SVD and Image Compression

Nov 17, 2020

CS 111

SVD AND DATA COMPRESSION

$$\underset{m\times n}{A} = \underset{m\times m}{U}\;\underset{m\times n}{S}\;\underset{n\times n}{V^T}$$

$S = \mathrm{diag}(\sigma_0, \sigma_1, \ldots)$

$$\underset{m\times n}{A_k} = \sum_{i=0}^{k-1} \sigma_i\, U[:,i]\, V[:,i]^T$$

$$= \underset{m\times k}{U_k}\;\underset{k\times k}{S_k}\;\underset{k\times n}{V_k^T}$$

$V_k^T = (V[:, :k])^T$

$S_k = \mathrm{diag}(\sigma_0, \ldots \sigma_{k-1})$

$U_k = U[:, :k]$

A_k is the same shape as A, $\|A_k - A\|_2 = \sigma_k$

A_k has mn elements, but we can store A_k as just U_k, S_k, V_k: $(m+n+1)k$ numbers.

If $k \ll m,n$ this is a big savings.