Error, Residual, Vector norms

CS111 Lecture

Oct 19, 2020

HOW BIG IS A VECTOR?

$$v = \begin{bmatrix} v_0 \\ v_1 \\ \vdots \\ v_{n-1} \end{bmatrix}$$

$$\text{norm}(v) \equiv \|v\|$$

$$\|v\| = \left(v_0^2 + v_1^2 + \cdots + v_{n-1}^2\right)^{1/2}$$

$$= \left(\sum_{i=0}^{n-1} v_i^2\right)^{1/2} \quad [2\text{-norm}]$$

Suppose $Ax_{exact} = b$

How good is some vector x?

DEF

error $\equiv x_{exact} - x$ $= 0$ if x is right (vector)

residual $\equiv b - Ax$ $= 0$ if x is right (vector)

residual norm $\equiv \|b - Ax\|$ $= 0$ if x is right (number)

relative residual norm $\equiv$

$$\|b - Ax\| \,/\, \|b\|$$

THERE ARE DIFFERENT NORMS

$$\|v\|_2 = \left(\sum_{i=0}^{n-1} v_i^2\right)^{1/2}$$ EUCLIDEAN NORM

$$\|v\|_1 = \sum_{i=0}^{n-1} |v_i|$$ MANHATTAN NORM

$$\|v\|_\infty = \max_{0 \le i \le n-1} |v_i|$$ MAX-NORM

$$\|v\|_p = \left(\sum_{i=0}^{n-1} v_i^p\right)^{1/p}$$ P-NORM

$\|v\|_0$ = # of nonzero v_i's.

¡(⚠ NOT A REAL NORM)!