MATH 151 Trigonometry Formula Sheet

Right Triangle Definition

$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{O}}{\text{H}}$$
$$\cos x = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{A}}{\text{H}}$$
$$\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{O}}{\text{A}}$$
$$\csc x = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{\text{H}}{\text{O}}$$
$$\sec x = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{\text{H}}{\text{A}}$$
$$\cot x = \frac{\text{adjacent}}{\text{opposite}} = \frac{\text{A}}{\text{O}}$$

Unit Circle Definition

$$\sin\theta = y \qquad \csc\theta = \frac{1}{y}$$
$$\cos\theta = x \qquad \sec\theta = \frac{1}{x}$$
$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \csc x = \frac{1}{\sin x}$$
$$\cos x = \frac{1}{\sec x} \qquad \sec x = \frac{1}{\cos x}$$
$$\tan x = \frac{1}{\cot x} \qquad \cot x = \frac{1}{\tan x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$\cot^2 x + 1 = \csc^2 x$$

Double Angle

$$\sin 2x = 2\sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Triple Angle

$$\sin 3x = 3\sin x - 4\sin^3 x$$
$$\cos 3x = -3\cos x + 4\cos^3 x$$
$$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

Half Angle

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$
$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$
$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$$

Sum and Difference

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Power Reduction

$$\sin^2 x = \frac{1-\cos(2x)}{2}$$
$$\cos^2 x = \frac{1+\cos(2x)}{2}$$
$$\tan^2 x = \frac{1-\cos(2x)}{1+\cos(2x)}$$
$$\sin^3 x = \frac{3\sin x - \sin(3x)}{4}$$
$$\cos^3 x = \frac{3\cos x + \cos(3x)}{4}$$
$$\tan^3 x = \frac{3\sin x - \sin(3x)}{3\cos x + \cos(3x)}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Even/Odd Identities

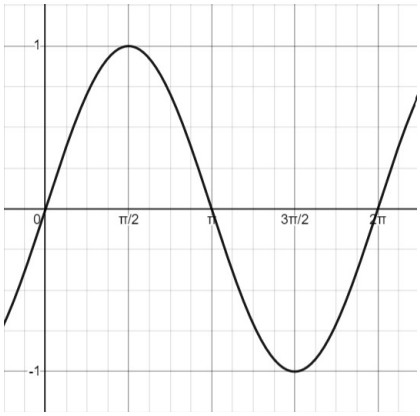
$$\sin(-x) = -\sin(x)$$
$$\cos(-x) = \cos(x)$$
$$\tan(-x) = -\tan(x)$$
$$\csc(-x) = -\csc(x)$$
$$\sec(-x) = \sec(x)$$
$$\cot(-x) = -\cot(x)$$

Domain/Range

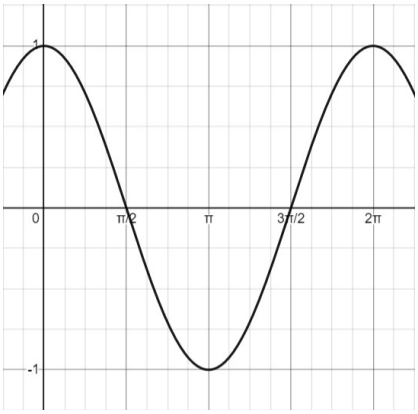
	Domain	Range
$\sin x$	$(-\infty, \infty)$	[-1,1]
$\cos x$	$(-\infty, \infty)$	[-1,1]
$\tan x$	$\mathbb{R} - (k\pi + \frac{\pi}{2})$	$(-\infty, \infty)$
$\csc x$	$\mathbb{R} - k\pi$	$(-\infty, -1] \cup [1, \infty)$
$\sec x$	$\mathbb{R} - (k\pi + \frac{\pi}{2})$	$(-\infty, -1] \cup [1, \infty)$
$\cot x$	$\mathbb{R} - k\pi$	$(-\infty, \infty)$
$\sin^{-1} x$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	$(-\infty, \infty)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$\csc^{-1} x$	$\lvert x\rvert \geq 1$	$[\frac{-\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$
$\sec^{-1} x$	$\lvert x\rvert \geq 1$	$[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$
$\cot^{-1} x$	$(-\infty, \infty)$	$(0, \pi)$

Some Unit Circle Values

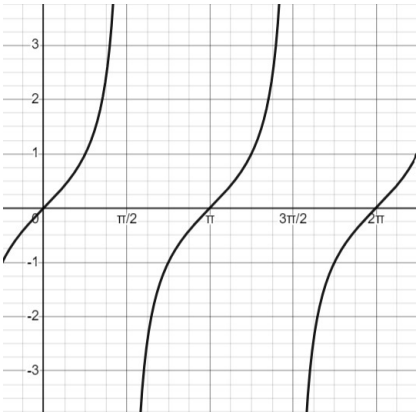
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	–
$\csc\theta$	–	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1
$\sec\theta$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	–
$\cot\theta$	–	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0



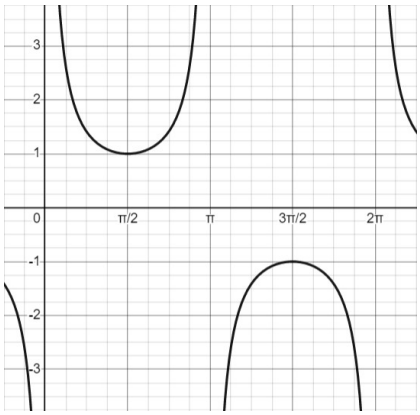
$\sin(x)$ parent graph



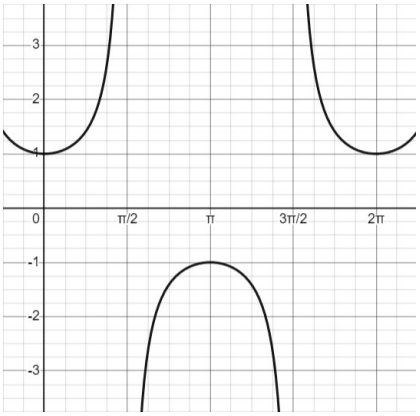
$\cos(x)$ parent graph



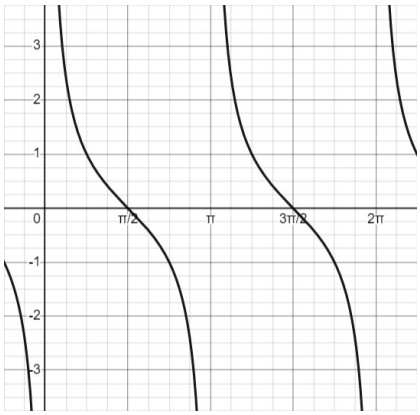
$\tan(x)$ parent graph



$\csc(x)$ parent graph



$\sec(x)$ parent graph



$\cot(x)$ parent graph

$A\sin(Bx+C)+D$	amplitude	period	phase shift	midline
	A	$\frac{2\pi}{B}$	$\frac{-C}{B}$	D