DESIGN AND VERIFICATION OF AN FIR LOW PASS FILTER USING A BLACKMAN WINDOW

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Contents

1. OVERVIEW

This project implements and verifies a linear-phase FIR low-pass filter under a set of fixed design constraints. The goal is not to explore an open-ended filter design, but to correctly derive the filter coefficients, implement the filter in MATLAB, and verify its behavior in both the time and frequency domains.

Several key parameters were assigned as part of the project, including the filter length and window type. In particular, a Blackman window was specified, which largely determines the filter's sidelobe attenuation and transition-band behavior. While these constraints fix the overall filter structure, they still require proper analytical formulation and careful validation of the final result.

Starting from the ideal low-pass impulse response, the FIR coefficients are derived and then windowed to produce a realizable, linear-phase filter. The filter is implemented in MATLAB, where the impulse response and frequency response are examined to confirm expected properties such as coefficient symmetry, stopband attenuation, and transition-band width.

Overall, this project demonstrates a standard FIR filter design and verification workflow under imposed constraints. The results illustrate how window choice and filter length shape practical filter performance, and how analytical expectations translate into simulated behavior.

2. DESIGN CONSTRAINTS AND SPECIFICATIONS

The FIR filter in this project was designed under a fixed set of constraints provided as part of the assignment. These constraints define the overall structure and expected behavior of the filter, while leaving the coefficient derivation and verification as the primary tasks.

The filter is a discrete-time, linear-phase FIR low-pass filter operating on a real-valued input signal. The sampling frequency and cutoff frequency were specified in advance, along with the filter length NNN and the window function to be applied. In this case, the Blackman window was assigned, fixing the general tradeoff between sidelobe attenuation and transition-band width.

The relevant design specifications are summarized below:

- **Filter type:** Low-pass FIR
- **Phase response:** Linear phase
- **Window function:** Blackman
- **Filter length:** $N = 21$
- **Sampling frequency:** $f_s = 12\ kHz$
- **Cutoff frequency:** $f_{cs} = 1500\ Hz$

These constraints determine the class of filter being implemented and strongly influence its frequency-domain characteristics. In particular, the choice of window and filter length limits how sharp the transition band can be and how much attenuation is achievable in the stopband. Within these limits, the objective is to correctly derive the filter coefficients and verify that the resulting filter behaves as expected.

3. FIR FILTER FORMULATION

An FIR filter produces its output as a finite weighted sum of input samples. In discrete time, the filter output $y[n]$ is given by the convolution of the input signal $x[n]$ with the filter's impulse response $h[n]$:

$$y[n] = \sum_{k=0}^{N-1} h[k]x[n-k]$$

Equation 1. Discrete Convolution

where N is the filter length and $h[k]$ are the FIR coefficients. Because the impulse response has finite duration, the filter is linear and time-invariant, and its behavior is fully determined by the coefficient sequence.

In this project, the filter is designed to have linear phase. For a real-valued FIR filter, linear phase is achieved when the impulse response is symmetric about its midpoint. This symmetry ensures that all frequency components experience the same group delay, preserving waveform shape in the passband. The assigned filter length N fixes the location of this midpoint at $\frac{N-1}{2}$.

The design begins with the ideal low-pass frequency response, which is unity for frequencies below the cutoff frequency f_c and zero above it. Taking the inverse discrete-time Fourier transform of this ideal frequency response yields the corresponding impulse response, which has a $sinc$ shape. Expressed over a finite index range and centered about the midpoint of the filter, the truncated ideal impulse response is given by:

$$h_{ideal}[n] = \begin{cases} \dfrac{sin\left(2\pi f_c \dfrac{\left(n - \frac{N-1}{2}\right)}{f_s}\right)}{\pi\left(n - \frac{N-1}{2}\right)}, & n \neq \dfrac{N-1}{2} \\ \dfrac{2f_c}{f_s}, & n = \dfrac{N-1}{2}, \end{cases} \qquad 0 \leq n \leq N-1$$

Equation 2. Impulse Response of an FIR Filter

where f_s is the sampling frequency. This expression defines the unwindowed FIR coefficients prior to applying the window function. While this impulse response has the desired low-pass shape, truncation alone introduces significant sidelobes in the frequency domain. To control these effects and obtain a practical FIR filter, the impulse response is modified using a window function, as described in the next section.

4. WINDOWING AND COEFFICIENT DERIVATION

The truncated ideal impulse response derived in the previous section is finite in length but exhibits undesirable frequency-domain artifacts due to abrupt truncation. To reduce sidelobe levels and shape the frequency response in a controlled way, the impulse response is multiplied by a window function. In this project, the window type was assigned as part of the design constraints, with a Blackman window applied to the ideal impulse response.

The Blackman window is defined as:

$$w[n] = 0.42 - 0.5\ cos\frac{2\pi n}{N-1} + 0.08\,cos\frac{4\pi n}{N-1}\ , \qquad 0 \le n \le N-1$$

Equation 3. Blackman Window Function

and is known for providing strong sidelobe attenuation in the frequency domain. This improved stopband performance comes at the cost of a wider transition band compared to other common window functions, reflecting the tradeoff imposed by the assigned window choice.

Applying the window directly to the truncated impulse response produces the final FIR coefficient sequence used in implementation. Combining the ideal impulse response with the Blackman window yields

$$h[n] = h_{ideal}[n]w[n]$$

Equation 4. Windowed Impulse Response

This operation preserves the symmetry of the impulse response about the midpoint $\frac{N-1}{2}$, ensuring linear-phase behavior, while shaping the frequency response according to the characteristics of the Blackman window. The resulting coefficient sequence is finite, real-valued, and fully defines the FIR filter.

Once computed, these coefficients are used directly in the MATLAB implementation. The impulse response and frequency response are then evaluated to confirm that the filter exhibits the expected symmetry, transition-band behavior, and stopband attenuation associated with the assigned design constraints.

5. MATLAB IMPLEMENTATION

MATLAB was used to implement the derived FIR filter coefficients and to verify the filter’s behavior in both the time and frequency domains. Rather than serving as a black-box design tool, MATLAB was used to confirm that the analytically derived coefficients produce the expected results under the assigned design constraints.

5.1 Implementation overview

The windowed FIR coefficients derived in Sections 3 and 4 were computed numerically and stored as a discrete coefficient sequence in MATLAB. These coefficients fully define the filter and were used directly for all subsequent analysis.

The MATLAB script performs the following steps:

- computes the ideal impulse response
- applies the assigned Blackman window
- evaluates the impulse response and frequency response
- applies the filter to a noisy test signal

The full MATLAB implementation used to generate the results in this section is provided in **Appendix A**.

5.2 Frequency Domain Analysis

The frequency response of the FIR filter was evaluated by computing the discrete-time Fourier transform of the coefficient sequence. The resulting magnitude response confirms the expected low-pass behavior imposed by the design constraints.

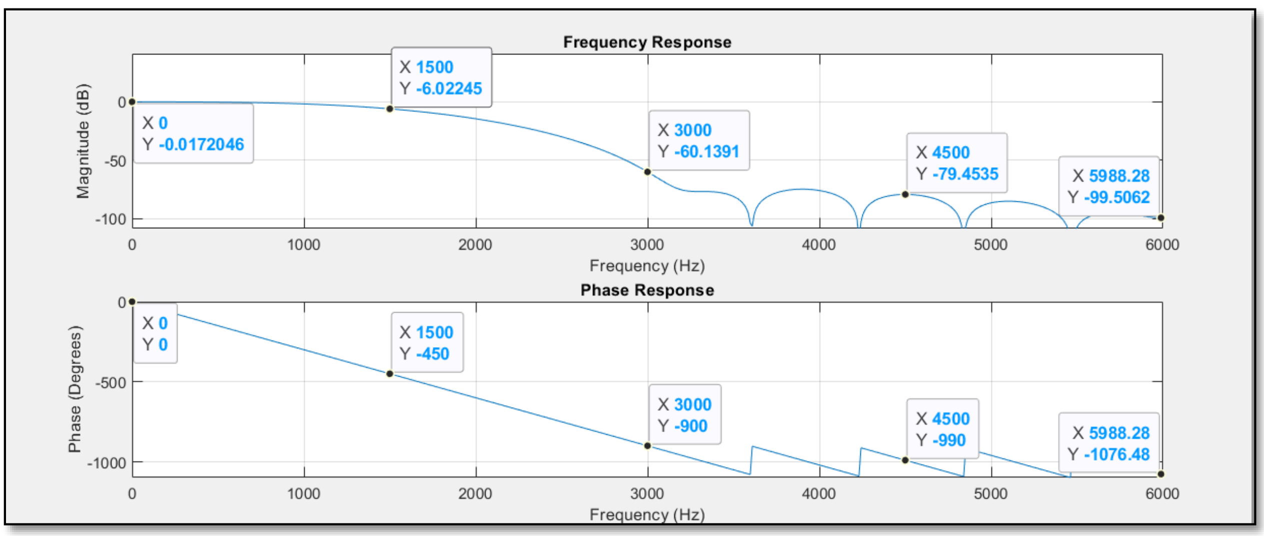


Figure 1. Bode Plot (Magnitude and Phase)

The magnitude response exhibits a smooth transition from passband to stopband, with strong sidelobe attenuation characteristic of the Blackman window. The cutoff frequency occurs near the specified design value, and the width of the transition band reflects the fixed filter length and window choice.

5.3 Time Domain Signal Filtering

To demonstrate the effect of the filter on a practical signal, a test signal containing high-frequency noise was passed through the FIR filter. The filtered output was then compared to both the original clean signal and the noisy input

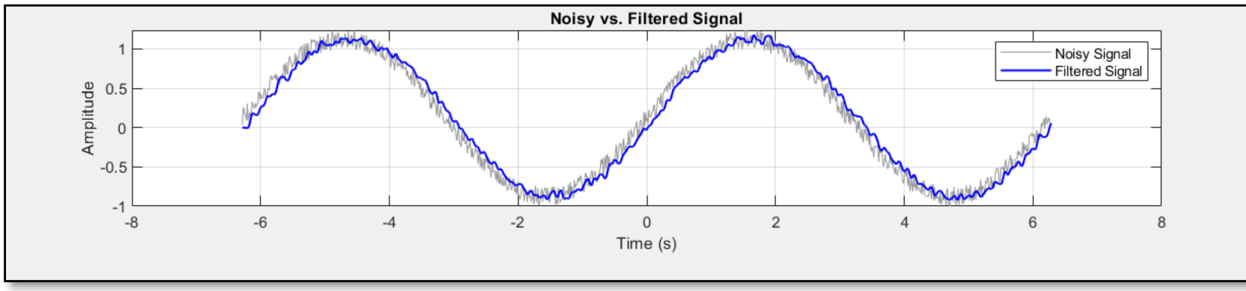


Figure 2. Time-Domain Signal: Noisy vs. Filtered (Overlayed)

The filtered output shows a clear reduction in high-frequency noise while preserving the underlying low-frequency signal content. This behavior is consistent with the expected response of a low-pass FIR filter and demonstrates the practical impact of the derived coefficient sequence.

5.4 Summary of Measured Results

Key performance metrics were extracted from the MATLAB analysis and are summarized in Table 5.1.

$\Omega\ (rad)$	$f = \Omega\ \frac{f_s}{2\pi}\ (Hz)$	$H(e^{j\Omega})$	$\lvert H(e^{j\Omega})\rvert$	$\lvert H(e^{j\Omega})\rvert_{dB}$	$\angle H(e^{j\Omega})^{\circ}$
0	0	0.997	0.997	-0.026	0°
$\frac{\pi}{4}$	1500	$-j0.05$	0.5	-6.021	-450°
$\frac{\pi}{2}$	3000	$-j0.001$	0.001	-60	-900°
$\frac{3\pi}{4}$	4500	$-0.0006 - j0.0006$	0.0004	-67.959	-1350°
π	6000	0.001	0.001	-60	-1800°

Figure 3. Summary Table (Magnitude and Phase at Selected Frequencies)

The measured cutoff frequency and passband gain closely match theoretical expectations based on the assigned design parameters. Stopband attenuation levels reflect the sidelobe suppression introduced by the Blackman window, confirming that the filter behaves as predicted.

6. RESULTS AND INTERPRETATION

The MATLAB results confirm that the derived FIR filter behaves as expected under the assigned design constraints. Both time-domain and frequency-domain analyses are consistent with theoretical predictions based on the windowed-sinc formulation.

The impulse response is symmetric about the midpoint of the filter, indicating linear-phase behavior and a constant group delay across frequencies. This symmetry is preserved by the Blackman window and confirms that the windowing operation does not alter the filter's phase characteristics.

In the frequency domain, the magnitude response exhibits strong stopband attenuation with a relatively wide transition band. This behavior reflects the known properties of the Blackman window and the fixed filter length, highlighting the tradeoff between sidelobe suppression and transition-band sharpness.

Filtering a noisy test signal further demonstrates the practical effect of these design choices. High-frequency noise is attenuated while low-frequency signal content is preserved, consistent with the intended low-pass behavior.

Overall, the results show close agreement between analytical expectations and simulated behavior, confirming the correctness of the coefficient derivation and implementation.

7. LIMITATIONS AND EXTENSIONS

This filter design was completed under fixed constraints, with the filter length and window type specified in advance. As a result, key performance characteristics such as transition-band width and stopband attenuation were largely determined by these parameters and could not be optimized further.

A narrower transition band would require either a longer filter or a different window function, each introducing tradeoffs in computational cost or sidelobe behavior. Exploring alternative windows, such as Hamming or Kaiser windows, would provide a useful comparison under the same filter length. Increasing the filter order would also improve frequency selectivity at the expense of increased computation and latency.

These extensions fall outside the scope of this project but would be natural next steps in a less constrained design setting.

8. **CONCLUSION**

This project implemented and verified a linear-phase FIR low-pass filter under fixed design constraints. Starting from the ideal impulse response, the filter coefficients were derived analytically, windowed using the assigned Blackman window, and implemented in MATLAB.

Simulation results confirmed the expected time- and frequency-domain behavior, including coefficient symmetry, strong stopband attenuation, and a transition band consistent with the imposed filter length and window choice. Filtering of a noisy test signal further demonstrated the practical effect of the design.

Overall, the results show close agreement between analytical formulation and simulated performance, illustrating a standard and reliable workflow for FIR filter implementation and verification under constrained design condition

9. APPENDIX A – MATLAB CODE

```
% JOHN RENNER - 00269141
% DSP I
% FINAL PROJECT
% MATLAB program to plot frequency response

clear all;
close all;
clc;

N = 21;             % filter length
Ftype = 1;          % LPF
fc = 1500;          % cutoff freq
fs = 12000;         % sampling freq
WnL = 2*pi*fc/fs;   % normalized angular cutoff (rad/sample)
WnH = 0;            % not used for LPF
Wtype = 5;          % Blackman window

%% ______________ Filter Calculation___________________________________

B = firwd(N, Ftype, WnL, WnH, Wtype);  % get b coefficients
[h, f] = freqz(B, 1, 512, fs);         % h = H(f), f=freq. vector
mag_db = 20* log10(abs(h));

%% ________________ Filter Plots _________________________________________
figure;

subplot(3,1,1);     % Create plot for freq. response
plot(f, mag_db);
xlabel('Frequency (Hz)');
ylabel('Magnitude (dB)');
title('Frequency Response');
grid on;

subplot(3,1,2);     % create plot for phase response
plot(f, unwrap(angle(h))*(180/pi));
xlabel('Frequency (Hz)');
ylabel('Phase (Degrees)');
title('Phase Response');
grid on;

%% _________Filtering a Noisy Signal ___________________________________
t = linspace(-2*pi,2*pi,1000);   % time vector for noisy signal
x = sin(t) + 0.25*rand(size(t)); % define noisy signal
y = conv(x,B);                   % apply filter to signal
y = y(1:length(x));              % needed to adjust length to avoid error

subplot(3,1,3);                  % create noisy signal plot
plot(t, x, 'Color', [0.6, 0.6, 0.6]); hold on;
plot(t, y, 'b', 'LineWidth', 1.2);
legend('Noisy Signal', 'Filtered Signal');
xlabel('Time (s)');
ylabel('Amplitude');
title('Noisy vs. Filtered Signal');
grid on;
```

Figure 4. This script implements the windowed-sinc FIR filter derived in Sections 3 and 4 and produces the frequency-domain and time-domain results discussed in Sections 5 and 6.

10. **APPENDIX B – HAND CALCULATIONS**

The following pages contain handwritten derivations used to compute the ideal impulse response, center tap value, and windowed FIR coefficients prior to MATLAB implementation. These calculations support the analytical development presented in Sections 3 and 4.

FIR FILTER DESIGN – FINAL PROJECT

a.) CALCULATE FILTER COEFFICIENTS

$f_c = 1500$ Hz $T_s = \frac{1}{12000}$ sec.

$f_s = 12000$ Hz. $N = 21 = 2M + 1$, $M = 10$

$$\Omega_c = 2\pi \cdot f_c \cdot T_s = (2\pi)(1500)\left(\frac{1}{12000}\right) = 0.25\pi \text{ rad}$$

$$h(n) = \begin{cases} \frac{\Omega_c}{\pi} & \text{for } n = 0 \\ \frac{\sin(\Omega_c n)}{n\pi} & \text{for } -M \le n \le M,\ n \ne 0 \end{cases}$$

$$h(0) = \frac{0.25\pi}{\pi} = 0.25$$

$$h(1) = \frac{\sin(0.25\pi)}{\pi} = \cancel{0.1592}\ 0.2251$$

$$h(2) = \frac{\sin(\pi/2)}{2\pi} = \cancel{0.15915}\ 0.1592$$

$$h(3) = \frac{\sin(3\pi/4)}{3\pi} = 0.0750$$

$$h(4) = \frac{\sin(\pi)}{4\pi} = 0$$

$$h(5) = \frac{\sin(5\pi/4)}{5\pi} = -0.045$$

$$h(6) = \frac{\sin(3\pi/2)}{6\pi} = -0.0531$$

$$h(7) = \frac{\sin(7\pi/4)}{7\pi} = -0.0322$$

$$h(8) = \frac{\sin(2\pi)}{7\pi} = 0$$

$$h(9) = \frac{\sin(9\pi/4)}{9\pi} = 0.025 \qquad h(10) = \frac{\sin(10\pi/4)}{10\pi} = 0.0318$$

AND BY SYMMETRY:

$h(-1) = h(1) = 0.2251$

$h(-2) = h(2) = 0.1592$

$h(-3) = h(3) = 0.0750$

$h(-4) = h(4) = 0$

$h(-5) = h(5) = -0.045$

$h(-6) = h(6) = -0.0531$

$h(-7) = h(7) = -0.0322$

$h(-8) = h(8) = 0$

$h(-9) = h(9) = 0.025$

$h(-10) = h(10) = 0.0318$

BLACKMAN WINDOW FUNCTION

$W_b(n) = 0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right)$

$W_b(0) = 0.42 - 0.5\cos(0) + 0.08\cos(0) = 0$

$W_b(1) = 0.42 - 0.5\cos\left(\frac{\pi}{10}\right) + 0.08\cos(\pi/5) = 0.0092$

$W_b(2) = 0.42 - 0.5\cos(\pi/5) + 0.08\cos(2\pi/5) = 0.0402$

$W_b(3) = 0.42 - 0.5\cos\left(\frac{3\pi}{10}\right) + 0.08\cos(3\pi/5) = 0.1014$

$W_b(4) = 0.42 - 0.5\cos(2\pi/5) + 0.08\cos(4\pi/5) = 0.2008$

$W_b(5) = 0.42 - 0.5\cos(\pi/2) + 0.08\cos(\pi) = 0.34$

$W_b(6) = 0.42 - 0.5\cos(3\pi/5) + 0.08\cos(6\pi/5) = 0.5098$

$W_b(7) = 0.42 - 0.5\cos(7\pi/10) + 0.08\cos(7\pi/5) = 0.6892$

$W_b(8) = 0.42 - 0.5\cos(4\pi/5) + 0.08\cos(8\pi/5) = 0.8492$

$W_b(9) = 0.42 - 0.5\cos(9\pi/10) + 0.08\cos(9\pi/5) = 0.9603$

$W_b(10) = 0.42 - 0.5\cos(\pi) + 0.08\cos(2\pi) = 1$

AND BY SYMMETRY

$W_b(11) = W_b(9) = 0.9603$

$W_b(12) = W_b(8) = 0.8492$

$W_b(13) = W_b(7) = 0.6892$

$W_b(14) = W_b(6) = 0.5098$

$W_b(15) = W_b(5) = 0.34$

$W_b(16) = W_b(4) = 0.2008$

$W_b(17) = W_b(3) = 0.1014$

$W_b(18) = W_b(2) = 0.0402$

$W_b(19) = W_b(1) = 0.0092$

$W_b(20) = W_b(0) = 0$

OUR FILTER GOES FROM $n = -10$ to $n = 10$, BUT OUR WINDOW FUNCTION GOES FROM $n=0$ to $n=20$. SO WE WILL SHIFT THE WINDOW FUNCTION BY M UNITS to GET A NEW WINDOW FUNCTION $W_b'(n)$ WHERE $W_b'(n) = W_b(n-M)$

APPLYING WINDOW FUNCTION TO FILTER:

$h_w(n) = h(n) \cdot W_{BL}'(n)$

$h_w(0) = h(0) \cdot W_b(0) = (0.25)(1) = 0.25$

$h_w(1) = h(1) \cdot W_b'(1) = 0.2161$

$h_w(2) = h(2) \cdot W_b'(2) = 0.1352$

$h_w(3) = h(3) \cdot W_b'(3) = 0.0517$

$h_w(4) = h(4) \cdot W_b'(4) = 0$

$h_w(5) = h(5) \cdot W_b'(5) = -0.0153$

$h_w(6) = h(6) \cdot W_b'(6) = -0.0107$

$h_w(7) = h(7) \cdot W_b'(7) = -0.0033$

$h_w(8) = h(8) \cdot W_b'(8) = 0$

$h_w(9) = h(9) \cdot W_b'(9) = -0.0002$

$h_w(10) = h(10) \cdot W_b'(10) = 0$

$\cancel{h_w(11) = h(11) \cdot W_b'(11) = -0.0002}$

AND BY SYMMETRY

$h_w(-1) = h_w(1) = 0.2161$

$h_w(-2) = h_w(2) = 0.1352$

$h_w(-3) = h_w(3) = 0.0517$

$h_w(-4) = h_w(4) = 0$

$h_w(-5) = h_w(5) = -0.0153$

$h_w(-6) = h_w(6) = -0.0107$

$h_w(-7) = h_w(7) = -0.0033$

$h_w(-8) = h_w(8) = 0$

$h_w(-9) = h_w(9) = -0.0002$

$h_w(-10) = h_w(10) = 0$

CALCULATING b COEFFICIENTS:

$$b_n = h_w(n-M)$$

$b_0 = h_w(0-10) = h_w(-10) = 0$

$b_1 = h_w(-9) = -0.0002$

$b_2 = h_w(-8) = 0$

$b_3 = h_w(-7) = -0.0033$

$b_4 = h_w(-6) = -0.0107$

$b_5 = h_w(-5) = -0.0153$

$b_6 = h_w(-4) = 0$

$b_7 = h_w(-3) = 0.0517$

$b_8 = h_w(-2) = 0.1352$

$b_9 = h_w(-1) = 0.2161$

$b_{10} = h_w(0) = 0.25$

$b_{11} = h_w(1) = 0.2161$

$b_{12} = h_w(2) = 0.1352$

$b_{13} = h_w(3) = 0.0517$

$b_{14} = h_w(4) = 0$

$b_{15} = h_w(5) = -0.0153$

$b_{16} = h_W(6) = -0.0107$

$b_{17} = h_w(7) = -0.0033$

$b_{18} = h_w(8) = 0$

$b_{19} = h_w(9) = -0.0002$

$b_{20} = h_w(10) = 0$

$$\therefore \; -0.0002\, z^{-19} X(z) + 0 z^{-20} X(z)$$

$$y(n) = \mathcal{Z}^{-1}\{Y(z)\} = -0.0002\, x(n-1) - 0.0033\, x(n-3)$$
$$- 0.0107\, x(n-4) - 0.0153\, x(n-5) + 0.0517\, x(\ldots$$
$$+ 0.1352\, x(n-8) + 0.2161\, x(n-9) + 0.25\, x(n\ldots$$
$$+ 0.216\, x(n-11) + 0.1352\, x(n-12) + 0.0517\, x(\ldots$$
$$- 0.0153\, x(n-15) - 0.0107\, x(n-16)$$
$$- 0.0033\, x(n-17) - 0.0002\, x(n-19)$$

C.) COMPUTE + PLOT MAG. OF FREQ RESPONSE @ $\Omega = 0, \pi/4, \pi/2, 3\pi/4, \ldots$

$$H(z)\Big|_{z=e^{j\Omega}} = b_0 e^{0j\Omega} + b_1 e^{-j\Omega} + b_2 e^{-j2\Omega} \ldots + b_{20} e^{-20j\Omega}$$

$$H(e^{j\Omega}) = e^{-j10\Omega}\left(b_0 e^{j10\Omega} + b_1 e^{j9\Omega} + b_2 e^{j8\Omega} \ldots + b_{10} e^{j0\Omega} \ldots + b_{2\ldots}\right.$$

RECALL $\cos(\theta) = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$ AND ALSO $b_0 = b_{20},\; b_1 = b_{19},\; b_2 = b_{18}$

$$H(e^{j\Omega}) = e^{-j10\Omega}\big(2b_0 \cos(10\Omega) + 2b_1 \cos(9\Omega) + 2b_2 \cos(8\Omega) \ldots$$
$$+ b_{10} \cos(0\Omega)\big)$$

$$H(e^{j\Omega}) = e^{-j10\Omega}\big(-0.0004 \cos(9\Omega) - 0.0066 \cos(7\Omega) - 0.0214 \cos(6\ldots$$
$$- 0.0306 \cos(5\Omega) + 0.1034 \cos(3\Omega)$$
$$+ 0.2704 \cos(2\Omega) + 0.4322 \cos(\Omega)$$
$$+ 0.25\big)$$

for $\Omega = 0$ rad.

$$H(e^{j(0)}) = e^{-j\,10(0)}\Big(-0.0004\cos(0) - 0.0066\cos(0) - 0.0214\cos(0) - 0.0306\cos(0) + 0.1034\cos(0) + 0.2704\cos(0) + .4322\cos(0) + 0.25\Big)$$

$$= 0.997$$

for $\Omega = \frac{\pi}{4}$ rad

$$H(e^{j\frac{\pi}{4}}) = e^{-j10(\frac{\pi}{4})}\Big(-0.0004\cos\left(\tfrac{9\pi}{4}\right) - 0.0066\cos\left(\tfrac{7\pi}{4}\right) - 0.0214\left(\tfrac{3}{}\right. - 0.0306\cos\left(\tfrac{5\pi}{4}\right) + 0.1034\cos\left(3\pi/4\right) + 0.2704\cos(\pi/2) + 0.4322\cos\left(\tfrac{\pi}{4}\right) + 0.25\Big)$$

$$= e^{-j\frac{5\pi}{2}} \cdot (0.5) = \cos\left(\tfrac{5\pi}{2}\right) - j\sin\left(\tfrac{5\pi}{2}\right)\cdot(0.5) = -j0.5$$

for $\Omega = \pi/2$ rad

$$H(e^{j\frac{\pi}{2}}) = e^{-j10(\frac{\pi}{2})}\Big(-0.0004\cos\left(\tfrac{9\pi}{2}\right) - 0.0066\cos\left(\tfrac{7\pi}{2}\right) - 0.0214\left(3\right. - 0.0306\cos\left(\tfrac{5\pi}{2}\right) + 0.1034\left(3\pi/2\right) + 0.2704\cos(\pi) + 0.4322\cos(\pi/2) + 0.25\Big)$$

$$= e^{-j10(\pi/2)}(.001) = j0.001$$

for $\Omega = 3\pi/4$

$$H(e^{j\frac{3\pi}{4}}) = e^{-j10\left(\frac{3\pi}{4}\right)}\Big(-0.0004\cos\left(\tfrac{27\pi}{4}\right) - 0.0066\cos\left(\tfrac{21\pi}{4}\right)$$
$$- 0.0214\cos(9\pi/2) - 0.0306\cos(15\pi/4)$$
$$+ 0.1034\cos(9\pi/4) + 0.2704\cos(3\pi/2)$$
$$+ 0.4322\cos(3\pi/2) + 0.25\Big)$$
$$= e^{-j10(3\pi/4)}(0.0008)$$
$$= \cos(3\pi/4) - j\sin(3\pi/4)(0.0008)$$
$$= -0.707 - j\,0.707(0.0008) = -0.0006 - j0.0006$$

for $\Omega = \pi$

$$H(e^{j\pi}) = e^{-j10\pi}\big(-0.0004\cos(9\pi) - 0.0066\cos(7\pi)$$
$$- 0.0214\cos(6\pi) - 0.0306\cos(5\pi)$$
$$+ .1034\cos(3\pi) + .2704\cos(2\pi)$$
$$+ 0.4322\cos(\pi) + 0.25\big)$$
$$= e^{-j10\pi}(0.001) = \cos(10\pi) - j\sin(10\pi)(0.001) = 0.001$$

FOR PHASE RESPONSE: $\angle H(e^{j\Omega}) = \begin{cases} -\Omega & \text{if } H(e^{j\Omega}) > 0 \\ -\Omega + \pi & \text{if } H(e^{j\Omega}) > 0 \end{cases}$

or $= -M\Omega$ + possible phase $180°$

$\angle H(e^{j0}) = 0$

$\angle H(e^{j\frac{\pi}{4}})$: $A(\omega) > 0$, so $\angle = -10\omega = -7.85$ rad

or $-450° \approx 270°$

$\angle H(e^{j\pi/2})$: $A(\omega) > 0$, so $\angle = -10\omega = -15.708$ rad

or -900 deg. $\approx 180°$

$\angle H(e^{j3\pi/4})$: $A(\omega) > 0$, so $\angle = -10\omega = -23.562$ rad

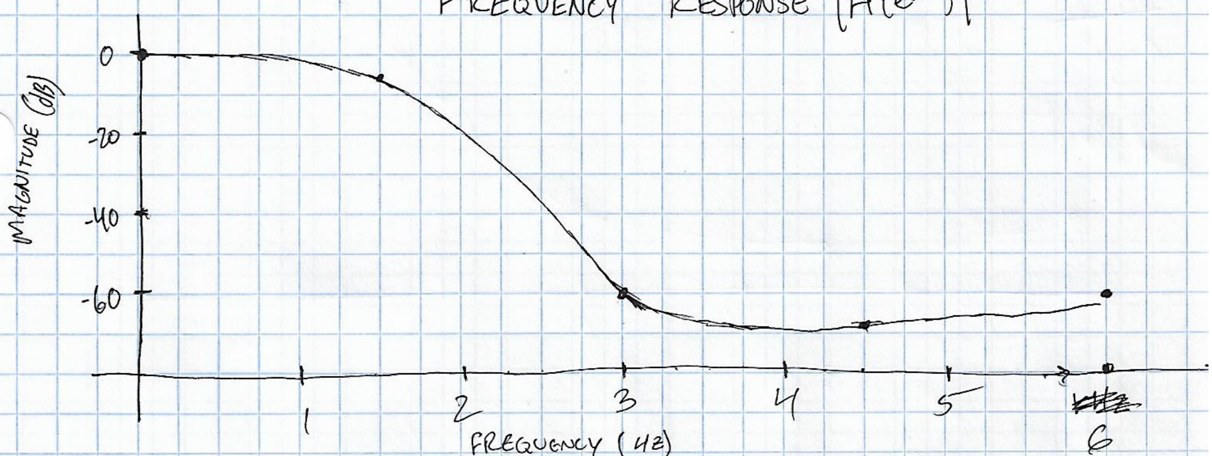
or $-1350° \approx 90°$

$\angle H(e^{j\pi})$: $A(\omega) > 0$, so $\angle = -10\omega = -31.46$ rad

or $-1800° \approx 0°$

Ω	$f = \frac{\Omega f_s}{2\pi}$ (Hz)	$H(e^{j\Omega})$	$\lvert H(e^{j\Omega})\rvert$	$\lvert H(e^{j\Omega})\rvert_{dB}$	$\angle H(e^{j\Omega})$
0	0	0.997	0.997	−0.026	0°
$\pi/4$	1500	$-j0.5$	0.5	−6.021	−450°
$\pi/2$	3000	$-j0.001$	0.001	−60	−900°
$3\pi/4$	4500	$-0.0006-j0.0006$	0.0004	−67.96	−1350° (
π	6000	0.001	0.001	−60	−1800° (

FREQUENCY RESPONSE $\lvert H(e^{j\Omega})\rvert$



PHASE RESPONSE $\angle H(e^{j\Omega})$