CS 292F Project Proposal : Spectral (Directed) Graph Theory

Aditya Dendukuri

Abstract

The goal of this project is to conduct a thorough review of research directions and results regarding spectral graph theory (SGT) for directed graphs that can be used to study biological reaction networks.

1 Problem Statement

Un-directed graphs can be defined as symmetric positive semi-definite matrices via the discreet graph Laplacian matrix. These properties have been exploited to develop a vast array of results that are used to study complex systems across natural sciences and engineering. However, directed graphs are trickier to work with as they cannot be represented as symmetric positive semi-definite matrices. Hence, Laplacian matrices for directed graphs may not be diagonalizable, and the eigenspaces of diagonalizable graphs may not exist in the real number space. Directed graphical models are ubiquitous in natural sciences and engineering as they can be used in modelling atomic structures to chemical reaction networks. Hence, developing a general graph theory for arbitrary graphs is an important research problem.

1.1 Starting Point and Goals

The starting point of this review project is these following papers.

- Laplacians and the Cheeger inequality for directed graphs (Chung 2005).
- Normalized graph Laplacians for directed graphs (Bauer 2012).
- A Graph-Theoretical Approach for the Analysis and Model Reduction of Complex-Balanced Chemical Reaction Networks (Rao, Schaft, and Jayawardhana 2013).

My goal is to compile a comprehensive review for spectral graph theory literature for directed graphs, with a specific application focus towards chemical reaction networks in systems biology.

References

Bauer, Frank (2012). "Normalized graph Laplacians for directed graphs". In: *Linear Algebra and its Applications* 436.11, pp. 4193–4222.

Chung, Fan (2005). "Laplacians and the Cheeger inequality for directed graphs". In: *Annals of Combinatorics* 9.1, pp. 1–19.

Rao, Shodhan, Arjan van der Schaft, and Bayu Jayawardhana (2013). "A graph-theoretical approach for the analysis and model reduction of complex-balanced chemical reaction networks". In: *Journal of Mathematical Chemistry* 51.9, pp. 2401–2422.