Project 2

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Contents

1 Introduction

1.1 Background and dataset

The objective of this report was to determine which covarites that can be used to predict if a US county has a low or high crime rate (per 1000 inhabitants). Data used to do this was county demographic information (CDI) for 440 of the most populous counites in the US 1990-1992. The record for each county includes data on the 14 variables listed below in table **??**. Counties with missing data has been removed from the dataset.

Table 1: iCDI dataset columns

Variable	Description
id	identification number, 1–440
county	county name
state	state abbreviation
area	land area (square miles)
popul	estimated 1990 population
pop1834	percent of 1990 CDI population aged 18–34
pop65plus	percent of 1990 CDI population aged 65 years old or older
phys	number of professionally active nonfederal physicians during 1990
beds	total number of beds, cribs and bassinets during 1990
crimes	total number of serious crimes in 1990
higrads	percent of adults (25 yrs old or older) who completed at least 12 years of school
bachelors	percent of adults (25 yrs old or older) with bachelor's degree
poors	Percent of 1990 CDI population with income below poverty level
unemployed	percent of 1990 CDI labor force which is unemployed
percapitaincome	per capita income of 1990 CDI population (dollars)
totalincome	total personal income of 1990 CDI population (in millions of dollars)
region	Geographic region classification used by the U.S. Bureau of the Census, including Northeast, Midwest, South and West

In order to measure crime rate, another varible called `crm1000` was added to the data set, descibing the number of serious crimes per 1000 inhabitants. Same thing with `phys`, with `phys1000`. Using `crm1000`, counties were divided into counties with high or non-high crime rate, using the median of `crm1000`, where counties with crime rate higher than the median were categorized as having a high crime rate. This crime status of the county was stored in another column called `hircrm`, which takes the value 1 if the county is a high crime county and zero if it is a low crime county. In this paper, this binary varible will be used as the dependent varible. This binary dependent value was then modelled using **logistic regression**.

1.2 Model

The logistic regression model used models the log-odds of a certain observation i as a linear combination of its covariates $X_{j,i}$ and parameters β_i, with together with an additive error ϵ_i. These error terms are assumed to follow a normal distribution and be indipendent, i.e. $\epsilon \sim N(0, \sigma)$ i.i.d.

$$\ln \frac{p_i}{1 - p_i} = \beta_0 + \sum_j \beta_j \cdot X_{j,i} + \epsilon_i \tag{1}$$

2 Analysis

2.1 The higrad model

2.1.1 Introduction

The first model considered had `higrads` as the sole covariate. In order to determine if there is a relationship between `hicrm` and `higrads` they are plotted against each other, see figure 1. Because `hicrm` is a binary varible it is very difficult to determine if there is a relationship by the pattern of the plot. In order to circumvent this problem a kernel smoother was added to the plot. The most smooth looking line was attained by setting the bandwidth to 20. The kernel curve was approximately of S-shaped, implying that a logistic model may be appropriate. Further, the S-shape is "downward facing", implying a negative β_1 . Furthermore the fitted model along with its 95 % confidence interval was added to the plot in figure 1.

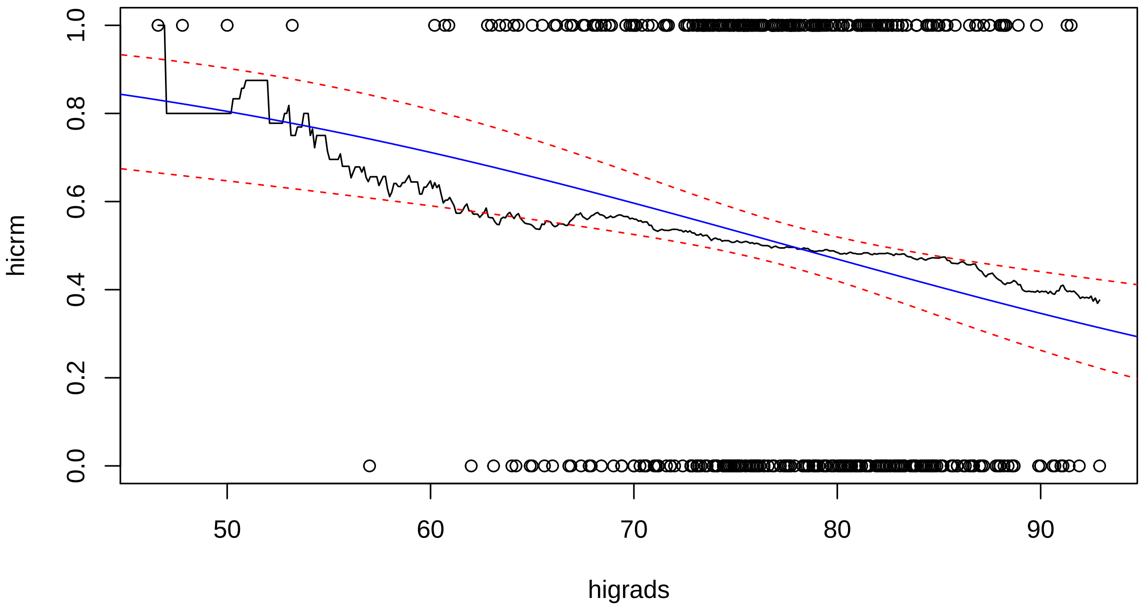


Figure 1: Plot of `hicrm` against `higrads`, including kernel smoothing and prediction of fitted model with 95 % confidence interval

NÅGOT OM ATT DEN INTE BESKRIVER JÄTTEBRA, BARA FRÅN 60% till 40% DÄR DET FINNS DATA?

As can be seen in figure 1, a higher number of higrads seems to make a county less probable to qualify as high crime county. Logically this makes sense. THe B!! values together with their 95 % confidence inteval is presented in table 2. Neither one of the B! confidence interval cover zero meaning that they are statistically significant at $\alpha = 0.05$. This is verfied by the very small P value.

The model becomes

$$\ln \frac{p_i}{1 - p_i} = \beta_0 + \beta_{higrads} \cdot X_{higrads,i} + \epsilon_i \quad (2)$$

2.1.2 Fitted model and significance

Table 2: β-values of `higrad` model, with 95 % confidence inteval

	Estimate	2.5 %	97.5 %	P-value
β_0	3.980	1.805	6.250	0.00044
$\beta_{higrads}$	-0.051	-0.080	-0.023	0.00041

If higrads increases 1%, odd decreases by 5% If higrads increases 10%, odd decreases by 40.1%

Table XX shows the odds ratio of having a high crime rate when the amount of higrads is increased by 1 and 10 % respectively. As can be seen in the table there is a substantial effect on the mentioned odds when the number of higrads is increased by 10 %.

TODO make table of odds ratio!!!! ODDS RATIO TYP INTE LIKA INTRESSANT HÄR, FÖR DEN ÄR SAMMA SOM decrease.one.percent

2.1.3 Model predictions

Using the higrads model,the probability, with confidence interval, of having a high crime rate in a county where the amount of higrads is 65 (percent), and where it is 85 (percent) is predicted. The result can be found in table XX.

Table 3: Test

Higrads	Probability (%)	2.5 %	97.5 %
6500	65.6	55.9	74.2
8500	40.6	34.1	47.6

2.1.4 Model performance analysis

Use the model to predict, for each of the counties, whether it would be expected to have a low or a high crime rate (predicted probability below or above 0.5) and calculate the sensitivity and specificity for this model.'

The sensitivity and specificity of the model is shown in table XX. (TODO make the table). As can be seen the higrad model does a rather bad job at correctly clasifying the the crime level status of the counties

Sensitivity is the proportion of the true successes that have been correctly classified as successes (true positive). Specificity is the proportion of the true failures that have been correctly classified as failures (true negatives).

Sensitivity was 55.5%

Specificity was 57.3%

2.2 The region model

2.2.1 Introduction

Next, a logistic model was adopted based on `region`. Since `region` is not continous , but categorial, it is modelled using "dummy variables" X_i. In order to implement this effectively, one of the categories is chosen as a reference variable, and the effects of other categories are measured in comparison to it.

In order to determine this reference variable - a cross-tabulation of the data between `region` and `hirm` is studied, see table 4.

Table 4: Cross-tabulation between `region` and `hicrm`

	Low crime	High crime
Northeast	82	21
Midwest	64	44
South	44	108
West	30	47

As a reference region, the one that has the largest number of counties in it's smallest low/high category was chosen. As a tie-breaker, the other low/high category was used. This approach produces the lowest standard error, and therefore highest significance. As seen in table 4, the above given condition results in choosing South as reference region.

Using this reference region, the logistic model becomes

$$\ln \frac{p_i}{1-p_i} = \beta_0 + \beta_{Northeast} \cdot X_{Northeast,i} + \beta_{Midwest} \cdot X_{Midwest,i} + \beta_{West} \cdot X_{West,i} + \epsilon_i \tag{3}$$

The β coefficients are measured relative to South and β_0 is log-odds coefficient for South.

2.2.2 Fitted model and significance

The model was fit with the given data set, estimating β_i, shown together with its 95 % confidence interval and P-value, in table 5.

Table 5: β-estimates for the `region` model, together with 95 % confidence interval and P-values

	Estimate	2.5 %	97.5 %	P-value
β_0	0.898	0.555	1.258	0.00044
$\beta_{Northeast}$	-2.260	-2.874	-1.682	0.00041
$\beta_{Midwest}$	-1.273	-1.800	-0.758	0.00044
β_{West}	-0.449	-1.025	0.131	0.00041

As may be seen in 5, the P-values for all of the β-estimates are less than 0.05, indicating statistical significance on a 95 % level.

Next, the odds-ratios for the different categories where determined. The odds-ratios measure the odds of a particular category in relation to the reference category. These may be calculated as $OR_i = e^{\beta_i}$ and are seen in table 6.

Table 6: Odds-ratios for the `region` model, together with 95 % confidence interval

	OR	2.5 %	97.5 %
Northeast	0.10	0.06	0.19
Midwest	0.28	0.17	0.47
West	0.64	0.36	1.14

As seen in table 6, the odds-ratios are less than 1 for all categories but the reference region. This implies that the odds for all regions are lower compared to the reference region, i.e. that the probability of a high crime rate is lower in all regions compared to the reference region. This can also be seen in table 4.

2.2.3 Model predictions

Using the fitted model, the probabilies of having a high crime rate, with confidence interval, for the different regions was determined, shown in table 7.

Table 7: Probability of high crime rate (%), together with 95 % confidence interval for each of the regions

	Probability (%)	2.5 %	97.5 %
Northeast	20.4	12.6	28.2
Midwest	40.7	31.5	50.0
South	71.1	63.8	78.3
West	61.0	50.1	71.9

2.2.4 Model performance analysis

In order to analyze model performance, the sensitivity and specificity of the model was calculated. The sensitivity of a model is the ratio of predicted positives to real positives in the dataset, while the specificity of a model is the ratio of predicted negatives to real negatives in the dataset. As such, the higher the value of the sensitivity and specificity, the better.

For the `region`, the sensitivity was 70.5%, while the specificity was 66.4%.

Comparing the two models analyzed so far, the `region` model performs better measured on sensitivity and specificity, as seen in table 8

Table 8: Comparison of sensitivity and specificity of `higrad` and `region` model

Covariate	Sensitivity (%)	Specificity (%)
Higrads	55.5	57.3
Region	70.5	66.4

2.3 Combined model and comparison

2.3.1 Introduction

Next a model that uses both `higrads` and `region` is analyzed. As such, this model becomes

$$\ln \frac{p_i}{1-p_i} = \beta_0 + \beta_{Northeast} \cdot X_{Northeast,i} + \beta_{Midwest} \cdot X_{Midwest,i} + \beta_{West} \cdot X_{West,i} + \beta_{higrads} \cdot X_{higrads,i} + \epsilon_i \tag{4}$$

2.3.2 Model comparison

In order to compare the models, metrics other than sensitivity and specificity may be studied. Some of these are AIC and BIC, which describe **WHAT?** and Nagelkerke psuedo R^2, which decribes **WHAT?**. As they are defined, AIC and BIC should be as low as possible for a model to be performant, while psuedo R^2 should be as high as possible.

Comparison between the models in regards to AIC, BIC and Psuedo R^2 are seen in figure 2, while comparison of sensitivity and specificity is seen in table 9.

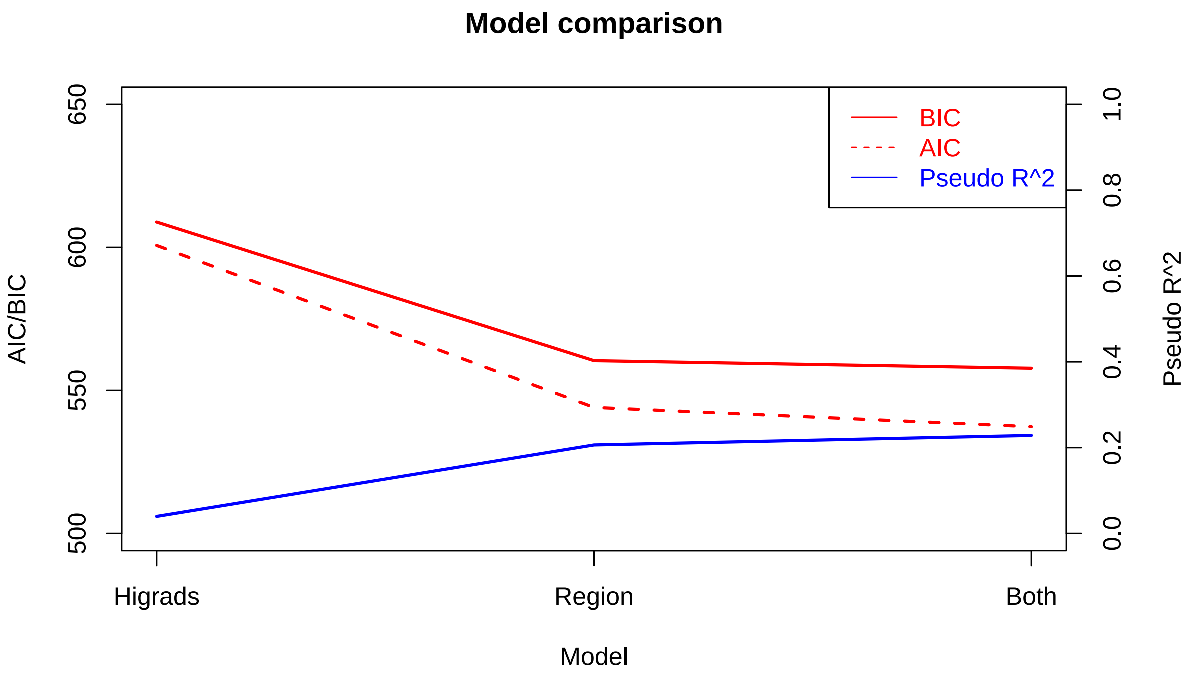


Figure 2: Comparison of AIC and BIC and Nagelkerke psuedo R^2 for the different models

Table 9: Comparison of sensitivity and specificity of models

Covariate	Sensitivity (%)	Specificity (%)
Higrads	55.5	57.3
Region	70.5	66.4
Both	70.5	67.3

As seen in figure 2 and table 9, the combined model with both the covariates performs the best on all the studied metrics.

2.3.3 Combined model performance

Performance of the combined model can be analyzed by studying a QQ-plot (see figure 3) the squared standardized Pearson residuals and the standardized deviance residuals against the linear predictor x^β (see figure 4). As well as the Cook's distance against the linear predictor, and against `higrads` and against `region` (see figure 5).

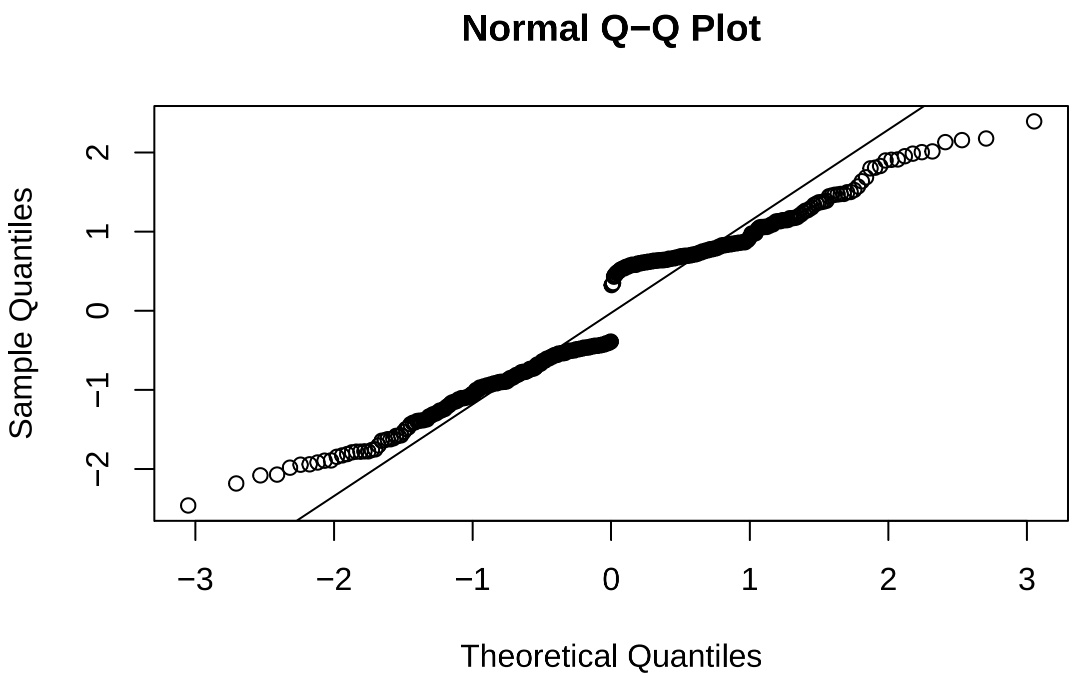


Figure 3: QQ-plot for the combined model

ANALYS HäR

Anything alarmin? Any interesting finds?

2.4 Interaction model

2.4.1 Introduction

As a forth model, interaction terms are also considered, building in that the effect of higrads may be different in different regions, where the log-odds in the model includes interaction terms such as $\beta_{Northeast*higrads} \cdot X_{higrads,i}$.

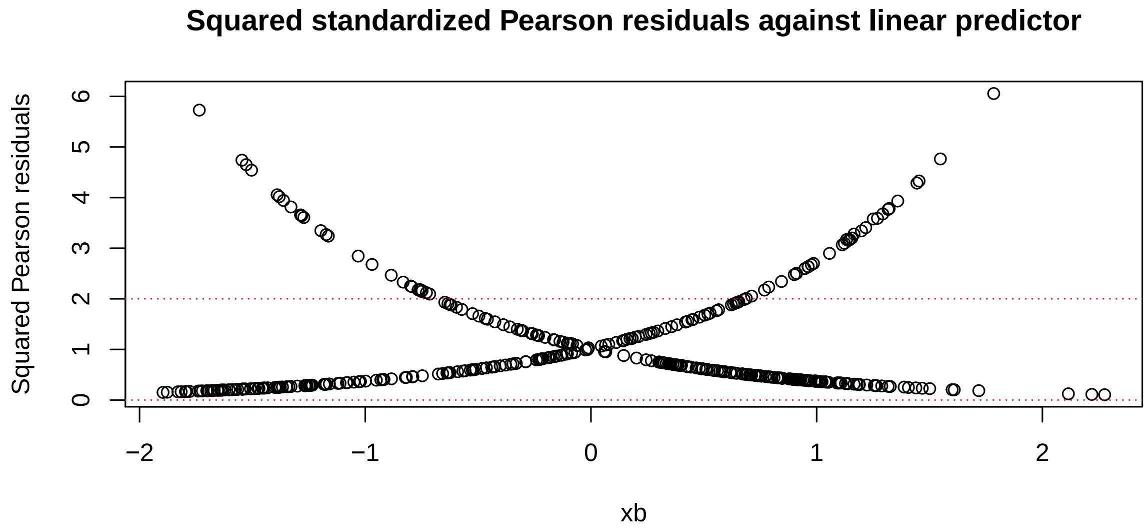
2.4.2 Model performance

The performance of the interaction model compared to the combined model may be analyzed by the likelihood test. This test is similar to a partial F-test, but adapted for logistical regression, using likelihoods since sums of squares are not applicable. The likelihood test results in the interaction model being significantly better than the combined model, with a P-value of 0.019.

In addition, the AIC, BIC, Nagelkerke, sensitivity and specificity is compared to the combined model, in table 10. Performance of the interaction model can be analyzed by studying a QQ-plot (see figure 6) the squared standardized Pearson residuals and the standardized deviance residuals against the linear predictor x^{β} (see figure 7). As well as the Cook's distance against the linear predictor, and against `higrads` and against `region` (see figure 8).

Table 10: Comparison of sensitivity and specificity of models

Covariate	AIC	BIC	Sensitivity (%)	Specificity (%)	Pseudo R2
Combined model	533	566	72	68	0.25
Interaction model	537	558	70	67	0.23



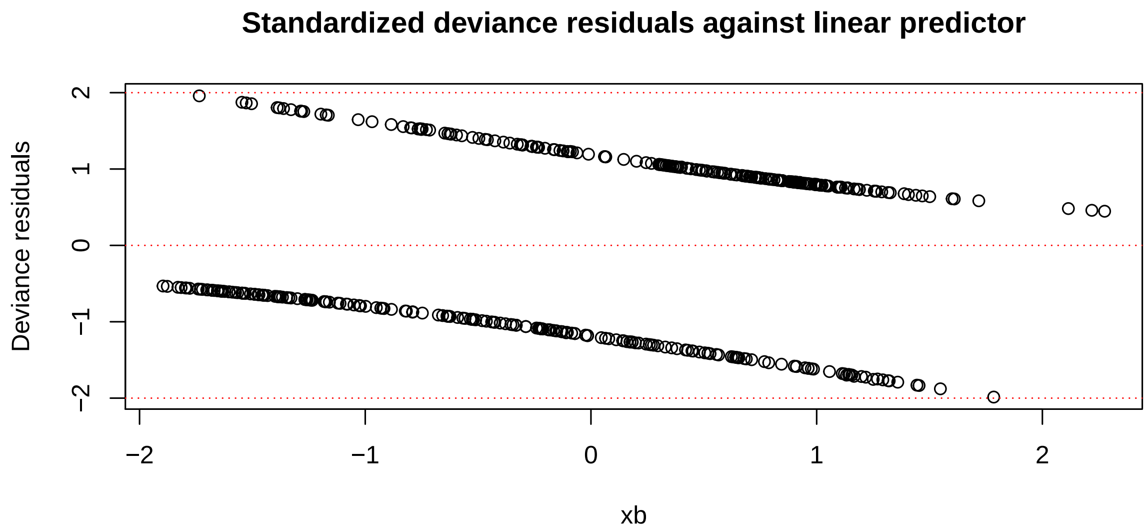


Figure 4: Squared standardized Pearson residuals as well as standardized deviance residuals for the combined model, against the linear predictor x^β

Both models perform better on some metrics, while performing worse on other. The interaction model has a worse AIC-value, but a lower BIC-value (KOMMENTERA). The sensitivity, specificity and Pseudo R^2 values are worse for the interaction model.

NÅGOT OM COOKS MM?

VILKEN ÄR BÄST?

2.5 Finding the optimal model

2.5.1 Methology

Next, an attempt to fit an optimal model to predict high crime rates is made, using the previous covariates, as well as `poors` and `pshys1000`. Interaction terms are ignored.

Models of increasingly complexity, adding more covariates are compared to each other on the used metrics, i.e. AIC, BIC, Pseudo R^2, sensitivity and specificity. In addition, the result of automatic selection using

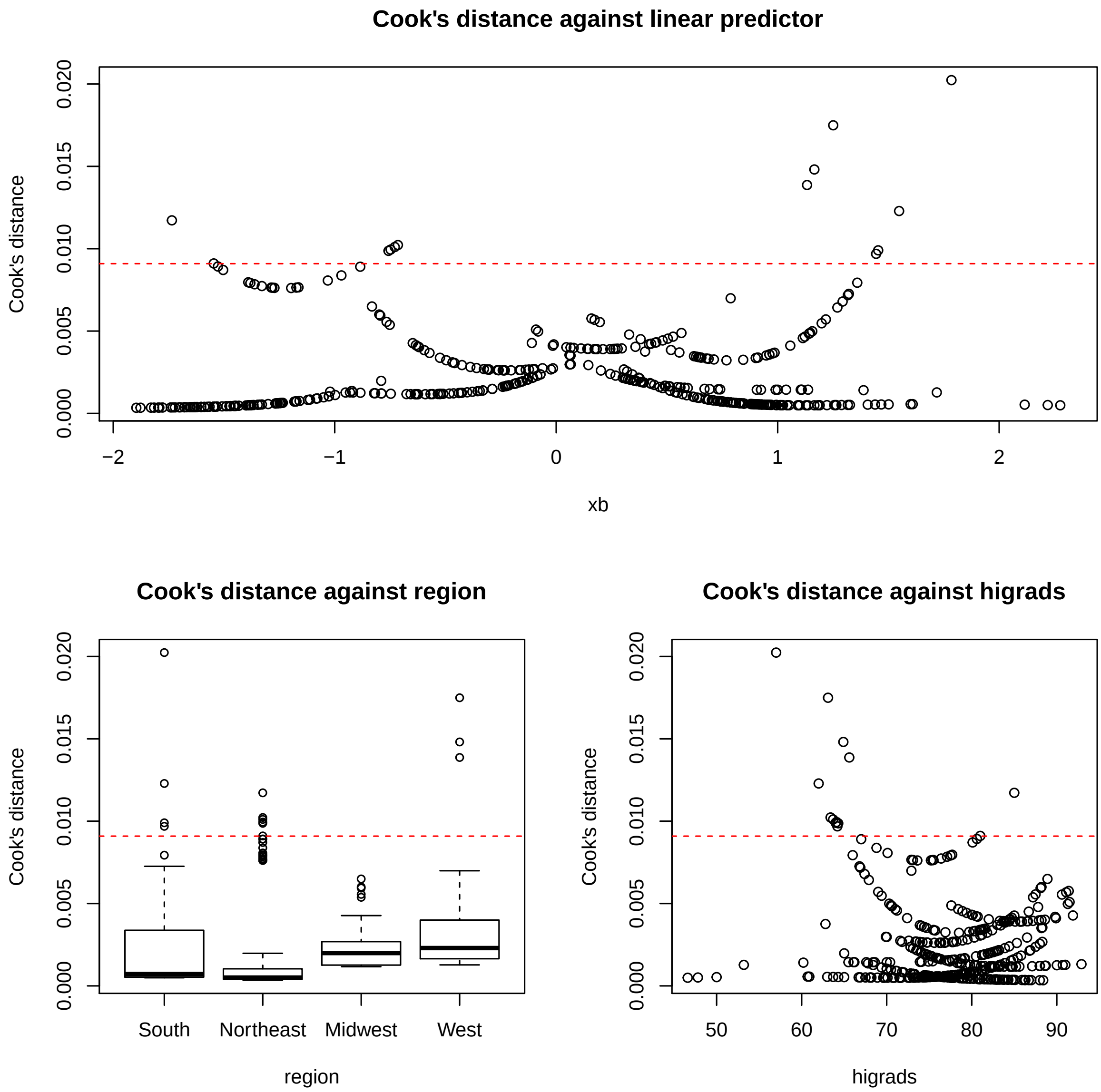


Figure 5: Cook's distance, for the combined model, against linear predictor, region as well as higrads

R `step` function is studyied.

2.5.2 Model comparison

AIC, BIC and Pseudo R^2 of the studied model are shown in figure 9. In addition, table 11 includes sensitivity and specificity for the different models.

Table 11: Comparison of sensitivity and specificity of models. Key: H = `higrads`, R = `region`, Po = `poors`, Phy = `phys1000`

Model	AIC	BIC	Sensitivity (%)	Specificity (%)	Pseudo R2
H	601	609	55	57	0.04
H + R	537	558	70	67	0.23
H + R + Po	494	518	73	75	0.34
H + R + Po + Phy	481	509	74	75	0.37
R + Po + Phy	480	504	75	74	0.37

Model	AIC	BIC	Sensitivity (%)	Specificity (%)	Pseudo R2
H + R + Phy	508	532	72	72	0.30

The results in 9 and 11 show that the `region` + `poors` + `phys1000` model performs best on most of the metrics. This result is also consistent with the `step` algorithm results. As such, this model is considered the **optimal model** for this problem.

2.5.3 Model performance

Performance of the optimal model is then analyzed by studying a QQ-plot (see figure 10) the squared standardized Pearson residuals and the standardized deviance residuals against the linear predictor x^{β} (see figure 11). As well as the Cook's distance against the linear predictor, and against `higrads` and against `region` (see figure 12).

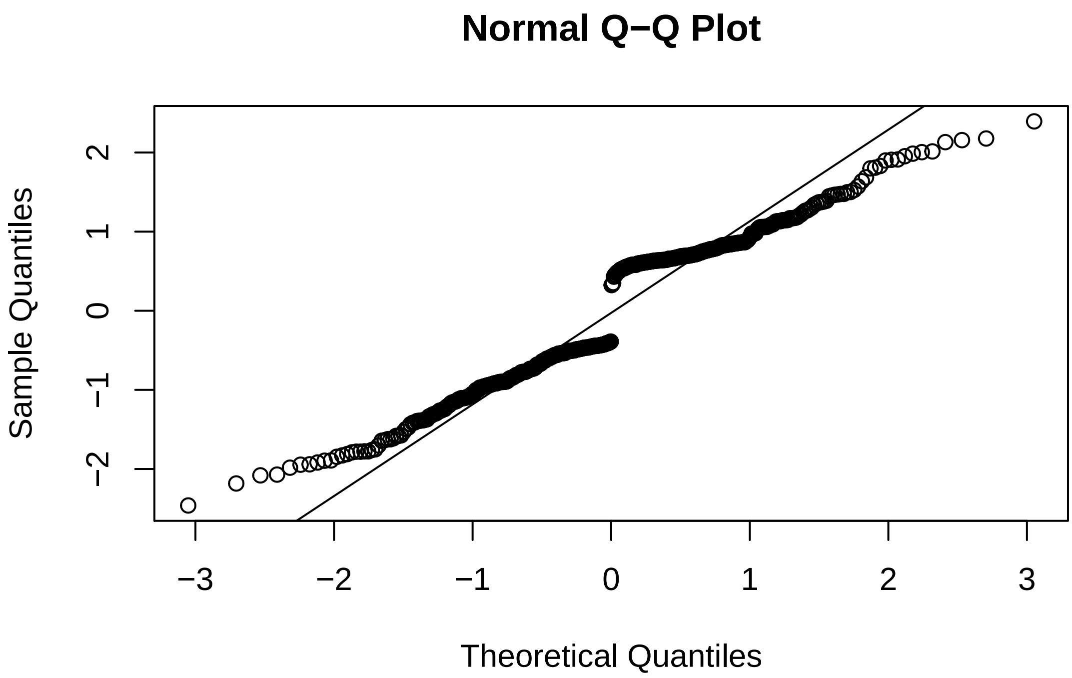


Figure 6: QQ-plot for the integration model

The outlier is Olmsted

2.5.4 Discussion

In order to first get a view on the different covariates and how they relate to each other, they are plotted against eachother in figure 13.

The optimal model includes the previously studied covariate `region`, but discards the `higrads` covariate. In addition, it includes the new `poors` and `phys1000` covariates. One explaination why `higrads` is not used in the optimal may be seen in figure 13, where there seems to be a high correlation between `higrads` and `poors`.

This hypothesis is tested in figure 2.5.4, where a linear regression model has been fit. Studying the P-value of the model reveals that the β-values are highly significant. \begin{figure}[h]

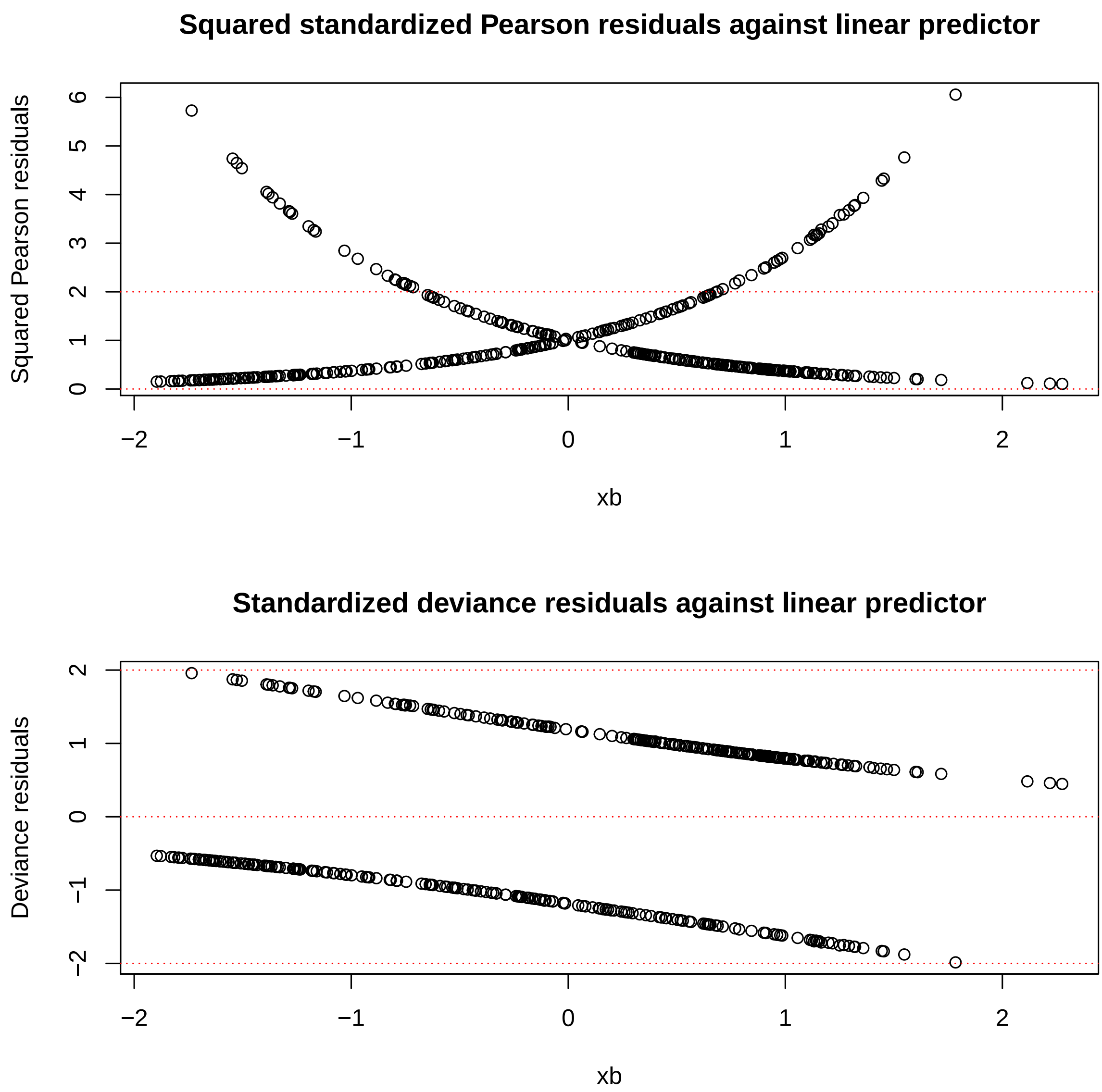
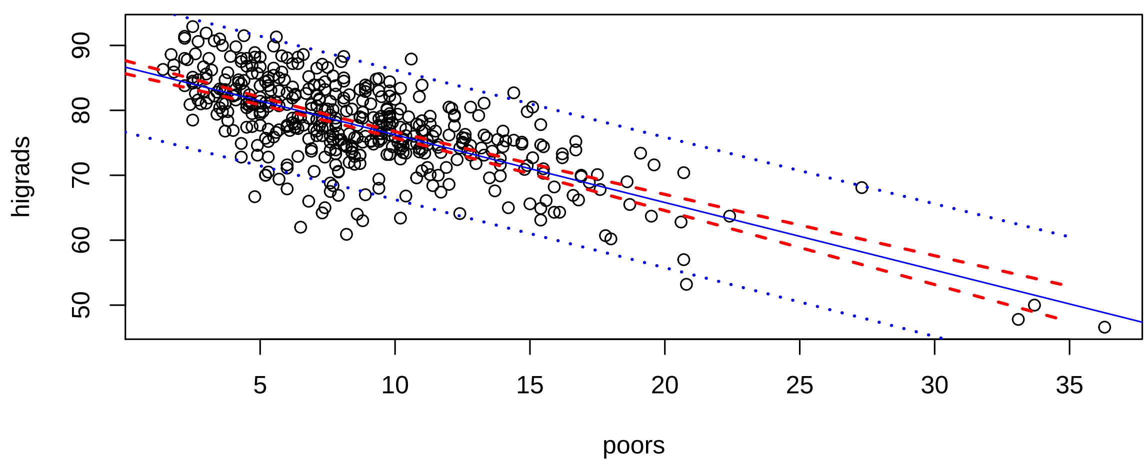


Figure 7: Squared standardized Pearson residuals as well as standardized deviance residuals for the interaction model, against the linear predictor x^{β}



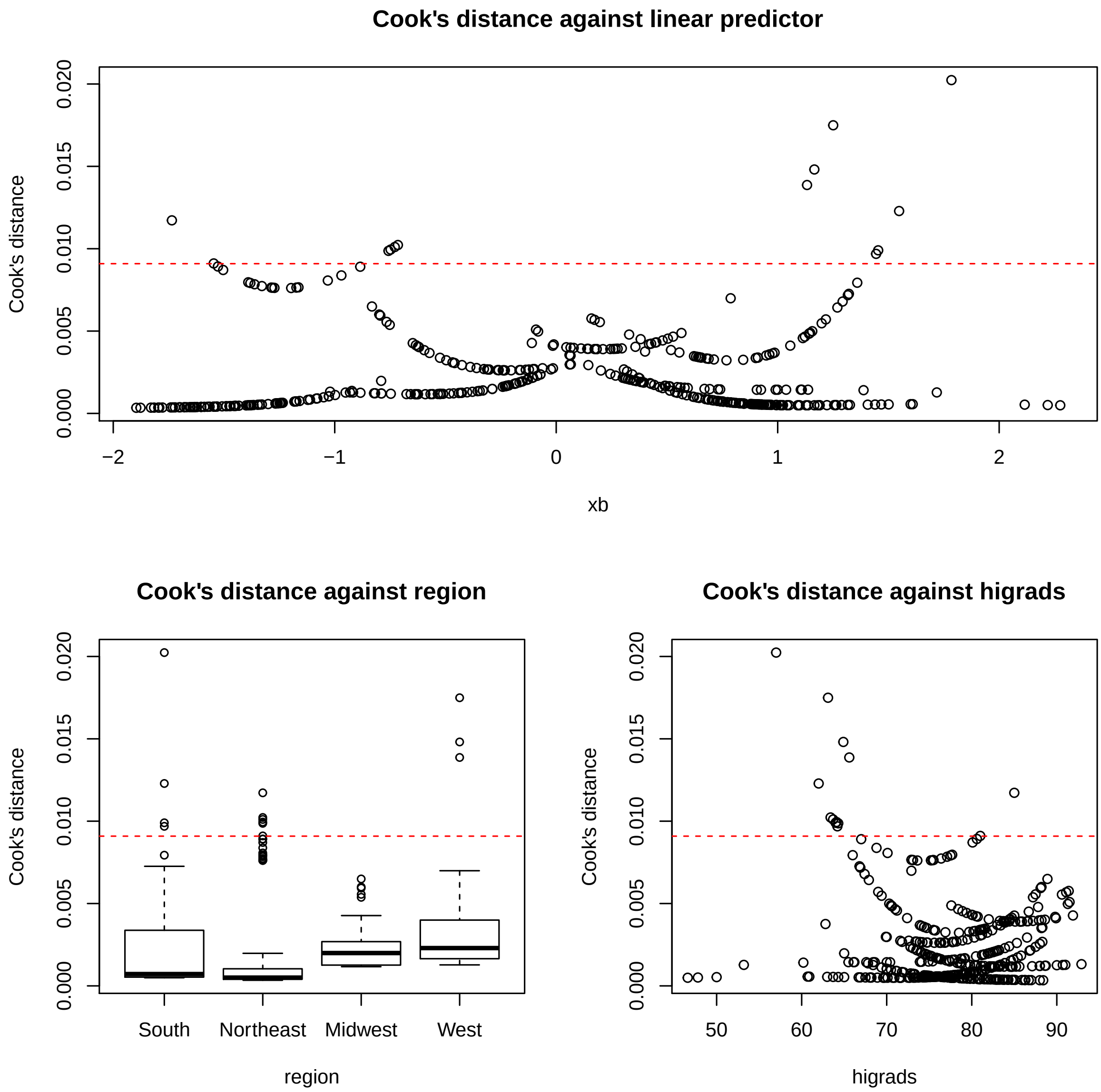


Figure 8: Cook's distance, for the interaction model, against linear predictor, region as well as higrads

\caption{Plot of `higrads` against `poors`, together with linear regression line, with 95 % confidence and prediction intervals} \end{figure}

Looking at how well `poors` predicts `hicrm` may be seen in figure 14. Here it may be seen that `poors` follow a more distinct S-shape, and that `poors` seems to split the dataset more distinctly between high and non-high crime rate, as it varies from ≈ 15, rather than the low separation discussed previously. As such,it seems that `higrads` and `poors` are highly correlated, but that `poors` better predict `hicrm` and is therefore better left in the model.

```
#>
#> Call:
#> glm(formula = hicrm ~ poors, family = "binomial", data = cdi)
#>
#> Deviance Residuals:
#>     Min       1Q   Median       3Q      Max
#> -2.4611  -0.9885  -0.2648   1.0397   1.6873
#>
```

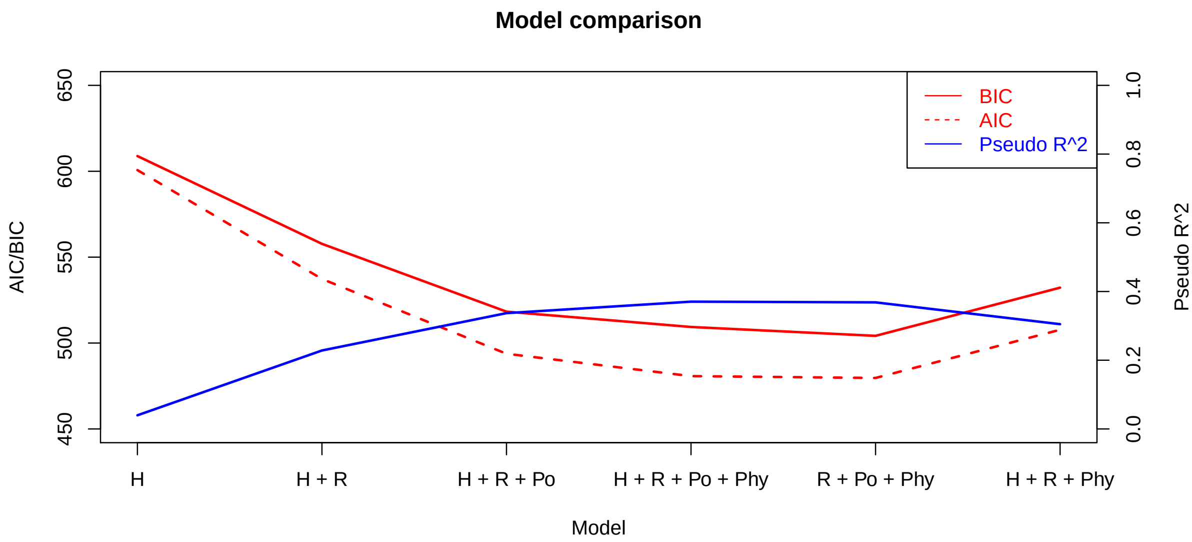


Figure 9: Comparison of AIC and BIC and Nagelkerke psuedo R^2 for the different models. Key: H = `higrads`, R = `region`, Po = `poors`, Phy = `phys1000`

```
#> Coefficients:
#>             Estimate Std. Error z value Pr(>|z|)
#> (Intercept) -2.04623    0.27545  -7.429 1.10e-13 ***
#> poors        0.24276    0.03126   7.765 8.14e-15 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#>
#> (Dispersion parameter for binomial family taken to be 1)
#>
#>     Null deviance: 609.97  on 439  degrees of freedom
#> Residual deviance: 526.19  on 438  degrees of freedom
#> AIC: 530.19
#>
#> Number of Fisher Scoring iterations: 4
```

Regarding `phys1000`, it appears in 13 that it does not have an as clear relationship to the other covariates and therefore provides more information to the model. Looking at how well `phys1000` predicts `hicrm`, seen in figure 15, it seems to follow an approximate S-shape and therefor contributes to the model.

```
#>
#> Call:
#> glm(formula = hicrm ~ phys1000, family = "binomial", data = cdi)
#>
#> Deviance Residuals:
#>     Min       1Q   Median       3Q      Max
#> -3.1344  -1.0989  -0.3058   1.1853   1.3977
#>
#> Coefficients:
#>             Estimate Std. Error z value Pr(>|z|)
#> (Intercept) -0.67608    0.19474  -3.472 0.000517 ***
#> phys1000     0.32757    0.08466   3.869 0.000109 ***
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#>
```

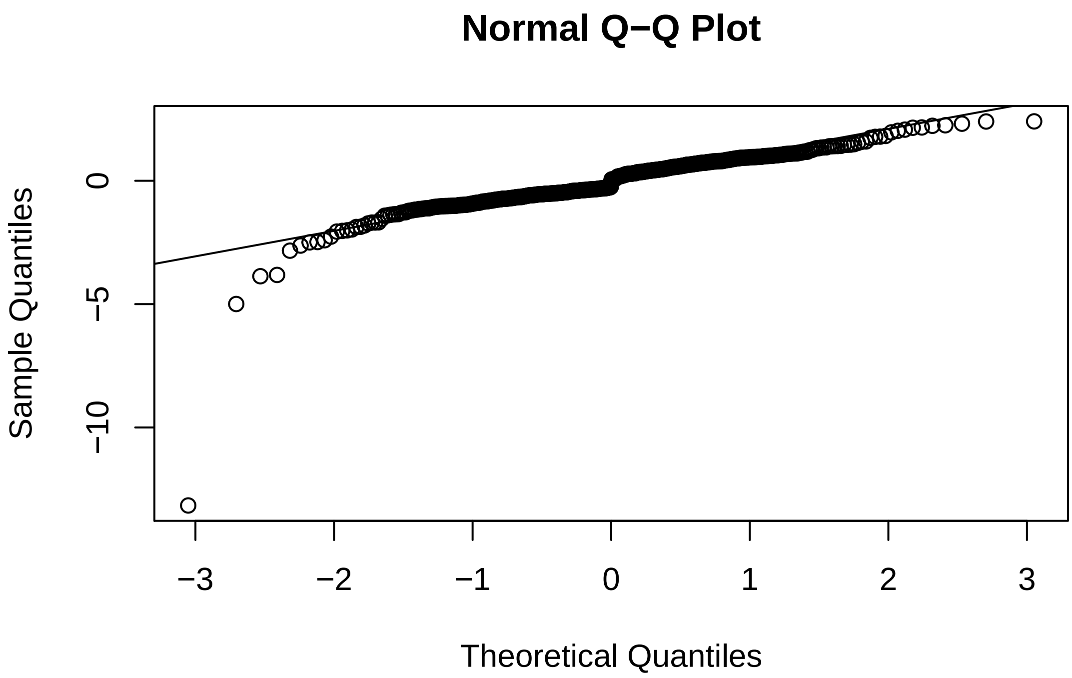


Figure 10: QQ-plot for the optimal model

```
#> (Dispersion parameter for binomial family taken to be 1)
#>
#>     Null deviance: 609.97  on 439  degrees of freedom
#> Residual deviance: 591.10  on 438  degrees of freedom
#> AIC: 595.1
#>
#> Number of Fisher Scoring iterations: 4
```

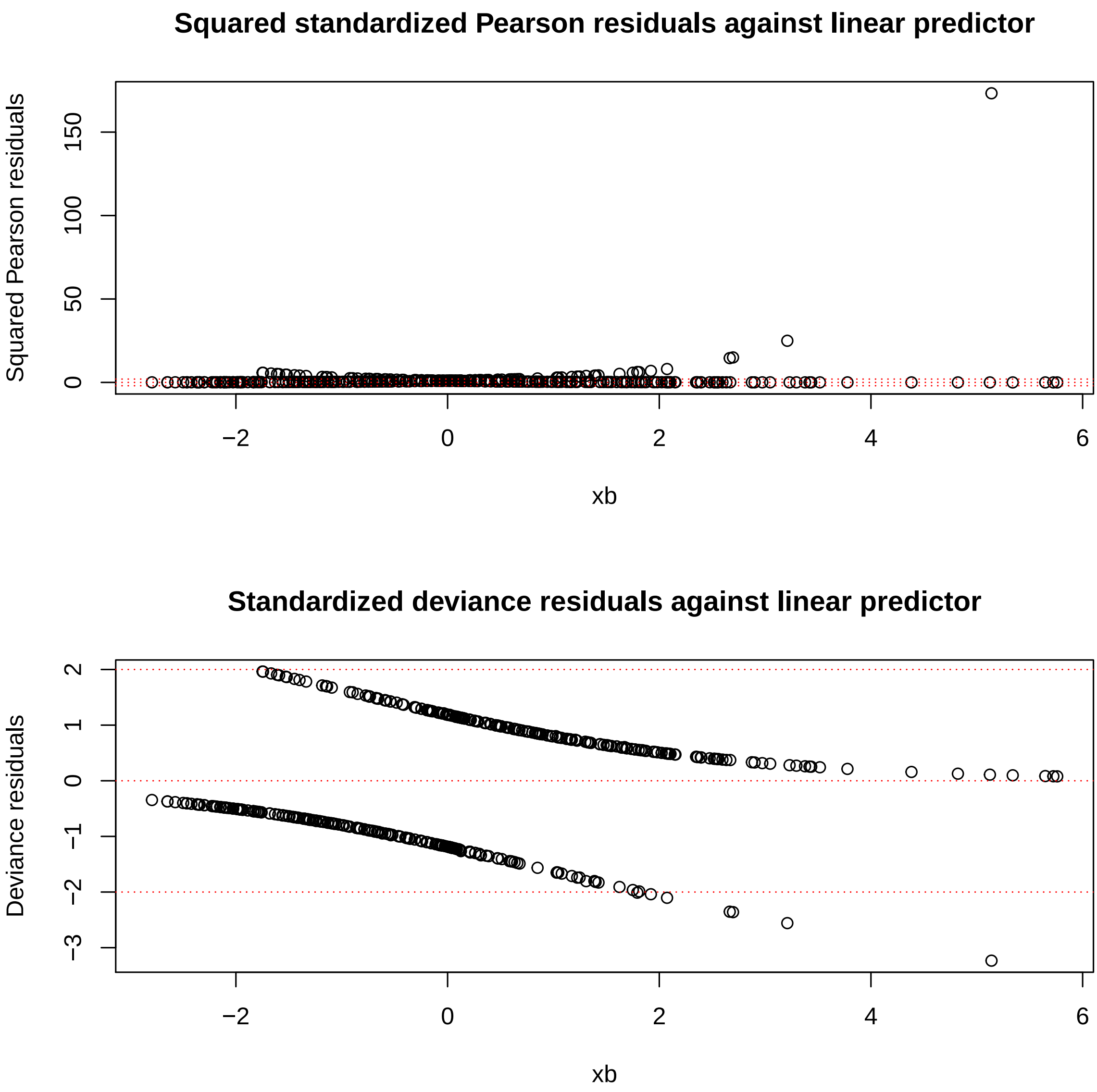


Figure 11: Squared standardized Pearson residuals as well as standardized deviance residuals for the optimal model, against the linear predictor x^{β}

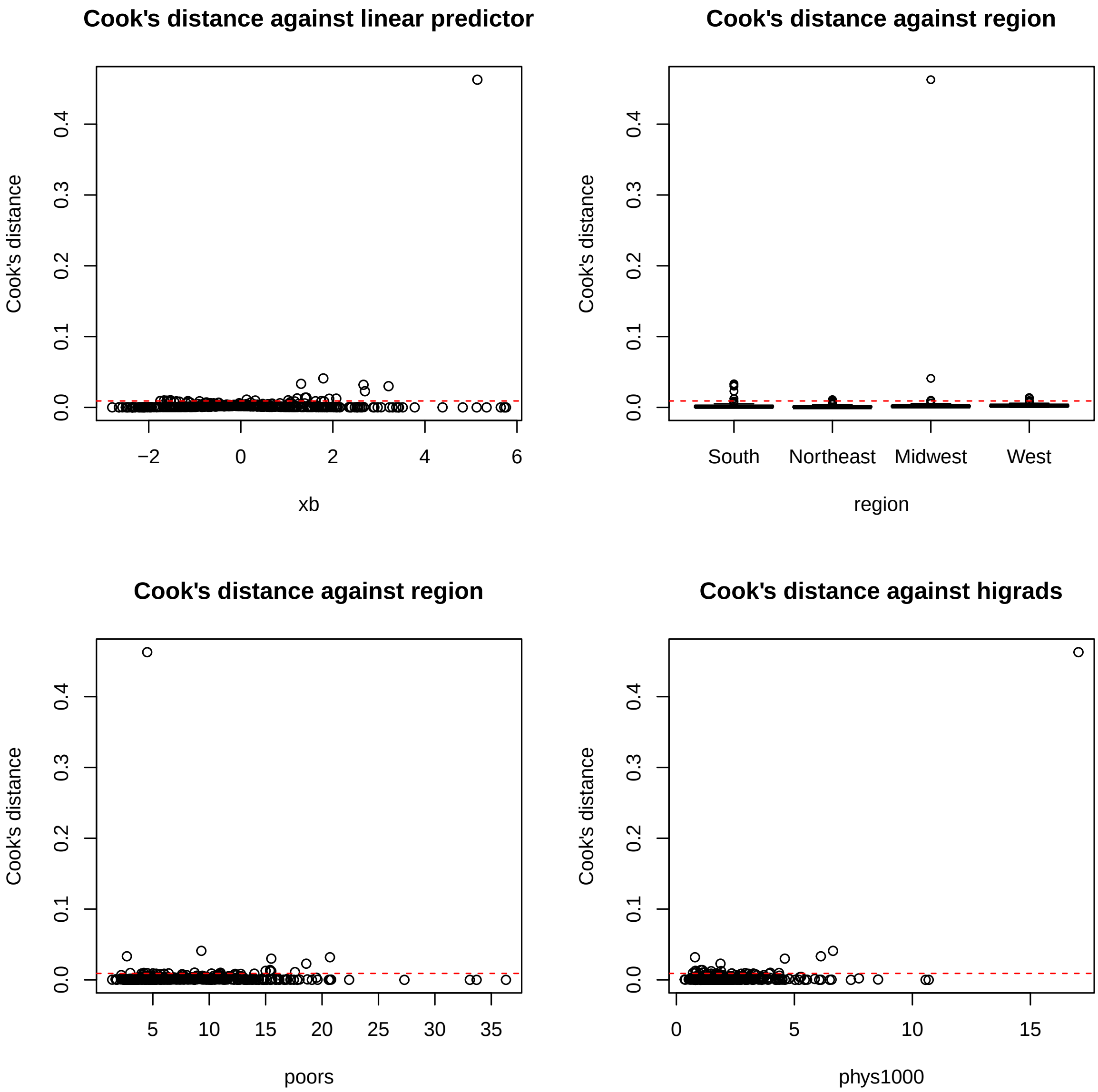


Figure 12: Cook's distance, for the optimal model, against linear predictor, region as well as higrads

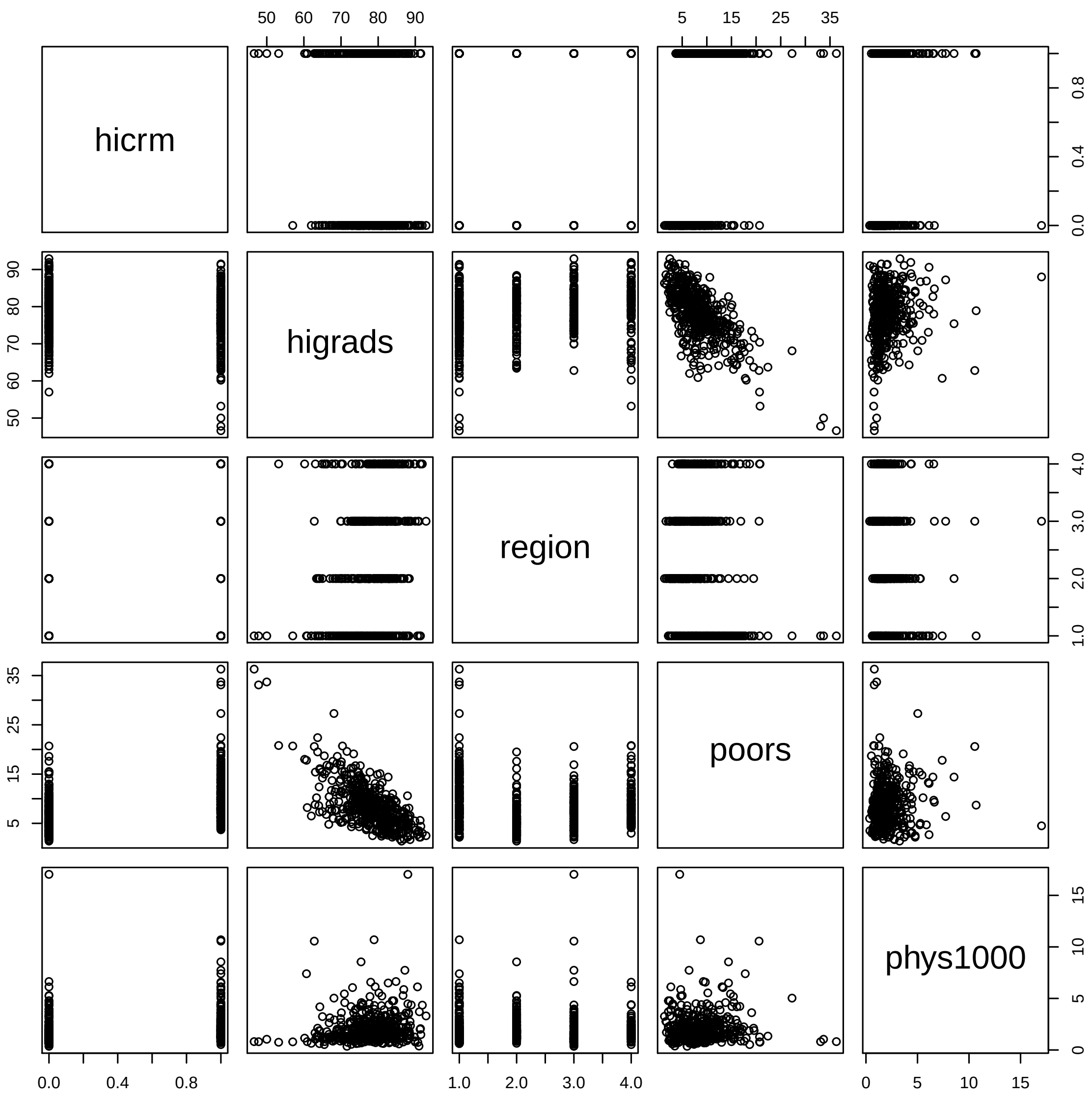


Figure 13: Plot of covariates against eachother

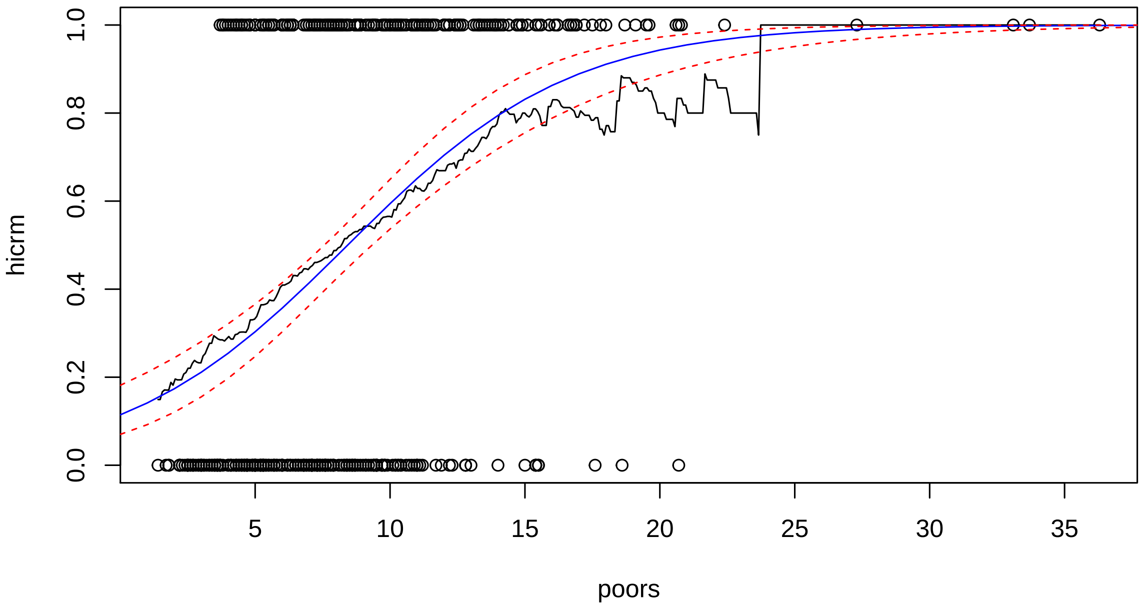


Figure 14: Plot of `hicrm` against `poors`, including kernel smoothing and prediction of fitted model with 95 % confidence interval

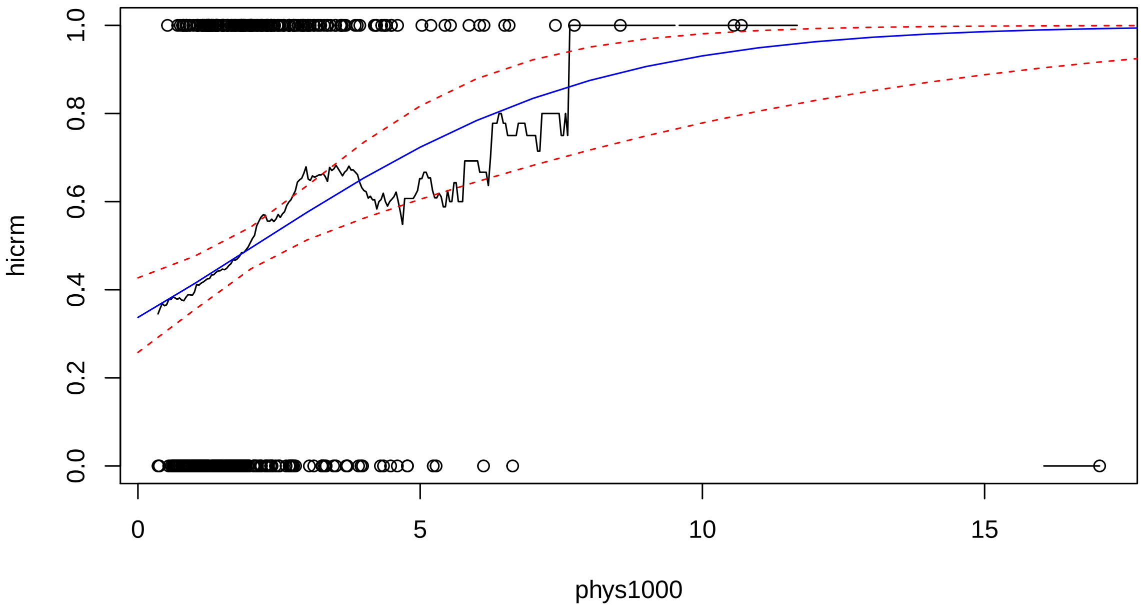


Figure 15: Plot of `hicrm` against `phys1000`, including kernel smoothing and prediction of fitted model with 95 % confidence interval