

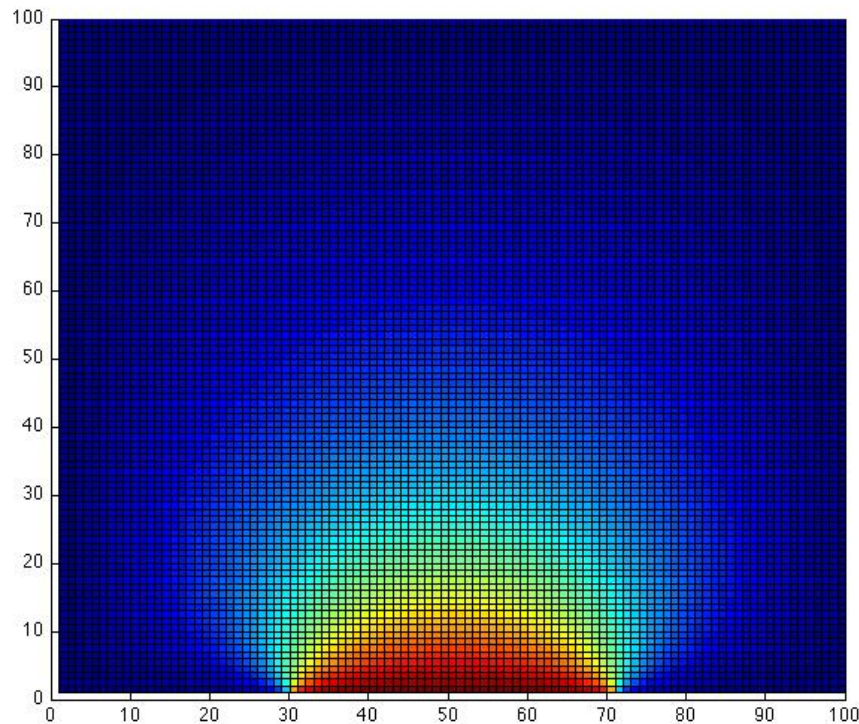
## ***Example: The Temperature Problem***

- A cabin in the snow
- Wall temperature is  $32^{\circ}$  except for a radiator at  $212^{\circ}$
- What is the temperature in the interior?



## Example: The Temperature Problem

- A cabin in the snow (the unit square ☺)
- Wall temperature is  $32^{\circ}$  except for a radiator at  $212^{\circ}$
- What is the temperature in the interior?



## ***The physics: Poisson's equation***

$$\nabla^2 u(x, y) \equiv \frac{\partial^2 u}{\partial x^2}(x, y) + \frac{\partial^2 u}{\partial y^2}(x, y) = f(x, y)$$

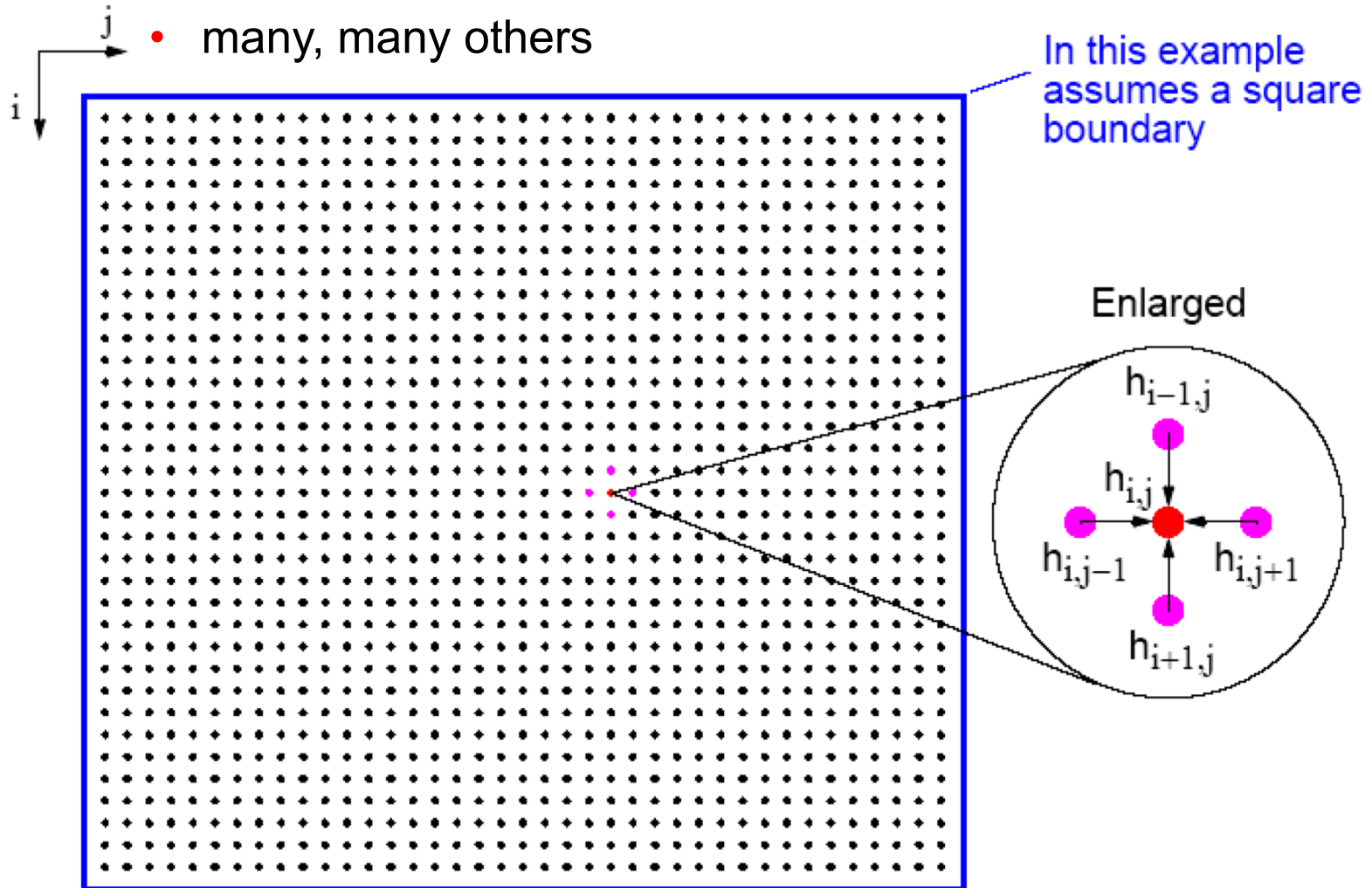
for  $(x, y) \in R = \{ (x, y) \mid a < x < b, \ c < y < d \}$ , and

$$u(x, y) = g(x, y)$$

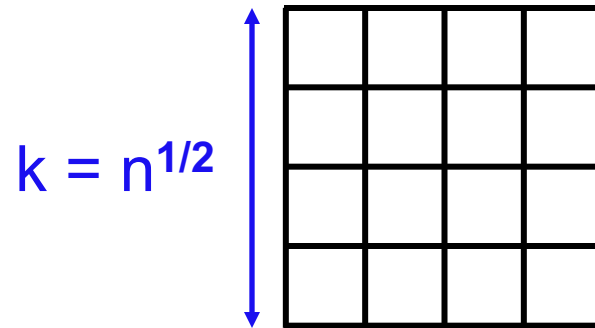
for  $(x, y)$  on the boundary of  $R$ .

# Many Physical Models Use Stencil Computations

- PDE models of heat, fluids, structures, ...
- Weather, airplanes, bridges, bones, ...
- Game of Life
- many, many others



## *Model Problem: Solving Poisson's equation for temperature*



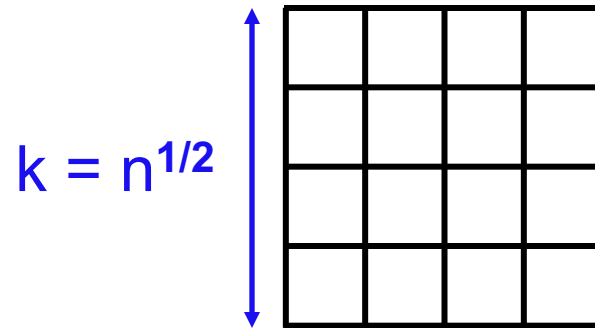
- Discrete approximation to Poisson's equation:

$$t(i) = \frac{1}{4} ( t(i-k) + t(i-1) + t(i+1) + t(i+k) )$$

- Intuitively:

Temperature at a point is the average  
of the temperatures at surrounding points

## ***Model Problem: Solving Poisson's equation for temperature***



- For each  $i$  from 1 to  $n$ , except on the boundaries:  
$$-t(i-k) - t(i-1) + 4*t(i) - t(i+1) - t(i+k) = 0$$
- $n$  equations in  $n$  unknowns:  $A*t = b$
- Each row of  $A$  has at most 5 nonzeros
- In three dimensions,  $k = n^{1/3}$  and each row has at most 7 nzs

# A Stencil Computation Solves a System of Linear Equations

- Solve  $Ax = b$  for  $x$
- Matrix  $A$ , right-hand side vector  $b$ , unknown vector  $x$
- $A$  is *sparse*: most of the entries are 0

The diagram illustrates the structure of a sparse matrix  $A$  and its multiplication with a vector  $x$  to solve the system  $Ax = b$ .

**Matrix  $A$ :** The matrix is shown as a large vertical column with a dashed diagonal line. A specific row, labeled " $i$ th equation" in pink, is highlighted. This row contains non-zero entries at positions  $i-n$ ,  $i-1$ ,  $i$ ,  $i+1$ , and  $i+n$ . The values are:  $1$  at  $i-n$ ,  $a_{i,i-1}$  at  $i-1$ ,  $1$  at  $i$ ,  $-4$  at  $i+1$ ,  $1$  at  $i+2$ , and  $1$  at  $i+n$ . The matrix is symmetric, with the same pattern of non-zero entries along the diagonal.

**Annotations for Matrix  $A$ :**

- An arrow points to the dashed diagonal with the text: "Those equations with a boundary point on diagonal unnecessary for solution".
- An arrow points to the top of the matrix with the text: "To include boundary values and some zero entries (see text)".

**Equation:** The matrix  $A$  is multiplied by the vector  $x$  to equal the right-hand side vector  $b$ .

**Vector  $x$ :** The vector  $x$  is shown as a vertical column with entries  $x_1, x_2, \dots, x_{N-1}, x_N$ . The first two entries,  $x_1$  and  $x_2$ , are highlighted with a dashed line.

**Right-hand side vector  $b$ :** The vector  $b$  is shown as a vertical column with entries  $0, 0, \dots, 0, 0$ . The first two entries,  $0$  and  $0$ , are highlighted with a dashed line.

The equation is written as:

$$A \times x = b$$