Next-Next-Gen Notes

Object-Oriented Maths

JP Guzman

March 5, 2018

undefined terms: $(:=),(=),(,),(arg\_(\_)),(.)$

1 Logic and Set Theory

1.1 Logical Truths and Operators

$$truth[t][] := \left(t = \begin{cases} T \\ F \end{cases} \right) \tag{1}$$

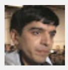
$$operatorOR[\vee][x,y] :=_1 \left(truth[x][]\right),_1 \left(truth[y][]\right),_1 \left(truth[x \vee y][] = \begin{cases} F & x=F, y=F \\ T & x=F, y=T \\ T & x=T, y=F \\ T & x=T, y=T \end{cases} \right) ._1 \tag{2}$$

$$operatorAND[\wedge][x,y] :=_1 \left(truth[x][]\right),_1 \left(truth[y][]\right),_1 \left(truth[x \wedge y][] = \begin{cases} F & x=F, y=F \\ F & x=F, y=T \\ F & x=T, y=F \\ T & x=T, y=T \end{cases} \right) ._1 \tag{3}$$

$$operatorNOT[\neg][x] :=_1 \left(truth[x][]\right),_1 \left(truth[\neg x][] = \begin{cases} T & x=F \\ F & x=T \end{cases} \right) ._1 \tag{4}$$

$$\begin{aligned} booleanAlgebra[\{T,F\},\wedge,\vee,\neg][] :=_1 {}^{POS-LCom}\left((x \wedge y = y \wedge x),_1 (x \vee y = y \vee x)\right) \ \# \text{ Commutative},_1 \\ {}^{POS-LDis}\Big(\left(x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)\right),_1 \left(x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)\right)\Big) \ \# \text{ Distributive},_1 \\ {}^{POS-LIdn}\left((x \wedge T = x),_1 (x \vee F = x)\right) \ \# \text{ Identity},_1 \\ {}^{POS-LCmp}\left((x \wedge \neg x = F),_1 (x \vee \neg x = T)\right) \ \# \text{ Complement}._1 \end{aligned}$$

Note: I sometimes get too lazy to refer to $POS-LCom$.



(5)

$$operatorXOR[\veebar][x,y] :=_1 \left(truth[x][]\right),_1 \left(truth[y][]\right),_1 \left(truth[x \veebar y][] = \begin{cases} F & x=F, y=F \\ T & x=F, y=T \\ T & x=T, y=F \\ F & x=T, y=T \end{cases} \right) ._1 \tag{6}$$

$$operatorIF[\Longrightarrow][x,y]:=_{1}\left(truth[x][]\right),_{1}\left(truth[y][]\right),_{1}\left(truth[x\Longrightarrow y][]=(\neg x)\lor y=\begin{cases}T & x=F,y=F\\ T & x=F,y=T\\ F & x=T,y=F\\ T & x=T,y=T\end{cases}\right)._{1} \tag{7}$$

$$\begin{aligned}&{}^{THM-LExp-1}_{POS-LCmp}(F=x\land\neg x)\Longrightarrow_{1}\\&{}^{THM-LExp-2}_{THM-LExp-1}(x),_{1}\\&{}^{THM-LExp-3}_{THM-LExp-1}(\neg x),_{1}\\&{}^{THM-LExp-4}_{THM-LExp-2}(x\lor y),_{1}\\&{}^{THM-LExp-5}_{THM-LExp-4,\ THM-LExp-3}(y)._{1}\\&{}^{THM-LExp}_{THM-LExp-1,\ THM-LExp-2,\ THM-LExp-3,\ THM-LExp-4,\ THM-LExp-5}(F\Longrightarrow y)\\&\#\ \text{The Principle of Explosion, anything follows from a false } (F) \text{ premise}\end{aligned} \tag{8}$$

$$operatorOIF[\Longleftarrow][x,y]:=_{1}\left(truth[x][]\right),_{1}\left(truth[y][]\right),_{1}\left(truth[x\Longleftarrow y][]=(\neg y)\lor x=\begin{cases}T & x=F,y=F\\ F & x=F,y=T\\ T & x=T,y=F\\ T & x=T,y=T\end{cases}\right)._{1} \tag{9}$$

$$\begin{aligned}&operatorIIF[\Longleftrightarrow][x,y]:=_{1}\left(truth[x][]\right),_{1}\left(truth[y][]\right),_{1}\\&\left(truth[x\Longleftrightarrow y][]=(x\Longrightarrow y)\land(y\Longrightarrow x)=\begin{cases}T & x=F,y=F\\ F & x=F,y=T\\ F & x=T,y=F\\ T & x=T,y=T\end{cases}\right)._{1}\end{aligned} \tag{10}$$

1.2 Boolean Algebra Properties

$$\begin{aligned}&{}^{THM-Dual-1}\Big(booleanAlgebra[\{T,F\},\land,\lor,\neg][]\Longleftrightarrow_{1}\\&\left((x\land y=y\land x),_{1}(x\lor y=y\lor x)\right)\ \#\ \text{Commutative},_{1}\\&\left(\left(x\land(y\lor z)=(x\land y)\lor(x\land z)\right),_{1}\left(x\lor(y\land z)=(x\lor y)\land(x\lor z)\right)\right)\ \#\ \text{Distributive},_{1}\\&\left((x\land T=x),_{1}(x\lor F=x)\right)\ \#\ \text{Identity},_{1}\\&\left((x\land\neg x=F),_{1}(x\lor\neg x=T)\right)\ \#\ \text{Complement}._{1}\Longleftrightarrow_{2}\\&\left((x\lor y=y\lor x),_{2}(x\land y=y\land x)\right)\ \#\ \text{Reordered Commutative},_{2}\\&\left(\left(x\lor(y\land z)=(x\lor y)\land(x\lor z)\right),_{2}\left(x\land(y\lor z)=(x\land y)\lor(x\land z)\right)\right)\ \#\ \text{Reordered Distributive},_{2}\\&\left((x\lor F=x),_{2}(x\land T=x)\right)\ \#\ \text{Reordered Identity},_{2}\\&\left((x\lor\neg x=T),_{2}(x\land\neg x=F)\right)\ \#\ \text{Reordered Complement}._{2}\Longleftrightarrow\\&booleanAlgebra[\{F,T\},\lor,\land,\neg][]\Big)\\&{}^{THM-Dual}_{THM-Dual-1}\left(booleanAlgebra[\{T,F\},\land,\lor,\neg][]\Longleftrightarrow booleanAlgebra[\{F,T\},\lor,\land,\neg][]\right)\\&\#\ \text{Boolean Algebra Duality follows from the swap symmetry of } (\land,T) \text{ and } (\lor,F) \text{ within the axioms}\end{aligned} \tag{11}$$

$$\substack{THM{-}LUNt{-}1}\Big((x\lor y=T=x\lor z)\land(x\land y=F=x\land z)\Big)\Longrightarrow{}_{1}$$

$$\substack{THM{-}LUNt{-}2\\ POS{-}LIdn}\big(y=y\land T\big),_{1}$$

$$\substack{THM{-}LUNt{-}3\\ THM{-}LUNt{-}1}\big(y\land T=y\land(x\lor z)\big),_{1}$$

$$\substack{THM{-}LUNt{-}4\\ POS{-}LDis}\big(y\land(x\lor z)=(y\land x)\lor(y\land z)\big),_{1}$$

$$\substack{THM{-}LUNt{-}5\\ POS{-}LCom\\ THM{-}LUNt{-}4}\big((y\land x)\lor(y\land z)=(x\land z)\lor(y\land z)\big),_{1}$$

$$\substack{THM{-}LUNt{-}6\\ POS{-}LCom\\ POS{-}LDis}\big((x\land z)\lor(y\land z)=z\land(x\lor y)\big),_{1}$$

$$\substack{THM{-}LUNt{-}7\\ THM{-}LUNt{-}1}\big(z\land(x\lor y)=z\land T\big),_{1}$$

$$\substack{THM{-}LUNt{-}8\\ POS{-}LIdn}\big(z\land T=z\big)._{1}$$

$$\substack{THM{-}LUNt\\ THM{-}LUNt{-}1\\ THM{-}LUNt{-}2\\ THM{-}LUNt{-}3\\ THM{-}LUNt{-}4\\ THM{-}LUNt{-}5\\ THM{-}LUNt{-}6\\ THM{-}LUNt{-}7\\ THM{-}LUNt{-}8}\Big(\big((x\lor y=T=x\lor z)\land(x\land y=F=x\land z)\big)\Longrightarrow(y=z)\Big)$$

Uniqueness of Complements (12)

$$\substack{THM{-}LDom{-}1\\ POS{-}LIdn}\big(x\lor T=(x\lor T)\land T\big)$$

$$\substack{THM{-}LDom{-}2\\ POS{-}LCmp}\big((x\lor T)\land T=(x\lor T)\land(x\lor\neg x)\big)$$

$$\substack{THM{-}LDom{-}3\\ POS{-}LDis}\big((x\lor T)\land(x\lor\neg x)=x\lor(T\land\neg x)\big)$$

$$\substack{THM{-}LDom{-}4\\ POS{-}LIdn}\big(x\lor(T\land\neg x)=x\lor\neg x\big)$$

$$\substack{THM{-}LDom{-}5\\ POS{-}LCmp}\big(x\lor\neg x=T\big)$$

$$\substack{THM{-}LDom{-}6\\ THM{-}LDom{-}1\\ THM{-}LDom{-}2\\ THM{-}LDom{-}3\\ THM{-}LDom{-}4\\ THM{-}LDom{-}5}\big(x\lor T=T\big)$$

$$\substack{THM{-}LDom\\ THM{-}LDom{-}6\\ THM{-}Dual}\big((x\lor T=T),(x\land F=F)\big)$$

Domination (13)

$$\substack{THM{-}LIdm{-}1\\ POS{-}LIdn}\big(x\lor x=(x\lor x)\land T\big)$$

$$\substack{THM{-}LIdm{-}2\\ POS{-}LCmp}\big((x\lor x)\land T=(x\lor x)\land(x\lor\neg x)\big)$$

$$\substack{THM{-}LIdm{-}3\\ POS{-}LDis}\big((x\lor x)\land(x\lor\neg x)=x\land(x\lor\neg x)\big)$$

$$\substack{THM{-}LIdm{-}4\\ POS{-}LCmp}\big(x\land(x\lor\neg x)=x\land T\big)$$

$$\substack{THM{-}LIdm{-}5\\ POS{-}LIdn}\big(x\land T=x\big)$$

$$\substack{THM{-}LIdm{-}6\\ THM{-}LIdm{-}1\\ THM{-}LIdm{-}2\\ THM{-}LIdm{-}3\\ THM{-}LIdm{-}4\\ THM{-}LIdm{-}5}\big(x\lor x=x\big)$$

$$\substack{THM{-}LIdm\\ THM{-}LIdm{-}6\\ THM{-}Dual}\big((x\lor x=x),(x\land x=x)\big)$$

Idempotent (14)

$$\substack{THM{-}LInv{-}1\\ POS{-}LIdn}\big(\neg\neg x=\neg\neg x\lor F\big)$$

$$\substack{THM{-}LInv{-}2\\ POS{-}LCmp}\big(\neg\neg x\lor F=\neg\neg x\lor(x\land\neg x)\big)$$

$$\substack{THM{-}LInv{-}3\\ POS{-}LDis}\big(\neg\neg x\lor(x\land\neg x)=(\neg\neg x\lor x)\land(\neg\neg x\lor\neg x)\big)$$

$$\substack{THM{-}LInv{-}4\\ POS{-}LCmp}\big((\neg\neg x\lor x)\land(\neg\neg x\lor\neg x)=(\neg\neg x\lor x)\land T\big)$$

$$\substack{THM{-}LInv{-}5\\ POS{-}LCmp}\big((\neg\neg x\lor x)\land T=(\neg\neg x\lor x)\land(x\lor\neg x)\big)$$

$$\substack{THM{-}LInv{-}6\\ POS{-}LDis}\big((\neg\neg x\lor x)\land(x\lor\neg x)=x\lor(\neg\neg x\land\neg x)\big)$$

$$\substack{THM-LInv-7\\POS-LCmp}\left(x\vee(\neg\neg x\wedge\neg x)=x\vee F\right)$$

$$\substack{THM-LInv-8\\POS-LIdn}\left(x\vee F=x\right)$$

$$\substack{THM-LInv\\THM-LInv-1\\THM-LInv-2\\THM-LInv-3\\THM-LInv-4\\THM-LInv-5\\THM-LInv-6\\THM-LInv-7\\THM-LInv-8}\left(\neg\neg x=x\right)$$

$$\#\ \text{Involution} \tag{15}$$

$$\substack{THM-LAbs-1\\POS-LIdn}\left(x\vee(x\wedge y)=(x\wedge T)\vee(x\wedge y)\right)$$

$$\substack{THM-LAbs-2\\POS-LDis}\left((x\wedge T)\vee(x\wedge y)=x\wedge(T\vee y)\right)$$

$$\substack{THM-LAbs-3\\THM-LDom}\left(x\wedge(T\vee y)=x\wedge T\right)$$

$$\substack{THM-LAbs-4\\POS-LIdn}\left(x\wedge T=x\right)$$

$$\substack{THM-LAbs-5\\THM-LAbs-1\\THM-LAbs-2\\THM-LAbs-3\\THM-LAbs-4}\left(x\vee(x\wedge y)=x\right)$$

$$\substack{THM-LAbs\\THM-LAbs-5\\THM-Dual}\Big(\big(x\vee(x\wedge y)=x\big),\big(x\wedge(x\vee y)=x\big)\Big)$$

$$\#\ \text{Absorption} \tag{16}$$

$$\substack{THM-LAsc-1}\Big(\big(A=x\vee(y\vee z)\big),\big(B=(x\vee y)\vee z\big)\Big)\Longrightarrow_1$$

$$\substack{THM-LAsc-2\\THM-LAsc-1}\Big(x\wedge A=x\wedge\big(x\vee(y\vee z)\big)\Big),_1$$

$$\substack{THM-LAsc-3\\THM-LAbs}\Big(x\wedge\big(x\vee(y\vee z)\big)=x\Big),_1$$

$$\substack{THM-LAsc-4\\THM-LAsc-1}\Big(x\wedge B=x\wedge\big((x\vee y)\vee z\big)\Big),_1$$

$$\substack{THM-LAsc-5\\POS-LDis}\Big(x\wedge\big((x\vee y)\vee z\big)=\big(x\wedge(x\vee y)\big)\vee(x\wedge z)\Big),_1$$

$$\substack{THM-LAsc-6\\THM-LAbs}\Big(\big(x\wedge(x\vee y)\big)\vee(x\wedge z)=x\vee(x\wedge z)\Big),_1$$

$$\substack{THM-LAsc-7\\THM-LAbs}\big(x\vee(x\wedge z)=x\big),_1$$

$$\substack{THM-LAsc-8\\THM-LAbs}\Big(\big(x\wedge(x\vee y)\big)\vee(x\wedge z)=x\vee(x\wedge z)\Big),_1$$

$$\substack{THM-LAsc-9\\THM-LAsc-2\\THM-LAsc-3\\THM-LAsc-4\\THM-LAsc-5\\THM-LAsc-6\\THM-LAsc-7\\THM-LAsc-8}(x\wedge A=x=x\wedge B),_1$$

$$\substack{THM-LAsc-10\\THM-LAsc-1}\Big(\neg x\wedge A=\neg x\wedge\big(x\vee(y\vee z)\big)\Big),_1$$

$$\substack{THM-LAsc-11\\POS-LDis}\Big(\neg x\wedge\big(x\vee(y\vee z)\big)=(\neg x\wedge x)\vee\big(\neg x\wedge(y+z)\big)\Big),_1$$

$$\substack{THM-LAsc-12\\POS-LCmp}\Big((\neg x\wedge x)\vee\big(\neg x\wedge(y\vee z)\big)=F\vee\big(\neg x\wedge(y\vee z)\big)\Big),_1$$

$$\substack{THM-LAsc-13\\POS-LIdn}\Big(F\vee\big(\neg x\wedge(y+z)\big)=\neg x\wedge(y\vee z)\Big),_1$$

$$\substack{THM-LAsc-14\\THM-LAsc-1}\Big(\neg x\wedge B=\neg x\wedge\big((x\vee y)\vee z\big)\Big),_1$$

$$\substack{THM-LAsc-15\\POS-LDis}\Big(\neg x\wedge\big((x\vee y)\vee z\big)=\big(\neg x\wedge(x\vee y)\big)\vee(\neg x\wedge z)\Big),_1$$

$$\substack{THM-LAsc-16\\POS-LDis}\Big(\big(\neg x\wedge(x\vee y)\big)\vee(\neg x\wedge z)=\big((\neg x\wedge x)\vee(\neg x\wedge y)\big)\vee(\neg x\wedge z)\Big),_1$$

$$\substack{THM-LAsc-17\\POS-LCmp}\Big(\big((\neg x\wedge x)\vee(\neg x\wedge y)\big)\vee(\neg x\wedge z)=\big(F\vee(\neg x\wedge y)\big)\vee(\neg x\wedge z)\Big),_1$$

$$\substack{THM-LAsc-18\\POS-LIdn}\Big(\big(F\vee(\neg x\wedge y)\big)\vee(\neg x\wedge z)=(\neg x\wedge y)\vee(\neg x\wedge z)\Big),_1$$

$$\substack{THM-LAsc-19\\POS-LDis}\big((\neg x\wedge y)\vee(\neg x\wedge z)=\neg x\wedge(y\vee z)\big),_1$$

$$\begin{matrix}THM{-}LAsc{-}20\\THM{-}LAsc{-}10\\THM{-}LAsc{-}11\\THM{-}LAsc{-}12\\THM{-}LAsc{-}13\\THM{-}LAsc{-}14\\THM{-}LAsc{-}15\\THM{-}LAsc{-}16\\THM{-}LAsc{-}17\\THM{-}LAsc{-}18\\THM{-}LAsc{-}19\end{matrix}\left(\neg x\wedge A=\neg x\wedge(y\vee z)=\neg x\wedge B\right),_1$$

$$\begin{matrix}THM{-}LAsc{-}21\\POS{-}LDis\end{matrix}\left(A=A\wedge T\right),_1$$

$$\begin{matrix}THM{-}LAsc{-}22\\POS{-}LCmp\end{matrix}\left(A\wedge T=A\wedge(x\vee\neg x)\right),_1$$

$$\begin{matrix}THM{-}LAsc{-}23\\POS{-}LDis\end{matrix}\left(A\wedge(x\vee\neg x)=(x\wedge A)\vee(\neg x\wedge A)\right),_1$$

$$\begin{matrix}THM{-}LAsc{-}24\\THM{-}LAsc{-}9\end{matrix}\left((x\wedge A)\vee(\neg x\wedge A)=(x\wedge B)\vee(\neg x\wedge A)\right),_1$$

$$\begin{matrix}THM{-}LAsc{-}25\\THM{-}LAsc{-}20\end{matrix}\left((x\wedge B)\vee(\neg x\wedge A)=(x\wedge B)\vee(\neg x\wedge B)\right),_1$$

$$\begin{matrix}THM{-}LAsc{-}26\\POS{-}LDis\end{matrix}\left((x\wedge B)\vee(\neg x\wedge B)=B\wedge(x\vee\neg x)\right),_1$$

$$\begin{matrix}THM{-}LAsc{-}27\\POS{-}LCmp\end{matrix}\left(B\wedge(x\vee\neg x)=B\wedge T\right),_1$$

$$\begin{matrix}THM{-}LAsc{-}27\\POS{-}LIdn\end{matrix}\left(B\wedge T=B\right),_1$$

$$\begin{matrix}THM{-}LAsc{-}28\\THM{-}LAsc{-}21\\THM{-}LAsc{-}22\\THM{-}LAsc{-}23\\THM{-}LAsc{-}24\\THM{-}LAsc{-}25\\THM{-}LAsc{-}26\\THM{-}LAsc{-}27\end{matrix}\left(A=B\right),_1$$

$$\begin{matrix}THM{-}LAsc{-}29\\THM{-}LAsc{-}28\\THM{-}LAsc{-}1\end{matrix}\left(x\vee(y\vee z)=(x\vee y)\vee z\right)._1$$

$$\begin{matrix}THM{-}LAsc\\THM{-}LAsc{-}29\\THM{-}Dual\end{matrix}\Big(\left(x\vee(y\vee z)=(x\vee y)\vee z\right),\left(x\wedge(y\wedge z)=(x\wedge y)\wedge z\right)\Big)$$

\# Associative (17)

$$\begin{matrix}THM{-}LDMr{-}1\\POS{-}LDis\end{matrix}\Big((x\vee y)\vee(\neg x\wedge\neg y)=\big((x\vee y)\vee\neg x\big)\wedge\big((x\vee y)\vee\neg y\big)\Big)$$

$$\begin{matrix}THM{-}LDMr{-}2\\POS{-}LCom\\THM{-}LAsc\end{matrix}\Big(\big((x\vee y)\vee\neg x\big)\wedge\big((x\vee y)\vee\neg y\big)=\big((x\vee\neg x)\vee y\big)\wedge\big((\neg y\vee y)\vee x\big)\Big)$$

$$\begin{matrix}THM{-}LDMr{-}3\\POS{-}LCmp\end{matrix}\Big(\big((x\vee\neg x)\vee y\big)\wedge\big((\neg y\vee y)\vee x\big)=(T\vee y)\wedge(T\vee x)\Big)$$

$$\begin{matrix}THM{-}LDMr{-}4\\THM{-}LDom\end{matrix}\left((T\vee y)\wedge(T\vee x)=T\wedge T\right)$$

$$\begin{matrix}THM{-}LDMr{-}5\\THM{-}LIdm\end{matrix}\left(T\wedge T=T\right)$$

$$\begin{matrix}THM{-}LDMr{-}6\\THM{-}LDMr{-}1\\THM{-}LDMr{-}2\\THM{-}LDMr{-}3\\THM{-}LDMr{-}4\\THM{-}LDMr{-}5\end{matrix}\left((x\vee y)\vee(\neg x\wedge\neg y)=T\right)$$

$$\begin{matrix}THM{-}LDMr{-}7\\POS{-}LDis\end{matrix}\left((x\vee y)\wedge(\neg x\wedge\neg y)=(x\wedge\neg x\wedge\neg y)\vee(y\wedge\neg x\wedge\neg y)\right)$$

$$\begin{matrix}THM{-}LDMr{-}8\\POS{-}LCom\\THM{-}LAsc\end{matrix}\Big((x\wedge\neg x\wedge\neg y)\vee(y\wedge\neg x\wedge\neg y)=\big((x\wedge\neg x)\wedge\neg y\big)\vee\big((y\wedge\neg y)\wedge\neg x\big)\Big)$$

$$\begin{matrix}THM{-}LDMr{-}9\\POS{-}LCmp\end{matrix}\Big(\big((x\wedge\neg x)\wedge\neg y\big)\vee\big((y\wedge\neg y)\wedge\neg x\big)=(F\wedge\neg y)\vee(F\wedge\neg x)\Big)$$

$$\begin{matrix}THM{-}LDMr{-}10\\THM{-}LDom\end{matrix}\left((F\wedge\neg y)\vee(F\wedge\neg x)=F\vee F\right)$$

$$\begin{matrix}THM{-}LDMr{-}11\\THM{-}LIdm\end{matrix}\left(F\vee F=F\right)$$

$$\begin{matrix}THM{-}LDMr{-}12\\THM{-}LDMr{-}7\\THM{-}LDMr{-}8\\THM{-}LDMr{-}9\\THM{-}LDMr{-}10\\THM{-}LDMr{-}11\end{matrix}\left((x\vee y)\wedge(\neg x\wedge\neg y)=F\right)$$

$$\begin{matrix}THM{-}LDMr{-}13\\THM{-}LDMr{-}6\\THM{-}LDMr{-}12\\POS{-}LCmp\end{matrix}\Big(\big((x\vee y)\vee(\neg x\wedge\neg y)=T=(x\vee y)\vee\neg(x\vee y)\big),\big((x\vee y)\wedge(\neg x\wedge\neg y)=F=(x\vee y)\wedge\neg(x\vee y)\big)\Big)$$

$$\begin{matrix}THM{-}LDMr{-}14\\THM{-}LDMr{-}13\\THM{-}LUNt\end{matrix}\left(\neg x\wedge\neg y=\neg(x\vee y)\right)$$

$$\begin{matrix}THM{-}LDMr\\THM{-}LDMr{-}14\\THM{-}Dual\end{matrix}\Big(\left(\neg x\wedge\neg y=\neg(x\vee y)\right),\left(\neg x\vee\neg y=\neg(x\wedge y)\right)\Big)$$

\# Boolean De Morgan's Laws (18)

$$\overset{THM-CtrP-1}{\underset{operatorIF}{}}\left(x \implies y = (\neg x) \vee y\right)$$

$$\overset{THM-CtrP-2}{\underset{POS-LCom,\ THM-LInv}{}}\Big((\neg x) \vee y = \big((\neg\neg y) \vee (\neg x)\big)\Big)$$

$$\overset{THM-CtrP-3}{\underset{operatorIF}{}}\left((\neg\neg y) \vee (\neg x) = (\neg y) \implies (\neg x)\right)$$

$$\overset{THM-CtrP}{\underset{THM-CtrP-1,\ THM-CtrP-2,\ THM-CtrP-3}{}}\left(x \implies y = (\neg y) \implies (\neg x)\right)$$

Contrapositive Law (19)

MISC IMPLICATION LAWS:

$$(T \implies x = x)$$

$$(F \implies x = T)$$

$$(x \implies T = T)$$

$$(x \implies F = \neg x)$$

$$\big((x \vee y) \implies z\big) = (x \implies z) \wedge (y \implies z)$$

$$\big(x \implies (y \wedge z) = (x \implies y) \wedge (x \implies z)\big) \quad (20)$$

1.3 Predicate Logic

$$predicate[P][] := truth[P(v_{free})][] \quad (21)$$

$$universalQuantifier[\forall][Q,P] :=_1 (predicate[Q][]),_1 (predicate[P][]),_1$$
$$(\forall_{arg_x(Q(x))}(P(x)) = Q(y_{free}) \implies P(y_{free})).{}_1 \quad (22)$$

$$existentialQuantifier[\exists][Q,P] := (\exists_{arg_x(Q(x))}(P(x)) = \neg\forall_{arg_x(Q(x))}(\neg P(x))) \quad (23)$$

$$uniquenessQuantifier[\exists!][Q,P] := (\exists!_{arg_x(Q(x))}(P(x)) = \exists_{arg_x(Q(x))}(P(x)) \wedge \forall_{arg_y(P(y))}\forall_{arg_z(P(z))}(y = z)) \quad (24)$$

$$0 \quad (25)$$

2 Dry Run

2.1 NaiveMaster

undefined terms: $:=$, $set, tuple, element, nnumber, \in, \subseteq, =, \not/, \subset, \cup, \cap, \emptyset, \{,\}, \langle,\rangle, |, \frown, \times, relation, property, binaryRelation, domain, range, field, \forall. \exists, \wedge, \vee, \implies, \impliedby, \iff,$

(26)

$$element[x][] \in set[y][]$$

x belongs to y (27)

$$set[x][] \subseteq set[y][]$$

x is included in y (28)

$$(set[x][] = set[y][]) := (set[x][] \subseteq set[y][] \wedge set[y][] \subseteq set[x][])$$
x is the same set as y (29)

$$(set[x][] \subset set[y][]) := (set[x][] \subseteq set[y][] \wedge set[x][] \neq set[y][])$$
x is a proper subset of y (30)

$$set[x][] \cup set[y][]$$
all elements in x or y (31)

$$set[x][] \cap set[y][]$$
all elements in x and y (32)

$$disjoint[x, y][] := set[x][] \cap set[y][] = \emptyset$$
disjoint sets do not intersect (33)

$$set[E][] = \{e_1, e_2, e_3, \cdots, e_n\}$$
unordered set containing $e_1, e_2, e_3, \cdots, e_n$
$$\{e_1, e_2, e_3\} = \{e_3, e_1, e_2\} \tag{34}$$

$$tuple[E][] = \langle e_1, e_2, e_3, \cdots, e_n\rangle$$
ordered tuple containing $e_1, e_2, e_3, \cdots, e_n$
$$\langle e_1, e_2, e_3\rangle \neq \langle e_2, e_3, e_1\rangle \tag{35}$$

$$set[X][]\hat{\ } nnumber[k][]$$
set of all ordered k-tuples from the elements in X
$$X\hat{\ }1 = \{e_1, e_2, e_3, \cdots, e_n\}\hat{\ }1 = \{\langle e_1\rangle, \langle e_2\rangle, \langle e_3\rangle, \cdots, \langle e_n\rangle\} = \{e_1, e_2, e_3, \cdots, e_n\} = X \tag{36}$$

$$set[Y][] \times set[Z][]$$
Cartesian product (37)

$$relation[R][S, k] := R \subseteq set[S][]\hat{\ } nnumber[k][]$$
k-tuple relation R on the set S takes only tuples that satisfy some relation (38)

$$property[P][S] := relation[P][S, 1] \subseteq set[S][]\hat{\ }1 = S$$
property P of the set S (39)

---

use defeq to keep the object typing instead of
ref-eq which reduces it to a propositional truth value.
But, how do I chain ref-eq tho? (40)

$$binaryRelation[B][S] = relation[B][S, 2] \subseteq set[S][]\hat{\ }2$$
$$xBy = \langle x, y\rangle \in B \tag{41}$$

$$domain[X][B, S] = \{x \mid \langle x, y\rangle \in binaryRelation[B][S]\} \tag{42}$$

$$range[Y][B, S] = \{y \mid \langle x, y\rangle \in binaryRelation[B][S]\} \tag{43}$$

$$field[F][B,S] = domain[X][B,S] \cup range[Y][B,S] \tag{44}$$

$$inverseRelation[B^{-1}][B,S] := \{\langle y,x\rangle \mid \langle x,y\rangle \in binaryRelation[B][S]\} \tag{45}$$

$$reflexive[B][S] := \forall_{x \in field[F][B,S]}(xBx) \tag{46}$$

$$symmetric[B][S] := \forall_{x,y \in S}(xBy \implies yBx) \tag{47}$$

$$transitive[B][S] := \forall_{x,y,z \in S}((xBy \land yBz) \implies xBz) \tag{48}$$

$$equivalenceRelation[B][S] := (reflexive[B][S] \land symmetric[B][S] \land transitive[B][S]) \tag{49}$$

$$\begin{gathered} equivalenceClass[[y]][y,B,S] := \{z \in field[F][B,S] \mid yBz\} \\ (equivalenceClass[[u]][u,B,S] = equivalenceClass[[v]][v,B,S]) \iff (uBv) \\ (equivalenceClass[[u]][u,B,S] \neq [[v]][v,B,S]) \implies ([u] \cap [v] = \emptyset) \\ \#\ \text{a set can be partitioned by equivalence classes} \end{gathered} \tag{50}$$

$$function[f][S] := (\forall_{x,y,z \in S}(\langle x,y\rangle \in f \land \langle x,y\rangle \in f) \implies (y = z)) \tag{51}$$