CS 292F Project Proposal

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I would like to write a survey that explores random walks on graphs and their algorithmic applications. In Spielman's book, he bounds the distance of a walk to its stationary distribution in terms of the second-smallest eigenvalue of the "walk matrix." Because we know the relationship between the eigenstructure of the walk matrix and the normalized Laplacian of a graph, we can express this in terms of the graph's Fiedler value.

With an eye towards more general algorithmic applications, I would like to explore how the mixing properties of finite state-space Markov chains can be understood by performing spectral analysis on its underlying graph. Any time-reversible Markov chain can be analyzed as a walk on an undirected, weighted graph, and any finite state-space Markov chain can be analyzed as a walk on a directed, weighted graph, so this is actually not a significant generalization.

In '87, Jerrum and Sinclair defined the conductance of a time-reversible Markov chain just as we defined it for weighted Laplacians (they differ by a factor of $|V|$), and bounded mixing time in terms of this conductance. This result has since been generalized to non-reversible Markov chains. Generalizations of the bounds obtained by Jerrum and Sinclair have been used in order to obtain approximation algorithms for approximating the volume of convex bodies and approximating the permanent of certain classes of matrices.

More recently, Spielman and Teng have used random walks in order to find good clusters in graphs. Among other things, the insight that a set with small conductance will remain fairly isolated under a random walk is leveraged. This work uses results developed for the volume-approximation problem.

My aim is to formulate an intuitive interpretation of the many ways that fundamental spectral insights about conductance can appear in practice.

Potential Sources:

Approximate Counting, Uniform Generation and Rapidly Mixing Markov Chains (Sinclair, Jerrum)

https://people.eecs.berkeley.edu/~sinclair/approx.pdf

A Local Clustering Algorithm for Massive Graphs and Its Application to Nearly Linear Time Graph Partitioning (Spielman, Teng)

http://cs-www.cs.yale.edu/homes/spielman/PAPERS/74488.pdf

The Mixing Rate of Markov Chains, an Isoperimetric Inequality, and Computing the Volume (Lovasz, Simonvitz)

https://old.renyi.hu/~miki/LovaszSimMixN16.pdf
Conductance and Rapidly Mixing Markov Chains (King):
https://www.cs.mcgill.ca/~jking/papers/conductance.pdf