Condition Number and Accuracy of $Ax=b$.

Condition number and $Ax=b$

$$\kappa_2(A) = \|A\|_2 \cdot \|A^{-1}\|_2 \geq 1$$

Thm CG converges to a fixed tolerance (on SPD A) in at most

$$O\left(\sqrt{\kappa_2(A)}\right)$$

THE RUNTIME FOR CG IS

$$O\left(\text{nnz}(A) \cdot \sqrt{\kappa_2(A)}\right)$$

If $Ax_{exact} = b$ and x is a "proposed solution"

error: $x_{exact} - x = e$

residual: $b - Ax = r$

error $= 0 \iff$ residual $= 0$

Notice: $Ae = r$

THM

$$\frac{\|e\|}{\|x_{exact}\|} \leq K(A) \frac{\|r\|}{\|b\|}$$

- $\frac{\|e\|}{\|x_{exact}\|}$: relative error norm
- $K(A)$: any K: K_2, K_1, K_∞
- $\frac{\|r\|}{\|b\|}$: relative residual norm