

## CS 111: Review Quiz: Answer key

1.

$$A^T = \begin{pmatrix} 3 & 0 & 1 \\ -1 & 1 & 0 \\ 2 & 2 & -1 \end{pmatrix}, A^2 = \begin{pmatrix} 11 & -4 & 2 \\ 2 & 1 & 0 \\ 2 & -1 & 3 \end{pmatrix}, A^T A = \begin{pmatrix} 10 & -3 & 5 \\ -3 & 2 & 0 \\ 5 & 0 & 9 \end{pmatrix}.$$

2.  $\|(3, 1, 4, 1, 5)^T\|_2 = \sqrt{52} \approx 7.2111$

3.

$$\begin{pmatrix} 2 & -3 & 1 \\ 0 & 2 & 3 \\ 1 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 4 \end{pmatrix}$$

In numpy, `A = np.array([[2, -3, 1], [0, 2, 3], [1, 0, 1]])` and `b = np.array([1, 7, 4])`. The 1-dimensional numpy array `b` can represent either a column vector or a row vector; Python's matrix-vector multiplication operator `@` will do the right thing.

4.

$$x = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

5. There are many answers to this. Here's one: Let `A = np.array([[1, 2], [2, 4]])` and `b = np.array([3, 3])`. (In math notation we write  $b = (3, 3)^T$ , which is a column vector because of the transpose.) Explanation 1 (column view): Matrix  $A$  is singular, so the space spanned by its columns is only one-dimensional, and it consists of multiples of the vector  $(1, 2)^T$ , which do not include  $b$ . Explanation 2 (row view): The two lines described by the rows of  $Ax = b$  are parallel and hence do not intersect. Explanation 3 (brute force view): No matter what  $x$  is, the second entry of  $Ax$  will be equal to twice the first entry of  $Ax$ , which rules out  $b$ .

6. There are many answers to this. Here's one: Take  $A$  to be the same matrix as in the previous problem, and  $b = (3, 6)^T$ . Two solutions are  $x = (1, 1)^T$  and  $x = (3, 0)^T$ .

7. No, it's not possible to have exactly two solutions to  $Ax = b$ . If  $x$  and  $y$  are two different solutions, then there are infinitely many solutions:  $x + \alpha(y - x)$  is a solution for every  $\alpha$ .

8.  $A$  has two eigenvalues, 3 and 5. Any multiple of  $(1, 1)^T$  is an eigenvector corresponding to 3, and any multiple of  $(1, -1)^T$  is an eigenvector corresponding to 5.

9.  $f'(x) = 21x^2 - 4x + 4$ .

10.  $\partial z / \partial x = e^{y/2}$ , and  $\partial z / \partial y = (x/2)e^{y/2}$ .

11.  $f(x) = x^3/3 - \cos x + c$  for some constant  $c$  (any constant will do).

12. The height is maximum when the derivative  $dh/dt$  is zero.  $dh/dt = 1280 - 32t$ , which is zero when  $t = 40$ , at which time the height is  $h = 25600$  feet. The bullet hits the ground when  $h = 0$ , which means  $1280t = 16t^2$ , which means  $t = 1280/16 = 80$  seconds after firing. (The other solution to  $h = 0$  is of course  $t = 0$ .)

13.  $y = e^{x^2/2}$ .