## **FPL Optimization Formulation**

The given data consists of:

Our core table, a  $525 \times 31$  dataframe consisting of player information including player name (cols 1, 2, and 3), total points, points per game, minutes played, and cost. The remaining twenty-four are dummy columns indicating which team (20 cols) and position (4 cols) a player played for in the 2021-2022 EPL Season.

The overall goal here is to maximize our expected points next season while staying within our cost constraint (under 800 cost units), team limit constraint (no more than three players from a given team), and position constraint (1 goalie, 3-5 defenders, 3-5 midfielders, and 1-3 forwards). The actual cost constraint is 1,000 units, with a total of 2 goalies, 5 defenders, 5 midfielders, and 3 forwards. But since we only field a lineup of 11 players most weeks in FPL, I thought it made sense to allocate 200 units for the 4 bench players which I discuss in part 2.

Of the given data, note that 'x' below represents the player total points, and 'p' represents the player total cost or price of a given player. A represents the goalkeeper column, b represents the defender column, d represents the midfielder column, e represents the forward column, and t represents each of the twenty columns of each team.

The problem formulation is the following:

Maximize 
$$c^T x$$

s.t:

 $c \in (0,1)$ 
 $c^T a = 1$ 
 $3 \le c^T b \le 5$ 
 $3 \le c^T d \le 5$ 
 $1 \le c^T e \le 3$ 
 $c^T t \le 3$  for each team  $t$ 
 $c^T p \le 800$ 

The decision variable is:

1. *C:* whether or not we select player *I* or not.

In plainer English, what is happening in the above formulation is:

1. We are maximizing the binary choice of selecting player *I* times the total point value player *I* provided over the last season.

- 2. We are subject to the constraint that we must choose one goalies (vector *a*), three to five defenders (vector *b*), three to five midfielders (vector *c*), and one to three forwards (*vector d*) in our starting XI. Note this will impact our bench choices later
- 3. We are also constrained by having no more than 3 players per team, and the total cost of the selected players must be less than or equal to 800 units (80m in the game currency).

Ultimately, I got the following player outputs from this:

‡ first ▼	second_name	web_name	total_points	points_per_game	minutes	▼ cost	team_name	position
Son	Heung-min	Son	258	7.40000	3009	120	Spurs	MID
Jarrod	Bowen	Bowen	206	5.70000	2987	85	West Ham	MID
Bukayo	Saka	Saka	179	4.70000	2978	80	Arsenal	MID
James	Maddison	Maddison	181	5.20000	2454	80	Leicester	MID
Trent	Alexander-Arnold	Alexander-Ar	208	6.50000	2853	75	Liverpool	DEF
Ivan	Toney	Toney	139	4.20000	2908	70	Brentford	FWD
João	Cancelo	Cancelo		5.60000	3227	70	Man City	DEF
Virgil	van Dijk	Van Dijk	183	5.40000	3060	65	Liverpool	DEF
Alisson	Ramses Becker	Alisson	176	4.90000	3240	55	Liverpool	GKP
Gabriel	dos Santos Maga	Gabriel	146	4.20000	3063	50	Arsenal	DEF
Matty	Cash	Cash	147	3.90000	3377	50	Aston Villa	DEF

Then, I did essentially the same formulation, except now I excluded the same players from the optimization problem, and had a budget constraint of 200 units. I also had to change the player constraints, since my starting XI already had max defenders. So I needed two forwards, one midfielder, and one goalkeeper. This problem was:

$$S.t:$$

$$c \in (0,1)$$

$$c^{T}a = 1$$

$$c^{T}b = 0$$

$$c^{T}d = 1$$

$$c^{T}e = 2$$

$$c^{T}t \le 3 \text{ for each team } t$$

$$c^{T}p \le 200$$

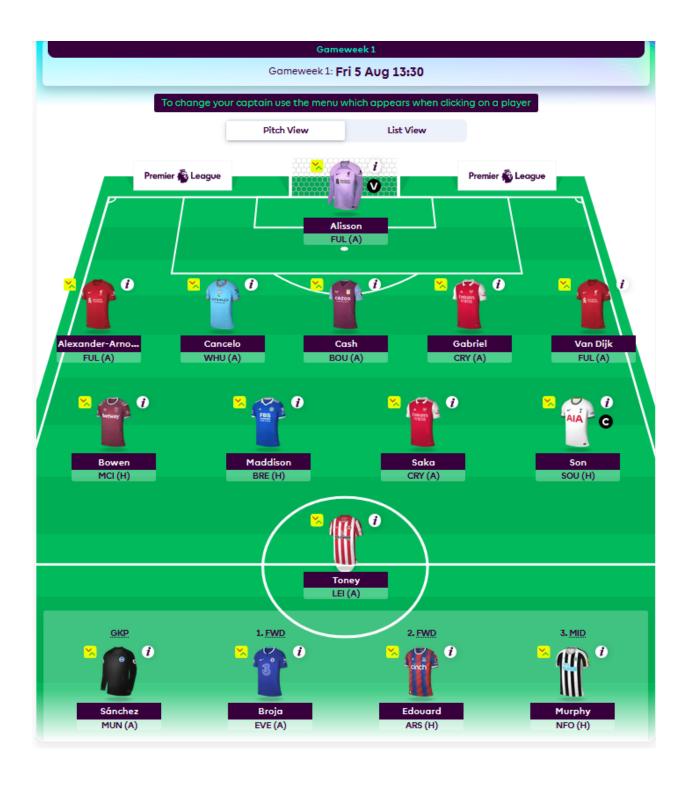
From this, my bench 4 are:

<sup>\*\*</sup>Technically the above formulation's team constraint could cause a problem, since I could have three players from the same team on my bench and starting XI in the optimizer, though this isn't allowed in the game. This wasn't a problem in my run, but it's a bug that could cause issues.

first_name	second_name	web_name	total_points	points_per_game	minutes	cost	team_name	position
Robert	Sánchez	Sánchez	126	3.40000	3330	45	Brighton	GKP
Armando	Broja	Broja	92	2.90000	1969	55	Chelsea	FWD
Odsonne	Edouard	Edouard	87	3.10000	1554	55	Crystal Palace	FWD
Jacob	Murphy	Murphy	62	1.90000	1457	45	Newcastle	MID

Pretty intuitive! Though I think Broja might not get a ton of time for Chelsea, despite his points last year.

So my final FPL lineup for now is:



Of course, this has plenty of limitations. It assumes that past performance is the best way to pick your team. And it also ignores some nuance in the season, like how flexible the team is to week-over-week changes in opponent difficulty. It also assumes that a \$20M bench budget is sufficient (I could have technically optimized that decision, but didn't for now). But overall, I think this is a decent-looking FPL team to start 2022!