

# STAT 221 Final Project - Mammoth Snow Depth

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# 1 Introduction

## 1.1 Background

Mammoth Mountain is situated in Northern California and is known for its great alpine ski and snowboarding conditions. For this purpose, the mountain features a ski resort with the same name. This resort has more than 3,500 acres of ski-able terrain and is serviced by 28 lifts, receiving over 1 million annual visitors. For Southern Californian residents especially, the mountain is of interest as it is one of the closest high-quality ski resorts, about a 4-6 hour drive away from Los Angeles. [1]

Most people familiar with alpine sports consider snow depth one of the most important conditions for the sport, as this affects which runs are open and how “good” the skiing is considered. As decisions about travelling to a ski resort generally are done in advance and require some planning (e.g. booking a cabin), being able to predict future conditions are useful to the alpine skier. In addition, the entire ski economy of a mountain such as Mammoth, including the resort, workers as well as restaurants and stores in the surrounding city, face uncertainty over how many visitors the mountain will get a given week or year as this drives revenue. As visitorship is likely correlated with snow depth, forecasting it would be an important tool also for those stakeholders. This paper will attempt to do just that: **model the snow depth at Mammoth mountain in order to make predictions on future skiing conditions.**

## 1.2 Data set

Data on historic snow depth at Mammoth were obtained from the reporting of Mammoth Mountain Ski Area, through a third party website [2]. The website does not provide the data easily in a downloadable format, hence the data was obtained through injecting JavaScript into a browser client that extracted the data from the browser’s JavaScript environment and printed it in a JSON format. This raw data is shown in figure 1, featuring 1791 recordings from 2011-12-01 through 2020-03-02 of daily snow depth measured in inches. Upon looking at the graph, two issues with the raw data are found: First, the dates in the off-season (outside of the winter months) are not included, rather the years are concatenated together in a single time-frame. Second, some values within the recorded period are missing and reported as 0. Hence, the data needed to be further processed and cleaned before fitting any models on it.

## 1.3 Cleaning data

The first step in cleaning the data was to include the missing dates in order to capture the full time-frame of the series. The next step was handling the missing values, both in the off-season as well as the missing recorded values. In order to deal with the missing recorded values, as well as to make lower variance predictions on snow depth further in the future than a couple of days, the data was aggregated and averaged (disregarding the missing values) per week, resulting in a weekly time series. This week was defined as starting a Saturday, as this is a day of interest for many weekend skiers. The off-season missing values were replaced by 0s, as this is an accurate description of the snow depth during those months – there rarely is any snow on Mammoth during the summer. The resulting data after cleaning is shown in figure 2, featuring 431 weeks. The availability of data per month is displayed in figure 3, showing that data exists for the most part Dec-May, with less than 50% Jun-Oct, again reaching ~80% in November.

# 2 Analysis

Looking at the graph in figure 2, it is clear that the ski season of 2020 has a far lower snow depth than prior years, an unfortunate fact for skiers this season. In order to study other properties than this obvious observation, time-series methods were applied.

## 2.1 Series properties

Figure 4 plots the ACF and PACF of the series. The ACF shows periodic behaviour with a 1 year period that appears to be slowly tailing off, while the PACF has 2 (barely) significant values and then cuts off, followed by a significant value at lag 1 year. This implying that a seasonal AR model may be a good description of the series. The 1 year period is easily seen also in the periodogram shown in figure 5, together with a 4 year period, both being significant peaks. The 4 year period may be an artifact of the data being recorded for 8 years and these years having a pattern of yearly depth by random chance.

The data does not appear to be stationary as it has an obvious yearly trend and a clear pattern of snowing and thawing in the beginning and end of seasons. As such, the data must first be detrended before ARIMA models can be applied.

## 2.2 Detrending

The most obvious trend in the data that may be removed is the 1 year seasonal trend. This trend can be seen in figure 6 which shows the average snow depth during the different weeks of the year (week 1 through week 53). This graph has a clear sinusoidal shape. As such, a first attempt at detrending the data was made by subtracting the average snow depth of the week of year to every data point. The result of this may be seen in figure 7. Looking at the graph, it does however not appear to be stationary. One artifact of this detrending method is that the different years experience different levels of snow, causing years with less snow to have a clear negative valley and years with more snow to have a positive peak. Looking at the snow depth by week of the year for all years simultaneously, seen in figure 8, makes it clear that the snow level is very different each year and that the average hence is not a good predictor.

One way to mitigate this fact is to study values in relation to the peak snow level of the year. The assumption one makes is that the dynamics that determine what the peak snow depth, or how high the snow peak will be a given year, is different from the dynamics that govern how its snowing and thawing during the year, dividing the analysis into a macro and micro model. With this approach, one model could be used to predict peak snow depth a given year, e.g. assuming that they are independent and follow a certain distribution. Under this assumption, predicting the snow depth for the rest of a season may be done by only looking at how the current and past values have been in relation to the peak snow depth during the year, which is the approach implemented in this paper. Figure 9 shows the snow depth relative to the peak value of the season, from this the average of the week of the year of every data point is subtracted in order to remove seasonality. This model assumes that the dynamics of when and how much it snows is similar from year to year. The resulting detrended graph is shown in figure 10 both with the off-seasons as 0s, as well as with with missing values omitted, all ski seasons shown next to each other. This series finally looks fairly stationary, while still having some features that can modelled with time-series methods. As the series however appears to still have an upwards linear trend, the final detrended data is created by removing this linear trend, shown in figure 11.

As for the missing values in the data set, there are multiple ways to handle them, the most obvious being either setting them to 0 or removing those dates from the data. As the length of the seasons differ from year to year, removing the off-seasons from the data makes seasonality measures such as the periodogram harder to interpret. On the other hand, including the 0s makes the variance in the data vary significantly as it is 0 in the off-season, which is not consistent with the ARIMA assumptions.

## 2.3 Detrended series properties

Figure 12 shows the ACF and PACF of the detrended series. The ACF is tailing off faster for this series, however still with a peak after one year, implying that the yearly feature has not been fully taken out. The PACF only has a single (barely) significant value then cuts off. This implies that a low order AR model may be appropriate, again potentially with a yearly seasonal component. The periodogram, seen in figure 13, has 5 significant peaks (in blue), with period  $\sim 2.1$ ,  $\sim 0.69$ ,  $\sim 0.44$ ,  $\sim 0.28$  and  $\sim 0.20$  years. Figure 14 shows

that the peaks has a clear pattern, the first having frequency  $\sim 0.48$ , and then being fairly evenly spaced. This means that the detrended series still has periodic components to it, however the frequencies are not easily interpretable as to what they describe. Figure 15 plots the parametric periodogram, showing the periodogram of the fitting the best fitting AR(p) model. For the detrended series, an AR(2) fit the best, having no clear peaks but a low frequency spectrum. These two periodograms both tell us is that the noise in the data is mostly low frequency, and therefore fairly smooth.

## 2.4 Model fitting

In order to fit a (seasonal) ARIMA model to the detrended data, different models with different values of  $p$ ,  $q$  and  $d$  as well as seasonal components were compared and evaluated based on their AIC-value, starting with bigger more complex models and lowering the complexity as long as terms are insignificant. This process yielded an SARIMA(2,0,1)(0,0,1)[52] model as the optimal model choice. This yearly seasonal model fitting better than a simpler ARMA(2,0,1) implies that there is still some seasonality in the data after detrending. In mathematical terms, this SARIMA(2,0,1)(0,0,1)[52] model can be written as:

$$\begin{aligned}\phi(B)x_t &= \theta(B)\Theta(B^{52})w_t \implies \\ (1 - \phi_1 B - \phi_2 B^2)x_t &= (1 + \theta B)(1 + \Theta B^{52})w_t \implies \\ x_t - \phi_1 x_{t-1} - \phi_2 x_{t-2} &= (1 + \Theta B^{52} + \theta B + \theta \Theta B^{53})w_t \implies \\ x_t &= w_t + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \theta w_{t-1} + \Theta w_{t-52} + \theta \Theta w_{t-53}\end{aligned}$$

For the fitted optimal model we have coefficients shown in table 1, all being significant.

	$\phi_1$	$\phi_2$	$\theta$	$\Theta$
Estimate	1.4321	-0.5351	-0.3756	0.1261
S.E.	0.1595	0.1381	0.1766	0.0549

Table 1: The coefficients of the fitted optimal SARIMA(2,0,1)(0,0,1)[52] model

## 2.5 Model interpretation

The low order of the AR and MA components indicated that the process has a “short memory”, i.e. that the snow depth a given week primarily depends on the snow depths the two weeks prior, as well as the snow depth last year at that time. The  $\phi_1$  coefficient being positive  $> 1$ , while the  $\phi_2$  being negative can be interpreted as the momentum of the previous week being preserved, while the momentum of the week before counteracts it. Additionally, the  $\phi_1$  and  $\phi_2$  coefficient adding to one can be interpreted as the snow being a weighted average of the last two weeks. The  $\theta$  coefficient being negative implies that shocks, e.g. snow falls/thaws one week will have the opposite effect the previous week.  $\Theta$  being positive and fairly small means that snow depth across years are connected and positively correlated.

## 2.6 Model evaluation

The performance and goodness of fit of the model can be evaluated in two main ways: studying the residuals to see how well the data conforms to the assumptions of the model, and looking at predictions of the model.

### 2.6.1 Residual analysis

In figure 16, several plots showing different characteristics of the residuals are shown: the standardized residuals, the ACF of residuals, a normal Q-Q plot and p-values for the Ljung-Box statistic, for the detrended data with missing values set to 0. Looking at the standardized residuals in the plot, it is clear that the

residuals does not fully fulfil the assumptions, as many have an absolute value greater than 2, and multiple even greater than 4 which would be improbable if they were normally distributed. There also seems to be some pattern left in them. The ACF has no significant peaks and hence there should be no clear dependence between residuals, this is confirmed by the p-values of the Ljung-Box statistic, where only one dips below the significance level. The normal Q-Q plot shows that the residuals do not appear to be normal, as many are diverging from the normal quantile line.

One reason for the standardized residuals diverging so far from the normal assumptions is that the data, and with it the residuals, contains a lot of 0s which can be seen in figure 17, showing a histogram of the residuals compared to a sampled normal distribution. These zeroes lower the variance estimate, making the standardized residuals bigger. One way to mitigate this problem is to train the model on a data set excluding the missing values, concatenating the seasons together. The residual statistics of this model is shown in figure 18. For this model, the standardized residuals are still bigger than expected by the normal assumption but to a lesser degree. The ACF has one significant value, but this could be a statistical fluke. The Ljung-Box looks better and the QQ plot similar to the previous model. In figure 19 plotting a histogram of the absolute values of the residuals compared to a sample normal distribution, the residuals more closely resemble a normal distribution than the model with missing values included. In the histogram, it is clear that the residuals have more “low” values as well as “high” values and fewer in-between, compared to the normal distribution. This could reflect the two dynamics seen in the data in figure 2, where it snows and thaws aggressively in the beginning and end of the season, with smaller differences within the season.

Looking at the periodogram of the residuals, shown in figure 20, it has some peaks however they do not appear significant. Further, a parametric periodogram results in an AR(0) model, shown in figure 21. Hence, the periodogram resembles that of white noise, implying that the model captures the features in the data and that there are few features left in the residuals.

In conclusion, the model does not completely meet the assumptions of the model, but is close to doing so. As a result, it should make fairly accurate predictions, however the uncertainty is likely bigger than predicted by the model.

### 2.6.2 Prediction

Using the trained model, predictions were made for the coming 30 weeks, results (after undoing the detrending) shown in figure 22. The model predicts that the snow depth will increase, which doesn’t seem consistent with the trajectory of the current season at first glance. However, looking at this time of year previous years (shown in dashed gray lines), the snow level tends to increase after this time of year. After the peak, the snow depth is expected to decrease quickly, consistent with the dynamics of the ski season. The standard errors are fairly small, implying a confident prediction, however as previously discussed this may not actually be the case.

Figure ?? shows the prediction of the model looking back 10 weeks, i.e. how it would predict this time of year if we were looking at data from earlier in the season, with the actual outcome shown in a dashed line. The model does not accurately predict the outcome of the season, as it predicts it going down rather than up. However the actual observation mostly fits within the two standard deviation area.

## 3 Conclusion

In this paper, time series methods were used to analyze and predict the snow depth at Mammoth mountain. An issue with the data-set that had to be handled in order to successfully apply the methods on it was missing values, primarily in the off-season but also to some extent within the season. This issue was dealt with by aggregating and averaging the data by week, as well as setting missing values to 0, assuming there was no snow in the off-season. Next, the data was detrended by normalizing the depth according to the yearly peak, as well as removing a yearly trend. This approach assumes a macro model which predicts the peak snow depth a given year, as well as a micro model which models the snowing and thawing dynamics

during the season. The micro model is implemented in this paper to predict the snow levels of the current year. The detrended data appears stationary, implying a SARIMA model may be appropriate. An optimal model was chosen based on the optimizing for lowest AIC, resulting in a SARIMA(2,0,1)(0,0,1)[52] model. The model performed best when fed with missing values removed from the data set, rather than set to 0. The resulting model appears to fairly well describe the dynamics of the data, based on the residual and forecast analysis. However, as always, the model should be used as an indication rather than the truth, as its results are uncertain.

Next steps could be to try other models on the data to more accurately represent it. As can be seen in plots (figure 2) of the data, the variance of the snow-depth varies significantly during the year: being 0 in the off-season, followed by fast snowing and thawing in the beginning and end of seasons and lower variance in between. This problem is partially mitigated by the detrending process, as this feature is less prevalent in the detrended data (as seen in 11), however the detrended data also shows this pattern as indicated by the histogram of the residuals shown in figure 19. A model that has a variable variance, e.g. GARCH model could hence be more appropriate. In order to forecast the snow depth of next year, with this approach, a macro model would also have to be developed. A primitive such would be to assume that the peak level follows e.g. a normal distribution with the sample mean and variance of the recorded history. As there are only 8 years on the recorded history the variance of this estimate would probably be very high however. Additionally, the histogram of the peaks shown in figure ?? does not appear to be normally distributed.

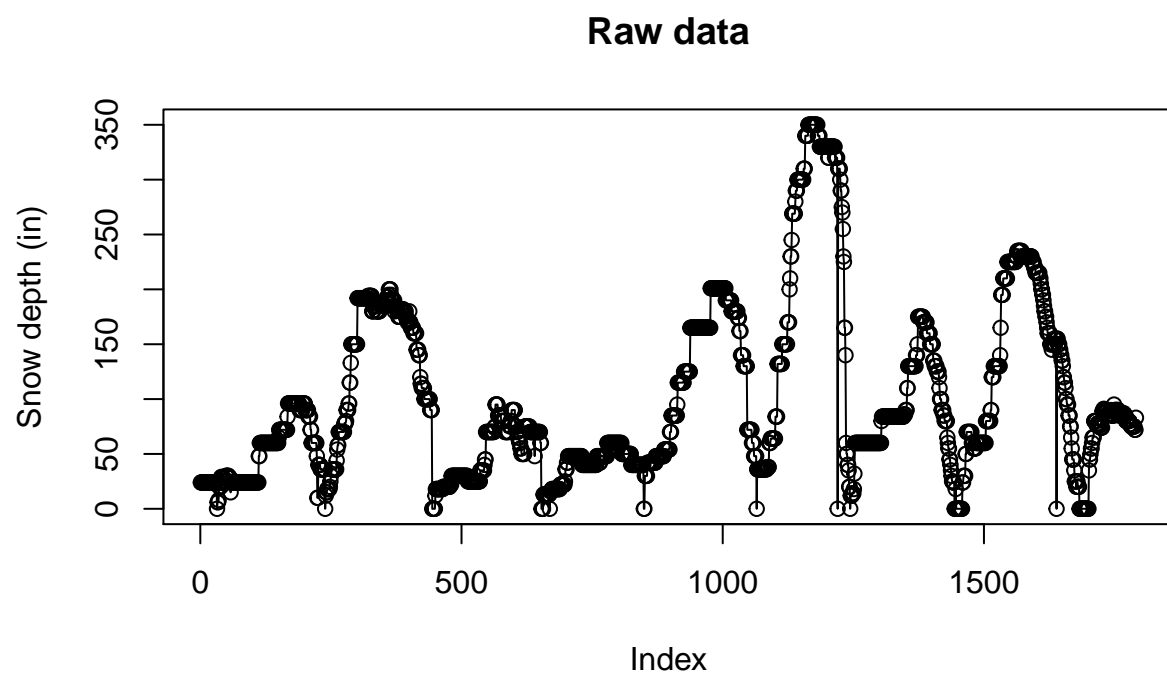


Figure 1: Raw Mammoth snow depth data

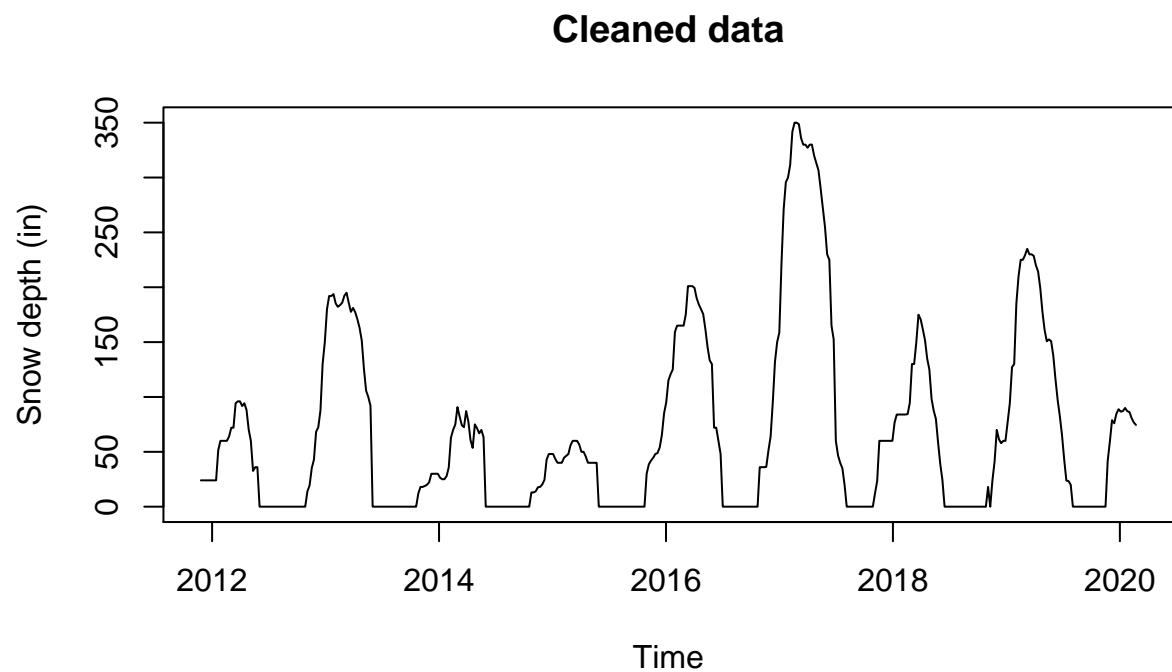


Figure 2: Mammoth snow depth data after initial cleaning

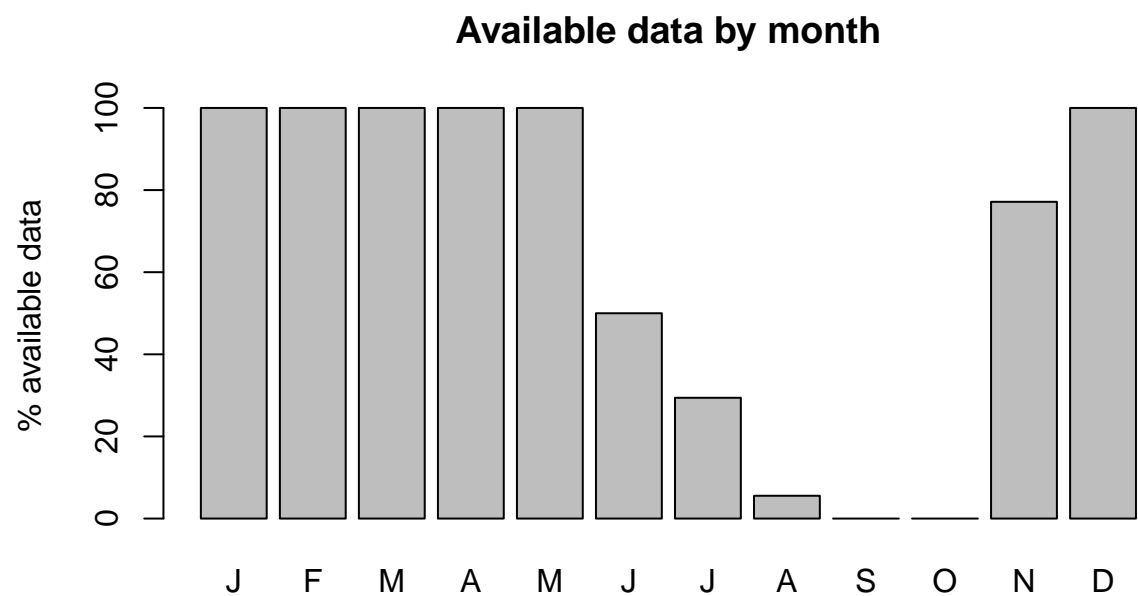


Figure 3: Percent of weeks where data is available, by month



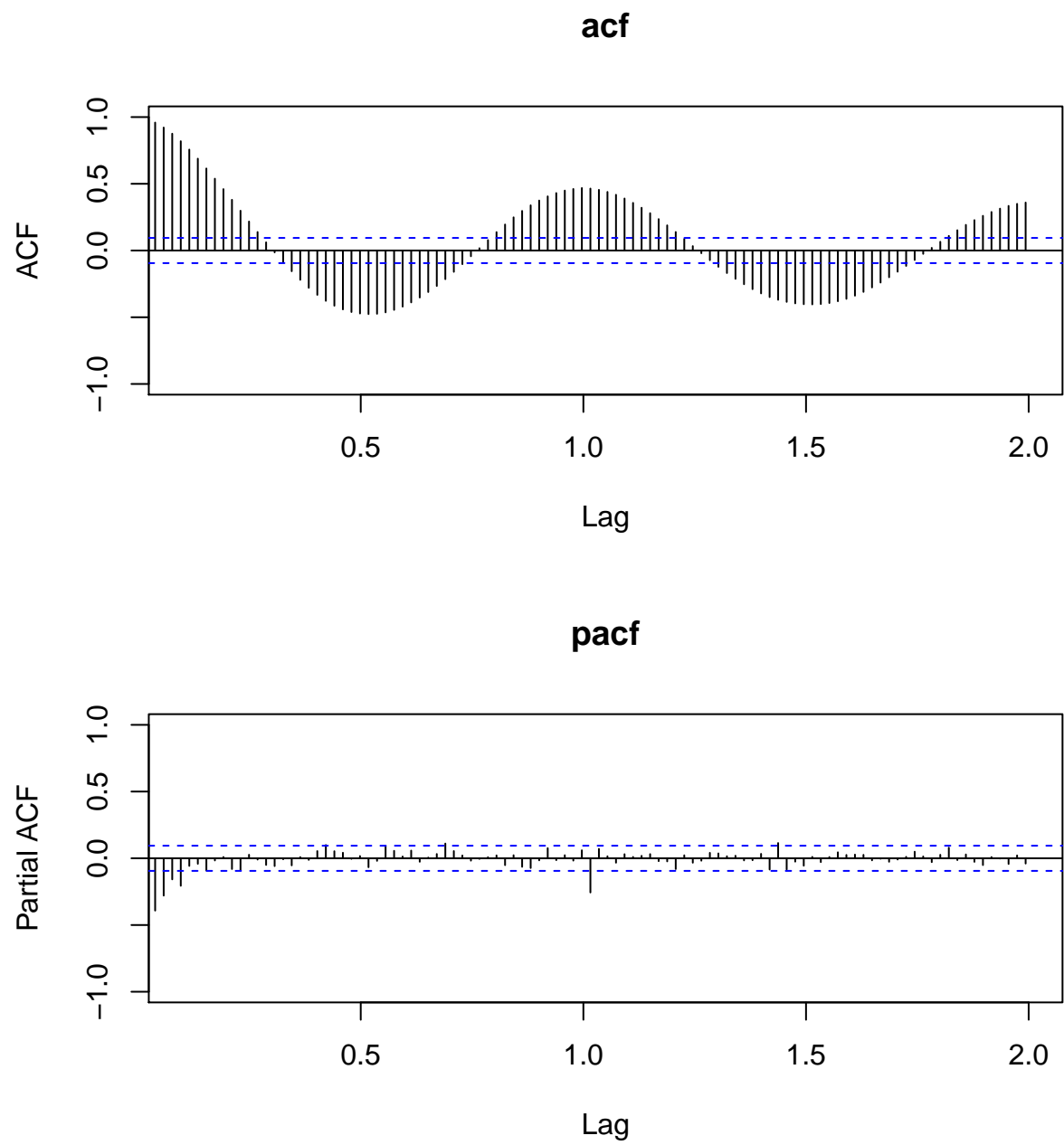


Figure 4: ACF and PACF for the cleaned time-series

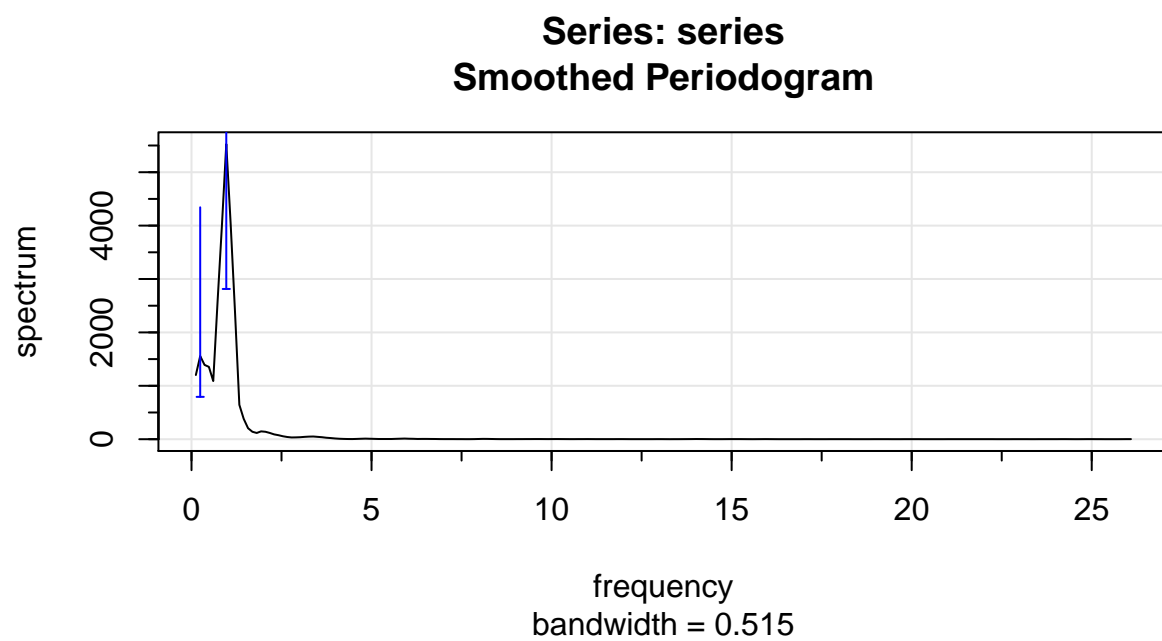
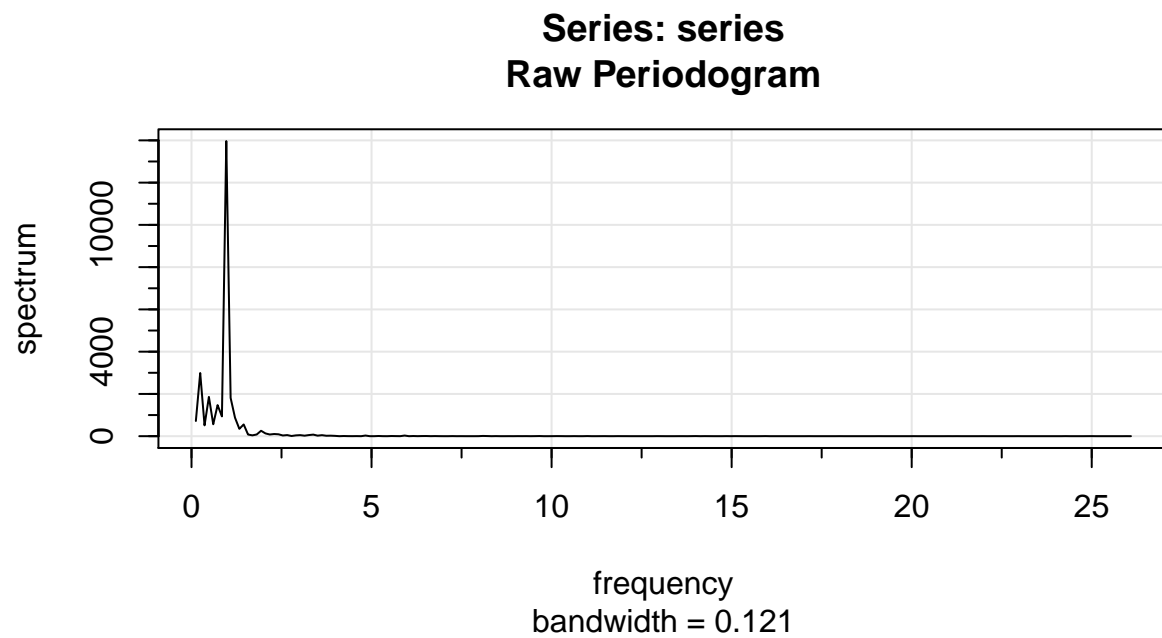


Figure 5: Periodogram for the cleaned data, peaks at period 1 and 4 years shown with a 95% significance interval

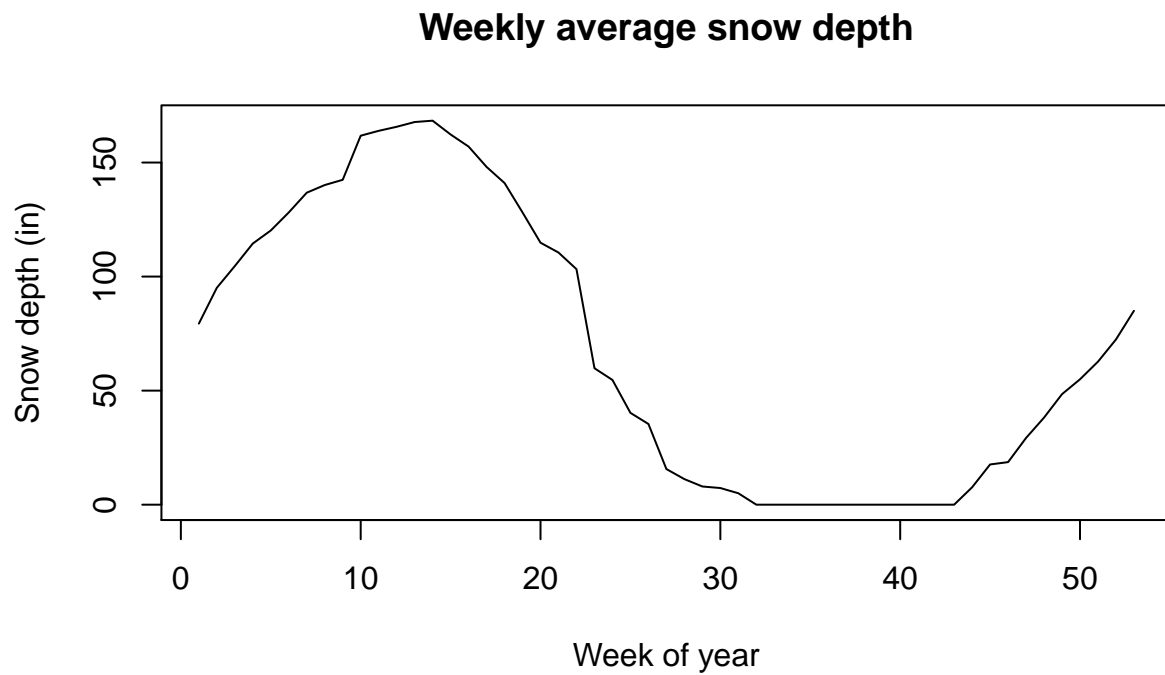


Figure 6: Average snow depth by week of the year

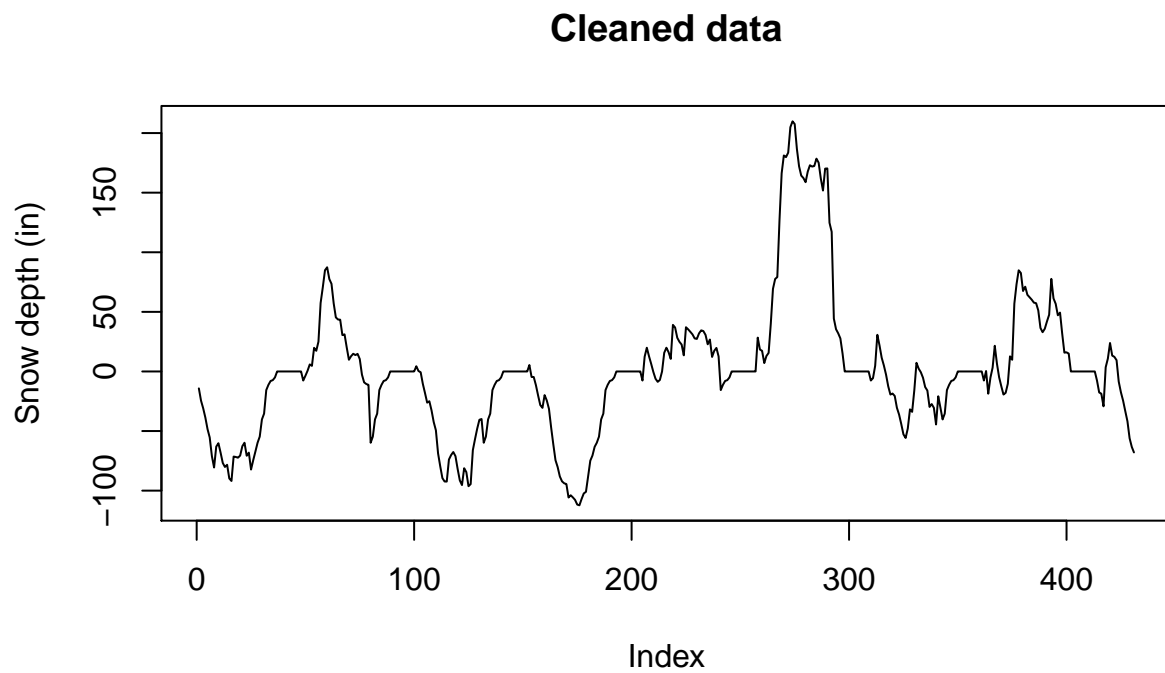


Figure 7: Snow depth detrended by subtracting average of week of the year

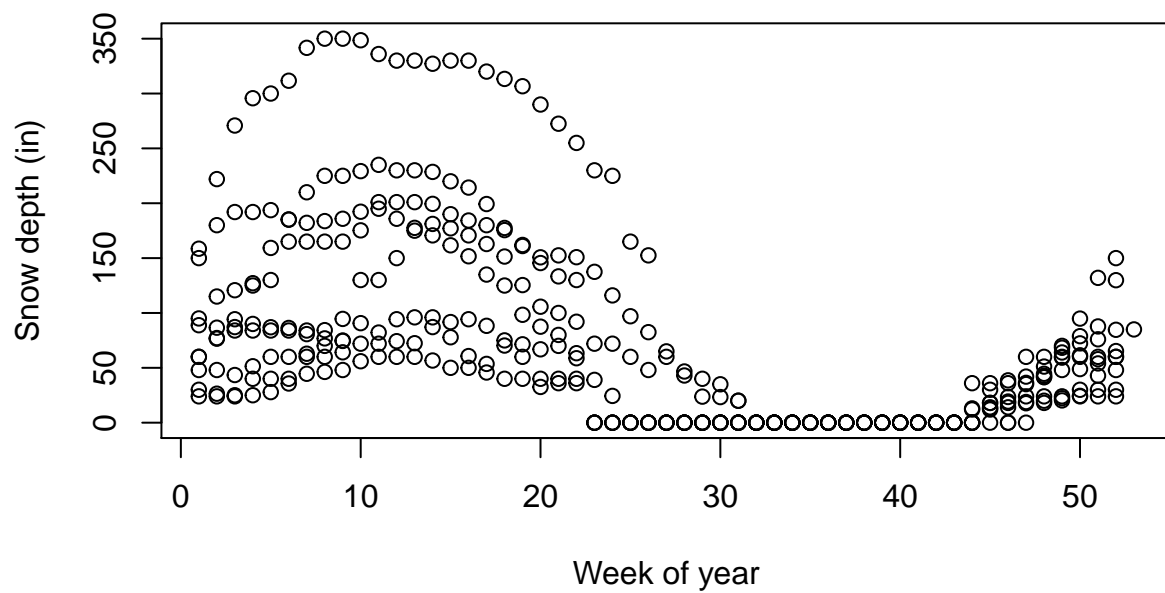


Figure 8: Weekly snow depth of all years

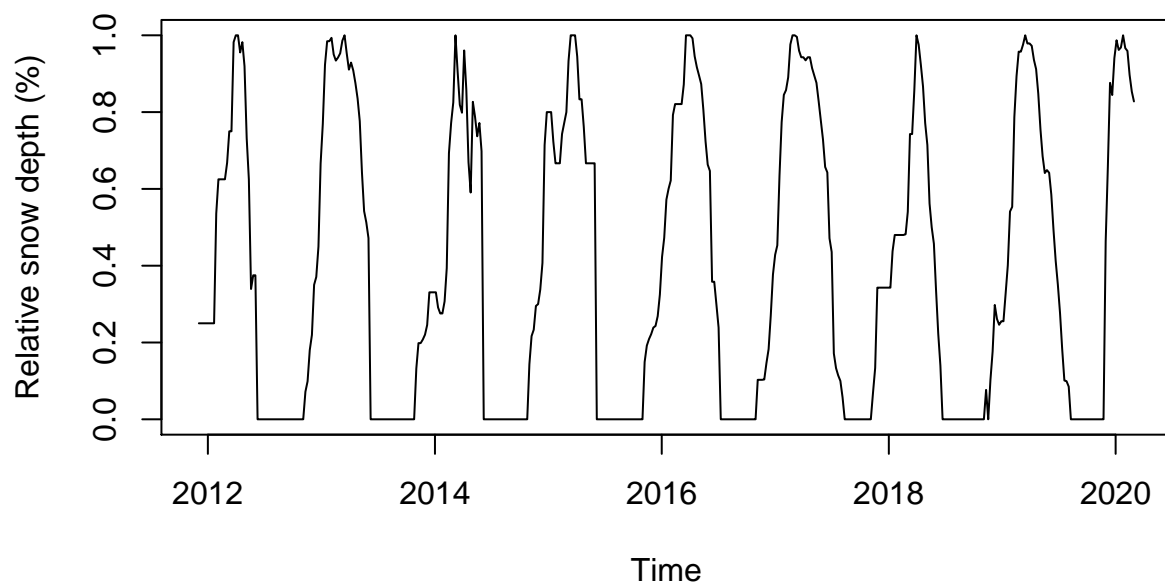


Figure 9: First step of detrending the data, taking the values relative to the peak of the season

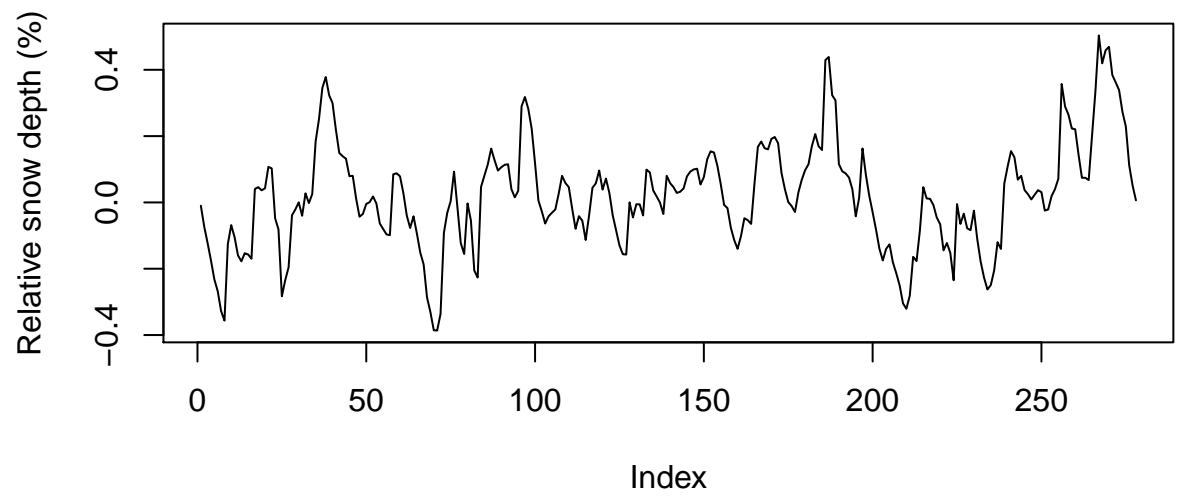
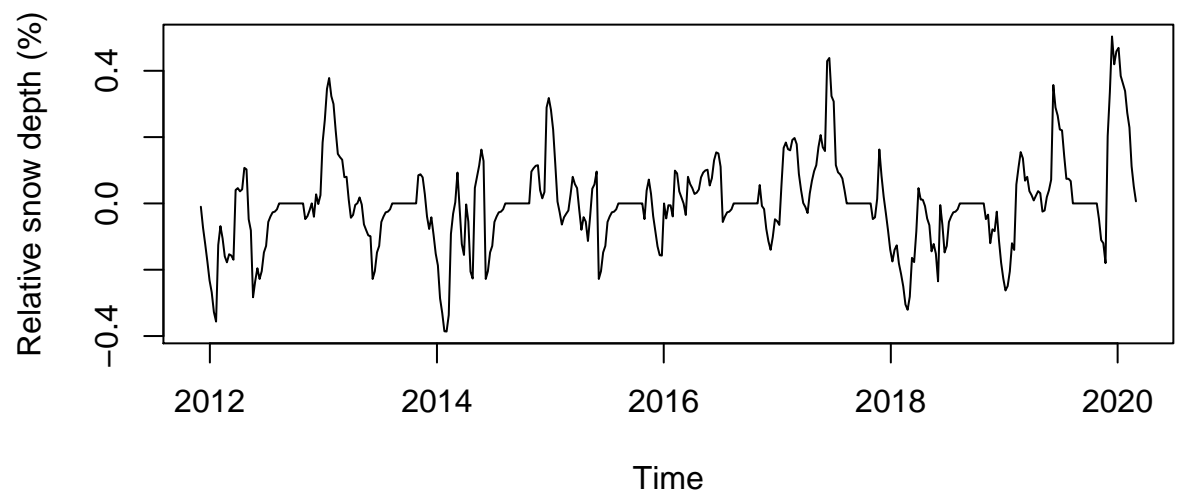


Figure 10: Detrended data after subtracting the yearly trend

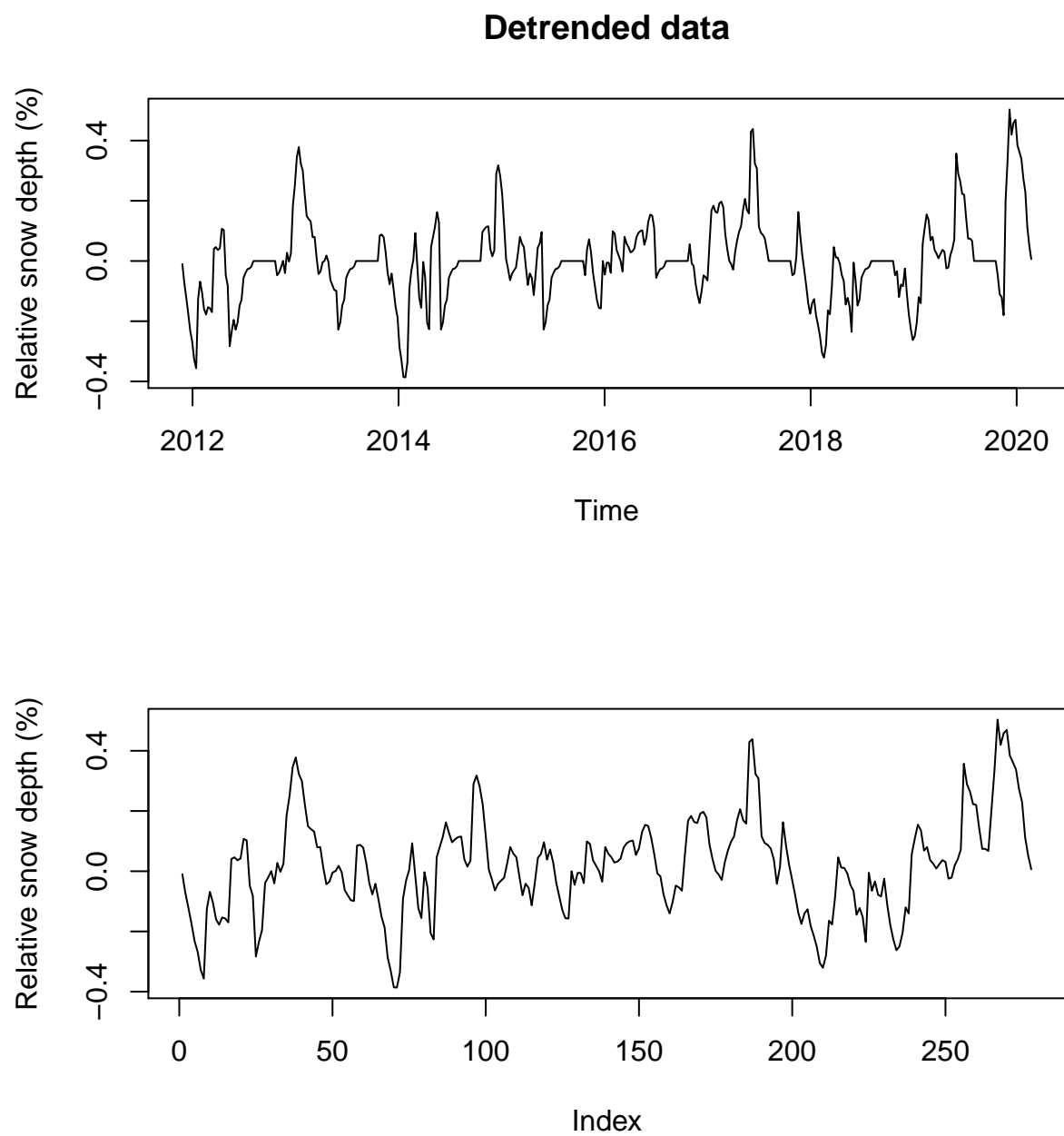


Figure 11: Final detrended data

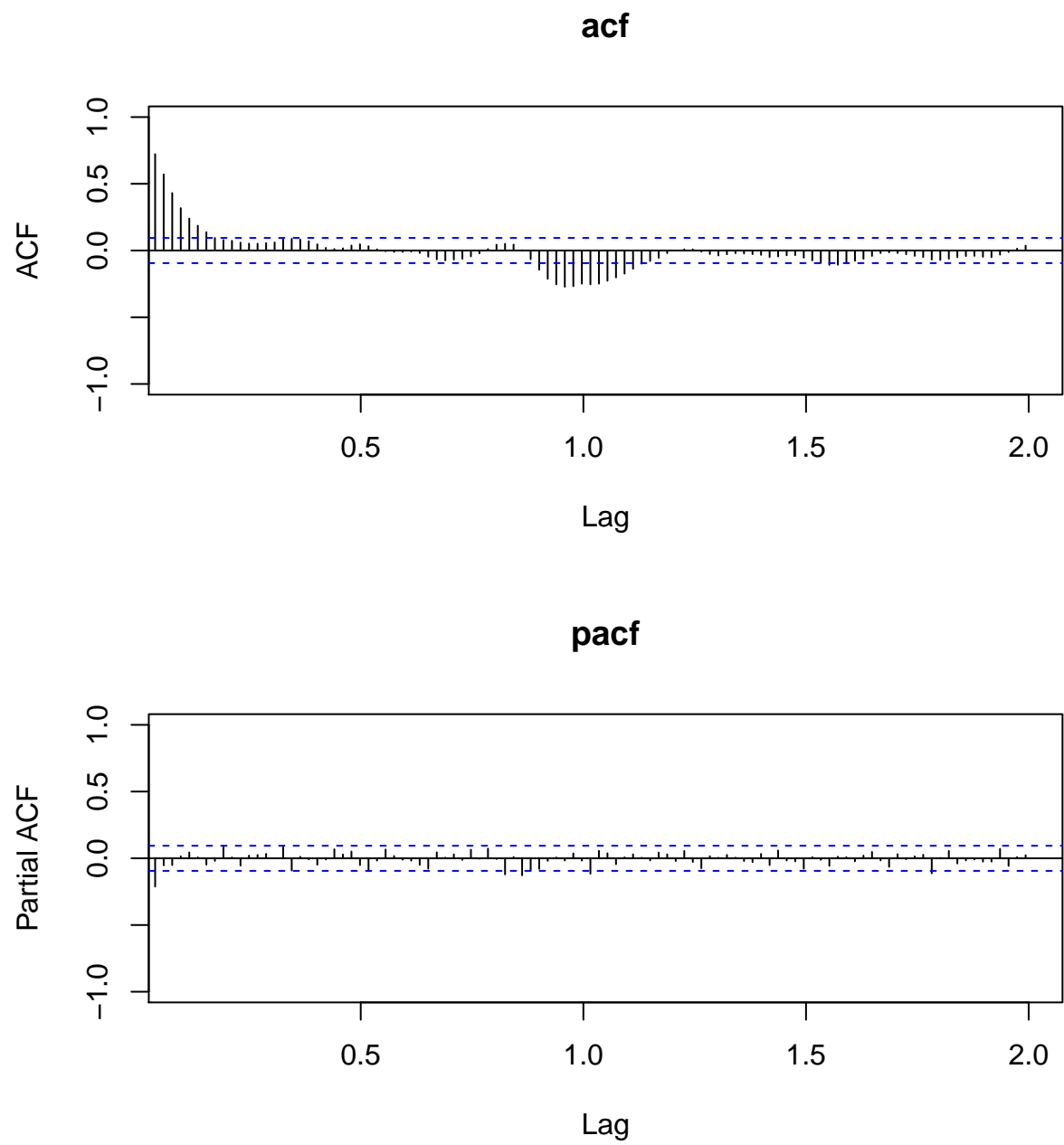
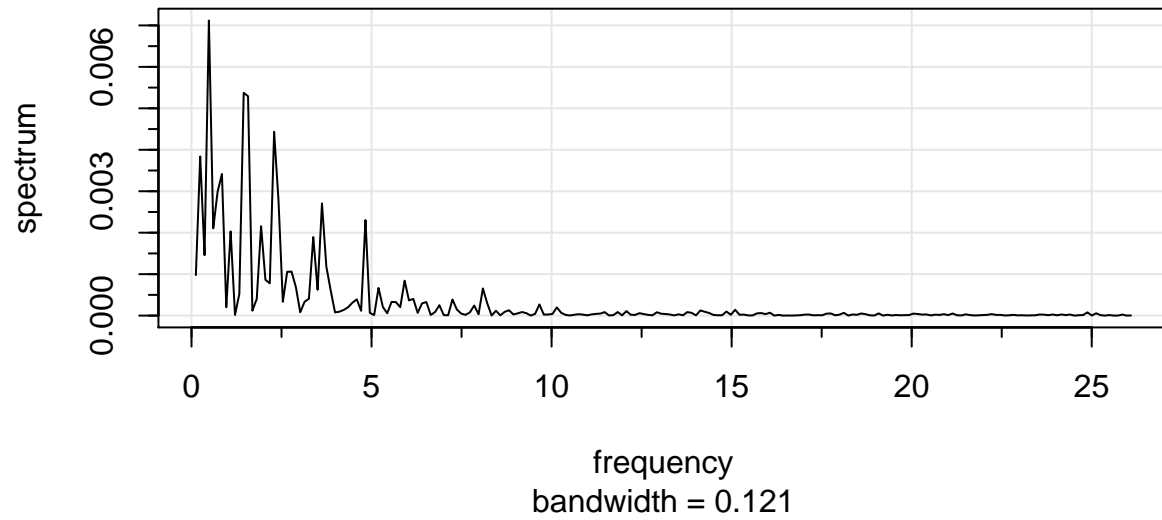


Figure 12: ACF and PACF for the detrended time-series

**Series: detrended  
Raw Periodogram**



**Series: detrended  
Smoothed Periodogram**

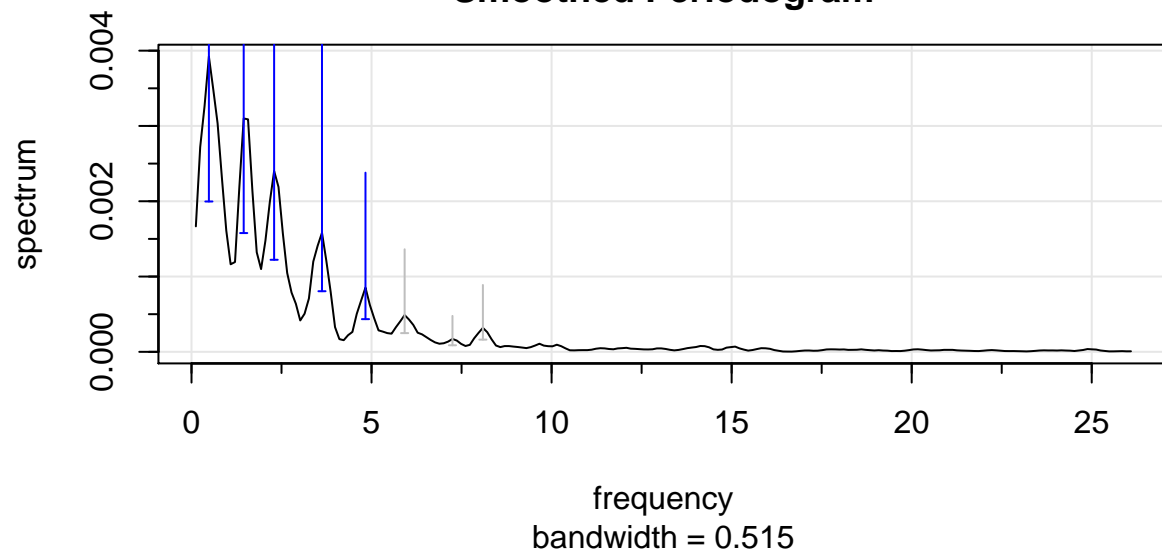


Figure 13: Periodogram for the detrended data, peaks shown with a 95% significance interval, significant peaks marked in blue



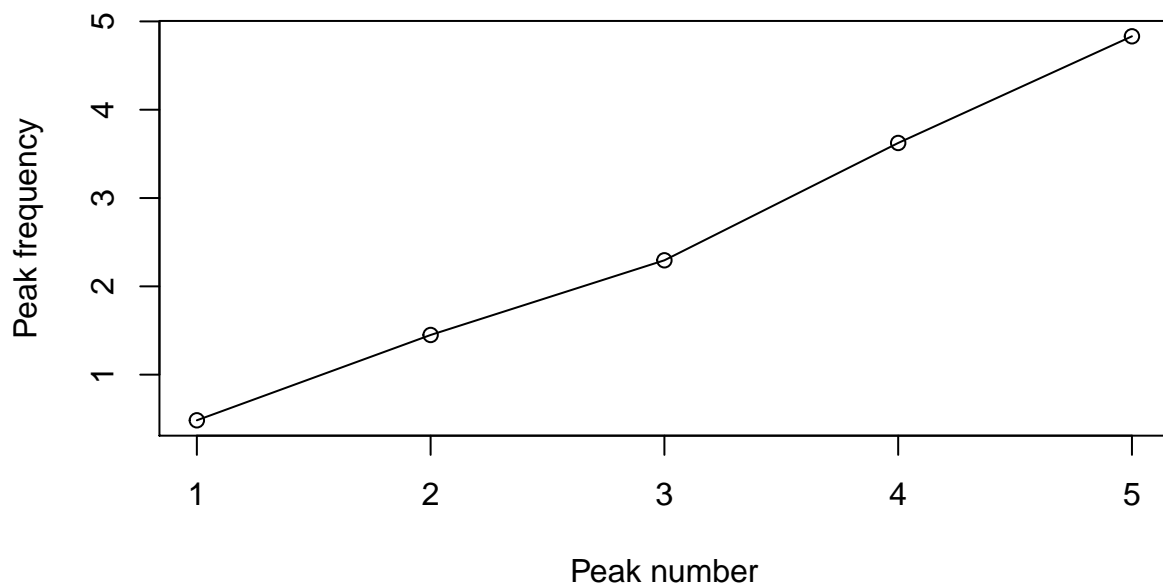


Figure 14: Frequencies of the significant peaks in the periodogram of the detrended data

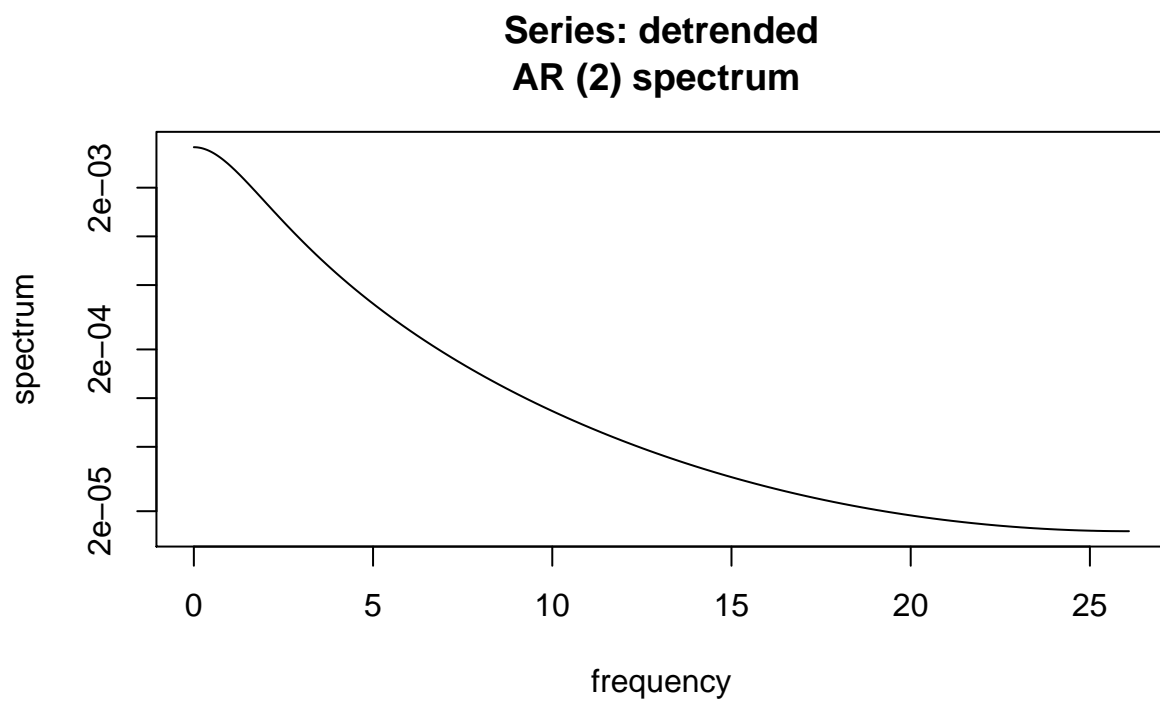


Figure 15: Parametric periodogram of the detrended data, showing the periodogram of an AR(2)

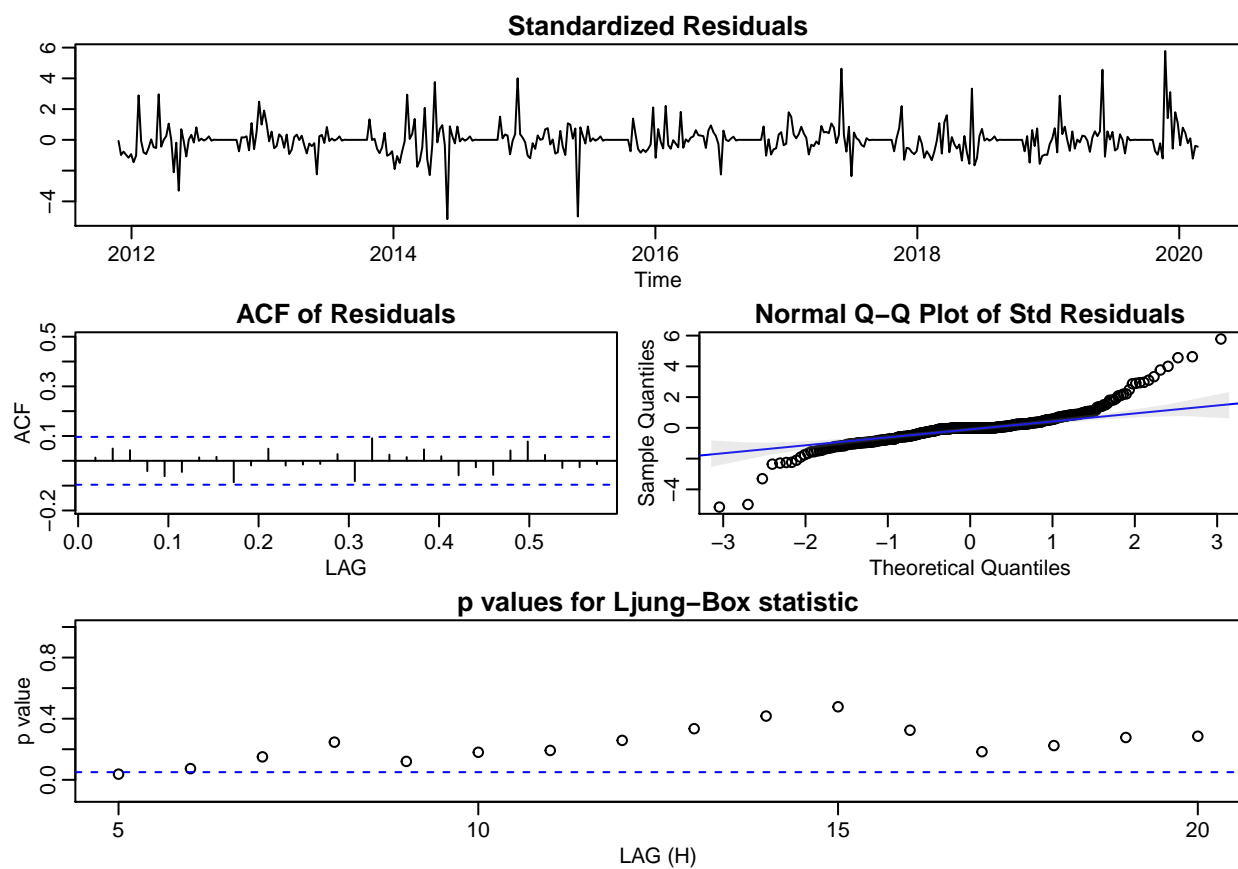


Figure 16: Plots of different characteristics of the best model's residuals fitted on the detrended data with missing values included

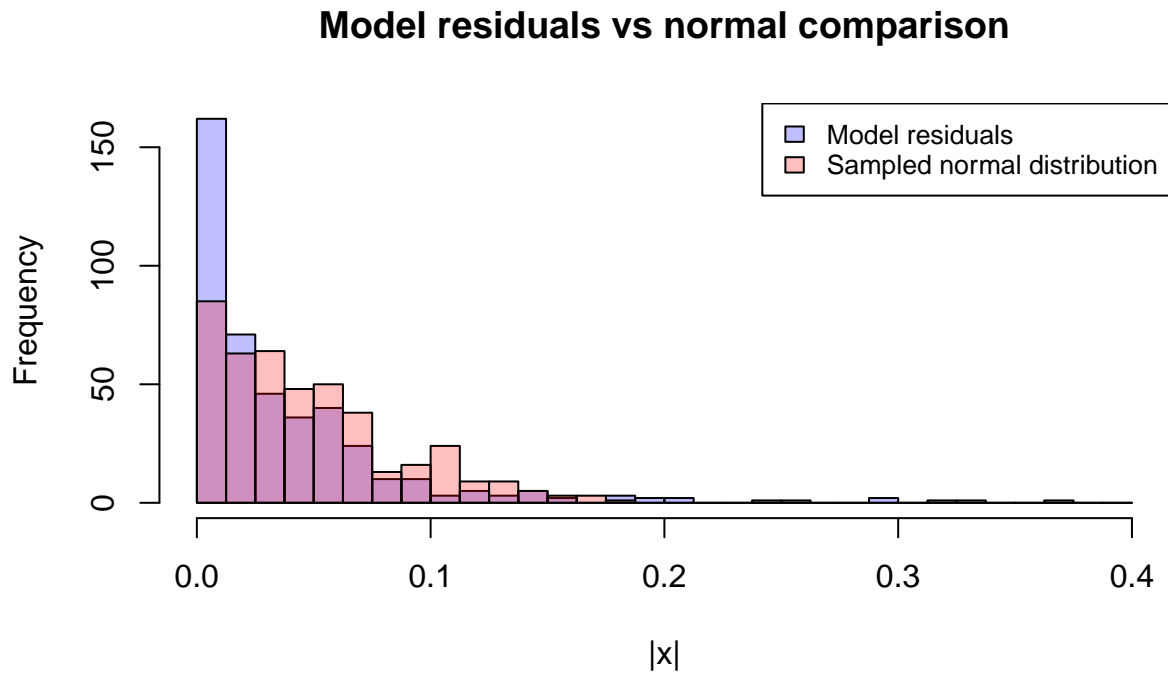
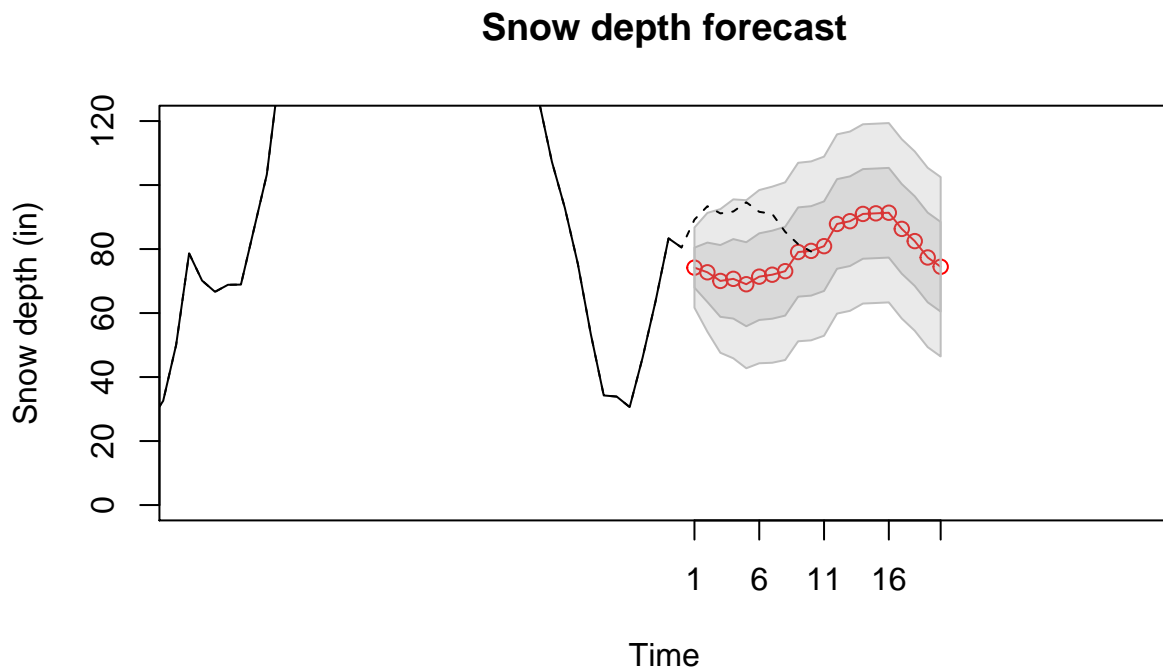


Figure 17: Histogram of residuals with model fitted on detrended data set including missing data, compared to a sampled normal distribution with same mean and standard deviation

## 4 Appendix - Figures and graphs



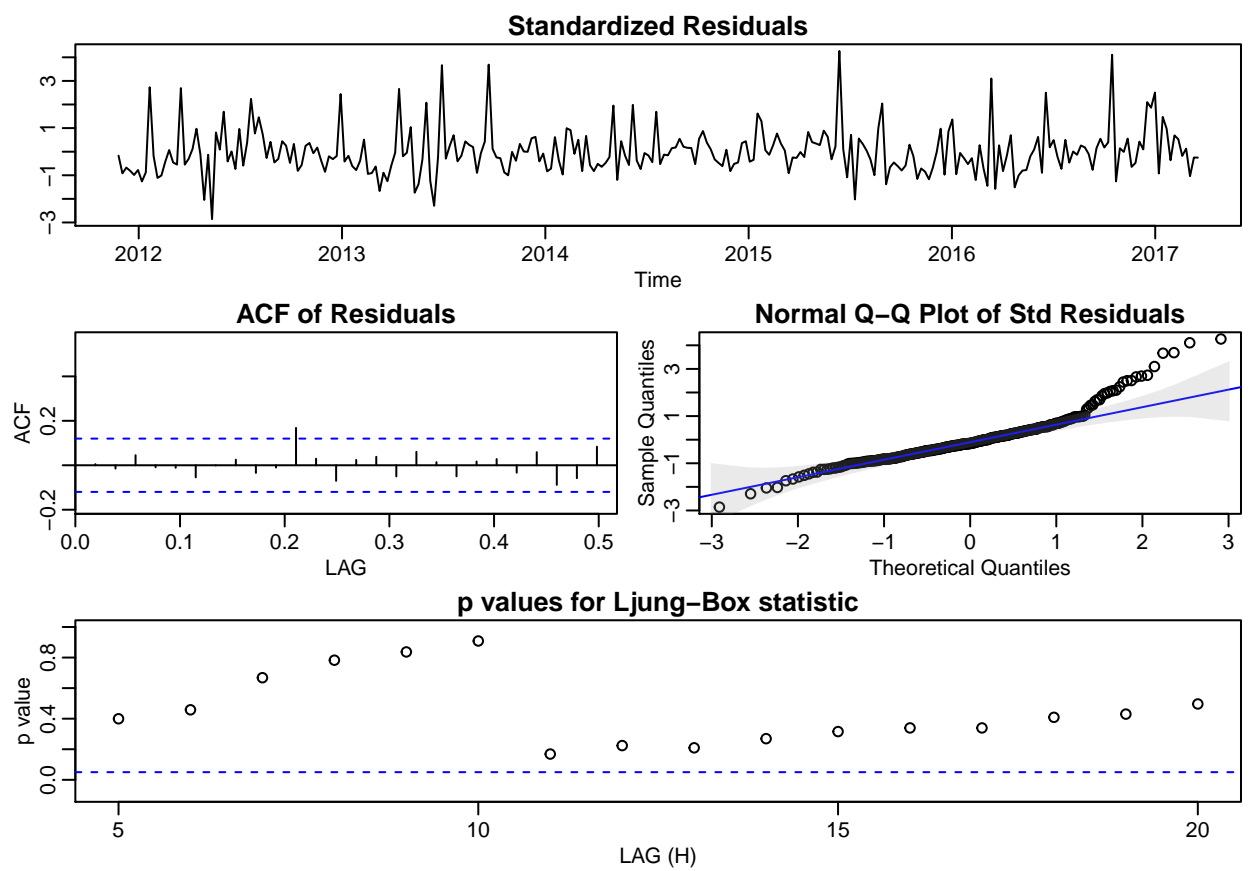


Figure 18: Plots of different characteristics of the best model's residuals fitted on the detrended data with missing values excluded

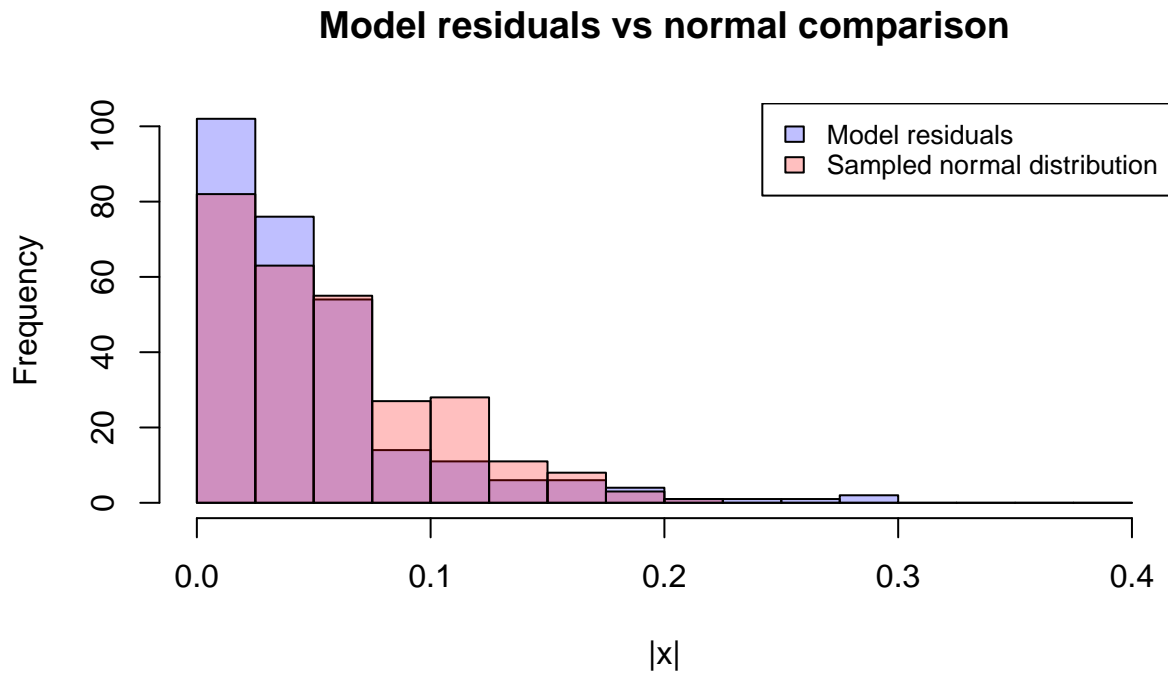


Figure 19: Histogram of residuals with model fitted on detrended data set excluding missing data, compared to a sampled normal distribution with same mean and standard deviation

## References

- [1] Mammoth Mountain,  
<https://www.mammothmountain.com/>
- [2] Mammoth Mountain, On The Snow  
<https://www.onthesnow.com/california/mammoth-mountain-ski-area/historical-snowfall.html>

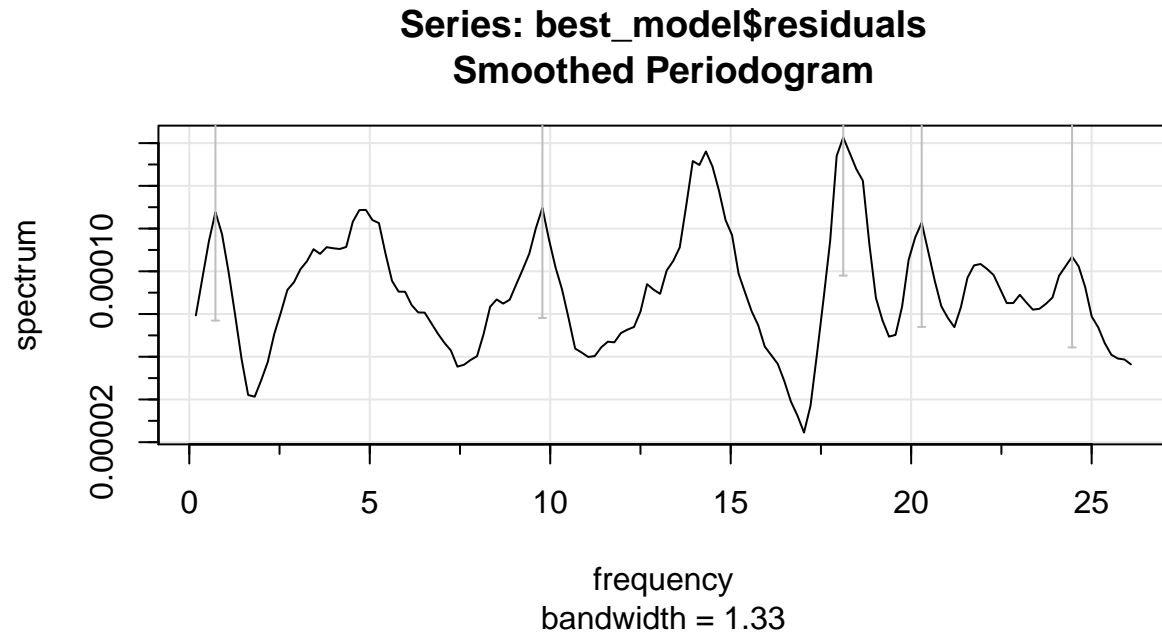
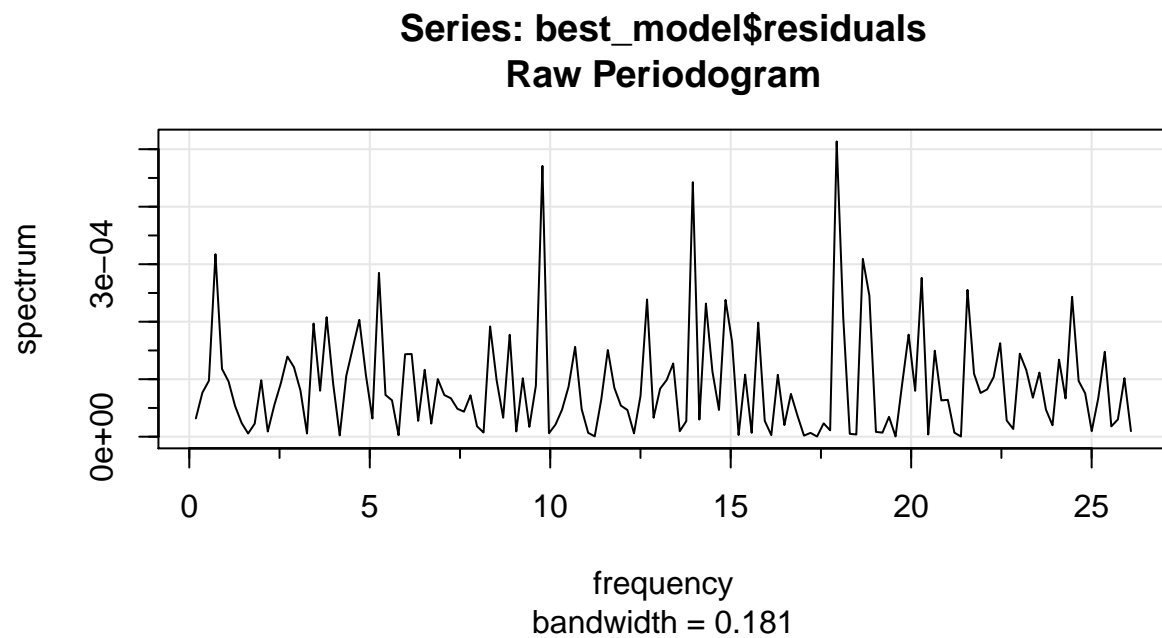


Figure 20: Periodogram for the residuals data, peaks shown with a 95% significance interval

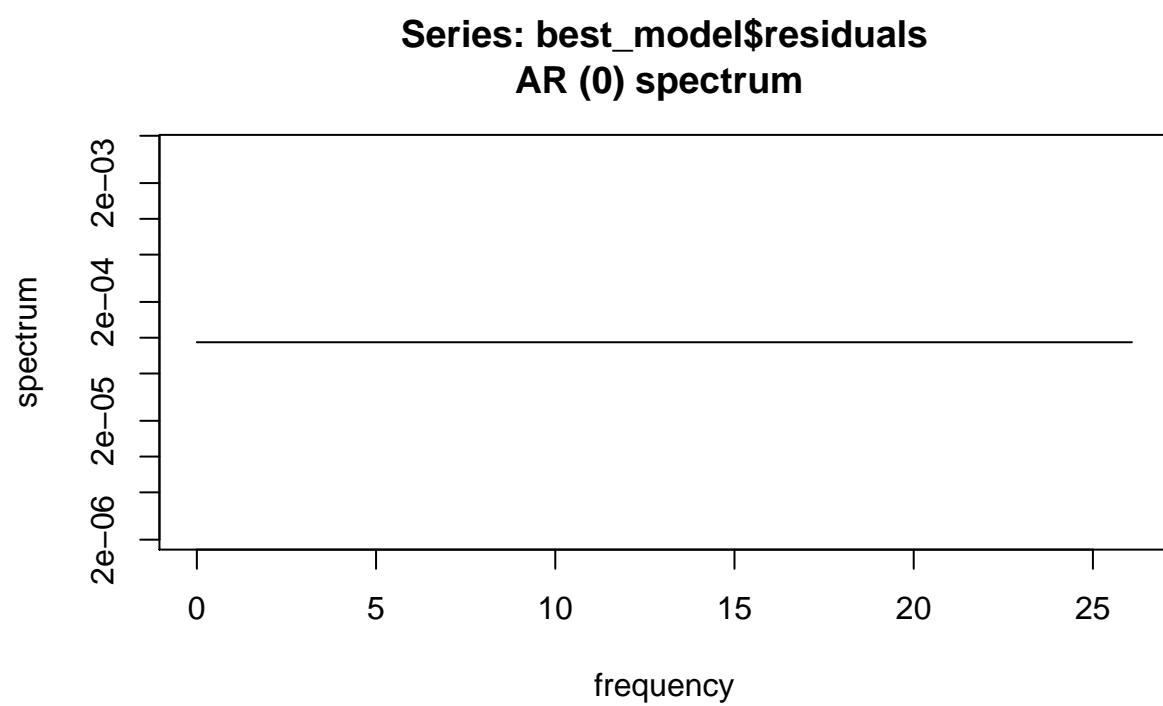


Figure 21: Parametric periodogram of the residuals, showing the periodogram of an AR(0)

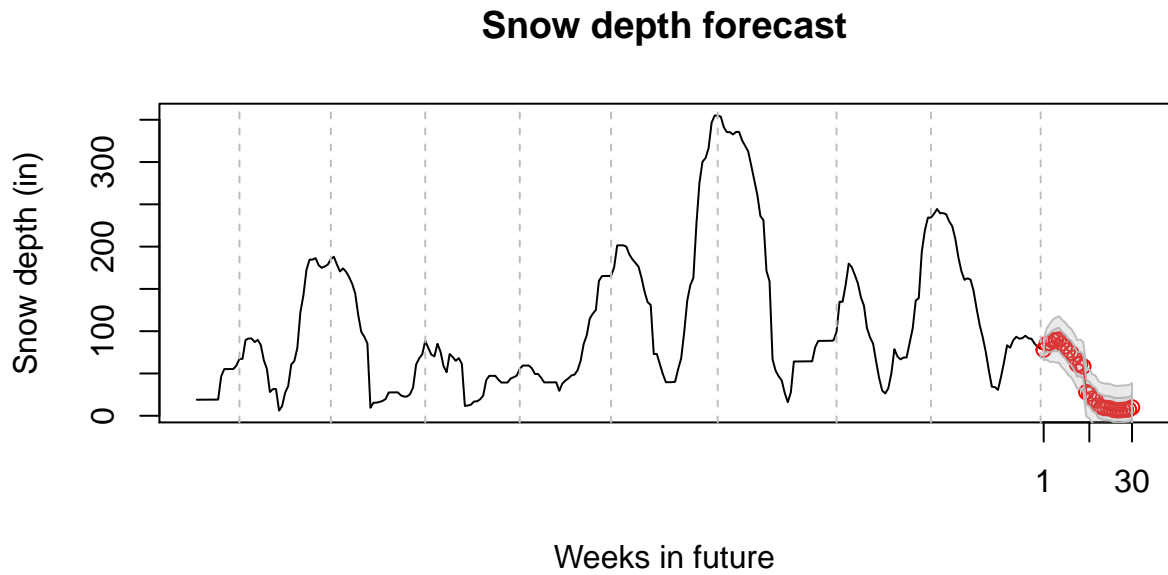
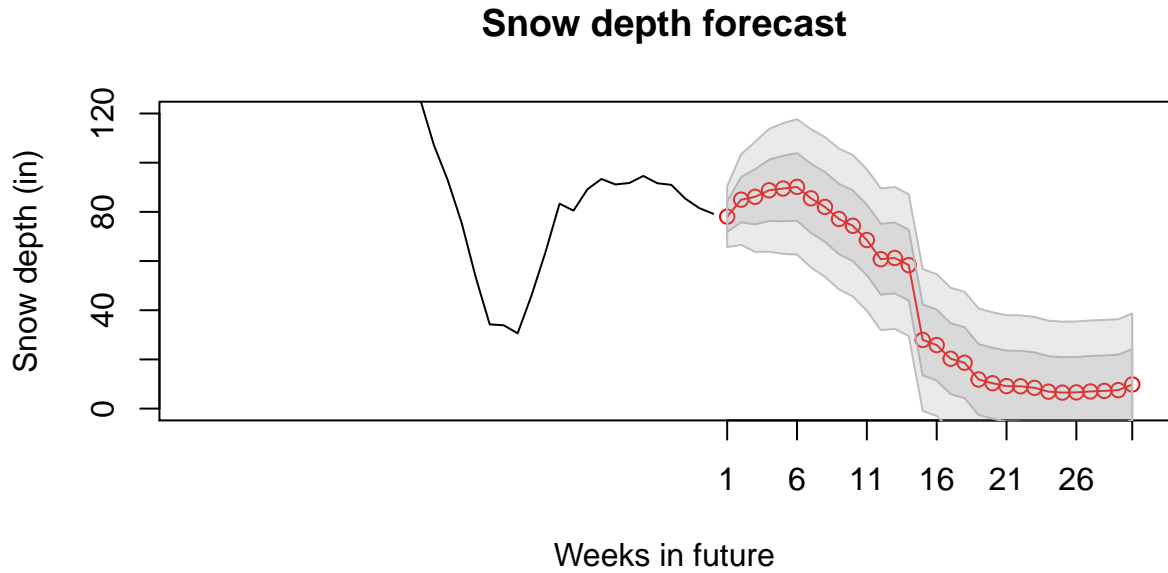


Figure 22: Forecast of the snow depth the coming 30 weeks, the same time of year as the forecast start of every year shown in dashed vertical lines. One and two standard deviations marked in gray.