Next-Next-Gen Notes

Object-Oriented Maths

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Format: $characteristic\big((subjects),(dependencies)\big) \iff \big(conditions(dependencies)\big) \wedge \big(conditions(subjects)\big)$
Note: All weaker objects automatically induces notions inherited from stronger objects.
TODO assign free variables as parameters
TODO define || abs cross-product and other missing refs
TODO distinguish new condition vs implied proposition - separate propositions into new line thms
TODO silent link expressions! - e.g. $backslashsilentPLPL_X$

1 Logic and Set Theory

1.1 Logical Truths and Operators

$$truth[t][] := t = \begin{cases} T \\ F \end{cases} \tag{1}$$

$$statement[s][] := correctSyntaxSemantics[s][] \tag{2}$$

$$proposition[s,t][] := \big(statement[s][]\big),\big(truth[t][]\big). \tag{3}$$

$$operatorOR[\vee][x,y] := \big(truth[x][]\big),\big(truth[y][]\big),\left(truth[x\vee y][] = \begin{cases} F & x=F,y=F \\ T & x=F,y=T \\ T & x=T,y=F \\ T & x=T,y=T \end{cases}\right). \tag{4}$$

$$operatorAND[\wedge][x,y] := \big(truth[x][]\big),\big(truth[y][]\big),\left(truth[x\wedge y][] = \begin{cases} F & x=F,y=F \\ F & x=F,y=T \\ F & x=T,y=F \\ T & x=T,y=T \end{cases}\right). \tag{5}$$

$$operatorNOT[\neg][x] := \big(truth[x][]\big),\left(truth[\neg x][] = \begin{cases} T & x=F \\ F & x=T \end{cases}\right). \tag{6}$$

$$\begin{aligned} booleanAlgebra[\{T,F\},\wedge,\vee,\neg][] := {}^{POS-LCom}\big((x\wedge y = y\wedge x),(x\vee y = y\vee x)\big) \;\#\text{ Commutative,} \\ {}^{POS-LDis}\Big(\big(x\wedge(y\vee z) = (x\wedge y)\vee(x\wedge z)\big),\big(x\vee(y\wedge z) = (x\vee y)\wedge(x\vee z)\big)\Big) \;\#\text{ Distributive,} \\ {}^{POS-LIdn}\big((x\wedge T = x),(x\vee F = x)\big) \;\#\text{ Identity,} \\ {}^{POS-LCmp}\big((x\wedge\neg x = F),(x\vee\neg x = T)\big) \;\#\text{ Complement.} \end{aligned} \tag{7}$$

$$operatorXOR[\veebar][x,y] := \left(truth[x][]\right), \left(truth[y][]\right), \left(truth[x \veebar y][] = \begin{cases} F & x=F, y=F \\ T & x=F, y=T \\ T & x=T, y=F \\ F & x=T, y=T \end{cases} \right). \tag{8}$$

$$operatorIF[\Longrightarrow][x,y] := \left(truth[x][]\right), \left(truth[y][]\right), \left(truth[x \Longrightarrow y][] = (\neg x) \vee y = \begin{cases} T & x=F, y=F \\ T & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{9}$$

$$\begin{aligned} &{}^{THM-LExp-1}_{POS-LCmp}(F = x \wedge \neg x) \Longrightarrow \\ &{}^{THM-LExp-2}_{THM-LExp-1}(x), \\ &{}^{THM-LExp-3}_{THM-LExp-1}(\neg x), \\ &{}^{THM-LExp-4}_{THM-LExp-2}(x \vee y), \\ &{}^{THM-LExp-5}_{\substack{THM-LExp-4 \\ THM-LExp-3}}(y). \\ &{}^{THM-LExp}_{\substack{THM-LExp-1 \\ THM-LExp-2 \\ THM-LExp-3 \\ THM-LExp-4 \\ THM-LExp-5}}(F \Longrightarrow y) \\ &\text{\# The Principle of Explosion, anything follows from a false } (F) \text{ premise} \end{aligned} \tag{10}$$

$$operatorOIF[\Longleftarrow][x,y] := \left(truth[x][]\right), \left(truth[y][]\right), \left(truth[x \Longleftarrow y][] = \begin{cases} T & x=F, y=F \\ F & x=F, y=T \\ T & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{11}$$

$$operatorIIF[\Longleftrightarrow][x,y] := \left(truth[x][]\right), \left(truth[y][]\right), \left(truth[x \Longleftrightarrow y][] = \begin{cases} T & x=F, y=F \\ F & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right). \tag{12}$$

1.2 Boolean Algebra Properties

$$\begin{aligned} &{}^{THM-Dual-1}_{POS-LCom}\Bigg(booleanAlgebra[\{T,F\}, \wedge, \vee, \neg][] \Longleftrightarrow \\ &\left((x \vee y = y \vee x), (x \wedge y = y \wedge x)\right) \text{ \# Reordered Commutative,} \\ &\Big(\left(x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)\right), \left(x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)\right)\Big) \text{ \# Reordered Distributive,} \\ &\left((x \vee F = x), (x \wedge T = x)\right) \text{ \# Reordered Identity,} \\ &\left((x \vee \neg x = T), (x \wedge \neg x = F)\right) \text{ \# Reordered Complement.} \Longleftrightarrow \\ &booleanAlgebra[\{F,T\}, \vee, \wedge, \neg][] \Bigg) \\ &{}^{THM-Dual}_{THM-Dual-1}\left(booleanAlgebra[\{T,F\}, \wedge, \vee, \neg][] \Longleftrightarrow booleanAlgebra[\{F,T\}, \vee, \wedge, \neg][]\right) \\ &\text{\# Boolean Algebra Duality follows from the swap symmetry of } (\wedge, T) \text{ and } (\vee, F) \text{ within the axioms} \end{aligned} \tag{13}$$

$$\substack{THM-LUNt-1}\Big((x\lor y=T=x\lor z)\land(x\land y=F=x\land z)\Big)\implies$$

$$\substack{THM-LUNt-2\\ POS-LIdn}\big(y=y\land T\big),$$

$$\substack{THM-LUNt-3\\ THM-LUNt-1}\big(y\land T=y\land(x\lor z)\big),$$

$$\substack{THM-LUNt-4\\ POS-LDis}\big(y\land(x\lor z)=(y\land x)\lor(y\land z)\big),$$

$$\substack{THM-LUNt-5\\ POS-LCom\\ THM-LUNt-4}\big((y\land x)\lor(y\land z)=(x\land z)\lor(y\land z)\big),$$

$$\substack{THM-LUNt-6\\ POS-LCom\\ POS-LDis}\big((x\land z)\lor(y\land z)=z\land(x\lor y)\big),$$

$$\substack{THM-LUNt-7\\ THM-LUNt-1}\big(z\land(x\lor y)=z\land T\big),$$

$$\substack{THM-LUNt-8\\ POS-LIdn}\big(z\land T=z\big).$$

$$\substack{THM-LUNt\\ THM-LUNt-1\\ THM-LUNt-2\\ THM-LUNt-3\\ THM-LUNt-4\\ THM-LUNt-5\\ THM-LUNt-6\\ THM-LUNt-7\\ THM-LUNt-8}\Big(\big((x\lor y=T=x\lor z)\land(x\land y=F=x\land z)\big)\implies(y=z)\Big)$$

\# Uniqueness of Complements (14)

$$\substack{THM-LDom-1\\ POS-LIdn}\big(x\lor T=(x\lor T)\land T\big),$$

$$\substack{THM-LDom-2\\ POS-LCmp}\big((x\lor T)\land T=(x\lor T)\land(x\lor\neg x)\big),$$

$$\substack{THM-LDom-3\\ POS-LDis}\big((x\lor T)\land(x\lor\neg x)=x\lor(T\land\neg x)\big),$$

$$\substack{THM-LDom-4\\ POS-LIdn}\big(x\lor(T\land\neg x)=x\lor\neg x\big),$$

$$\substack{THM-LDom-5\\ POS-LCmp}\big(x\lor\neg x=T\big).$$

$$\substack{THM-LDom-6\\ THM-LDom-1\\ THM-LDom-2\\ THM-LDom-3\\ THM-LDom-4\\ THM-LDom-5}\big(x\lor T=T\big),$$

$$\substack{THM-LDom\\ THM-LDom-6\\ THM-Dual}\big((x\lor T=T),(x\land F=F)\big).$$

\# Domination (15)

$$\substack{THM-LIdm-1\\ POS-LIdn}\big(x\lor x=(x\lor x)\land T\big),$$

$$\substack{THM-LIdm-2\\ POS-LCmp}\big((x\lor x)\land T=(x\lor x)\land(x\lor\neg x)\big),$$

$$\substack{THM-LIdm-3\\ POS-LDis}\big((x\lor x)\land(x\lor\neg x)=x\land(x\lor\neg x)\big),$$

$$\substack{THM-LIdm-4\\ POS-LCmp}\big(x\land(x\lor\neg x)=x\land T\big),$$

$$\substack{THM-LIdm-5\\ POS-LIdn}\big(x\land T=x\big),$$

$$\substack{THM-LIdm-6\\ THM-LIdm-1\\ THM-LIdm-2\\ THM-LIdm-3\\ THM-LIdm-4\\ THM-LIdm-5}\big(x\lor x=x\big),$$

$$\substack{THM-LIdm\\ THM-LIdm-6\\ THM-Dual}\big((x\lor x=x),(x\land x=x)\big).$$

\# Idempotent (16)

$$\substack{THM-LInv-1\\ POS-LIdn}\big(\neg\neg x=\neg\neg x\lor F\big),$$

$$\substack{THM-LInv-2\\ POS-LCmp}\big(\neg\neg x\lor F=\neg\neg x\lor(x\land\neg x)\big),$$

$$\substack{THM-LInv-3\\ POS-LDis}\big(\neg\neg x\lor(x\land\neg x)=(\neg\neg x\lor x)\land(\neg\neg x\lor\neg x)\big),$$

$$\substack{THM-LInv-4\\ POS-LCmp}\big((\neg\neg x\lor x)\land(\neg\neg x\lor\neg x)=(\neg\neg x\lor x)\land T\big),$$

$$\substack{THM-LInv-5\\ POS-LCmp}\big((\neg\neg x\lor x)\land T=(\neg\neg x\lor x)\land(x\lor\neg x)\big),$$

$$\substack{THM-LInv-6\\ POS-LDis}\big((\neg\neg x\lor x)\land(x\lor\neg x)=x\lor(\neg\neg x\land\neg x)\big),$$

$$\substack{THM-LInv-7\\POS-LCmp}\big(x \lor (\neg\neg x \land \neg x) = x \lor F\big),$$
$$\substack{THM-LInv-8\\POS-LIdn}(x \lor F = x),$$
$$\substack{THM-LInv\\THM-LInv-1\\THM-LInv-2\\THM-LInv-3\\THM-LInv-4\\THM-LInv-5\\THM-LInv-6\\THM-LInv-7\\THM-LInv-8}(\neg\neg x = x).$$
$$\# \text{ Involution} \tag{17}$$

$$\substack{THM-LAbs-1\\POS-LIdn}\big(x \lor (x \land y) = (x \land T) \lor (x \land y)\big),$$
$$\substack{THM-LAbs-2\\POS-LDis}\big((x \land T) \lor (x \land y) = x \land (T \lor y)\big),$$
$$\substack{THM-LAbs-3\\THM-LDom}\big(x \land (T \lor y) = x \land T\big),$$
$$\substack{THM-LAbs-4\\THM-LIdn}(x \land T = x),$$
$$\substack{THM-LAbs-5\\THM-LAbs-1\\THM-LAbs-2\\THM-LAbs-3\\THM-LAbs-4}\big(x \lor (x \land y) = x\big),$$
$$\substack{THM-LAbs\\THM-LAbs-5\\THM-Dual}\Big(\big(x \lor (x \land y) = x\big), \big(x \land (x \lor y) = x\big)\Big).$$
$$\# \text{ Absorption} \tag{18}$$

$$\substack{000\\000}()$$
$$\# \text{ Associative} \tag{19}$$

$$\substack{000\\000}()$$
$$\# \text{ Boolean De Morgan's Laws} \tag{20}$$

$$000 TODOIFPROPERTIES \tag{21}$$