

ITERATIVE METHODS FOR SOLVING $Ax=b$

(Jacobi and Conjugate
Gradient)

CS 111

Oct 22, 2020

$$\begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \end{bmatrix}$$

answer is
 $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$$\begin{aligned} 3x_0 + x_1 &= 9 & \Rightarrow x_0 &= (9 - x_1) / 3 \\ -2x_0 + 4x_1 &= 8 & \Rightarrow x_1 &= (8 + 2x_0) / 4 \end{aligned}$$

IDEA: Change x_i to satisfy equation # i
 (all at once)

MATRIX VIEW

$$\begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$$

$$A = C + D$$

← diagonal part of A.

$$x = (b - Cx) \cdot D^{-1}$$

$$x = (b - Cx) / d$$

↑ ↑ $d = \text{diagonal of } D$
 as a vector
 pointwise divide