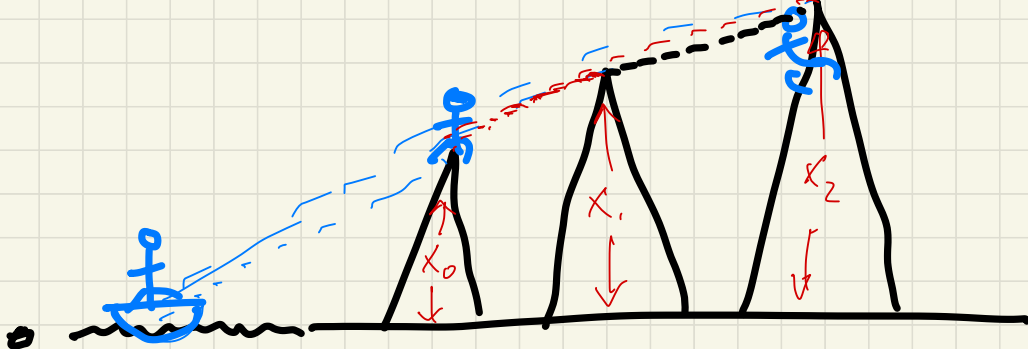


DATA FITTING BY LEAST SQUARES

CS 111

Nov. 5, 2020





First
surveyor

$$x_0 = 1237$$

$$x_1 = 1941$$

$$x_2 = 2417$$

they can't all
be right.

SL thinks

$$x_1 - x_0 = 704$$

Second
surveyor

$$x_1 - x_0 = 711$$

$$x_2 - x_0 = 1177$$

$$x_2 - x_1 = 475$$

overdetermined!

$Ax = b$
doesn't have
a solution.

$$A \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = b$$

x

b

$$\begin{matrix} m & n \\ \boxed{A} & \boxed{x} \end{matrix} \approx \begin{matrix} m \\ \boxed{b} \end{matrix} \quad m > n$$

Least squares solution: Find to find the x that minimizes $\|b - Ax\|_2$

$$A = QR \text{ (full size)} \quad \begin{matrix} m & n \\ \boxed{A} & = \begin{matrix} m & n \\ \boxed{Q} & \boxed{R} \end{matrix} \end{matrix}$$

$\min \|r\|_2 = \|b - Ax\|_2$ is the same
as $\min \|\tilde{Q}^T r\|_2 = \|\tilde{Q}^T b - \tilde{Q}^T A x\|_2$

$$\tilde{Q}^T A x = \underbrace{\tilde{Q}^T (Q R)}_I x = R x$$

$$\min_x \|Q^T b - R x\|_2$$

$$\min_x \|Q^T b - R x\|_2$$

$$\begin{bmatrix} Q \\ b \end{bmatrix} - \begin{bmatrix} Q \\ 0 \end{bmatrix} x$$

$$R x \approx Q^T b = y$$

$$\begin{bmatrix} R \\ 0 \end{bmatrix} x \approx \begin{bmatrix} y \\ 0 \end{bmatrix}$$

STILL
OVERDET.

EASY TO SEE BEST SOLN!

$$R x = \begin{bmatrix} \text{perfect} \\ \text{hopeless} \end{bmatrix} \approx \begin{bmatrix} \text{perfect} \\ \text{hopeless} \end{bmatrix}$$

WE ACTUALLY USE ECONOMY-SIZE.

$$A = Q_2 \begin{bmatrix} R \\ 0 \end{bmatrix}$$

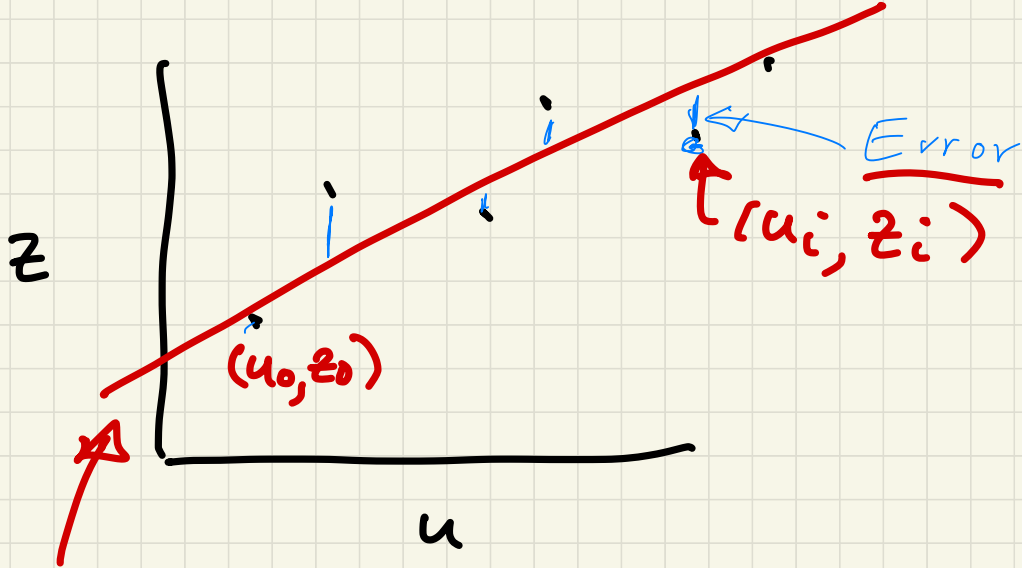
$$Ax \approx b \Leftrightarrow Rx \approx Q_2^T b$$

$$\begin{matrix} n \\ \boxed{\begin{matrix} R \\ 0 \end{matrix}} \end{matrix} \begin{matrix} n \\ x \end{matrix} \approx \begin{matrix} n \\ \boxed{} \end{matrix}$$

\uparrow
 $Q_2^T b$

$$\begin{matrix} n \\ \boxed{} \end{matrix} \begin{matrix} n \\ Q_2^T \end{matrix} \begin{matrix} n \\ y \end{matrix} = \begin{matrix} n \\ \boxed{} \end{matrix}$$

\uparrow
 y

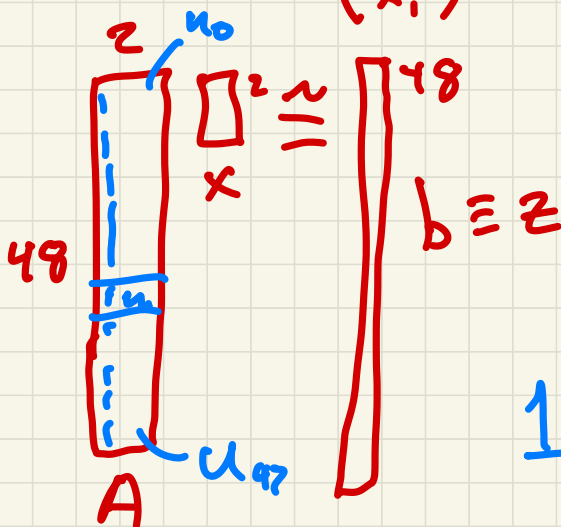


line: $y = x_0 + x_1 u$

want: $y(u_i) = z_i$

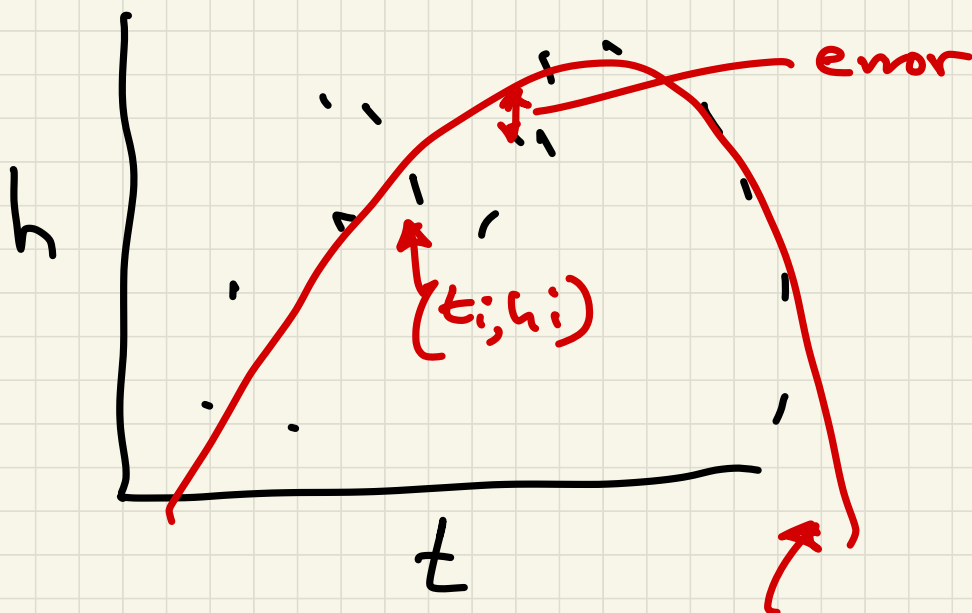
least sqs: $\min \left(\sum_{i=0}^{47} (y(u_i) - z_i)^2 \right)$

$$X = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$$



$$y(u_i) \approx z_i$$

$$1 x_0 + x_1 u_i \approx z_i$$



$$Ax = b$$

$$y = x_0 + x_1 t + x_2 t^2$$

$$y(t_i) = x_0 + x_1 t_i + x_2 t_i^2 \approx h_i$$

$A =$

$$\begin{bmatrix} 1 & t_0 & t_0^2 \\ 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}$$

\approx

$$\begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ \vdots \end{bmatrix}$$