Signal Processing

Homework Assignment 2 - Kalman filtering

PLEASE READ CAREFULLY: *In this lab you will implement a Kalman filter and test and compare its performance under various different scenarios. The software may be written in MATLAB. This lab can be done in groups of two or three. The submission of the lab report will contain an explanation of the design and implementation of the Kalman filter and a number of plots and answers to questions as requested. Students are also required to include a copy of their source code as a separate file with the submission. The submission due date is* **Dec. 7, 2015**.

Consider the following state space linear system:

$$x_{k+1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x_k + w_k. \tag{1}$$

where the noise w_k is i.i.d. Gaussian with mean $\begin{bmatrix} 0 & 0 \end{bmatrix}'$ and covariance matrix

$$Q = \mathbb{E}[w_k w_k'] = \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^2}{2} & T \end{bmatrix} q.$$

The system (1) can be used to model a target moving with approximately constant velocity, where the first component of x_k represents the position and the second component of x_k the velocity of the target [1].

In this lab we will consider the tracking of this target using multiple sensors, specifically the case of two sensors. Sensor 1 has scalar measurements

$$y_{1,k} = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k + v_{1,k} \tag{2}$$

and sensor 2 has scalar measurements

$$y_{2,k} = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k + v_{2,k} \tag{3}$$

i.e. each sensor has noisy measurements of the position only. The measurement noises $v_{1,k}$ and $v_{2,k}$ are assumed to be i.i.d. Gaussian and independent of each other, with zero means and variances $\mathbb{E}[v_{1,k}^2] = \sigma_1^2$ and $\mathbb{E}[v_{2,k}^2] = \sigma_2^2$.

Note: For this lab we will use the following values for the above parameters: $T = 0.1, q = 0.2, \sigma_1^2 = 0.2, \sigma_2^2 = 0.3$.

These two measurements are then sent to a fusion center which will combine them to estimate x_k. In this lab we consider two different ways in which these sensor measurements are combined.

Method 1: In method 1, the two measurements $y_{1,k}$ and $y_{2,k}$ are received perfectly by the fusion center, which then regards these two measurements as a single vector measurement

$$y_k = \begin{bmatrix} y_{1,k} \\ y_{2,k} \end{bmatrix}. \tag{4}$$

A Kalman filter is then used on y_k to estimate x_k.

Method 2: In method 2, the fusion center linearly combines $y_{1,k}$ and $y_{2,k}$ by adding them together to form y_k:

$$y_k = y_{1,k} + y_{2,k}. \tag{5}$$

A Kalman filter is then used on y_k to estimate x_k.

Question 1

Consider method 1, given by equations (1),(2),(3),(4).

(a) Write this system in the form

$$\begin{aligned} x_{k+1} &= Ax_k + w_k \\ y_k &= Cx_k + v_k \end{aligned}$$

i.e. specify what the A and C matrices are. Also specify the covariance matrices of the noise terms w_k and v_k.

(b) Implement and simulate the Kalman filter for this system in MATLAB. Assume that the initial condition x_0 is Gaussian with mean $\begin{bmatrix} 0 & 0 \end{bmatrix}'$ and covariance matrix Q. You may use the MATLAB function `mvnrnd` to generate multivariate Gaussian random vectors.

(c) On the same graph, plot $x_k^1, \hat{x}_{k|k-1}^1$, and $\hat{x}_{k|k}^1$, for $k = 1, \ldots, 100$, where the superscript 1 denotes the first component of the vectors.

(d) On the same graph, plot $x_k^2, \hat{x}_{k|k-1}^2$, and $\hat{x}_{k|k}^2$, for $k = 1, \ldots, 100$, where the superscript 2 denotes the second component of the vectors.

(e) On the same graph, plot the trace of the error covariance matrices, trace$(P_{k|k-1})$ and trace$(P_{k|k})$, for $k = 1, \ldots, 100$.

(f) What happens to $\hat{x}_{k|k-1}^1$ as $k \to \infty$?

(g) What happens to $P_{k|k-1}$ as $k \to \infty$? Write down the limiting value of the covariance matrix $P_{k|k-1}$, if it exists.

Question 2

Now consider method 2, given by equations (1),(2),(3),(5). Repeat Question 1(a)-(g) for this system.

Question 3

(a) Compare the trace of the error covariance matrices, trace$(P_{k|k-1})$, for methods 1 and 2.
(b) Now suppose that the fusion center does not receive the sensor measurements perfectly, but are corrupted in noise. Consider the following modified methods.

*Method 1**: In method 1*, the two measurements $y_{1,k}$ and $y_{2,k}$ are received by the fusion center together with additive noise. The fusion center thus has the single vector measurement:

$$y_k = \begin{bmatrix} y_{1,k} \\ y_{2,k} \end{bmatrix} + \begin{bmatrix} n_{1,k} \\ n_{2,k} \end{bmatrix}. \tag{6}$$

where $n_{1,k}$ and $n_{2,k}$ are i.i.d. Gaussians with zero means and variances $\mathbb{E}[n_{1,k}^2] = \mathbb{E}[n_{2,k}^2] = \sigma_n^2$. The system is thus given by the equations (1),(2),(3),(6). A Kalman filter is then used on y_k to estimate x_k.

*Method 2**: In method 2*, the fusion center receives the sum of $y_{1,k}$ and $y_{2,k}$ together with additive noise:

$$y_k = y_{1,k} + y_{2,k} + n_k \tag{7}$$

where n_k is i.i.d. Gaussian with zero mean and variance $\mathbb{E}[n_k^2] = \sigma_n^2$. The system is thus given by the equations (1),(2),(3),(7). A Kalman filter is then used on y_k to estimate x_k.

Suppose $\sigma_n^2 = 0.25$. Implement the Kalman filters for methods 1* and 2* and compare the trace of the error covariance matrices, trace$(P_{k|k-1})$. Does your answer differ from Question 3(a)?

References

[1] Y. Bar-Shalom, X. R. Li, and T. Kirubarajan, *Estimation with applications to tracking and navigation.* New York: John Wiley & Sons, 2001.