

SVD and Image Compression

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CS 111



SVD AND DATA COMPRESSION

$$\begin{matrix} n \\ \boxed{A} \\ m \end{matrix} = \begin{matrix} m \\ \boxed{U} \\ m \end{matrix} \begin{matrix} n \\ \boxed{S} \\ n \end{matrix} \begin{matrix} n \\ \boxed{V^T} \\ n \end{matrix}$$

$S = \text{diag}(\sigma_0, \sigma_1, \dots)$

$$\begin{matrix} n \\ \boxed{A_k} \\ m \end{matrix} = \sum_{i=0}^{k-1} \sigma_i U[:, i] V[:, i]^T$$

$$= \begin{matrix} k \\ \boxed{U_k} \\ m \end{matrix} \begin{matrix} k \\ \boxed{S_k} \\ k \end{matrix} \begin{matrix} n \\ \boxed{V_k} \\ k \end{matrix} \leftarrow V_k^T = (V[:, i, k])^T$$

$S_k = \text{diag}(\sigma_0, \dots, \sigma_{k-1})$

$U_k = U[:, :k]$

A_k is the same shape as A , $\|A_k - A\|_2 = \sigma_k$

A_k has mn elements, but we can store

A_k as just U_k, S_k, V_k :

$(m+n+1)k$ numbers.

If $k \ll m, n$ this is a big savings.