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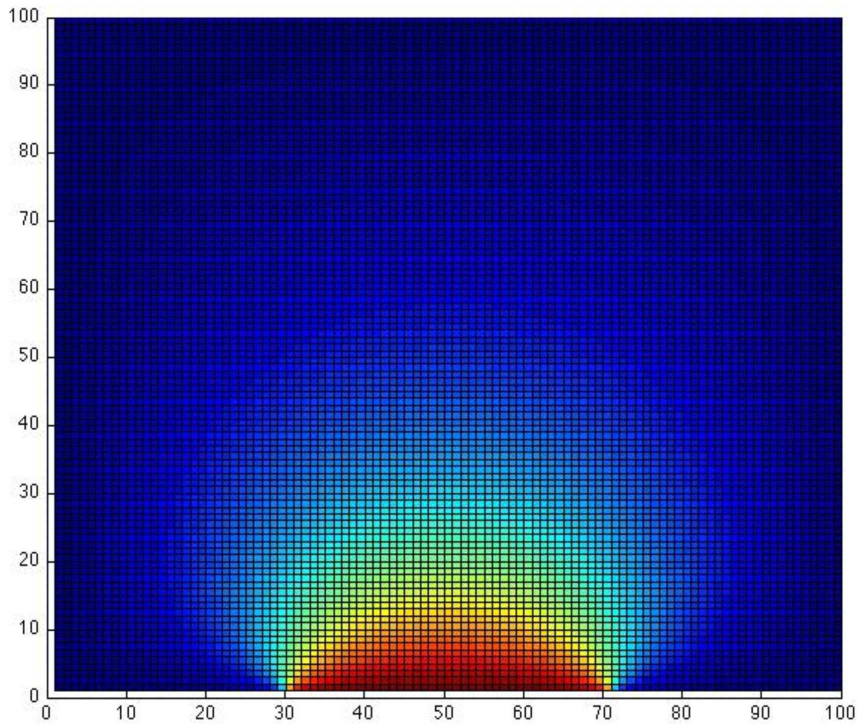


The physics: Poisson's equation

$$\nabla^2 u(x,y) \equiv \frac{\partial^2 u}{\partial x^2}(x,y) + \frac{\partial^2 u}{\partial y^2}(x,y) = f(x,y)$$

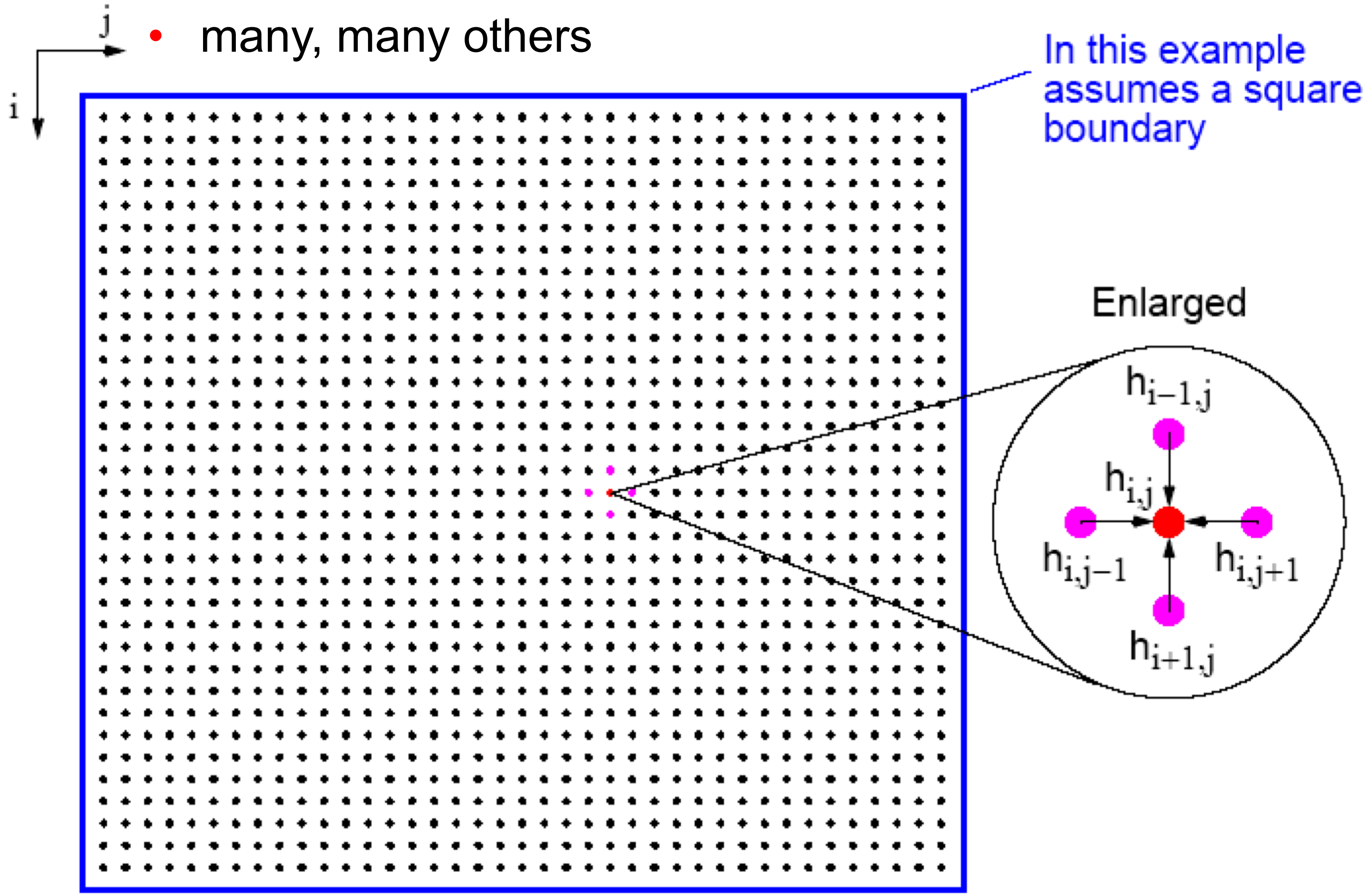
for $(x,y) \in R = \{ (x,y) \mid a<x<b,\ c<y<d \}$, and

$$u(x,y) = g(x,y)$$

for (x,y) on the boundary of R.

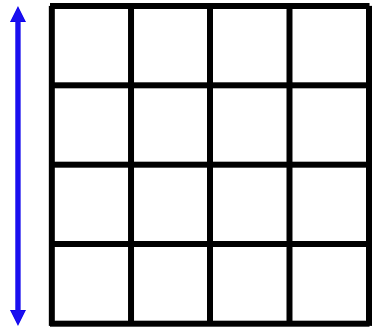
Many Physical Models Use Stencil Computations

- PDE models of heat, fluids, structures, …
- Weather, airplanes, bridges, bones, …
- Game of Life
- many, many others



Model Problem: Solving Poisson's equation for temperature

$k = n^{1/2}$



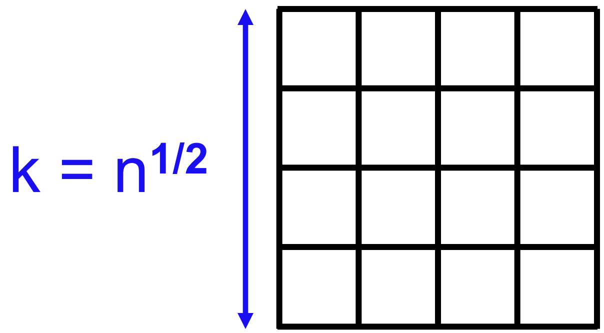
- Discrete approximation to Poisson's equation:

$$t(i) = \frac{1}{4} \left(t(i-k) + t(i-1) + t(i+1) + t(i+k) \right)$$

- Intuitively:

Temperature at a point is the average
of the temperatures at surrounding points

Model Problem: Solving Poisson's equation for temperature



- For each i from 1 to n, except on the boundaries:

$$-t(i-k) - t(i-1) + 4*t(i) - t(i+1) - t(i+k) = 0$$

- n equations in n unknowns: $A*t = b$
- Each row of A has at most 5 nonzeros
- In three dimensions, $k = n^{1/3}$ and each row has at most 7 nzs

A Stencil Computation Solves a System of Linear Equations

- Solve $Ax = b$ for x
- Matrix A, right-hand side vector b, unknown vector x
- A is *sparse*: most of the entries are 0