
Mathematical Foundations of the GraphBLAS and Big Data

Jeremy Kepner & Hayden Jananthan

SIAM Conference on Mathematics of Data Science (MDS20)

This material is based upon work supported by the National Science Foundation CCF-1533644.
Any opinions, findings, conclusions or recommendations expressed in this material are those of
the author(s) and do not necessarily reflect the views of the National Science Foundation.



**Massachusetts
Institute of
Technology**




Thanks!

MIT Team

William Arcand, Chansup Byun, David Bestor, William Bergeron, Alan Edelman, Vijay Gadepally, Matthew Hubbell, Michael Houle, Michael Jones, Anna Klein, Charles Leiserson, Peter Michaleas, Lauren Milechin, Julie Mullen, Andrew Prout, Albert Reuther, Antonio Rosa, Siddharth Samsi, Charles Yee, ...

and the

GraphBLAS Community



Cornell University

We
the Simons Four

arXiv.org > cs > arXiv:1606.05790

Search...

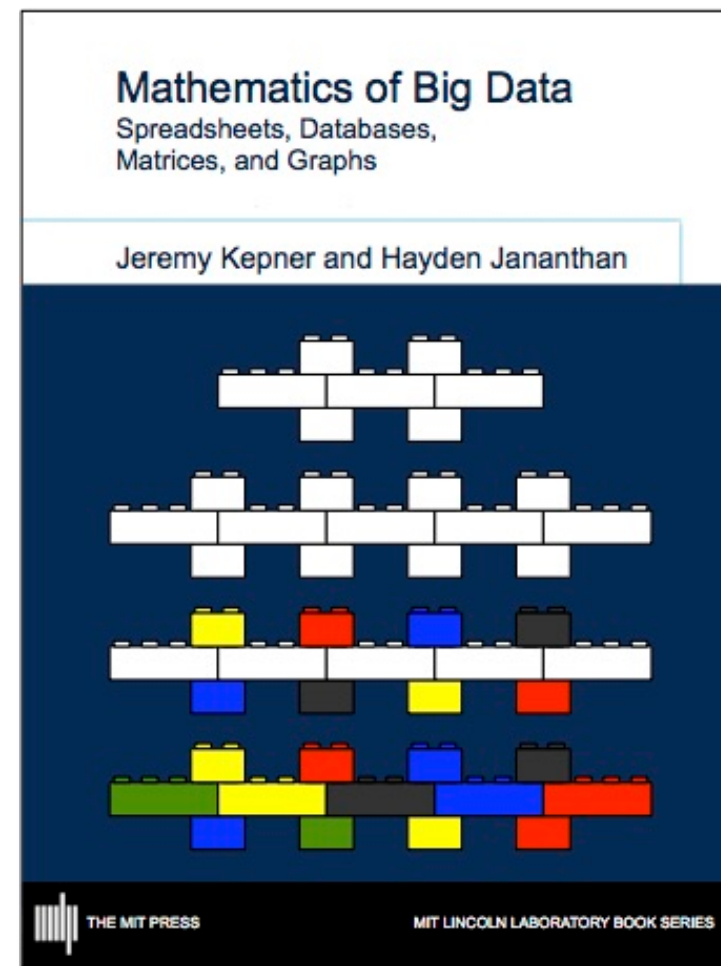
Help | Advanced S

Computer Science > Mathematical Software

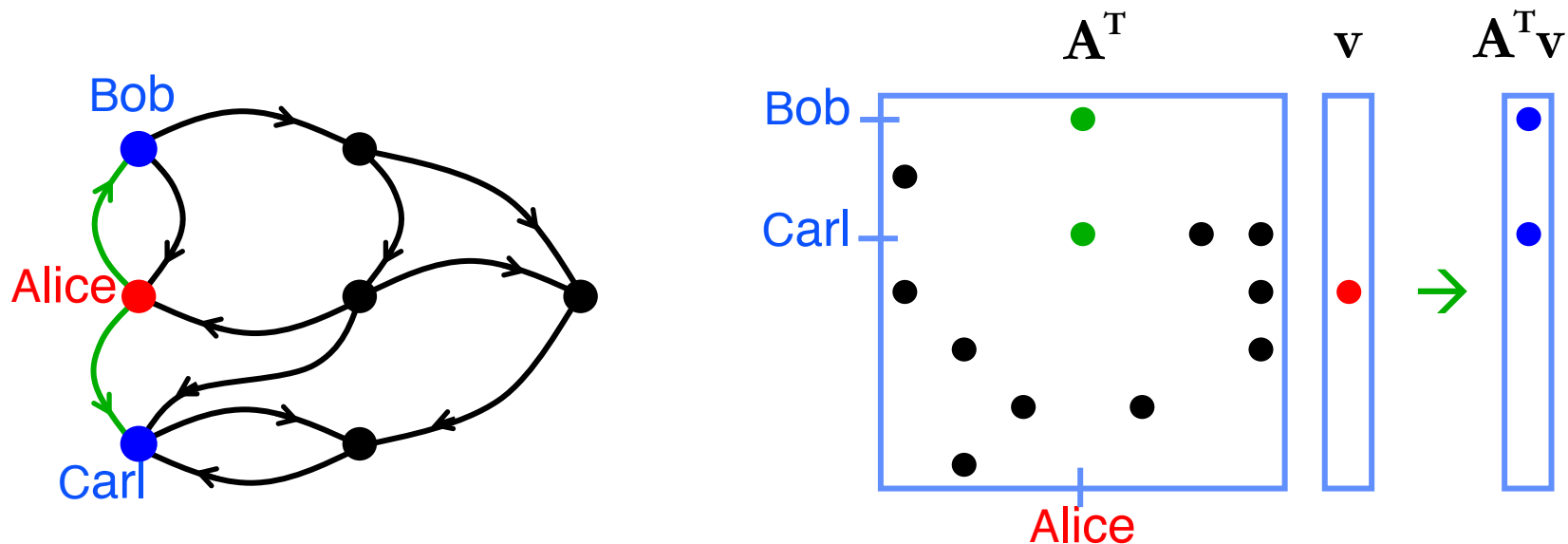
[Submitted on 18 Jun 2016 (v1), last revised 14 Jul 2016 (this version, v2)]

Mathematical Foundations of the GraphBLAS

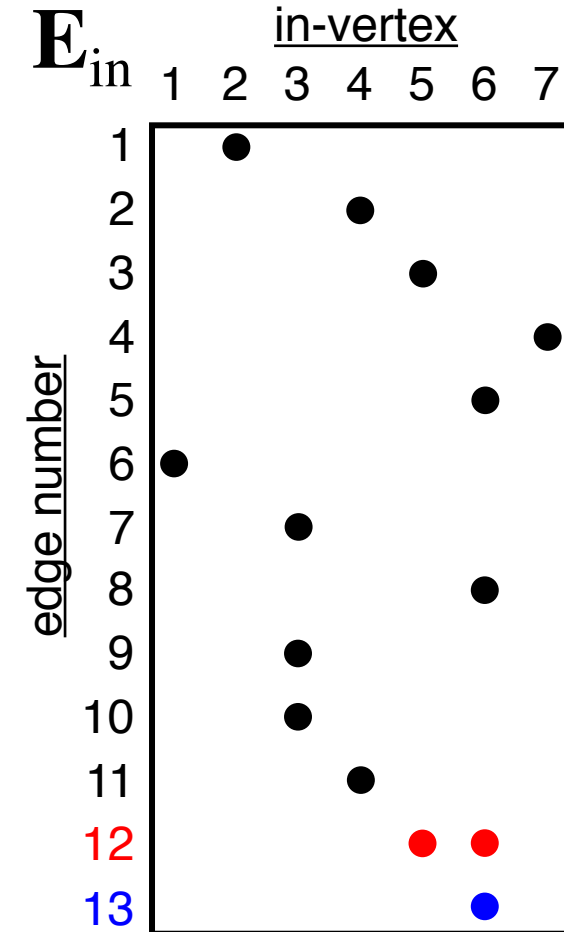
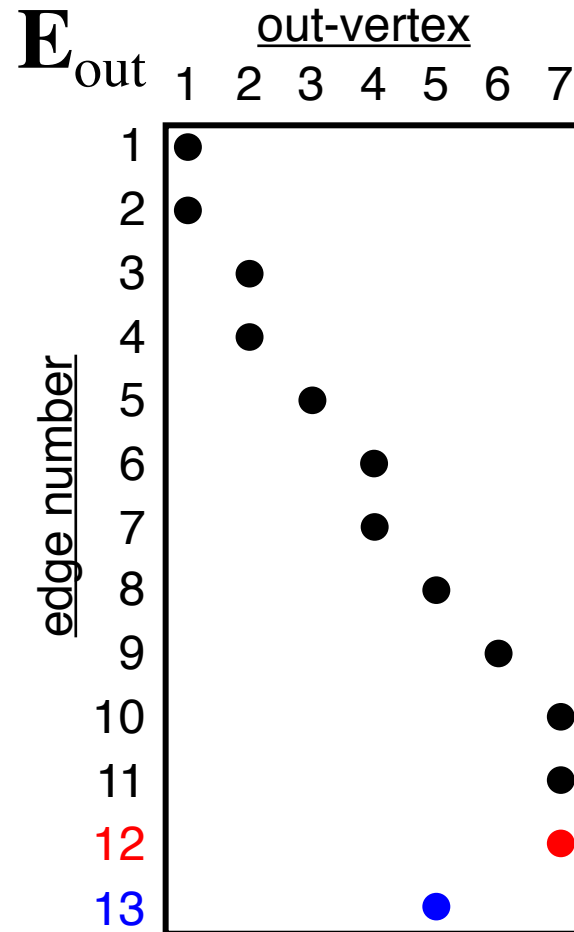
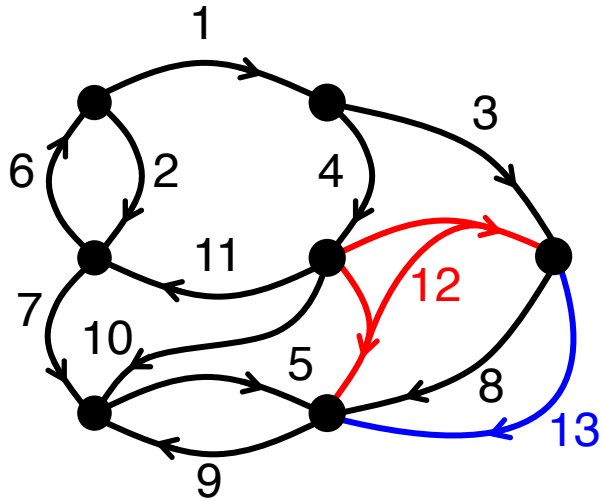
Jeremy Kepner, Peter Aaltonen, David Bader, Aydın Buluc, Franz Franchetti, John Gilbert, Dylan Hutchison, Manoj Kumar, Andrew Lumsdaine, Henning Meyerhenke, Scott McMillan, Jose Moreira, John D. Owens, Carl Yang, Marcin Zalewski, Timothy Mattson



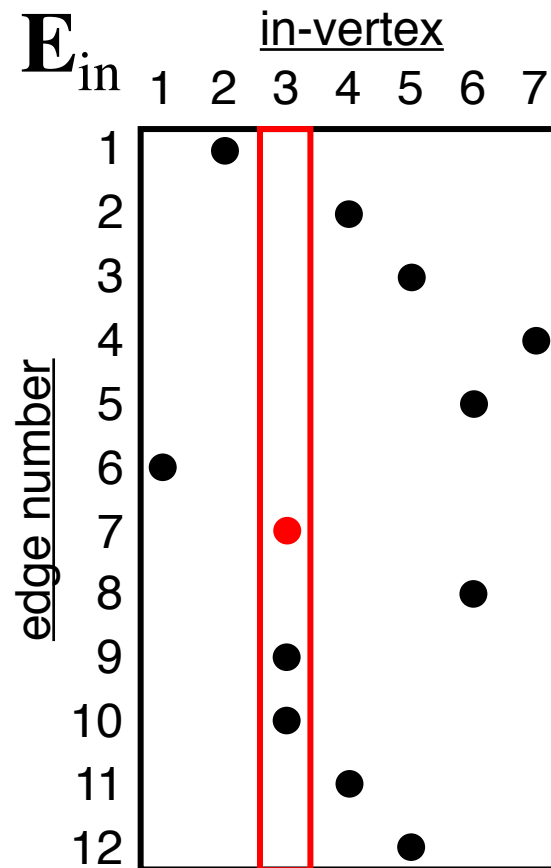
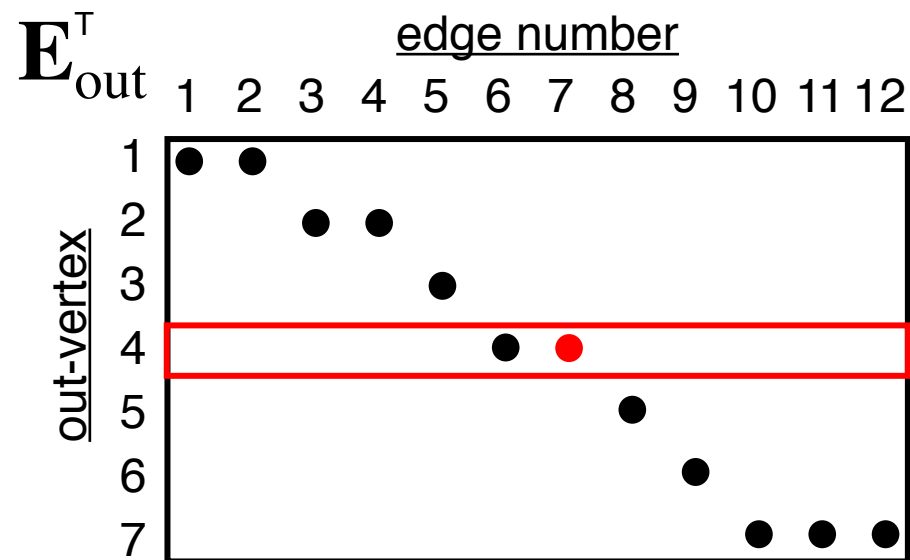
Foundational Graph Adjacency Duality



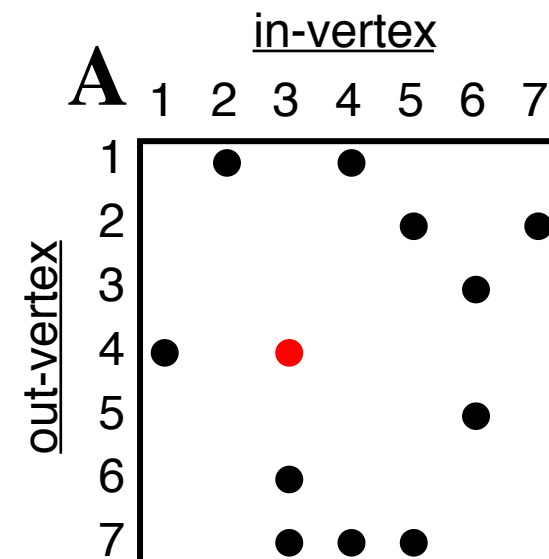
Edge Perspective



Edge-Adjacency Duality

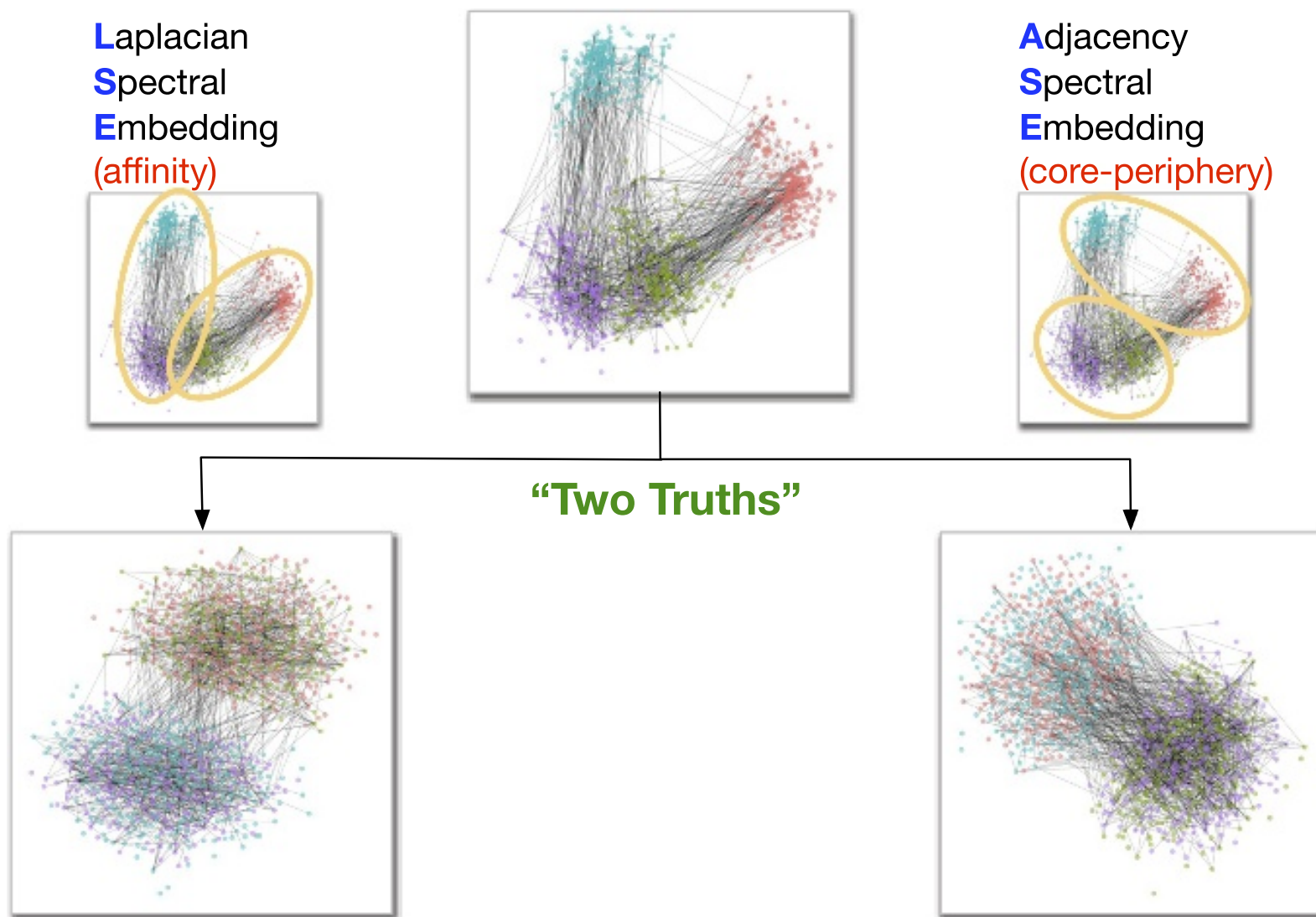


=



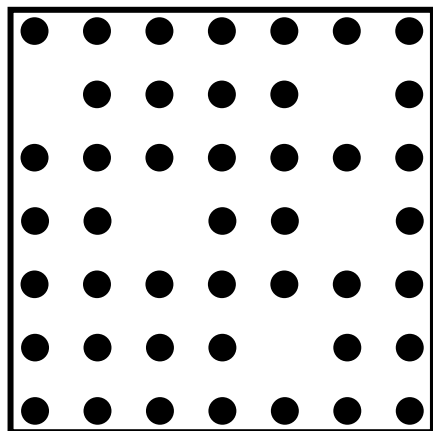
Adjacency vs Laplacian ($\text{diag}(\mathbf{d}) - \mathbf{A}$)

- Brain connectome depicting connectivity structure
- Eigenvalues of Laplacian shows affinity structure
 - Left hemisphere/right hemisphere
- Eigenvalues of Adjacency shows core-periphery structure
 - Gray matter/white matter



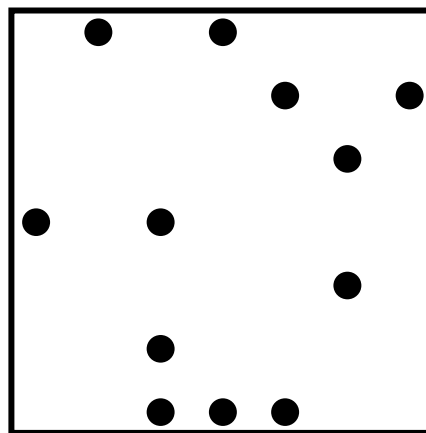
Dense, Sparse, Hypersparse (N x N)

$\mathbf{A}_{\text{dense}}$



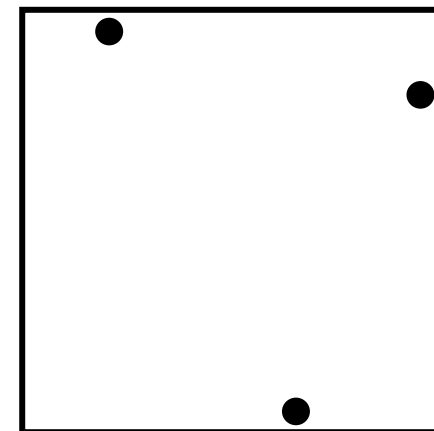
$$\text{nnz}(\mathbf{A}_{\text{dense}}) \sim N^2$$

$\mathbf{A}_{\text{sparse}}$

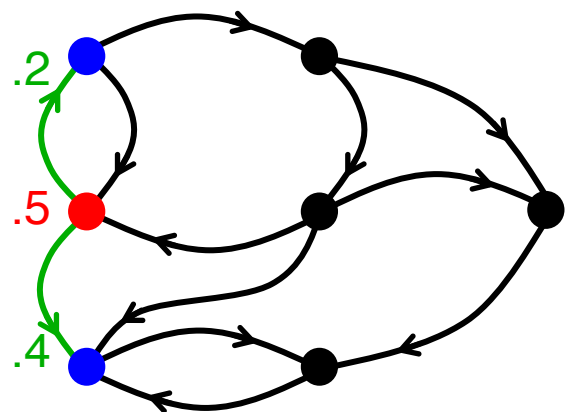


$$\text{nnz}(\mathbf{A}_{\text{sparse}}) \sim N$$

$\mathbf{A}_{\text{hypersparse}}$



$$\text{nnz}(\mathbf{A}_{\text{hypersparse}}) \ll N$$



		out-vertex							\mathbf{v}
\mathbf{A}^T		1	2	3	4	5	6	7	
in-vertex	1				.2				.5
	2								
	3								
	4				.4				
	5								
	6								
	7								

$$\mathbf{A}^T \mathbf{v} =$$

$+. \times$	max.+ min.+	max.min	min.max	max. \times min. \times
$\begin{bmatrix} .1 \\ .2 \end{bmatrix}$	$\begin{bmatrix} .7 \\ .9 \end{bmatrix}$	$\begin{bmatrix} .2 \\ .4 \end{bmatrix}$	$\begin{bmatrix} .5 \\ .5 \end{bmatrix}$	$\begin{bmatrix} .1 \\ .2 \end{bmatrix}$

Associative Array

A

	Artist	Date	Duration	Genre	Label	Release	Track	Type	Writer
031013ktnA1	Kitten	2013-03-10	3:57	Rock	Free	Yesterday	Yesterday (What's Love)	Single	Chad Anderson;Chloe Chaidez;Julian Chaidez
053013ktnA1	Bandayde	2013-05-30	5:14	Electronic	Free	Cut It Out Remixes	Sugar (Bandayde Remix)	Single	Julian Chaidez;Nicholas Johns
053013ktnA2	Kastle	2013-05-30	3:07	Electronic	Free	Cut It Out Remixes	Cut It Out (Kastle Remix)	Single	Barrett Rich
063012ktnA1	Kitten	2010-06-30	4:38	Rock	The Control Group	Kill The Light	Kill The Light	EP	Chad Anderson;Chloe Chaidez
063012ktnA1	Kitten	2012-06-30	3:28	Rock	Atlantic	Cut It Out/Sugar	Cut It Out	Single	Chad Anderson;Chloe Chaidez;Julian Chaidez
063012ktnA2	Kitten	2010-06-30	3:18	Rock	The Control Group	Kill The Light	Chinatown	EP	Chad Anderson;Chloe Chaidez
063012ktnA2	Kitten	2012-06-30	3:46	Rock	Atlantic	Cut It Out/Sugar	Sugar	Single	Chad Anderson;Chloe Chaidez
063012ktnA3	Kitten	2010-06-30	3:21	Rock	The Control Group	Kill The Light	Johnny;Johnny;Johnny	EP	Chad Anderson;Chloe Chaidez
063012ktnA4	Kitten	2010-06-30	2:43	Rock	The Control Group	Kill The Light	Allison Day	EP	Chad Anderson;Chloe Chaidez
063012ktnA5	Kitten	2010-06-30	4:07	Rock	The Control Group	Kill The Light	Kitten With A Whip	EP	Chad Anderson;Chloe Chaidez
082812ktnA1	Kitten	2012-08-28	3:25	Pop	Atlantic	Cut It Out	Cut It Out	EP	Chad Anderson;Chloe Chaidez;Julian Chaidez
082812ktnA2	Kitten	2012-08-28	3:13	Pop	Atlantic	Cut It Out	Japanese Eyes	EP	Chad Anderson;Chloe Chaidez
082812ktnA3	Kitten	2012-08-28	4:45	Pop	Atlantic	Cut It Out	G#	EP	Chad Anderson;Chloe Chaidez
082812ktnA4	Kitten	2012-08-28	3:46	Pop	Atlantic	Cut It Out	Sugar	EP	Chad Anderson;Chloe Chaidez
082812ktnA5	Kitten	2012-08-28	4:24	Pop	Atlantic	Cut It Out	Junk	EP	Chad Anderson;Chloe Chaidez;Julian Chaidez
082812ktnA6	Kitten	2012-08-28	6:10	Pop	Atlantic	Cut It Out	Christina	EP	Chad Anderson;Chloe Chaidez
093012ktnA1	Kitten	2013-09-30	4:55	Electronic;Pop	Elektra Records	Like A Stranger	Doubt	LP	Chad Anderson;Chloe Chaidez
093012ktnA2	Kitten	2013-09-30	3:34	Electronic;Pop	Elektra Records	Like A Stranger	I'll Be Your Girl	LP	Chad Anderson;Chloe Chaidez
093012ktnA3	Kitten	2013-09-30	4:33	Electronic;Pop	Elektra Records	Like A Stranger	Yesterday	LP	Chad Anderson;Chloe Chaidez;Julian Chaidez
093012ktnA4	Kitten	2013-09-30	5:13	Electronic;Pop	Elektra Records	Like A Stranger	Like A Stranger	LP	Chad Anderson;Chloe Chaidez
093012ktnA5	Kitten	2013-09-30	3:44	Electronic;Pop	Elektra Records	Like A Stranger	King Of Kings	LP	Chad Anderson;Chloe Chaidez
093012ktnA6	Kitten	2013-09-30	3:00	Electronic;Pop	Elektra Records	Like A Stranger	Graffiti Soul	LP	Chad Anderson;Chloe Chaidez
093012ktnA7	Kitten	2013-09-30	5:45	Electronic;Pop	Elektra Records	Like A Stranger	Cathedral	LP	Chad Anderson;Chloe Chaidez
093012ktnA8	Kitten	2012-09-16	3:13			Japanese Eyes	Japanese Eyes	Single	Chad Anderson;Chloe Chaidez

$$A_1 = A(11:13, 'Artist Date')$$

$$A_2 = A(12:14, 'Date Duration Genre')$$

	Artist	Date
082812ktnA1	Kitten	2012-08-28
082812ktnA2	Kitten	2012-08-28
082812ktnA3	Kitten	2012-08-28

	Date	Duration	Genre
082812ktnA2	2012-08-28	3:13	Pop
082812ktnA3	2012-08-28	4:45	Pop
082812ktnA4	2012-08-28	3:46	Pop

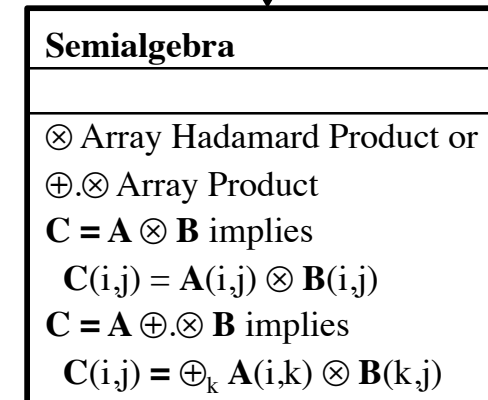
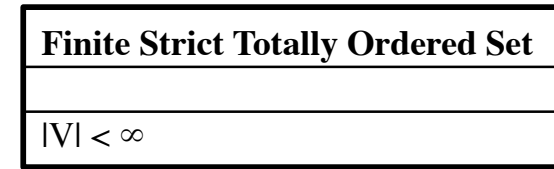
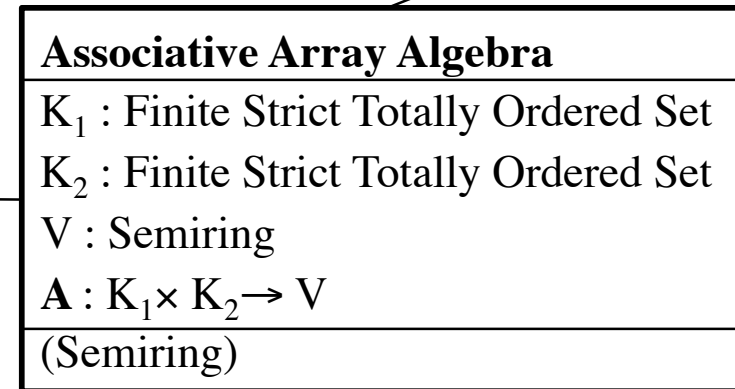
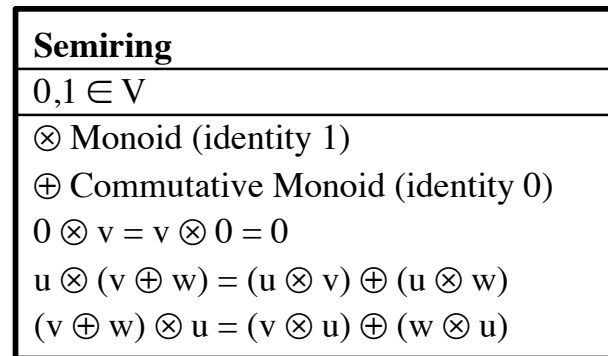
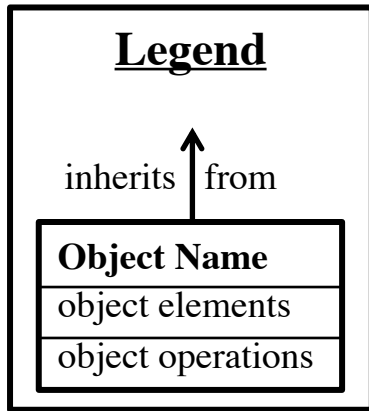
Associative Array Operations

A_1	+	A_2	=		Artist	Date	Duration	Genre
				082812ktnA1	Kitten	2012-08-28		
				082812ktnA2	Kitten	2012-08-28	3:13	Pop
A_1		A_2	=	082812ktnA3	Kitten	2012-08-28	4:45	Pop
				082812ktnA4		2012-08-28	3:46	Pop

					Artist	Date	Duration	Genre
				082812ktnA1	Kitten	2012-08-28		
A_1	-	A_2	=	082812ktnA2	Kitten		3:13	Pop
				082812ktnA3	Kitten		4:45	Pop
				082812ktnA4		2012-08-28	3:46	Pop

					Date
A_1	&	A_2	=	082812ktnA2	2012-08-28
				082812ktnA3	2012-08-28

Associative Array Algebra





Related Topics

Tropical Algebra

Tropical Geometry

Fuzzy Algebra

Bottleneck Algebra

Nonlinear Algebra

Matroids