\big),$$

$$\substack{\mathit{THM{-}LInv{-}4}\\ \mathit{POS{-}LCmp}}\big((\neg\neg x\vee x)\wedge(\neg\neg x\vee\neg x)=(\neg\neg x\vee x)\wedge T\big),$$

$$\substack{THM-LInv-5\\POS-LCmp}\big((\neg\neg x\vee x)\wedge T=(\neg\neg x\vee x)\wedge(x\vee\neg x)\big),$$
$$\substack{THM-LInv-6\\POS-LDis}\big((\neg\neg x\vee x)\wedge(x\vee\neg x)=x\vee(\neg\neg x\wedge\neg x)\big),$$
$$\substack{THM-LInv-7\\POS-LCmp}\big(x\vee(\neg\neg x\wedge\neg x)=x\vee F\big),$$
$$\substack{THM-LInv-8\\POS-LIdn}(x\vee F=x),$$
$$\substack{THM-LInv\\THM-LInv-1\\THM-LInv-2\\THM-LInv-3\\THM-LInv-4\\THM-LInv-5\\THM-LInv-6\\THM-LInv-7\\THM-LInv-8}(\neg\neg x=x).$$

$$\#\ \text{Involution} \tag{31}$$

$$\substack{THM-LAbs-1\\POS-LIdn}\big(x\vee(x\wedge y)=(x\wedge T)\vee(x\wedge y)\big),$$
$$\substack{THM-LAbs-2\\POS-LDis}\big((x\wedge T)\vee(x\wedge y)=x\wedge(T\vee y)\big),$$
$$\substack{THM-LAbs-3\\THM-LDom}\big(x\wedge(T\vee y)=x\wedge T\big),$$
$$\substack{THM-LAbs-4\\THM-LIdn}(x\wedge T=x),$$
$$\substack{THM-LAbs-5\\THM-LAbs-1\\THM-LAbs-2\\THM-LAbs-3\\THM-LAbs-4}\big(x\vee(x\wedge y)=x\big),$$
$$\substack{THM-LAbs\\THM-LAbs-5\\THM-Dual}\Big(\big(x\vee(x\wedge y)=x\big),\big(x\wedge(x\vee y)=x\big)\Big).$$

$$\#\ \text{Absorption} \tag{32}$$

$$\substack{THM-LAsc-1}((A=x\vee(y\vee z)),(B=(x\vee y)\vee z))\Longrightarrow$$
$$\substack{THM-LAsc-2\\THM-LAsc-1}(x\wedge A=x\wedge(x\vee(y\vee z))),,$$
$$\substack{THM-LAsc-3\\THM-LAbs}(x\wedge(x\vee(y\vee z))=x),,$$
$$\substack{THM-LAsc-4\\THM-LAsc-1}(x\wedge B=x\wedge((x\vee y)\vee z)),,$$
$$\substack{THM-LAsc-5\\POS-LDis}(x\wedge((x\vee y)\vee z)=(x\wedge(x\vee y))\vee(x\wedge z)),,$$
$$\substack{THM-LAsc-6\\THM-LAbs}((x\wedge(x\vee y))\vee(x\wedge z)=x\vee(x\wedge z)),,$$
$$\substack{THM-LAsc-7\\THM-LAbs}(x\vee(x\wedge z)=x),,$$
$$\substack{THM-LAsc-8\\THM-LAbs}((x\wedge(x\vee y))\vee(x\wedge z)=x\vee(x\wedge z)),,$$
$$\substack{THM-LAsc-9\\THM-LAsc-2\\THM-LAsc-3\\THM-LAsc-4\\THM-LAsc-5\\THM-LAsc-6\\THM-LAsc-7\\THM-LAsc-8}(x\wedge A=x=x\wedge B).,$$
$$\substack{THM-LAsc-10\\THM-LAsc-1}(\neg x\wedge A=\neg x\wedge(x\vee(y\vee z))),,$$
$$\substack{THM-LAsc-11\\POS-LDis}(\neg x\wedge(x\vee(y\vee z))=(\neg x\wedge x)\vee(\neg x\wedge(y+z))),,$$
$$\substack{THM-LAsc-12\\POS-LCmp}((\neg x\wedge x)\vee(\neg x\wedge(y\vee z))=F\vee(\neg x\wedge(y\vee z))),,$$
$$\substack{THM-LAsc-13\\POS-LIdn}(F\vee(\neg x\wedge(y+z))=\neg x\wedge(y\vee z)),,$$
$$\substack{THM-LAsc-14\\THM-LAsc-1}(\neg x\wedge B=\neg x\wedge((x\vee y)\vee z)),,$$
$$\substack{THM-LAsc-15\\POS-LDis}(\neg x\wedge((x\vee y)\vee z)=(\neg x\wedge(x\vee y))\vee(\neg x\wedge z)),,$$
$$\substack{THM-LAsc-16\\POS-LDis}((\neg x\wedge(x\vee y))\vee(\neg x\wedge z)=((\neg x\wedge x)\vee(\neg x\wedge y))\vee(\neg x\wedge z)),,$$
$$\substack{THM-LAsc-17\\POS-LCmp}(((\neg x\wedge x)\vee(\neg x\wedge y))\vee(\neg x\wedge z)=(F\vee(\neg x\wedge y))\vee(\neg x\wedge z)),,$$
$$\substack{THM-LAsc-18\\POS-LIdn}((F\vee(\neg x\wedge y))\vee(\neg x\wedge z)=(\neg x\wedge y)\vee(\neg x\wedge z)),,$$
$$\substack{THM-LAsc-19\\POS-LDis}((\neg x\wedge y)\vee(\neg x\wedge z)=\neg x\wedge(y\vee z)),,$$

$$\substack{THM-LAsc-20\\THM-LAsc-10\\THM-LAsc-11\\THM-LAsc-12\\THM-LAsc-13\\THM-LAsc-14\\THM-LAsc-15\\THM-LAsc-16\\THM-LAsc-17\\THM-LAsc-18\\THM-LAsc-19}(\neg x\wedge A=\neg x\wedge(y\vee z)=\neg x\wedge B).,$$

$$\substack{THM-LAsc-21\\POS-LDis}(A=A\wedge T),,$$

$$\substack{THM-LAsc-22\\POS-LCmp}(A\wedge T=A\wedge(x\vee\neg x)),,$$

$$\substack{THM-LAsc-23\\POS-LDis}(A\wedge(x\vee\neg x)=(x\wedge A)\vee(\neg x\wedge A)),,$$

$$\substack{THM-LAsc-24\\THM-LAsc-9}((x\wedge A)\vee(\neg x\wedge A)=(x\wedge B)\vee(\neg x\wedge A)),,$$

$$\substack{THM-LAsc-25\\THM-LAsc-20}((x\wedge B)\vee(\neg x\wedge A)=(x\wedge B)\vee(\neg x\wedge B)),,$$

$$\substack{THM-LAsc-26\\POS-LDis}((x\wedge B)\vee(\neg x\wedge B)=B\wedge(x\vee\neg x)),,$$

$$\substack{THM-LAsc-27\\POS-LCmp}(B\wedge(x\vee\neg x)=B\wedge T),,$$

$$\substack{THM-LAsc-27\\POS-LIdn}(B\wedge T=B),,$$

$$\substack{THM-LAsc-28\\THM-LAsc-21\\THM-LAsc-22\\THM-LAsc-23\\THM-LAsc-24\\THM-LAsc-25\\THM-LAsc-26\\THM-LAsc-27}(A=B),,$$

$$\substack{THM-LAsc-29\\THM-LAsc-28\\THM-LAsc-1}(x\vee(y\vee z)=(x\vee y)\vee z).,$$

$$\substack{THM-LAsc\\THM-LAsc-29\\THM-Dual}((x\vee(y\vee z)=(x\vee y)\vee z),(x\wedge(y\wedge z)=(x\wedge y)\wedge z))..$$

\# Associative (33)

$$\substack{THM-LDMr-1\\POS-LDis}((x\vee y)\vee(\neg x\wedge\neg y)=((x\vee y)\vee\neg x)\wedge((x\vee y)\vee\neg y)),$$

$$\substack{THM-LDMr-2\\POS-LCom\\THM-LAsc}(((x\vee y)\vee\neg x)\wedge((x\vee y)\vee\neg y)=((x\vee\neg x)\vee y)\wedge((\neg y\vee y)\vee x)),$$

$$\substack{THM-LDMr-3\\POS-LCmp}(((x\vee\neg x)\vee y)\wedge((\neg y\vee y)\vee x)=(T\vee y)\wedge(T\vee x)),$$

$$\substack{THM-LDMr-4\\THM-LDom}((T\vee y)\wedge(T\vee x)=T\wedge T),$$

$$\substack{THM-LDMr-5\\THM-LIdm}(T\wedge T=T),$$

$$\substack{THM-LDMr-6\\THM-LDMr-1\\THM-LDMr-2\\THM-LDMr-3\\THM-LDMr-4\\THM-LDMr-5}((x\vee y)\vee(\neg x\wedge\neg y)=T).$$

$$\substack{THM-LDMr-7\\THM-LDis}((x\vee y)\wedge(\neg x\wedge\neg y)=(x\wedge\neg x\wedge\neg y)\vee(y\wedge\neg x\wedge\neg y)),$$

$$\substack{THM-LDMr-8\\POS-LCom\\THM-LAsc}((x\wedge\neg x\wedge\neg y)\vee(y\wedge\neg x\wedge\neg y)=((x\wedge\neg x)\wedge\neg y)\vee((y\wedge\neg y)\wedge\neg x)),$$

$$\substack{THM-LDMr-9\\POS-LCmp}(((x\wedge\neg x)\wedge\neg y)\vee((y\wedge\neg y)\wedge\neg x)=(F\wedge\neg y)\vee(F\wedge\neg x)),$$

$$\substack{THM-LDMr-10\\THM-LDom}((F\wedge\neg y)\vee(F\wedge\neg x)=F\vee F),$$

$$\substack{THM-LDMr-11\\THM-LIdm}(F\vee F=F),$$

$$\substack{THM-LDMr-12\\THM-LDMr-7\\THM-LDMr-8\\THM-LDMr-9\\THM-LDMr-10\\THM-LDMr-11}((x\vee y)\wedge(\neg x\wedge\neg y)=F).$$

$$\substack{THM-LDMr-13\\THM-LDMr-6\\THM-LDMr-12\\POS-LCmp}(((x\vee y)\vee(\neg x\wedge\neg y)=T=(x\vee y)\vee\neg(x\vee y)),((x\vee y)\wedge(\neg x\wedge\neg y)=F=(x\vee y)\wedge\neg(x\vee y))),$$

$$\substack{THM-LDMr-14\\THM-LDMr-13\\THM-LUNt}(\neg x\wedge\neg y=\neg(x\vee y)),$$

$$\substack{THM-LDMr\\THM-LDMr-14\\THM-Dual}((\neg x\wedge\neg y=\neg(x\vee y)),(\neg x\vee\neg y=\neg(x\wedge y))).$$

\# Boolean De Morgan's Laws (34)

$$\substack{THM-CtrP-1\\\Longrightarrow}(x\Longrightarrow y=(\neg x)\vee y),$$

$$
\begin{aligned}
&{}^{THM-CtrP-2}_{\substack{POS-LCom\\THM-LInv}}((\neg x)\vee y=((\neg\neg y)\vee(\neg x))),\\
&\overset{THM-CtrP-3}{\Longrightarrow}((\neg\neg y)\vee(\neg x)=(\neg y)\Longrightarrow(\neg x)),\\
&{}^{THM-CtrP}_{\substack{THM-CtrP-1\\THM-CtrP-2\\THM-CtrP-3}}(x\Longrightarrow y=(\neg y)\Longrightarrow(\neg x)).
\end{aligned}
$$

\# Contrapositive Law (35)

$$\boxed{(T\Longrightarrow x=x)} \tag{36}$$

$$\boxed{(F\Longrightarrow x=T)} \tag{37}$$

$$\boxed{(x\Longrightarrow T=T)} \tag{38}$$

$$\boxed{(x\Longrightarrow F=\neg x)} \tag{39}$$

$$\boxed{((x\vee y)\Longrightarrow z)=(x\Longrightarrow z)\wedge(y\Longrightarrow z)} \tag{40}$$

$$\boxed{(x\Longrightarrow(y\wedge z)=(x\Longrightarrow y)\wedge(x\Longrightarrow z))} \tag{41}$$

2.3 Predicate Logic

$$\forall_{x\in\mathbb{N}}(2x/2=x) \tag{42}$$

$$P(x)\Longrightarrow D \tag{43}$$