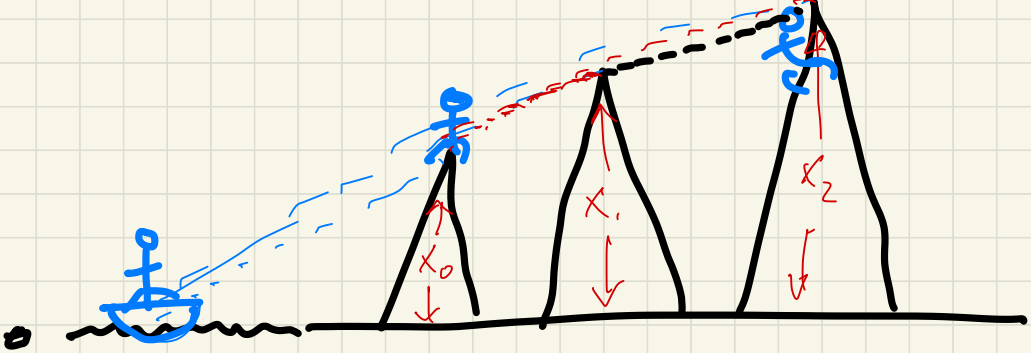
DATA FITTING BY LEAST SQUARES

CS 111

Nov. 5, 2020



First surveyor:

$x_0 = 1237.$

$x_1 = 1941$

$x_2 = 2417$

they can't all be right.

S1 thinks $x_1 - x_0 = 704$

Second surveyor:

$x_1 - x_0 = 711$ ✗

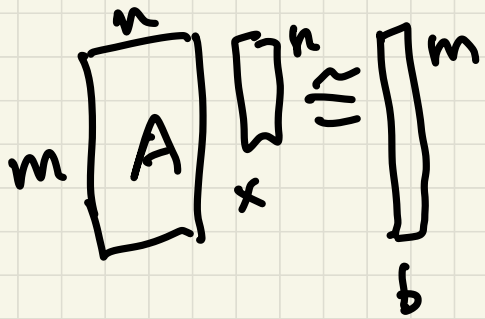
$x_2 - x_0 = 1177$

$x_2 - x_1 = 475$ ✗

Overdetermined!

$Ax = b$ doesn't have a solution.

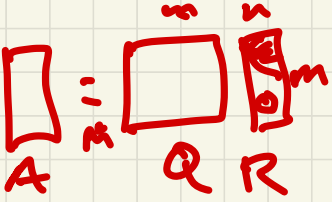
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \quad x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} \cong b = \begin{bmatrix} 1237 \\ 1941 \\ 2417 \\ 711 \\ 1177 \\ 475 \end{bmatrix}$$



$m > n$

Least squares solution: Find to find the x that minimizes $\|b - Ax\|_2$

$A = QR$ (full size)

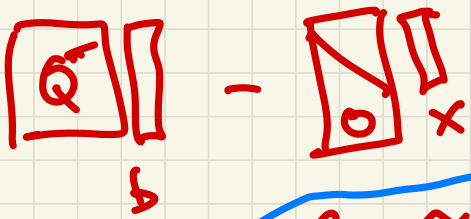


$\min \|r\|_2 = \|b - Ax\|_2$ is the same as $\min \|Q^T r\|_2 = \|Q^T b - Q^T A x\|_2$

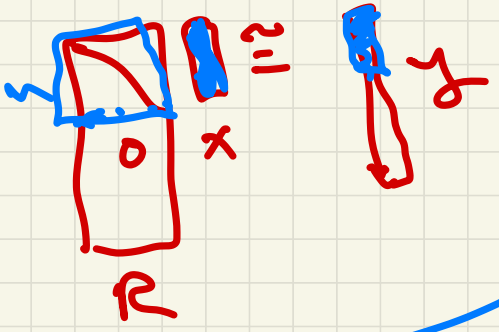
$$Q^T A x = \underbrace{Q^T (Q}_{I} R) x = Rx$$

$$\min_x \|Q^T b - Rx\|_2$$

$$\min_x \|Q^T b - Rx\|_2$$



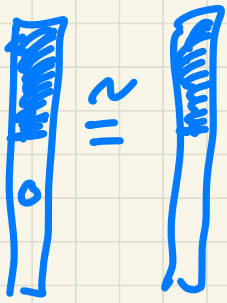
$Rx \cong Q^T b = y$



STILL OVERDET.

EASY TO SEE BEST SOLN !

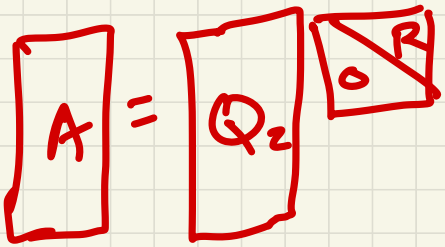
$Rx =$



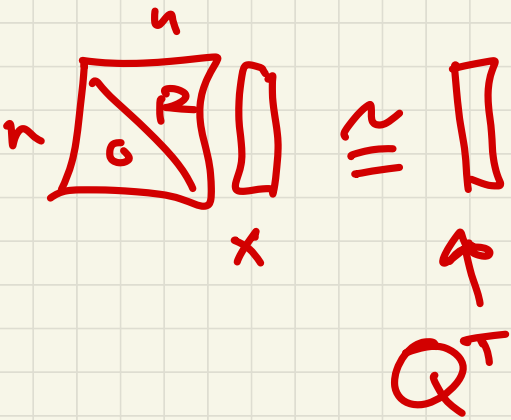
perfect ✓

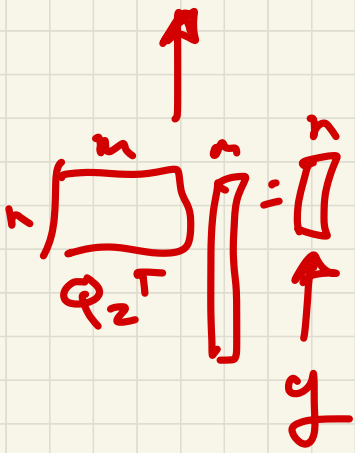
hopeless ✓

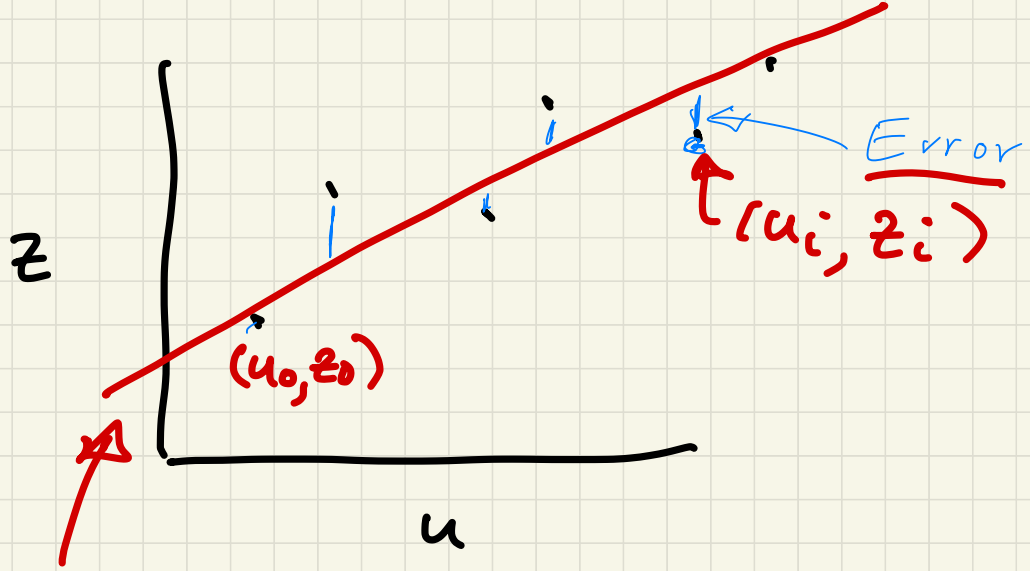
WE ACTUALLY USE ECONOMY-SIZE.



$$Ax \cong b \iff Rx \cong Q_2^T b$$





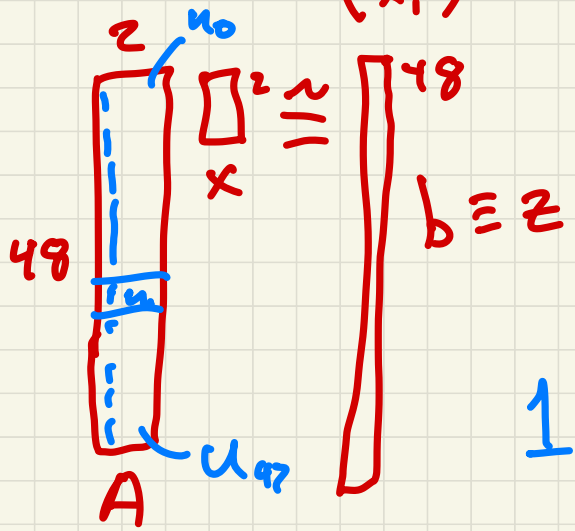


line: $y = x_0 + x_1 u$

want: $y(u_i) = z_i$

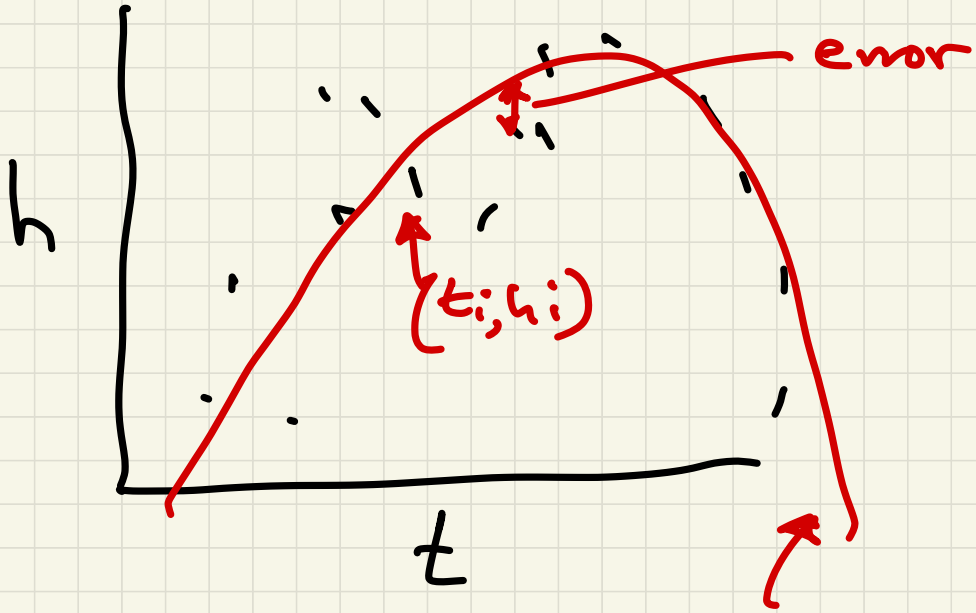
least sqs: $\min \left(\sum_{i=0}^{47} (y(u_i) - z_i)^2 \right)$

$X = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$



$y(u_i) \cong z_i$

$1 x_0 + x_1 u_i \cong z_i$



$$y = x_0 + x_1 t + x_2 t^2$$

$$Ax = b$$

$$y(t_i) = x_0 + x_1 t_i + x_2 t_i^2 \cong h_i$$

$$A = \begin{bmatrix} 1 & t_0 & t_0^2 \\ 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ \vdots & & \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} \cong \begin{bmatrix} h_0 \\ h_1 \\ h_2 \end{bmatrix}$$