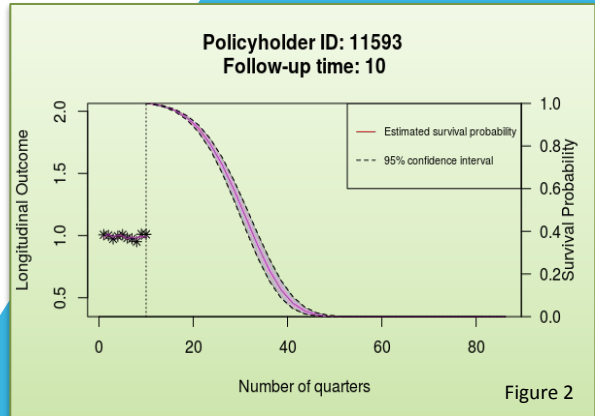


# A Joint Model for the Prediction of Policyholder Surrender

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## Abstract

Joint models for longitudinal and time-to-event data are of growing popularity in biomedical research. However, the utility of the overall approach is not limited to medical studies—in this project, we demonstrate a novel application of joint modeling techniques to a financial study, and present a joint model for policyholder surrender of variable annuity contracts.



## Joint Modeling

The prevailing approach for time-variant survival processes is to fit a time-dependent Cox model<sup>1</sup>. In 2012, Sweeting et al<sup>2</sup> showed that this approach under-estimates the effects of endogenous covariates. A viable alternative is to use these variables in longitudinal models of fixed and random effects, and to include the resultant fitted values for each subject's endogenous measure in a Cox Proportional Hazards model, yielding our Joint Model, which gives the hazard rate for subject  $i$  at time  $t$ :

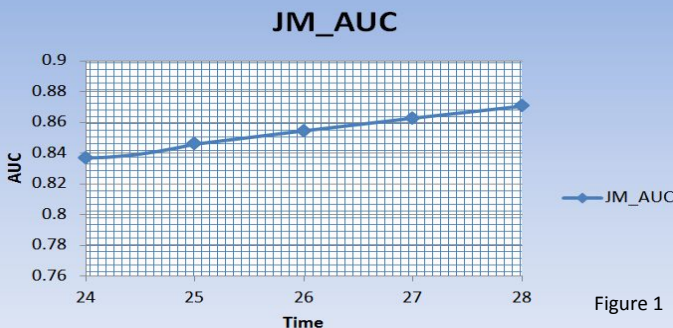
$$\lambda_i(t) = \lambda_0(t) \exp(x_i^T \cdot \beta + \alpha m_i(t))$$

Here,  $\lambda_0(t)$  is a ubiquitous baseline hazard rate,  $x_i^T$  are the observed values for subject  $i$  at censoring time,  $\beta$  is the associated coefficient vector,  $m_i(t)$  is the fitted value for subject  $i$ 's longitudinal response from the linear mixed model, and  $\alpha$  is the strength of effect of the longitudinal process on the hazard rate.

## Methods and Results

Using R packages “nlme,” and “survival”, a model of linear mixed effects using moneyness (guaranteed benefit/account value) as a response, and a Cox Proportional Hazards model for time-to-surrender were fitted, respectively.

Moneyness was determined in the LME to be a time-dependent longitudinal process ( $\beta = .01$ ,  $p = .0033$ ) over  $[0, 2]$ . Package “JM” was then used to consolidate the two into a joint model. The fitted values for moneyness demonstrate a negative effect on the surrender rate ( $\alpha = -.38$ ,  $p = .0138$ ) in the presence of other significant exogenous covariates.



## Discussion

Joint models allow us to evaluate each subject's survival probabilities dynamically as the longitudinal response fluctuates (Figure 2). However, moneyness is only one amongst many endogenous covariates that can be modeled in this fashion, and a multivariate approach is needed. Moreover, the intent of this project was to demonstrate a novel approach for modeling time-to-surrender.

## References and Acknowledgments

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