RANK, NORM, CONDITION NUMBER

HOW BIG IS A MATRIX?

RANK, NORM, CONDITION NUMBER

DEF The <u>rank</u> of a matrix is the number of linearly independent columns it has.

THEOREM :

The rank of any matrix is the number of linearly independent rows.

(Even if the matrix isn't square.)

DEF The _norm_ of A, written $\|A\|$

is   max $\|Av\| / \|v\|$

over
all nonzero
vectors v.   vector norms.

$$\|A\|_2 = \max \|Av\|_2 / \|v\|_2$$

$$\|A\|_1 = \max \|Av\|_1 / \|v\|_1$$

$$etc.$$

DEF Let $M = \max \dfrac{\|Av\|}{\|v\|}$ over nonzero v

$(M = \|A\|)$.

Let $m = \min \dfrac{\|Av\|}{\|v\|}$ over nonzero v.

The <u>condition number</u> of A

is M/m.

$\kappa_2(A)$
$\kappa_1(A)$
$\kappa_\infty(A)$

Written $\kappa(A)$

$\kappa(A) \geq 1$

well-conditioned, ill-conditioned

$\kappa(A)$ can be infinite
(if there is a null vector)

IF A is square + nonsingular,

$$\kappa(A) = \|A\| \cdot \|A^{-1}\| .$$