

Differential Calculus Formula Sheet

Derivative Rules

$$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x))$$

$$\frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx}(f(x))$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(f(x) \cdot g(x)) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\frac{d}{dx}(\tan(x)) = \sec^2(x)$$

$$\frac{d}{dx}(\sec(x)) = \sec(x) \cdot \tan(x)$$

$$\frac{d}{dx}(\csc(x)) = -\csc(x) \cdot \cot(x)$$

$$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$$

$$\frac{d}{dx}(\sinh(x)) = \cosh(x)$$

$$\frac{d}{dx}(\cosh(x)) = \sinh(x)$$

$$\frac{d}{dx}(|x|) = \frac{|x|}{x}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(b^x) = b^x \cdot \ln(b)$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_b(x)) = \frac{1}{x \cdot \ln(b)}$$

$$\frac{d}{dx}(f(x)^{g(x)}) = \left(g'(x) \cdot \ln(f(x)) + g(x) \cdot \frac{f'(x)}{f(x)} \right)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\csc^{-1}(x)) = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cot^{-1}(x)) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}(f^{-1}(x)) = \frac{1}{f'(f^{-1}(x))}$$

L'Hôpital's Rule

Only when the function $h(x) = \frac{f(x)}{g(x)}$ is in indeterminate form, then

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}$$

Indeterminate Form

There are 7 different kinds of indeterminate form:

$$\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, \infty - \infty, 0^0, 1^\infty, \text{ and } \infty^0.$$

Limit Definition of a Derivative

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{dy}{dx} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

Relevant Physics Formulas

Position:

Momentum and force:

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$\frac{dp}{dt} = F$$

$$\frac{dx}{dt} = v = v_0 + at$$

Work and force:

$$\frac{d^2x}{dt^2} = a$$

$$\frac{dW}{dx} = F$$

Relevant Geometry Formulas

Shape	Surface Area	Volume
Sphere	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Cylinder	$2\pi r h + 2\pi r^2$	$\pi r^2 h$
Cone	$\pi r^2 + \pi r \sqrt{h^2 + r^2}$	$\frac{1}{3}\pi r^2 h$

Epsilon-Delta Definition of a Derivative

The derivative of f at a point c is equal to L is defined as:

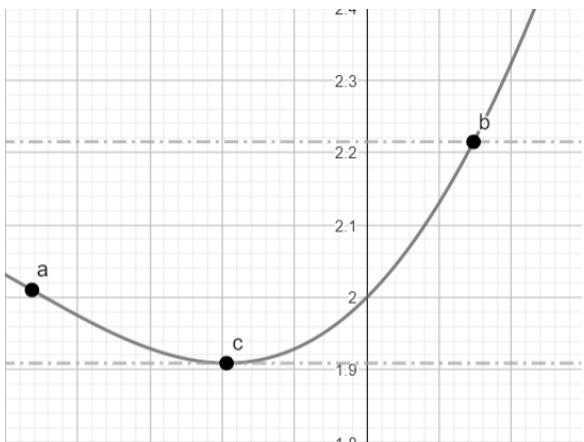
For all $\varepsilon > 0$, there exists a $\delta > 0$ such that for all x , if $|x - c| < \delta$, then $|\frac{f(x) - f(c)}{x - c} - L| < \varepsilon$

First Derivative Test

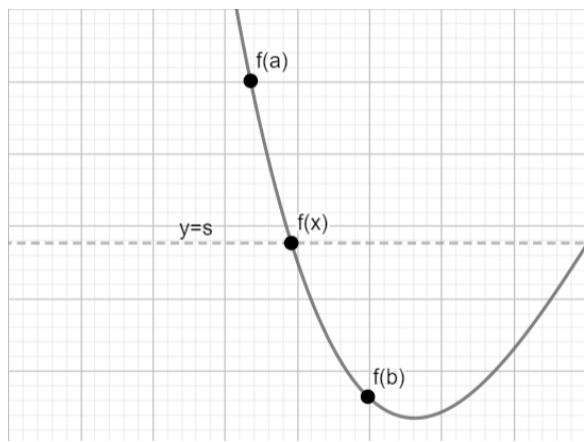
1. Find all critical points c where $f'(c) = 0$
2. Find $f'(x)$ for points to the left and the right of c
 - (a) If f' changes from negative $(-)$ to positive $(+)$ at c , then $f(c)$ is a local minimum
 - (b) If f' changes from positive $(+)$ to negative $(-)$ at c , then $f(c)$ is a local maximum
 - (c) If f' does not change sign at c , then $f(c)$ is neither a minimum nor a maximum

Second Derivative Test

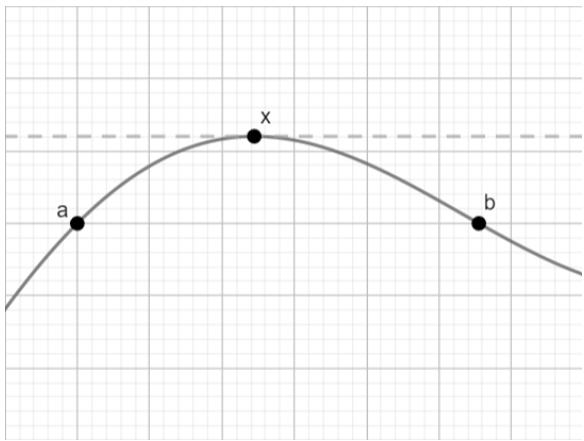
1. Find all critical points c where $f'(c) = 0$
2. Find $f''(c)$ for each c
 - (a) If $f''(c) > 0$ or f'' is concave up at c , then $f(c)$ is a local minimum
 - (b) If $f''(c) < 0$ or f'' is concave down at c , then $f(c)$ is a local maximum
 - (c) If $f''(c) = 0$ or f'' has no concavity at c , then the test is inconclusive



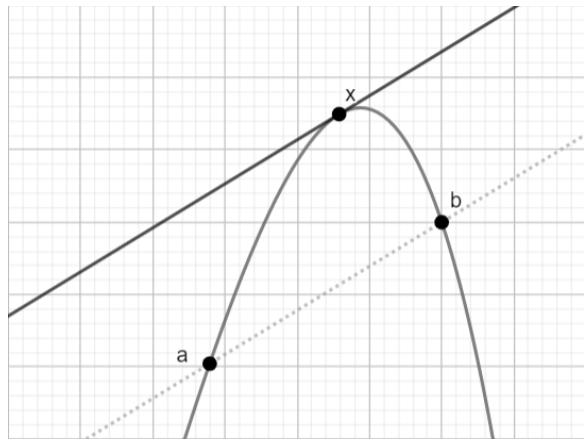
Extreme Value Theorem: If f is continuous from a to b , then f has a minimum and maximum value between a and b .



Intermediate Value Theorem: For any a and b , if f is continuous from a to b , then for any s with $f(a) < s < f(b)$, there exists an x between a and b such that $f(x) = s$.



Rolle's Theorem: If f is differentiable from a to b and if $f(a) = f(b)$, then there is an x with $a < x < b$ such that $f'(x) = 0$.



Mean Value Theorem: For any a and b , if f is differentiable from a to b , then there exists an x between a and b such that $f'(x) = \frac{f'(b)-f'(a)}{b-a}$.