FIR FILTER DESIGN – FINAL PROJECT

a.) CALCULATE FILTER COEFFICIENTS

$f_c = 1500$ Hz $\qquad T_s = \frac{1}{12000}$ sec.

$f_s = 12000$ Hz. $\qquad N = 21 = 2M + 1, \quad M = 10$

$$\Omega_c = 2\pi \cdot f_c \cdot T_s = (2\pi)(1500)\left(\frac{1}{12000}\right) = 0.25\pi \text{ rad}$$

$$h(n) = \begin{cases} \frac{\Omega_c}{\pi} & \text{for } n = 0 \\ \frac{\sin(\Omega_c n)}{n\pi} & \text{for } -M \le n \le M, \; n \neq 0 \end{cases}$$

$$h(0) = \frac{0.25\pi}{\pi} = 0.25$$

$$h(1) = \frac{\sin(0.25\pi)}{\pi} = \cancel{0.1592} \; 0.2251$$

$$h(2) = \frac{\sin(\pi/2)}{2\pi} = \cancel{0.15915} \; 0.1592$$

$$h(3) = \frac{\sin(3\pi/4)}{3\pi} = 0.0750$$

$$h(4) = \frac{\sin(\pi)}{4\pi} = 0$$

$$h(5) = \frac{\sin(5\pi/4)}{5\pi} = -0.045$$

$$h(6) = \frac{\sin(3\pi/2)}{6\pi} = -0.0531$$

$$h(7) = \frac{\sin(7\pi/4)}{7\pi} = -0.0322$$

$$h(8) = \frac{\sin(2\pi)}{7\pi} = 0$$

$$h(9) = \frac{\sin(9\pi/4)}{9\pi} = 0.025 \qquad h(10) = \frac{\sin(10\pi/4)}{10\pi} = 0.0318$$

AND BY SYMMETRY:

$h(-1) = h(1) = 0.2251$

$h(-2) = h(2) = 0.1592$

$h(-3) = h(3) = 0.0750$

$h(-4) = h(4) = 0$

$h(-5) = h(5) = -0.045$

$h(-6) = h(6) = -0.0531$

$h(-7) = h(7) = -0.0322$

$h(-8) = h(8) = 0$

$h(-9) = h(9) = 0.025$

$h(-10) = h(10) = 0.0318$

BLACKMAN WINDOW FUNCTION

$w_b(n) = 0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right)$

$w_b(0) = 0.42 - 0.5\cos(0) + 0.08\cos(0) = 0$

$w_b(1) = 0.42 - 0.5\cos\left(\frac{2\pi}{10}\right) + 0.08\cos(\pi/5) = 0.0092$

$w_b(2) = 0.42 - 0.5\cos(\pi/5) + 0.08\cos(2\pi/5) = 0.0402$

$w_b(3) = 0.42 - 0.5\cos\left(\frac{3\pi}{10}\right) + 0.08\cos(3\pi/5) = 0.1014$

$w_b(4) = 0.42 - 0.5\cos(2\pi/5) + 0.08\cos(4\pi/5) = 0.2008$

$w_b(5) = 0.42 - 0.5\cos(\pi/2) + 0.08\cos(\pi) = 0.34$

$w_b(6) = 0.42 - 0.5\cos(3\pi/5) + 0.08\cos(6\pi/5) = 0.5098$

$w_b(7) = 0.42 - 0.5\cos(7\pi/10) + 0.08\cos(7\pi/5) = 0.6892$

$w_b(8) = 0.42 - 0.5\cos(4\pi/5) + 0.08\cos(8\pi/5) = 0.8492$

$w_b(9) = 0.42 - 0.5\cos(9\pi/10) + 0.08\cos(9\pi/5) = 0.9603$

$w_b(10) = 0.42 - 0.5\cos(\pi) + 0.08\cos(2\pi) = 1$

AND BY SYMMETRY

$W_b(11) = W_b(9) = 0.9603$

$W_b(12) = W_b(8) = 0.8492$

$W_b(13) = W_b(7) = 0.6892$

$W_b(14) = W_b(6) = 0.5098$

$W_b(15) = W_b(5) = 0.34$

$W_b(16) = W_b(4) = 0.2008$

$W_b(17) = W_b(3) = 0.1014$

$W_b(18) = W_b(2) = 0.0402$

$W_b(19) = W_b(1) = 0.0092$

$W_b(20) = W_b(0) = 0$

OUR FILTER GOES FROM $n = -10$ to $n = 10$, BUT OUR WINDOW FUNCTION GOES FROM $n = 0$ to $n = 20$. SO WE WILL SHIFT THE WINDOW FUNCTION BY M UNITS TO GET A NEW WINDOW FUNCTION $W_b'(n)$ WHERE $W_b'(n) = W_b(n - M)$

APPLYING WINDOW FUNCTION TO FILTER:

$h_w(n) = h(n) \cdot w_b'(n)$

$h_w(0) = h(0) \cdot w_b(0) = (0.25)(1) = 0.25$

$h_w(1) = h(1) \cdot w_b'(1) = 0.2161$

$h_w(2) = h(2) \cdot w_b'(2) = 0.1352$

$h_w(3) = h(3) \cdot w_b'(3) = 0.0517$

$h_w(4) = h(4) \cdot w_b'(4) = 0$

$h_w(5) = h(5) \cdot w_b'(5) = -0.0153$

$h_w(6) = h(6) \cdot w_b'(6) = -0.0107$

$h_w(7) = h(7) \cdot w_b'(7) = -0.0033$

$h_w(8) = h(8) \cdot w_b'(8) = 0$

$h_w(9) = h(9) \cdot w_b'(9) = -0.0002$

$h_w(10) = h(10) \cdot w_b'(10) = 0$

~~$h_w(11) = h(11) \cdot w_b'(11) = -.0002$~~

AND BY SYMMETRY

$h_w(-1) = h_w(1) = 0.2161$

$h_w(-2) = h_w(2) = 0.1352$

$h_w(-3) = h_w(3) = 0.0517$

$h_w(-4) = h_w(4) = 0$

$h_w(-5) = h_w(5) = -0.0153$

$h_w(-6) = h_w(6) = -0.0107$

$h_w(-7) = h_w(7) = -0.0033$

$h_w(-8) = h_w(8) = 0$

$h_w(-9) = h_w(9) = -0.0002$

$h_w(-10) = h_w(10) = 0$

CALCULATING b COEFFICIENTS:

$$b_n = h_w(n-M)$$

$b_0 = h_w(0-10) = h_w(-10) = 0$

$b_1 = h_w(-9) = -0.0002$

$b_2 = h_w(-8) = 0$

$b_3 = h_w(-7) = -0.0033$

$b_4 = h_w(-6) = -0.0107$

$b_5 = h_w(-5) = -0.0153$

$b_6 = h_w(-4) = 0$

$b_7 = h_w(-3) = 0.0517$

$b_8 = h_w(-2) = 0.1352$

$b_9 = h_w(-1) = 0.2161$

$b_{10} = h_w(0) = 0.25$

$b_{11} = h_w(1) = 0.2161$

$b_{12} = h_w(2) = 0.1352$

$b_{13} = h_w(3) = 0.0517$

$b_{14} = h_w(4) = 0$

$b_{15} = h_w(5) = -0.0153$

$b_{16} = h_w(6) = -0.0107$

$b_{17} = h_w(7) = -0.0033$

$b_{18} = h_w(8) = 0$

$b_{19} = h_w(9) = -0.0002$

$b_{20} = h_w(10) = 0$

b.) DERIVE TRANSFER FUNCTION AND DIFFERENCE EQ.

TRANSFER FUNCTION

$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{2M} z^{-2M}$$

$$H(z) = 0z^0 - 0.0002z^{-1} + 0z^{-2} - 0.0033z^{-3}$$
$$- 0.0107z^{-4} - 0.0153z^{-5} + 0z^{-6}$$
$$+ 0.0517z^{-7} + 0.1352z^{-8} + 0.2161z^{-9}$$
$$+ 0.25z^{-10} + 0.216z^{-11} + 0.1352z^{-12}$$
$$+ 0.0517z^{-13} + 0z^{-14} - 0.0153z^{-15}$$
$$- 0.0107z^{-16} - 0.0033z^{-17} + 0z^{-18}$$
$$- 0.0002z^{-19} + 0z^{-20}$$

DIFFERENCE EQUATION

$$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(z) = H(z)X(z) = 0z^0X(z) - 0.0002z^{-1}X(z) + 0z^{-2}X(z)$$
$$- 0.0033z^{-3}X(z) - 0.0107z^{-4}X(z)$$
$$- 0.0153z^{-5}X(z) + 0z^{-6}X(z)$$
$$+ 0.0517z^{-7}X(z) + 0.1352z^{-8}X(z)$$
$$+ 0.2161z^{-9}X(z) + 0.25z^{-10}X(z)$$
$$+ 0.216z^{-11}X(z) + 0.1352z^{-12}X(z)$$
$$+ 0.0517z^{-13}X(z) + 0z^{-14}X(z) - 0.0153z^{-15}X(z)$$
$$- 0.0107z^{-16}X(z) + 0.0033z^{-17}X(z) + 0z^{-18}X(z) -$$

$$\therefore \; -0.0002\, z^{-19} X(z) + 0 z^{-20} X(z)$$

$$y(n) = \mathcal{Z}^{-1}\{Y(z)\} = -0.0002\, x(n-1) - 0.0033\, x(n-3)$$
$$- 0.0107\, x(n-4) - 0.0153\, x(n-5) + 0.0517\, x(n$$
$$+ 0.1352\, x(n-8) + 0.2161\, x(n-9) + 0.25\, x(n$$
$$+ 0.216\, x(n-11) + 0.1352\, x(n-12) + 0.0517\, x($$
$$- 0.0153\, x(n-15) - 0.0107\, x(n-16)$$
$$- 0.0033\, x(n-17) - 0.0002\, x(n-19)$$

C.) COMPUTE + PLOT MAG. OF FREQ RESPONSE @ $\Omega = 0, \pi/4, \pi/2, 3\pi/4,$

$$H(z)\Big|_{z=e^{j\Omega}} = b_0 e^{0j\Omega} + b_1 e^{-j\Omega} + b_2 e^{-j2\Omega} \ldots + b_{20} e^{-20j\Omega}$$

$$H(e^{j\Omega}) = e^{-j10\Omega}\left(b_0 e^{j10\Omega} + b_1 e^{j9\Omega} + b_2 e^{j8\Omega} \ldots + b_{10} e^{j0\Omega} \ldots + b_2\right.$$

RECALL $\cos(\theta) = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$ AND ALSO $b_0 = b_{20}$, $b_1 = b_{19}$, $b_2 = b_{18}$

$$H(e^{j\Omega}) = e^{-j10\Omega}\left(2b_0\cos(10\Omega) + 2b_1\cos(9\Omega) + 2b_2\cos(8\Omega) \ldots\right.$$
$$\left. + b_{10}\cos(0\Omega)\right)$$

$$H(e^{j\Omega}) = e^{-j10\Omega}\left(-0.0004\cos(9\Omega) + 0.0066\cos(7\Omega) - 0.0214\cos(6\right.$$
$$- 0.0306\cos(5\Omega) + 0.1034\cos(3\Omega)$$
$$+ 0.2704\cos(2\Omega) + 0.4322\cos(\Omega)$$
$$\left. + 0.25\right)$$

for $\Omega = 0$ rad.

$$H(e^{j(0)}) = e^{-j10(0)}\Big(-0.0004\cos(0) - 0.0066\cos(0) - 0.0214\cos(0) - 0.0306\cos(0) + 0.1034\cos(0) + 0.2704\cos(0) + .4322\cos(0) + 0.25\Big) = 0.997$$

for $\Omega = \frac{\pi}{4}$ rad

$$H(e^{j\frac{\pi}{4}}) = e^{-j10(\frac{\pi}{4})}\Big(-0.0004\cos(\tfrac{9\pi}{4}) - 0.0066\cos(\tfrac{7\pi}{4}) - 0.0214(\text{?}) - 0.0306\cos(\tfrac{5\pi}{4}) + 0.1034\cos(3\pi/4) + 0.2704\cos(\pi/2) + 0.4322\cos(\tfrac{\pi}{4}) + 0.25\Big)$$

$$= e^{-j\frac{5\pi}{2}} \cdot (0.5) = \cos(\tfrac{5\pi}{2}) - j\sin(\tfrac{5\pi}{2}) \cdot (0.5) = -j0.5$$

for $\Omega = \pi/2$ rad

$$H(e^{j\frac{\pi}{2}}) = e^{-j10(\frac{\pi}{2})}\Big(-0.0004\cos(\tfrac{9\pi}{2}) - 0.0066\cos(\tfrac{7\pi}{2}) - 0.0214(\text{B}) - 0.0306\cos(\tfrac{5\pi}{2}) + 0.1034(3\pi/2) + 0.2704\cos(\pi) + 0.4322\cos(\pi/2) + 0.25\Big)$$

$$= e^{-j10(\pi/2)}(.001) = -j0.001$$

for $\Omega = 3\pi/4$

$$H(e^{j\frac{3\pi}{4}}) = e^{-j10\left(\frac{3\pi}{4}\right)}\left(-0.0004\cos\left(\frac{27\pi}{4}\right) - 0.0066\cos\left(\frac{21\pi}{4}\right)\right.$$
$$- 0.0214\cos\left(9\pi/2\right) - 0.0306\cos\left(15\pi/4\right)$$
$$+ 0.1034\cos\left(9\pi/4\right) + 0.2704\cos\left(3\pi/2\right)$$
$$\left. + 0.4322\cos\left(3\pi/2\right) + 0.25\right)$$
$$= e^{-j10(3\pi/4)}\left(0.0008\right)$$
$$= \cos\left(3\pi/4\right) - j\sin\left(3\pi/4\right)(0.0008)$$
$$= -0.707 - j\,0.707(0.0008) = -0.0006 - j0.0006$$

for $\Omega = \pi$

$$H(e^{j\pi}) = e^{-j10\pi}\left(-0.0004\cos(9\pi) - 0.0066\cos(7\pi)\right.$$
$$- 0.0214\cos(6\pi) - 0.0306\cos(5\pi)$$
$$+ .1034\cos(3\pi) + .2704\cos(2\pi)$$
$$\left. + 0.4322\cos(\pi) + 0.25\right)$$
$$= e^{-j10\pi}\left(0.001\right) = \cos(10\pi) - j\sin(10\pi)(0.001) = 0.001$$

FOR PHASE RESPONSE: $\angle H(e^{j\Omega}) = \begin{cases} -\Omega & \text{if } H(e^{j\Omega}) > 0 \\ -\Omega + \pi & \text{if } H(e^{j\Omega}) > 0 \end{cases}$

or $= -M\Omega$ + possible phase 180°

$\angle H(e^{j0}) = 0$

$\angle H(e^{j\frac{\pi}{4}})$: $A(\omega) > 0$, so $\angle = -10\omega = -7.85$ rad

or $-450° \approx 270°$

$\angle H(e^{j\pi/2})$: $A(\omega) > 0$, so $\angle = -10\omega = -15.708$ rad

or -900 deg. $\approx 180°$

$\angle H(e^{j3\pi/4})$: $A(\omega) > 0$, so $\angle = -10\omega = -23.562$ rad

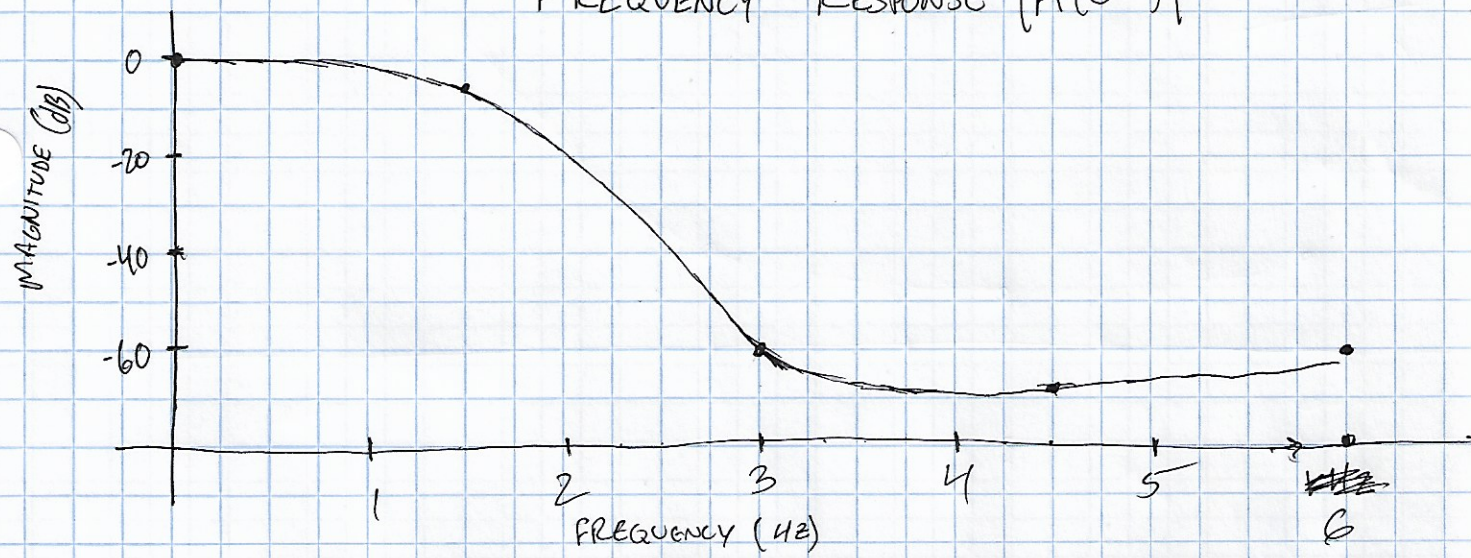
or $-1350° \approx 90°$

$\angle H(e^{j\pi})$: $A(\omega) > 0$, so $\angle = -10\omega = -31.46$ rad

or $-1800° \approx 0°$

Ω	$f = \frac{\Omega f_s}{2\pi}$ (Hz)	$H(e^{j\Omega})$	$\lvert H(e^{j\Omega}) \rvert$	$\lvert H(e^{j\Omega}) \rvert_{dB}$	$\angle H(e^{j\Omega})$
0	0	0.997	0.997	-0.026	0°
$\pi/4$	1500	$-j0.5$	0.5	-6.021	-450°
$\pi/2$	3000	$-j0.001$	0.001	-60	-900°
$3\pi/4$	4500	$-0.0006 - j0.0006$	0.0004	-67.96	-1350°
π	6000	0.001	0.001	-60	-1800°

FREQUENCY RESPONSE $|H(e^{j\Omega})|$



PHASE RESPONSE $\angle H(e^{j\Omega})$