# Next-Next-Gen Notes

Object-Oriented Maths

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## 1 Logic and Set Theory

### 1.1 D: Logical Truths and Operators

**undefined terms:** $:=,=,(\_),,,.,$

$$truth[t][] := \left( t = \begin{cases} T \\ F \end{cases} \right) \tag{1}$$

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$$operatorOR[\vee][x,y] :=_1 \left(truth[x][]\right),_1 \left(truth[y][]\right),_1 \left( truth[x \vee y][] = \begin{cases} F & x=F, y=F \\ T & x=F, y=T \\ T & x=T, y=F \\ T & x=T, y=T \end{cases} \right).{}_1 \tag{2}$$

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$$operatorAND[\wedge][x,y] :=_1 \left(truth[x][]\right),_1 \left(truth[y][]\right),_1 \left( truth[x \wedge y][] = \begin{cases} F & x=F, y=F \\ F & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right).{}_1 \tag{3}$$

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$$operatorNOT[\neg][x] :=_1 \left(truth[x][]\right),_1 \left( truth[\neg x][] = \begin{cases} T & x=F \\ F & x=T \end{cases} \right).{}_1 \tag{4}$$

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$$operatorXOR[\veebar][x,y] :=_1 \left(truth[x][]\right),_1 \left(truth[y][]\right),_1 \left( truth[x \veebar y][] = \begin{cases} F & x=F, y=F \\ T & x=F, y=T \\ T & x=T, y=F \\ F & x=T, y=T \end{cases} \right).{}_1 \tag{5}$$

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$$operatorIF[\Longrightarrow][x,y] :=_1 \left(truth[x][]\right),_1 \left(truth[y][]\right),_1 \left( truth[x \Longrightarrow y][] = (\neg x) \vee y = \begin{cases} T & x=F, y=F \\ T & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right).{}_1$$

$$\#\text{ a counterexample cannot follow from a false precedence, thus the conditional cannot be false} \tag{6}$$

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$$operatorOIF[\Longleftarrow][x,y]:=_1(truth[x][]),_1(truth[y][]),_1\left(truth[x\Longleftarrow y][]=(\neg y)\vee x=\begin{cases}T & x=F,y=F\\ F & x=F,y=T\\ T & x=T,y=F\\ T & x=T,y=T\end{cases}\right).{}_1 \quad (7)$$

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$$operatorIIF[\Longleftrightarrow][x,y]:=_1(truth[x][]),_1(truth[y][]),_1$$
$$\left(truth[x\Longleftrightarrow y][]=(x\Longrightarrow y)\wedge(y\Longrightarrow x)=\begin{cases}T & x=F,y=F\\ F & x=F,y=T\\ F & x=T,y=F\\ T & x=T,y=T\end{cases}\right).{}_1 \quad (8)$$

## 1.2 P: Boolean Algebra

$$booleanAlgebra[(\top,\bot,\otimes,\oplus,\ominus)][]:=_1{}^{POS-LCom}((x\otimes y=y\otimes x),_1(x\oplus y=y\oplus x)) \ \#\ \text{Commutative},_1$$
$${}^{POS-LDis}\Big((x\otimes(y\oplus z)=(x\otimes y)\oplus(x\otimes z)),_1(x\oplus(y\otimes z)=(x\oplus y)\otimes(x\oplus z))\Big) \ \#\ \text{Distributive},_1$$
$${}^{POS-LIdn}((x\otimes\top=x),_1(x\oplus\bot=x)) \ \#\ \text{Identity},_1$$
$${}^{POS-LCmp}\Big((x\otimes(\ominus x)=\bot),_1(x\oplus(\ominus x)=\top)\Big) \ \#\ \text{Complement}.{}_1$$
$$\#\ \text{Note: I sometimes get too lazy to refer to } POS-LCom. \quad (9)$$

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$${}^{INS-LBAl}(booleanAlgebra[T,F,\wedge,\vee,\neg][])$$
$$\#\ \text{Proven by way of cases or truth tables} \quad (10)$$

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$${}^{THM-Dual-1}\Big(booleanAlgebra[(T,F,\wedge,\vee,\neg)][]\Longleftrightarrow_1$$
$$((x\wedge y=y\wedge x),_1(x\vee y=y\vee x)) \ \#\ \text{Commutative},_1$$
$$\Big((x\wedge(y\vee z)=(x\wedge y)\vee(x\wedge z)),_1(x\vee(y\wedge z)=(x\vee y)\wedge(x\vee z))\Big) \ \#\ \text{Distributive},_1$$
$$((x\wedge T=x),_1(x\vee F=x)) \ \#\ \text{Identity},_1$$
$$((x\wedge\neg x=F),_1(x\vee\neg x=T)) \ \#\ \text{Complement}.{}_1\Longleftrightarrow_2$$
$$((x\vee y=y\vee x),_2(x\wedge y=y\wedge x)) \ \#\ \text{Reordered Commutative},_2$$
$$\Big((x\vee(y\wedge z)=(x\vee y)\wedge(x\vee z)),_2(x\wedge(y\vee z)=(x\wedge y)\vee(x\wedge z))\Big) \ \#\ \text{Reordered Distributive},_2$$
$$((x\vee F=x),_2(x\wedge T=x)) \ \#\ \text{Reordered Identity},_2$$
$$((x\vee\neg x=T),_2(x\wedge\neg x=F)) \ \#\ \text{Reordered Complement}.{}_2\Longleftrightarrow$$
$$booleanAlgebra[(F,T,\vee,\wedge,\neg)][]\Big)$$
$${}^{THM-Dual-2}_{THM-Dual-1}(booleanAlgebra[(T,F,\wedge,\vee,\neg)][]\Longleftrightarrow booleanAlgebra[(F,T,\vee,\wedge,\neg)][])$$
$${}^{THM-Dual}_{THM-Dual-2}{}^{}_{INS-LBAl}booleanAlgebra[(F,T,\vee,\wedge,\neg)][]()$$
$$\#\ \text{Boolean Algebra Duality follows from the swap symmetry of } (\wedge,T) \text{ and } (\vee,F) \text{ within the axioms} \quad (11)$$

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$${}^{THM-LUNt-1}((x\vee y=T=x\vee z)\wedge(x\wedge y=F=x\wedge z))\Longrightarrow_1$$
$${}^{THM-LUNt-2}_{INS-LBAl}{}^{}_{POS-LIdn}(y=y\wedge T),_1$$
$${}^{THM-LUNt-3}_{THM-LUNt-1}(y\wedge T=y\wedge(x\vee z)),_1$$

$$(y \wedge (x \vee z) = (y \wedge x) \vee (y \wedge z))_{,1} \quad {\scriptstyle THM-LUNt-4 \atop POS-LDis}$$

$$((y \wedge x) \vee (y \wedge z) = (x \wedge z) \vee (y \wedge z))_{,1} \quad {\scriptstyle THM-LUNt-5 \atop POS-LCom \atop THM-LUNt-4}$$

$$((x \wedge z) \vee (y \wedge z) = z \wedge (x \vee y))_{,1} \quad {\scriptstyle THM-LUNt-6 \atop POS-LCom \atop POS-LDis}$$

$$(z \wedge (x \vee y) = z \wedge T)_{,1} \quad {\scriptstyle THM-LUNt-7 \atop THM-LUNt-1}$$

$$(z \wedge T = z)_{,1} \quad {\scriptstyle THM-LUNt-8 \atop POS-LIdn}$$

$$(((x \vee y = T = x \vee z) \wedge (x \wedge y = F = x \wedge z)) \Longrightarrow (y = z)) \quad {\scriptstyle THM-LUNt \atop THM-LUNt-1 \atop THM-LUNt-2 \atop THM-LUNt-3 \atop THM-LUNt-4 \atop THM-LUNt-5 \atop THM-LUNt-6 \atop THM-LUNt-7 \atop THM-LUNt-8}$$

\# Uniqueness of Complements (12)

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$$(x \vee T = (x \vee T) \wedge T) \quad {\scriptstyle THM-LDom-1 \atop INS-LBAl \atop POS-LIdn}$$

$$((x \vee T) \wedge T = (x \vee T) \wedge (x \vee \neg x)) \quad {\scriptstyle THM-LDom-2 \atop POS-LCmp}$$

$$((x \vee T) \wedge (x \vee \neg x) = x \vee (T \wedge \neg x)) \quad {\scriptstyle THM-LDom-3 \atop POS-LDis}$$

$$(x \vee (T \wedge \neg x) = x \vee \neg x) \quad {\scriptstyle THM-LDom-4 \atop POS-LIdn}$$

$$(x \vee \neg x = T) \quad {\scriptstyle THM-LDom-5 \atop POS-LCmp}$$

$$(x \vee T = T) \quad {\scriptstyle THM-LDom-6 \atop THM-LDom-1 \atop THM-LDom-2 \atop THM-LDom-3 \atop THM-LDom-4 \atop THM-LDom-5}$$

$$((x \vee T = T), (x \wedge F = F)) \quad {\scriptstyle THM-LDom \atop THM-LDom-6 \atop THM-Dual}$$

\# Domination (13)

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$$(x \vee x = (x \vee x) \wedge T) \quad {\scriptstyle THM-LIdm-1 \atop INS-LBAl \atop POS-LIdn}$$

$$((x \vee x) \wedge T = (x \vee x) \wedge (x \vee \neg x)) \quad {\scriptstyle THM-LIdm-2 \atop POS-LCmp}$$

$$((x \vee x) \wedge (x \vee \neg x) = x \wedge (x \vee \neg x)) \quad {\scriptstyle THM-LIdm-3 \atop POS-LDis}$$

$$(x \wedge (x \vee \neg x) = x \wedge T) \quad {\scriptstyle THM-LIdm-4 \atop POS-LCmp}$$

$$(x \wedge T = x) \quad {\scriptstyle THM-LIdm-5 \atop POS-LIdn}$$

$$(x \vee x = x) \quad {\scriptstyle THM-LIdm-6 \atop THM-LIdm-1 \atop THM-LIdm-2 \atop THM-LIdm-3 \atop THM-LIdm-4 \atop THM-LIdm-5}$$

$$((x \vee x = x), (x \wedge x = x)) \quad {\scriptstyle THM-LIdm \atop THM-LIdm-6 \atop THM-Dual}$$

\# Idempotent (14)

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$$(\neg\neg x = \neg\neg x \vee F) \quad {\scriptstyle THM-LInv-1 \atop INS-LBAl \atop POS-LIdn}$$

$$(\neg\neg x \vee F = \neg\neg x \vee (x \wedge \neg x)) \quad {\scriptstyle THM-LInv-2 \atop POS-LCmp}$$

$$(\neg\neg x \vee (x \wedge \neg x) = (\neg\neg x \vee x) \wedge (\neg\neg x \vee \neg x)) \quad {\scriptstyle THM-LInv-3 \atop POS-LDis}$$

$$((\neg\neg x \vee x) \wedge (\neg\neg x \vee \neg x) = (\neg\neg x \vee x) \wedge T) \quad {\scriptstyle THM-LInv-4 \atop POS-LCmp}$$

$$((\neg\neg x \vee x) \wedge T = (\neg\neg x \vee x) \wedge (x \vee \neg x)) \quad {\scriptstyle THM-LInv-5 \atop POS-LCmp}$$

$$((\neg\neg x \vee x) \wedge (x \vee \neg x) = x \vee (\neg\neg x \wedge \neg x)) \quad {\scriptstyle THM-LInv-6 \atop POS-LDis}$$

$$(x \vee (\neg\neg x \wedge \neg x) = x \vee F) \quad {\scriptstyle THM-LInv-7 \atop POS-LCmp}$$

$$(x \vee F = x) \quad {\scriptstyle THM-LInv-8 \atop POS-LIdn}$$

$$\overset{THM-LInv}{\overset{THM-LInv-1}{\overset{THM-LInv-2}{\overset{THM-LInv-3}{\overset{THM-LInv-4}{\overset{THM-LInv-5}{\overset{THM-LInv-6}{\overset{THM-LInv-7}{\overset{THM-LInv-8}{(\neg\neg x = x)}}}}}}}}}$$

\# Involution (15)

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$$\overset{THM-LAbs-1}{\overset{INS-LBAl}{\overset{POS-LIdn}{(x \vee (x \wedge y) = (x \wedge T) \vee (x \wedge y))}}}$$

$$\overset{THM-LAbs-2}{\overset{POS-LDis}{((x \wedge T) \vee (x \wedge y) = x \wedge (T \vee y))}}$$

$$\overset{THM-LAbs-3}{\overset{THM-LDom}{(x \wedge (T \vee y) = x \wedge T)}}$$

$$\overset{THM-LAbs-4}{\overset{POS-LIdn}{(x \wedge T = x)}}$$

$$\overset{THM-LAbs-5}{\overset{THM-LAbs-1}{\overset{THM-LAbs-2}{\overset{THM-LAbs-3}{\overset{THM-LAbs-4}{(x \vee (x \wedge y) = x)}}}}}$$

$$\overset{THM-LAbs}{\overset{THM-LAbs-5}{\overset{THM-Dual}{\Big((x \vee (x \wedge y) = x), (x \wedge (x \vee y) = x)\Big)}}}$$

\# Absorption (16)

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$$\overset{THM-LAsc-1}{\Big((A = x \vee (y \vee z)), (B = (x \vee y) \vee z)\Big)} \Longrightarrow_1$$

$$\overset{THM-LAsc-2}{\overset{THM-LAsc-1}{\Big(x \wedge A = x \wedge (x \vee (y \vee z))\Big)}}_1$$

$$\overset{THM-LAsc-3}{\overset{THM-LAbs}{\Big(x \wedge (x \vee (y \vee z)) = x\Big)}}_1$$

$$\overset{THM-LAsc-4}{\overset{THM-LAsc-1}{\Big(x \wedge B = x \wedge ((x \vee y) \vee z)\Big)}}_1$$

$$\overset{THM-LAsc-5}{\overset{INS-LBAl}{\overset{POS-LDis}{\Big(x \wedge ((x \vee y) \vee z) = (x \wedge (x \vee y)) \vee (x \wedge z)\Big)}}}_1$$

$$\overset{THM-LAsc-6}{\overset{THM-LAbs}{\Big((x \wedge (x \vee y)) \vee (x \wedge z) = x \vee (x \wedge z)\Big)}}_1$$

$$\overset{THM-LAsc-7}{\overset{THM-LAbs}{(x \vee (x \wedge z) = x)}}_1$$

$$\overset{THM-LAsc-8}{\overset{THM-LAbs}{\Big((x \wedge (x \vee y)) \vee (x \wedge z) = x \vee (x \wedge z)\Big)}}_1$$

$$\overset{THM-LAsc-9}{\overset{THM-LAsc-2}{\overset{THM-LAsc-3}{\overset{THM-LAsc-4}{\overset{THM-LAsc-5}{\overset{THM-LAsc-6}{\overset{THM-LAsc-7}{\overset{THM-LAsc-8}{(x \wedge A = x = x \wedge B)}}}}}}}}_1$$

$$\overset{THM-LAsc-10}{\overset{THM-LAsc-1}{\Big(\neg x \wedge A = \neg x \wedge (x \vee (y \vee z))\Big)}}_1$$

$$\overset{THM-LAsc-11}{\overset{INS-LBAl}{\overset{POS-LDis}{\Big(\neg x \wedge (x \vee (y \vee z)) = (\neg x \wedge x) \vee (\neg x \wedge (y + z))\Big)}}}_1$$

$$\overset{THM-LAsc-12}{\overset{INS-LBAl}{\overset{POS-LCmp}{\Big((\neg x \wedge x) \vee (\neg x \wedge (y \vee z)) = F \vee (\neg x \wedge (y \vee z))\Big)}}}_1$$

$$\overset{THM-LAsc-13}{\overset{INS-LBAl}{\overset{POS-LIdn}{\Big(F \vee (\neg x \wedge (y + z)) = \neg x \wedge (y \vee z)\Big)}}}_1$$

$$\overset{THM-LAsc-14}{\overset{THM-LAsc-1}{\Big(\neg x \wedge B = \neg x \wedge ((x \vee y) \vee z)\Big)}}_1$$

$$\overset{THM-LAsc-15}{\overset{INS-LBAl}{\overset{POS-LDis}{\Big(\neg x \wedge ((x \vee y) \vee z) = (\neg x \wedge (x \vee y)) \vee (\neg x \wedge z)\Big)}}}_1$$

$$\overset{THM-LAsc-16}{\overset{INS-LBAl}{\overset{POS-LDis}{\Big((\neg x \wedge (x \vee y)) \vee (\neg x \wedge z) = ((\neg x \wedge x) \vee (\neg x \wedge y)) \vee (\neg x \wedge z)\Big)}}}_1$$

$$\overset{THM-LAsc-17}{\overset{INS-LBAl}{\overset{POS-LCmp}{\Big(((\neg x \wedge x) \vee (\neg x \wedge y)) \vee (\neg x \wedge z) = (F \vee (\neg x \wedge y)) \vee (\neg x \wedge z)\Big)}}}_1$$

$$\overset{THM-LAsc-18}{\overset{INS-LBAl}{\overset{POS-LIdn}{\Big((F \vee (\neg x \wedge y)) \vee (\neg x \wedge z) = (\neg x \wedge y) \vee (\neg x \wedge z)\Big)}}}_1$$

$$\overset{THM-LAsc-19}{\overset{INS-LBAl}{\overset{POS-LDis}{((\neg x \wedge y) \vee (\neg x \wedge z) = \neg x \wedge (y \vee z))}}}_1$$

$$\overset{THM-LAsc-20}{\overset{THM-LAsc-10}{\overset{THM-LAsc-11}{\overset{THM-LAsc-12}{\overset{THM-LAsc-13}{\overset{THM-LAsc-14}{\overset{THM-LAsc-15}{\overset{THM-LAsc-16}{\overset{THM-LAsc-17}{\overset{THM-LAsc-18}{\overset{THM-LAsc-19}{}}}}}}}}}}}\big(\neg x \wedge A = \neg x \wedge (y \vee z) = \neg x \wedge B\big)_{,1}$$

$$\overset{THM-LAsc-21}{\overset{INS-LBAl}{POS-LDis}}(A = A \wedge T)_{,1}$$

$$\overset{THM-LAsc-22}{\overset{INS-LBAl}{POS-LCmp}}\big(A \wedge T = A \wedge (x \vee \neg x)\big)_{,1}$$

$$\overset{THM-LAsc-23}{\overset{INS-LBAl}{POS-LDis}}\big(A \wedge (x \vee \neg x) = (x \wedge A) \vee (\neg x \wedge A)\big)_{,1}$$

$$\overset{THM-LAsc-24}{THM-LAsc-9}\big((x \wedge A) \vee (\neg x \wedge A) = (x \wedge B) \vee (\neg x \wedge A)\big)_{,1}$$

$$\overset{THM-LAsc-25}{THM-LAsc-20}\big((x \wedge B) \vee (\neg x \wedge A) = (x \wedge B) \vee (\neg x \wedge B)\big)_{,1}$$

$$\overset{THM-LAsc-26}{\overset{INS-LBAl}{POS-LDis}}\big((x \wedge B) \vee (\neg x \wedge B) = B \wedge (x \vee \neg x)\big)_{,1}$$

$$\overset{THM-LAsc-27}{\overset{INS-LBAl}{POS-LCmp}}\big(B \wedge (x \vee \neg x) = B \wedge T\big)_{,1}$$

$$\overset{THM-LAsc-27}{\overset{INS-LBAl}{POS-LIdn}}(B \wedge T = B)_{,1}$$

$$\overset{THM-LAsc-28}{\overset{THM-LAsc-21}{\overset{THM-LAsc-22}{\overset{THM-LAsc-23}{\overset{THM-LAsc-24}{\overset{THM-LAsc-25}{\overset{THM-LAsc-26}{THM-LAsc-27}}}}}}}(A = B)_{,1}$$

$$\overset{THM-LAsc-29}{\overset{THM-LAsc-28}{THM-LAsc-1}}\big(x \vee (y \vee z) = (x \vee y) \vee z\big)_{,1}$$

$$\overset{THM-LAsc}{\overset{THM-LAsc-29}{THM-Dual}}\Big(\big(x \vee (y \vee z) = (x \vee y) \vee z\big), \big(x \wedge (y \wedge z) = (x \wedge y) \wedge z\big)\Big)$$

\# Associative (17)

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$$\overset{THM-LDMr-1}{\overset{INS-LBAl}{POS-LDis}}\Big(\big(x \vee y\big) \vee \big(\neg x \wedge \neg y\big) = \big(\big(x \vee y\big) \vee \neg x\big) \wedge \big(\big(x \vee y\big) \vee \neg y\big)\Big)$$

$$\overset{THM-LDMr-2}{\overset{POS-LCom}{THM-LAsc}}\Big(\big(\big(x \vee y\big) \vee \neg x\big) \wedge \big(\big(x \vee y\big) \vee \neg y\big) = \big(\big(x \vee \neg x\big) \vee y\big) \wedge \big(\big(\neg y \vee y\big) \vee x\big)\Big)$$

$$\overset{THM-LDMr-3}{\overset{INS-LBAl}{POS-LCmp}}\Big(\big(\big(x \vee \neg x\big) \vee y\big) \wedge \big(\big(\neg y \vee y\big) \vee x\big) = \big(T \vee y\big) \wedge \big(T \vee x\big)\Big)$$

$$\overset{THM-LDMr-4}{THM-LDom}\big(\big(T \vee y\big) \wedge \big(T \vee x\big) = T \wedge T\big)$$

$$\overset{THM-LDMr-5}{THM-LIdm}\big(T \wedge T = T\big)$$

$$\overset{THM-LDMr-6}{\overset{THM-LDMr-1}{\overset{THM-LDMr-2}{\overset{THM-LDMr-3}{\overset{THM-LDMr-4}{THM-LDMr-5}}}}}\big(\big(x \vee y\big) \vee \big(\neg x \wedge \neg y\big) = T\big)$$

$$\overset{THM-LDMr-7}{\overset{INS-LBAl}{POS-LDis}}\big(\big(x \vee y\big) \wedge \big(\neg x \wedge \neg y\big) = \big(x \wedge \neg x \wedge \neg y\big) \vee \big(y \wedge \neg x \wedge \neg y\big)\big)$$

$$\overset{THM-LDMr-8}{\overset{POS-LCom}{THM-LAsc}}\Big(\big(x \wedge \neg x \wedge \neg y\big) \vee \big(y \wedge \neg x \wedge \neg y\big) = \big(\big(x \wedge \neg x\big) \wedge \neg y\big) \vee \big(\big(y \wedge \neg y\big) \wedge \neg x\big)\Big)$$

$$\overset{THM-LDMr-9}{\overset{INS-LBAl}{POS-LCmp}}\Big(\big(\big(x \wedge \neg x\big) \wedge \neg y\big) \vee \big(\big(y \wedge \neg y\big) \wedge \neg x\big) = \big(F \wedge \neg y\big) \vee \big(F \wedge \neg x\big)\Big)$$

$$\overset{THM-LDMr-10}{THM-LDom}\big(\big(F \wedge \neg y\big) \vee \big(F \wedge \neg x\big) = F \vee F\big)$$

$$\overset{THM-LDMr-11}{THM-LIdm}\big(F \vee F = F\big)$$

$$\overset{THM-LDMr-12}{\overset{THM-LDMr-7}{\overset{THM-LDMr-8}{\overset{THM-LDMr-9}{\overset{THM-LDMr-10}{THM-LDMr-11}}}}}\big(\big(x \vee y\big) \wedge \big(\neg x \wedge \neg y\big) = F\big)$$

$$\overset{THM-LDMr-13}{\overset{THM-LDMr-6}{\overset{THM-LDMr-12}{POS-LCmp}}}\Big(\big(\big(x \vee y\big) \vee \big(\neg x \wedge \neg y\big) = T = \big(x \vee y\big) \vee \neg\big(x \vee y\big)\big), \big(\big(x \vee y\big) \wedge \big(\neg x \wedge \neg y\big) = F = \big(x \vee y\big) \wedge \neg\big(x \vee y\big)\big)\Big)$$

$$\overset{THM-LDMr-14}{\overset{THM-LDMr-13}{THM-LUNt}}(\neg x \wedge \neg y = \neg(x \vee y))$$

$$\overset{THM-LDMr}{\overset{THM-LDMr-14}{THM-Dual}}\Big((\neg x \wedge \neg y = \neg(x \vee y)), (\neg x \vee \neg y = \neg(x \wedge y))\Big)$$

\# Boolean De Morgan's Laws (18)

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$$\overset{THM-CtrP-1}{operatorIF}(x \implies y = (\neg x) \vee y)$$

$$\overset{THM-CtrP-2}{\overset{POS-LCom}{THM-LInv}}\Big((\neg x) \vee y = ((\neg\neg y) \vee (\neg x))\Big)$$

$$\overset{THM-CtrP-3}{operatorIF}((\neg\neg y) \vee (\neg x) = (\neg y) \implies (\neg x))$$

$$\overset{THM-CtrP}{\overset{THM-CtrP-1}{\overset{THM-CtrP-2}{THM-CtrP-3}}}(x \implies y = (\neg y) \implies (\neg x))$$

\# Contrapositive Law (19)

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**MISC IMPLICATION LAWS:**

$$(T \implies x = x)$$

$$(F \implies x = T)$$

$$(x \implies T = T)$$

$$(x \implies F = \neg x)$$

$$((x \vee y) \implies z) = (x \implies z) \wedge (y \implies z)$$

$$(x \implies (y \wedge z) = (x \implies y) \wedge (x \implies z)) \quad (20)$$

## 1.3 Predicates, Sets, Tuples

**undefined terms:** $arg\_(\_), set, \in, \{\_\},$

$$predicate[P][] := truth[P(v_{free})][] \quad (21)$$

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$$universalQuantifier[\forall][Q,P] :=_1 (predicate[Q][]),_1 (predicate[P][]),_1 (\forall_{arg_x(Q(x))}(P(x)) = Q(y_{free}) \implies P(y_{free})).{}_1 \quad (22)$$

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$$existentialQuantifier[\exists][Q,P] := (\exists_{arg_x(Q(x))}(P(x)) = \neg\forall_{arg_x(Q(x))}(\neg P(x))) \quad (23)$$

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$$uniquenessQuantifier[\exists!][Q,P] := (\exists!_{arg_x(Q(x))}(P(x)) = \exists_{arg_x(Q(x))}(P(x) \wedge \neg\exists_{arg_y(Q(y))}(P(y) \wedge \neg(y = x)))) \quad (24)$$

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$$relationSetEq[=][X,Y] := (\forall_{arg_z(z \in X \vee z \in Y)}(z \in X \wedge z \in Y)) \quad (25)$$

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$$operatorIntersection[\bigcap][X] := (\bigcap(X) = \{arg_z(\forall_{x \in X}(z \in x))\}) \quad (26)$$

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$$operatorUnion[\bigcup][X] := (\bigcup(X) = \{arg_z(\exists_{x \in X}(z \in x))\}) \quad (27)$$

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$$orderedPair[<x,y>][] == <x,y> = <a,b> iff x = a and y = b == \{\{x\},\{x,y\}\} \quad (28)$$