UNIVERSITY OF
Waterloo

Department of Electrical and Computer Engineering
University of Waterloo

ECE671
Microwave and RF Engineering

Instructor
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Fall 2015

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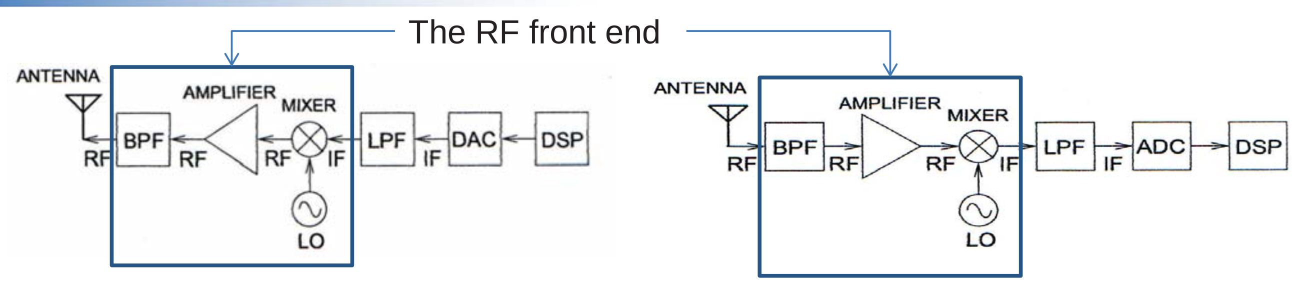
Radio Systems



- The transmitter/receiver includes several building blocks.
  - Filters for frequency selectivity and interference elimination
  - Amplifiers (Low noise, Power amplifier) to boost the received and transmitted signals
  - Mixers and oscillators to translate the modulation information from low (baseband) to radio frequencies and vice versa
  - Signal converters
  - Digital signal processors

Course Objective

- After successful completion of the course work, students are expected to be able to design and analyze various microwave
  - passive (filters, couplers, combiners/dividers)
  - and active circuits (linear amplifiers and oscillators).
- This course introduces transmission line theory, impedance matching techniques, and microwave circuit network analysis.
- It discusses the design of practical microwave circuits such as filters, couplers, low-noise and linear amplifiers, and oscillators.

Course Objective

- The course focuses on the fundamental methods for the analysis and design of microwave/RF passive and active circuits. The essentials of computer-aided design of microwave/RF circuits as well as major aspects of hardware implementations will be covered. Important RF applications for wireless communication systems will also be discussed.
- The course will include:
  - Transmission line theories and generalized matrix representation of RF circuits
  - Analysis of multi-port RF networks
  - Introduction to modern microwave planar technologies
  - Lumped and distributed microstrip circuits
  - Analysis of microstrip circuits
  - Microstrip couplers, hybrids and impedance matching networks
  - Microwave resonators and filters
  - Design of RF low noise amplifiers (LNAs)
  - Design of RF oscillators and mixers
  - Use of existing commercial CAD design tools for RF circuits
  - Hybrid and Monolithic RF circuits

Course Information

- Perquisites:
  - Basic knowledge in Electromagnetic and analog Electronic
  - Electrical and Computer Engineering course "Electromagnetic Fields"(ECE 370) or equivalent, or permission of the instructor
- The required textbook for this course is
  - Microwave Engineering, by David Pozar, Wiley
- Reference books
  - Michael Steer, Microwave and RF Design: A Systems Approach, 1st Edition, ISBN 9781891121883, SciTech Publishing, 2010
  - Guillermo Gonzalez, Microwave Transistor Amplifiers: Analysis and Design, Second Edition, Prentice Hall, ISBN: 0-13-254335-4
  - R. Ludwig, P. Bretchko, RF Circuit Design: Theory and Applications, Upper Saddle River, NJ: Prentice Hall, 2000

Marking Scheme

Component	Weight
Design Project	35%
Assignment	15%
Final Exam	50%

- Use of CALCULATORS in Examinations Programmable and/or scientific calculators without formulae storage and /or text display features may be used during examinations. Personnel computers may not be used in examinations

DESIGN PROJECT:

- Each student will have a design project dealing with the design of a particular RF passive or active circuit with realistic specifications. The design project requires that the student use RF CAD software Agilent's Advanced Design System.
- The project consists of:
  - library research on a topic in Microwave Engineering of your choosing;
  - presentation of your project to the class
  - formal, written Project Report.
  - Use the IEEE Xplore web site or Engineering Village 2 to find journal papers dealing with your topic.
  - Each project will be presented to the class in the last week of classes. You will have 15 minutes to present your project.
  - You must submit the Project Report on the last day of classes.

DESIGN PROJECT:

- Students shall go through a full real-world engineering design cycle for developing a working microwave circuit (amplifier). This cycle includes:
  - preliminary pencil and paper design work
  - simulation and optimization of the realistic implementation of the circuit using standard CAD tools (ADS)
  - layout in file of the artwork necessary for the fabrication of the circuit,
  - production of a high-quality full engineering technical report

DESIGN PROJECT:

- Paper and Pencil Design
  - Follow the procedures discussed in the class as well as in the several design examples included in course reference book.
  - Matching networks to be initially designed using analytical techniques (i.e. equations).
- Computer Simulation Work
  - The design of all circuits initially developed on paper is then simulated, refined and optimized using Agilent ADS software. This is a CAD software that aid to obtain the theoretical performance of a realistic microwave circuit before fabrication.
  - Please contact Phil Regier to set an account for ADS (EIT 4176, 36233).

DESIGN PROJECT:

- The project report must be properly referenced: Include a numbered list of references at the end of your report.

DESIGN PROJECT:

- The circuit to be designed simulated in a 50Ω system.
- In the case of active circuits,
  - the transistors shall be biased using two independent DC power supplies;
  - RF choke and DC block shall be incorporated at the input and output of the “circuit”.
  - The circuit must be DC isolated.

DESIGN PROJECT:

- Substrate:
  - The circuits will be fabricated using the popular Microwave Integrated Circuit (MIC) technology
  - all distributed elements will be micro-strip transmission lines.
  - The substrate used to fabricate your circuits is a high-quality low-loss microwave substrate that has the following characteristics:
    - Type: RT/Duroid5880
    - Thickness: 30 mils
    - Dielectric constant: $\varepsilon r = 2.20 \pm 0.04$.
    - Dielectric losses: $\tan\delta = 0.0009$ at 10 GHz.
    - Copper thickness: 1 ounce per square foot or 34 µm.

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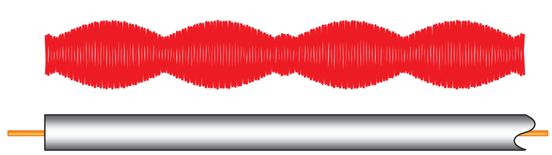
Transmission Line

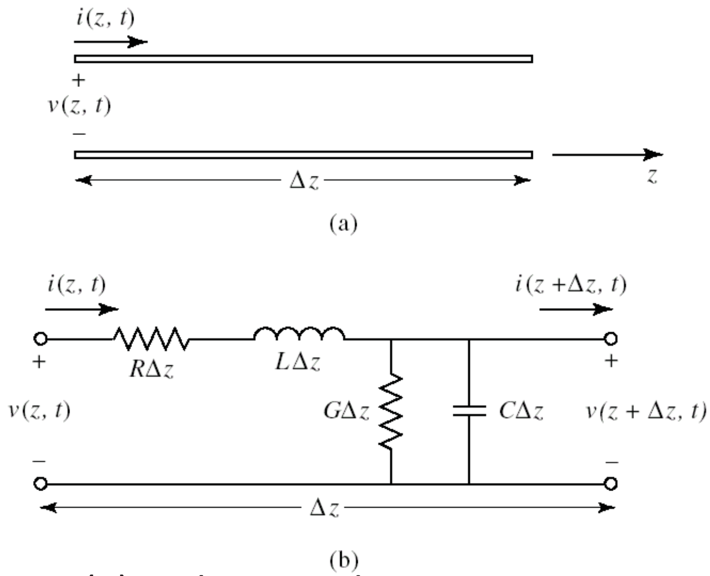
- **Low frequencies**
  - wavelengths >> wire length
  - current (I) travels down wires easily for efficient power transmission
  - measured voltage and current not dependent on position along wire

- **High frequencies**
  - wavelength is in same order than the length of transmission medium
  - measured envelope voltage dependent on position along line
  - need transmission lines for efficient power transmission
  - matching to characteristic impedance (Zo) is very important for low reflection and maximum power transfer



Transmission Line

- Voltage and current definitions and equivalent circuit for an incremental length of transmission line.
  - R: conductivity loss
  - G: Dielectric Loss
  - L: self inductance
  - C: capacitance



(a) Voltage and current definitions.

(b) Lumped-element equivalent circuit.

Transmission Line Equations

- Transmission line is usually used to connect a source and a load
- The voltage and current across the transmission line varies as a function of the time t and spatial positions z
- The application of the Kirtchoff's voltage law to the equivalent lumped model leads to

$$\begin{cases} v(z,t) - R\Delta z i(z,t) - L\Delta z \dfrac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0 \\ \dfrac{v(z+\Delta z,t) - v(z,t)}{\Delta z} = -Ri(z,t) - L\dfrac{\partial i(z,t)}{\partial t} \end{cases}$$

$$as\,\Delta z \to 0 \Rightarrow \frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

Transmission Line Equations

- Similarly the application of the Kirchhoff law on current leads to

$$i(z,t) - G\Delta z v(z+\Delta z,t) - C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

$$as\,\Delta z \to 0 \Rightarrow \frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

- A set of two first order differential transmission line equations called **telegrapher's equations** is then obtained

$$\Rightarrow \begin{cases} \dfrac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\dfrac{\partial i(z,t)}{\partial t} \\ \dfrac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\dfrac{\partial v(z,t)}{\partial t} \end{cases}$$

Transmission Line Equations

- Since the interest is in sinusoidal steady state condition, phasor notation will be used

$$\begin{cases} v(z,t) = \Re e[V(z)e^{j\omega t}] \\ i(z,t) = \Re e[I(z)e^{j\omega t}] \end{cases}$$

- In time domain $\frac{\partial}{\partial t}$ is equivalent to a multiplication by $j\omega$ in the phasor domain

- **Telegrapher's equations phasor form**

$$\Rightarrow \begin{cases} \dfrac{dV(z)}{dz} = -(R + j\omega L)I(z) \\ \dfrac{dI(z)}{dz} = -(G + j\omega C)V(z) \end{cases}$$

Transmission Line Equations

- the derivation of the Telegrapher's equations phasor form leads to second order differential equations

$$\Rightarrow \begin{cases} \dfrac{d^2V(z)}{dz^2} = -(R + j\omega L)\dfrac{dI(z)}{dz} \\ \dfrac{d^2I(z)}{dz^2} = -(G + j\omega C)\dfrac{dV(z)}{dz} \end{cases}$$

- Substituting telegrapher equation into the previous equation leads to

$$\Rightarrow \begin{cases} \dfrac{d^2V(z)}{dz^2} = (R + j\omega L)(G + j\omega C)V(z) \\ \dfrac{d^2I(z)}{dz^2} = (G + j\omega C)(R + j\omega L)I(z) \end{cases}$$

Transmission Line Equations

- Let $Y = G + j\omega C$ and $Z = R + j\omega L$
- then

$$\Rightarrow \begin{cases} \dfrac{d^2V(z)}{dz^2} = ZYV(z) \\ \dfrac{d^2I(z)}{dz^2} = ZYI(z) \end{cases}$$

- The differential equations solutions are

$$\Rightarrow \begin{cases} V(z) = Ae^{-\gamma z} + Be^{\gamma z} \\ I(z) = Ce^{-\gamma z} - De^{\gamma z} \end{cases}$$

$$A = V_o^+$$
$$B = V_o^-$$
$$C = I_o^+$$
$$D = I_o^-$$

- where $\gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$

Transmission Line Equations

- using $\dfrac{dV(z)}{dz} = (R + j\omega L)I(z)$ and $V(z) = Ae^{-\gamma z} + Be^{\gamma z}$

  $$\frac{dV(z)}{dz} = \gamma A e^{-\gamma z} - \gamma B e^{\gamma z} = (R + j\omega L)I(z)$$

  $$\text{So } I(z) = \frac{\gamma}{(R + j\omega L)}[Ae^{-\gamma z} - Be^{\gamma z}]$$

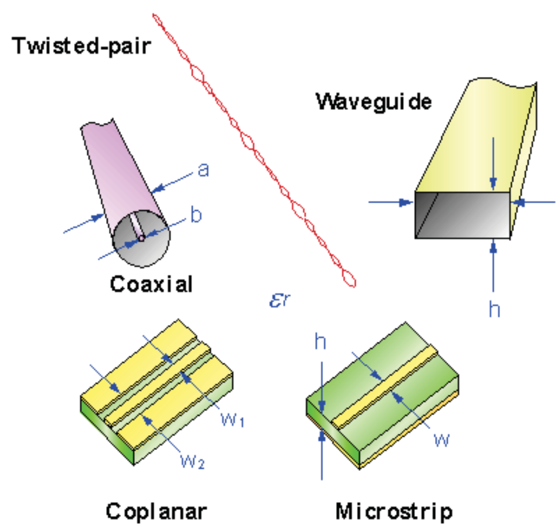
- Comparison of each term of the previous equation to

  $$I(z) = Ce^{\gamma z} - De^{-\gamma z}$$

  leads to $$\frac{A}{C} = \frac{B}{D} = Z_o = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Transmission Line Equations

- Z_o is called the characteristic impedance of the transmission line
- It is equal to the ratio voltage amplitude to the current amplitude of each of the traveling waves individually
- **It is not equal to the ratio of the total voltage and the total current**
- **Zo is a function of physical dimensions and ε_r**
- Zo is usually a real impedance



Transmission Line

$$\Rightarrow \begin{cases} V(z) = Ae^{-\gamma z} + Be^{\gamma z} \\ I(z) = \dfrac{A}{Z_o}e^{-\gamma z} - \dfrac{B}{Z_o}e^{\gamma z} \end{cases} \qquad \text{where } \gamma = \alpha + j\beta$$

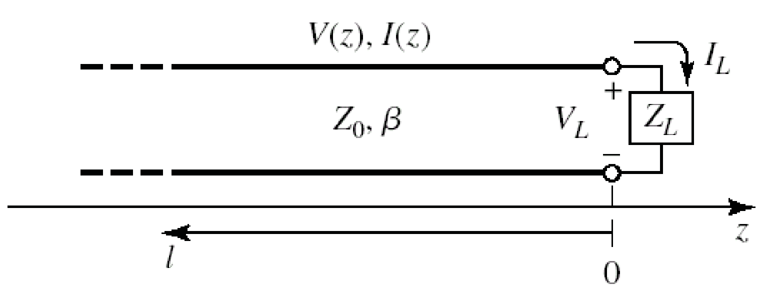
- For lossless transmission line (R=G=0) $\alpha = 0 \Rightarrow \gamma = j\beta = j\omega\sqrt{LC}$ *and* $Z_o = \sqrt{\dfrac{L}{C}}$
- For low loss transmission line $R << \omega L, G << \omega C \Rightarrow \gamma \approx \dfrac{1}{2}\sqrt{LC}\left(\dfrac{R}{L} + \dfrac{G}{C}\right) + j\omega\sqrt{LC}$
- In one time period, the wave travels distant λ, designated as wavelength.

$$\lambda = \frac{2\pi}{\beta}$$

Transmission line terminated in a load

- For lossless line with an impedance characteristic Zo and terminated with a load Z_L, the voltage and current at the load are described as

$$\begin{cases} V(z=0) = Ae^{-\gamma z} + Be^{\gamma z} = A + B \\ I(z=0) = \dfrac{A}{Z_o}e^{-\gamma z} - \dfrac{B}{Z_o}e^{\gamma z} = \dfrac{A}{Z_o} - \dfrac{B}{Z_o} \end{cases} \Rightarrow Z_L = Z_o\frac{A+B}{A-B}$$

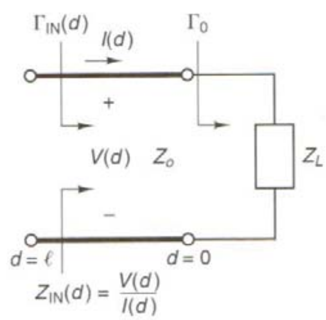


Transmission line terminated in a load

- The voltage reflection coefficient is defined as the amplitude of the reflected voltage normalized to the input voltage

$$\Gamma_L = \frac{V^-}{V^+} = \frac{B}{A} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

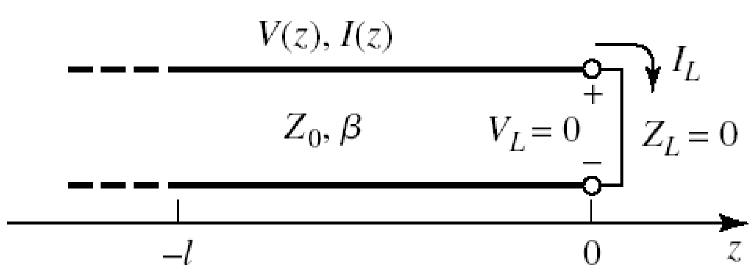
- The reflection coefficient can be determined as a function of the position along the line



$\Gamma_{in}(z = d) = ?$ and how the reflection coefficient varies along the line?

Transmission line terminated in a load

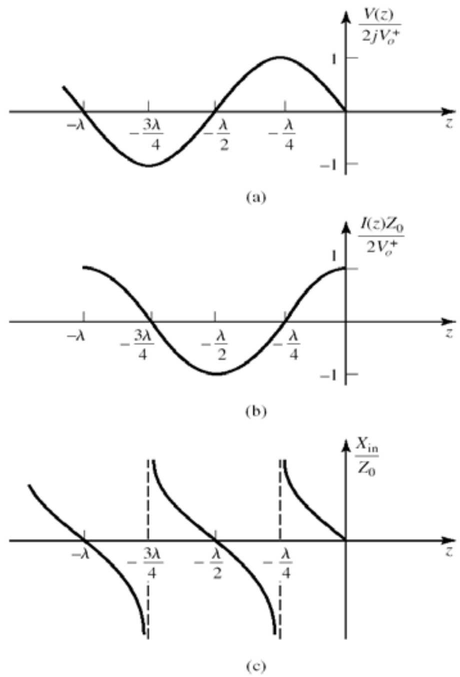
- As example lets consider the a lossless line terminated with a short circuit



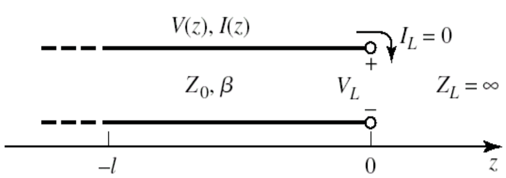
$$\Rightarrow V(0) = A + B = 0 \Rightarrow A = -B \Rightarrow \Gamma(0) = -1 \Rightarrow \Gamma(d) = \Gamma(0)e^{-j2\beta l} = -e^{-j2\beta l}$$

Transmission line terminated in a load

(a) Voltage,
(b) Current,
(c) Impedance (R_{in} = 0 or ∞) variation along a short-circuited transmission line.

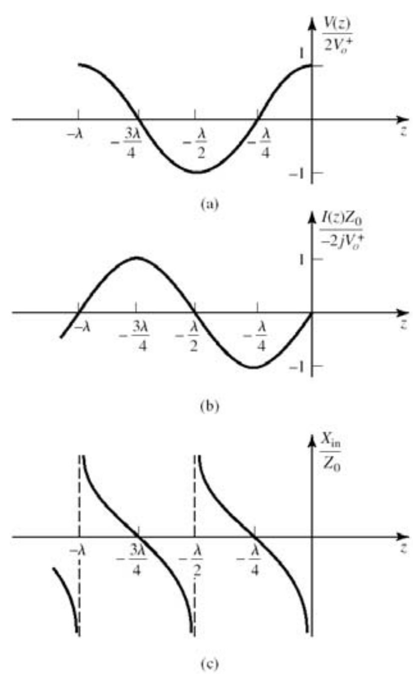


Transmission line terminated in a load



A transmission line terminated in an open circuit.

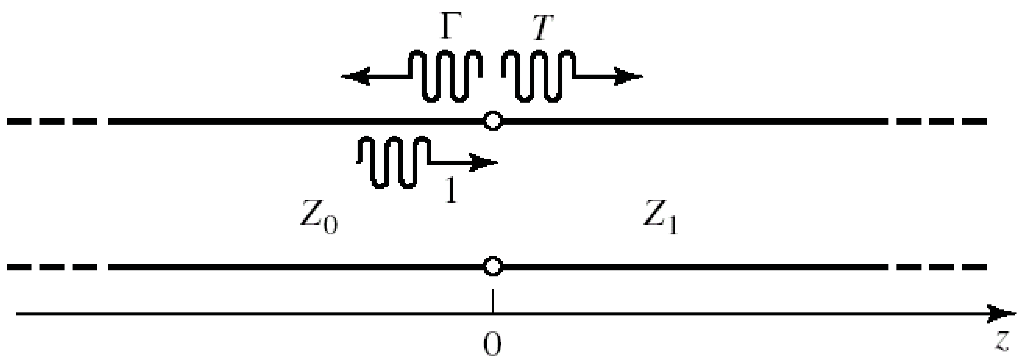
(a) Voltage,
(b) Current,
(c) Impedance (R_{in} = 0 or ∞) variation along an open-circuited transmission line.



$$\Rightarrow I(0) = A - B = 0 \Rightarrow A = B \Rightarrow \Gamma(0) = 1 \Rightarrow \Gamma(d) = \Gamma(0)e^{-j2\beta l} = e^{-j2\beta l}$$

Transmission line terminated in a load

- Reflection and transmission at the junction of two transmission lines with different characteristic impedances.
- If $Z_o=Z_1 \rightarrow \Gamma=0$, there is now reflection (no reverse wave)



Power Flow

- The power flow on the line can be determined as

$$P = \frac{1}{2}\mathrm{Re}\left\{V(z)I^*(z)\right\} = \frac{1}{2}\frac{|A|^2}{Z_o}(1-|\Gamma_o|^2)$$

$$V(z) = A(e^{-j\gamma z} + \Gamma_o e^{j\gamma z})$$

$$I(z) = \frac{A}{Z_o}(e^{-j\gamma z} - \Gamma_o e^{j\gamma z})$$

- The flow power is independent from z
- Power delivered to the load is equal to the *incident* one minus the reflected power

Return Loss

- If Γ=0, then no reflection, all the power is delivered to the load
- If $|\Gamma|$=1, then all the power is reflected and no power is delivered to the load
- If 0<$|\Gamma|$<1 then the load is mismatched, fraction of the incident wave is reflected and the rest is delivered to the load. The reflected portion of the incident power can be seen as loss "Return Loss"

$$RL=-20\log(|\Gamma|)\ \text{dB}$$

VSWR

Given $V(z) = Ae^{j\beta z}\left[1+\Gamma_o e^{-2j\beta z}\right]$

Then $|V(z)| = |A|\left|1+\Gamma_o e^{-2j\beta z}\right|$

The minimum of |V(z)| is equal to $V_{\min} = |A|\left(1-|\Gamma_o|\right)$

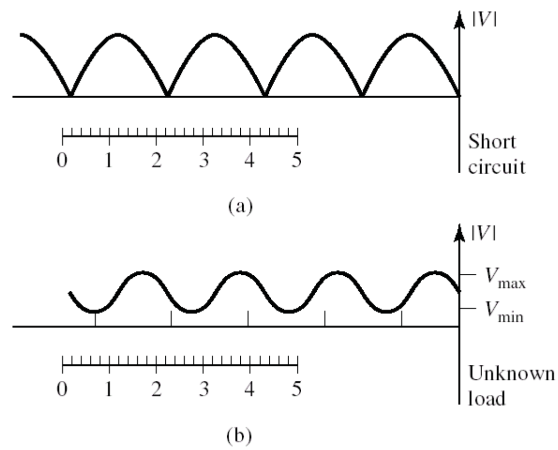
The maximum of |V(z)| is equal to $V_{\max} = |A|\left(1+|\Gamma_o|\right)$

The VSWR is defined as the ratio of the maximum to the minimum of the voltage standing wave

$$VSWR = \left|\frac{V_{\max}}{V_{\min}}\right| = \frac{1+|\Gamma_o|}{1-|\Gamma_o|}$$

VSWR

- The voltage across the TL has periodic maximums and minimums.
- The spacing between the maximums and minimums is equal to λ/4.
- the spacing between two maximums or two minimums is equal to λ/2

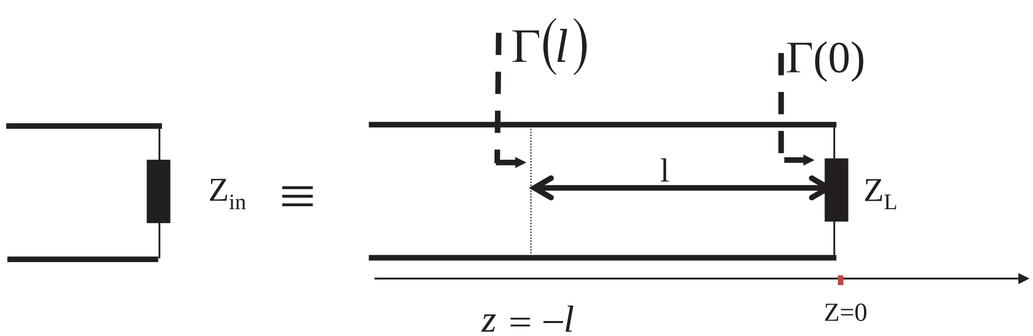


Voltage standing wave patterns (*a*) Standing wave for short-circuit load. (*b*) Standing wave for unknown load.

VSWR varies from   1 to ∞

Input impedance of a loaded TL

- As the voltage and current varies as function of z, the input impedance of the line varies on the position along the line



$$Z_{in} = \frac{V(-l)}{I(-l)} = Z_o \frac{A\left(e^{j\beta l} + \Gamma_o e^{-j\beta l}\right)}{A\left(e^{j\beta l} - \Gamma_o e^{-j\beta l}\right)} = Z_o \frac{1+\Gamma_o e^{-2j\beta l}}{1-\Gamma_o e^{-2j\beta l}}$$

Input impedance of a loaded TL

Given $\Gamma(l) = \dfrac{Be^{-\gamma l}}{Ae^{+\gamma l}} = \Gamma(0)e^{-2\gamma l}$ *and* $\Gamma(0) = \dfrac{Z_L - Z_o}{Z_L + Z_o}$

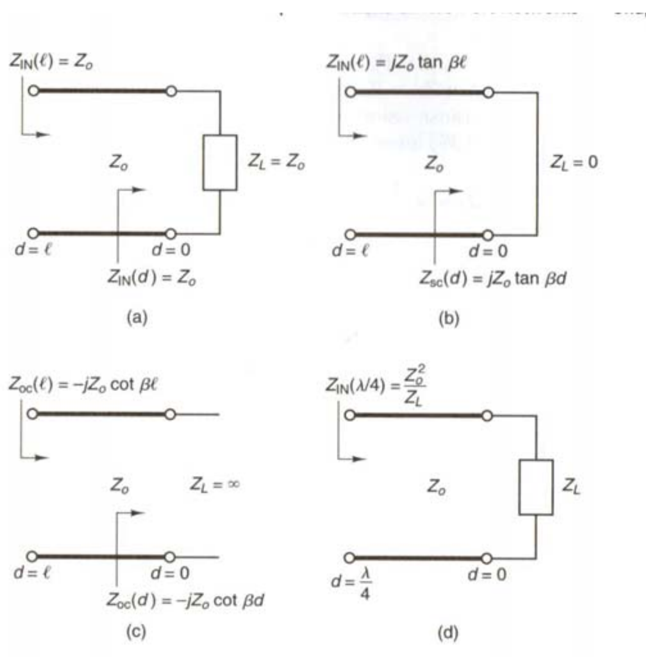
Then $$Z_{in} = Z_o \frac{Z_L + Z_o \tanh(\gamma l)}{Z_o + Z_L \tanh(\gamma l)}$$

For lossless case, we have $$Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)}$$

Often normalized impedances are used

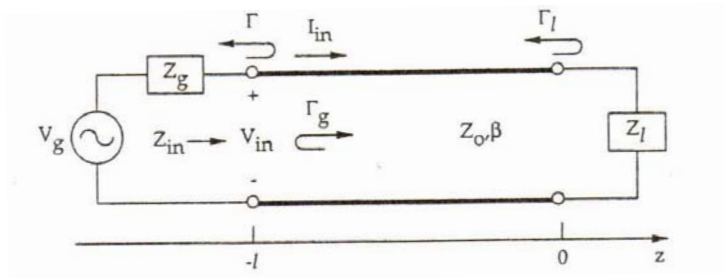
$$z_L = \frac{Z_L}{Z_o} \quad \Rightarrow \quad z_{in} = \frac{Z_{in}}{Z_o} = \frac{z_L + j\tan(\beta l)}{1 + jz_L \tan(\beta l)}$$

Input impedance of a loaded TL



General Transmission Line

- The study of transmission line so far assumed that the generator is ideal.
- In physical circuits that is not true, and as a result mismatch can exit between the transmission line and the generator in addition to the possible mismatch between the load and the transmission line.



- Three possible cases for the general transmission line:
  - Matched load
  - Matched generator
  - Maximum power transfer

General Transmission Line

- Before examining the three cases, we need to determine an expression for the power being delivered to the load. To begin, we define some variables from the previous figure:

$$\Gamma_\ell = \frac{Z_\ell - Z_o}{Z_\ell + Z_o} \qquad \Gamma_g = \frac{Z_g - Z_o}{Z_g + Z_o} \qquad \Gamma = \frac{Z_{in} - Z_g}{Z_{in} + Z_g}$$

$$\text{where} \qquad Z_{in} = Z_o \frac{Z_\ell + jZ_o \tan(\beta\ell)}{Z_o + jZ_\ell \tan(\beta\ell)}$$

- The voltage along the length of the transmission line can be written as:

$$V(z) = A\left(e^{-j\beta z} + \Gamma_\ell e^{j\beta z}\right)$$

General Transmission Line

The voltage at the input to the transmission line can be found be evaluating at $z = -l$.

$$V(-\ell) = V_g \frac{Z_{in}}{Z_{in} + Z_g} = A\left(e^{j\beta\ell} + \Gamma_\ell e^{-j\beta\ell}\right)$$

For lossless transmission line, the power delivered to the load is the same as the power input to the line.

$$P_{in} = \frac{1}{2}\text{Re}\left\{V_{in} I_{in}^*\right\} = \frac{1}{2}\left|V_g\right|^2 \left|\frac{Z_{in}}{Z_{in} + Z_g}\right|^2 \text{Re}\left\{\frac{1}{Z_{in}}\right\}$$

Thus the power delivered to the load is function of the input impedance of the line and the generator impedance.

General Transmission Line

If we now assume that $Z_{in} = r_{in} + jx_{in}$ and $Z_g = r_g + jx_g$

Then

$$P_{in} = \frac{1}{2}\left|V_g\right|^2 \left|\frac{Z_{in}}{Z_{in} + Z_o}\right|^2 \text{Re}\left\{\frac{1}{Z_{in}}\right\}$$

$$= \frac{1}{2}\left|V_g\right|^2 \frac{r_{in}^2 + x_{in}^2}{\left(r_{in} + r_g\right)^2 + \left(x_{in} + x_g\right)^2} \frac{r_{in}}{r_{in}^2 + x_{in}^2}$$

$$= \frac{1}{2}\left|V_g\right|^2 \frac{r_{in}}{\left(r_{in} + r_g\right)^2 + \left(x_{in} + x_g\right)^2}$$

This equation is a suitable tool in evaluating the three different load conditions.

Matched Load

- Assuming a fixed generator impedance, a load is matched to the transmission line when $Z_l = Z_o$. In this case $\Gamma_l = 0$ and there are no reflections from the load and the VSWR=1.

- The input impedance $Z_{in} = Z_o$, and since we can assume that Z_o is real then

$$P_{in} = \frac{1}{2}\left|V_g\right|^2 \frac{Z_o}{\left(Z_o + r_g\right)^2 + \left(x_g\right)^2}$$

Matched Generator

- The second case is when the transmission line is matched to the generator impedance. This is accomplished by adjusting the transmission line parameters until $Z_{in} = Z_g$. This ensure that $\Gamma = 0$ but does not imply that $\Gamma_l = 0$, so there may be a standing wave on the line resulting in a VSWR>1.

$$P_{in} = \frac{1}{2}\left|V_g\right|^2 \frac{r_g}{4\left(r_g^2 + x_g^2\right)}$$

Conjugate match

- The final case is matching for maximum power transfer. This condition is best determined by determining the maximum of the equation.

$$P_{in} = \frac{1}{2}\left|V_g\right|^2 \frac{r_{in}}{\left(r_{in}+r_g\right)^2+\left(x_{in}+x_g\right)^2}$$

$$\frac{\partial P}{\partial r_{in}} = \frac{1}{(r_{in}+r_g)^2+(x_{in}+x_g)^2} + \frac{-2r_{in}(r_{in}+r_g)}{\left[(r_{in}+r_g)^2+(x_{in}+x_g)^2\right]^2} = 0$$

$$\frac{\partial P}{\partial x_{in}} = \frac{-2x_{in}(x_{in}+x_g)}{\left[(r_{in}+r_g)^2+(x_{in}+x_g)^2\right]^2} = 0$$

- Simultaneous solution for r_{in} and x_{in} in results in

$$r_{in} = r_g \quad \text{and} \quad x_{in} = -x_g \quad \Rightarrow \quad Z_{in} = Z_g^*$$

Conjugate match

•So, maximum power is transferred to the load when the impedance seen by the generator is equal to the complex conjugate of the generator impedance. This is known as the conjugate match.

$$P_{in} = \frac{1}{2}\left|V_g\right|^2 \frac{1}{4r_g}$$

•If the generator impedance is real, then the matched generator case and the conjugate match case are equal. Maximum power transfer does not imply that there is no standing wave on the line, but rather that the line has been adjusted such that all of the wave on the line add up at the load.

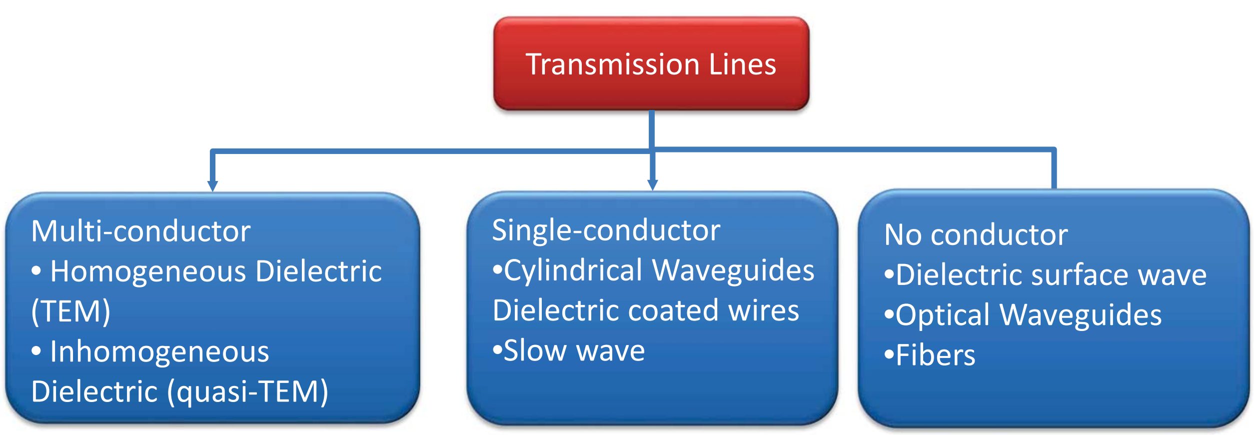
Transmission Lines: Circuit approach

Advantages versus disadvantages of electric circuit representation

- Clear intuitive physical picture
- yields a standardized two-port network representation
- serves as building blocks to go from microscopic to macroscopic forms

- Basically a one dimensional Representation (cannot take into account interferences)
- Material nonlinearities, hysteresis, and temperature effects are not accounted for

Transmission line

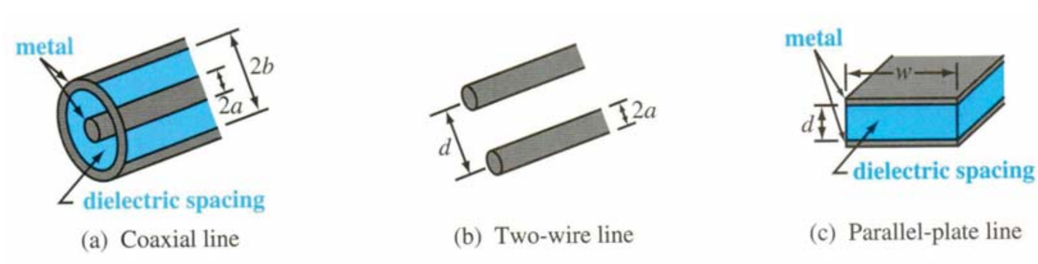


Transmission line

	Coax	*Waveguide*	*Microstrip*
Modes: preferred Other	TEM TM,TE	TE_{10} TM,TE	Quasi TEM Other EH
Dispersion	none	medium	low
Bandwidth	High	low	high
Power capacity	Medium	High	low
Integration with other components	Hard	Hard	easy
fabrication	medium	medium	easy
Physical size	Large	Large	small
Loss	Medium	low	high

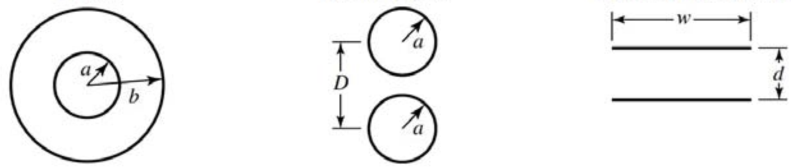
Example of Transmission Line

- The wave propagating in a Transverse Electromagnetic (TEM) transmission line are characterized by electrical and magnetic fields that are entirely transverse to the direction of propagation



Coaxial Cable Parameters

TABLE 2.1 Transmission Line Parameters for Some Common Lines



	COAX	TWO-WIRE	PARALLEL PLATE
L	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	$\frac{\mu}{\pi}\cosh^{-1}\left(\frac{D}{2a}\right)$	$\frac{\mu d}{w}$
C	$\frac{2\pi\epsilon'}{\ln b/a}$	$\frac{\pi\epsilon'}{\cosh^{-1}(D/2a)}$	$\frac{\epsilon' w}{d}$
R	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{R_s}{\pi a}$	$\frac{2R_s}{w}$
G	$\frac{2\pi\omega\epsilon''}{\ln b/a}$	$\frac{\pi\omega\epsilon''}{\cosh^{-1}(D/2a)}$	$\frac{\omega\epsilon'' w}{d}$

$\epsilon = \epsilon' - j\epsilon''$

μ, ε and σ pertain to the insulting material between the conductor

$Rs = \sqrt{\pi f \mu_c / \sigma_c}$ where μ_c and σ_c pertain to the conductors

Microstrip Transmission Line

Ease of fabrication: printed circuit boards!

Substrates: duroid, quartz, alumina, silicon

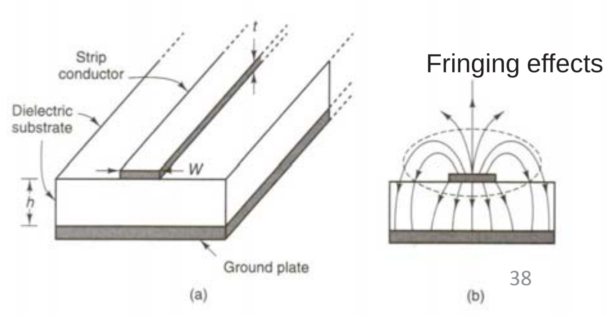
Propagation only quasi-TEM since not all E-M field lines are entirely in substrate; problem at higher freq.

Phase velocity for quasi-TEM: $v_p = c/\sqrt{\varepsilon_{ff}}$

c is speed of light, ε_{eff} = effective relative dielectric constant of the substrate. (not = to ε_r).

Characteristic impedance: $Z_0 = 1/(v_p C)$ with C=capacitance per unit length of the microstrip.

Wavelength: $\lambda = v_p / f = c/(f\sqrt{\varepsilon_{ff}}) = \lambda_o / \sqrt{\varepsilon_{ff}}$



Microstrip Transmission Line

How to find ε_{eff} and Z_0 ? Preferably in analytical form.

The QuasiTEM mode is only accurate at low frequency but for high freq the longitudinal component of the fields are non negligible and quais-TEM assumption is not valid

Assuming zero (negligible) thickness of the strip conductor (t/h<0.005):

- For $W/h \le 1$:

$$Z_0 = \frac{60}{\sqrt{\varepsilon_{ff}}} \ln(8\frac{h}{W} + 0.25\frac{W}{h}) \quad (1)$$

Where

$$\varepsilon_{ff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[\left(1 + 12\frac{h}{W}\right)^{-1/2} + 0.04\left(1 - \frac{W}{h}\right)^2\right] \quad (2)$$

Microstrip Transmission Line

- For $W/h \ge 1$:

$$Z_0 = \frac{120\pi/\sqrt{\varepsilon_{ff}}}{W/h + 1.393 + 0.667\ln(W/h + 1.444)} \quad (3)$$

Where

$$\varepsilon_{ff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + 12\frac{h}{W}\right)^{-1/2} \quad (4)$$

Based on the previous expressions, the wavelength in microstrip line is given by

- For $W/h \ge 0.6$:

$$\lambda = \frac{\lambda}{\sqrt{\varepsilon_r}}\left[\frac{\in_r}{1 + 0.63(\varepsilon_r - 1)(W/h)^{0.1255}}\right]^{1/2} \quad (5)$$

- For $W/h < 0.6$:

$$\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}}\left[\frac{\in_r}{1 + 0.6\left(\varepsilon_r - 1\right)\left(W/h\right)^{0.0297}}\right]^{1/2} \quad (6)$$

Microstrip Transmission Line

For design: eqs. Relating Z_0 with ε_r and W/h:

- For W/h $\leq$ 2:

$$\frac{W}{h} = \frac{8e^A}{e^{2A} - 2} \quad (7)$$

- For W/h $\geq$ 2:

$$\frac{W}{h} = \frac{2}{\pi}\left\{B - 1 - \ln(2B-1) + \frac{\varepsilon_r - 1}{2\varepsilon_r}\left[\ln(B-1) + 0.39 - \frac{0.61}{\varepsilon_r}\right]\right\} \quad (8)$$

Microstrip Transmission Line

Where

$$A = \frac{Z_0}{60}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right) \quad (9)$$

And $B = 377\pi / \left(2Z_0\sqrt{\varepsilon_r}\right)$

Non-negligible thickness of conductor $\Rightarrow$ included via increased capacitance $\Rightarrow$ replace strip width W with W_{eff} , e.g.

For $W/h \geq 1/2\pi$

$$\frac{W_{eff}}{h} = \frac{W}{h} + \frac{t}{\pi h}\left(1 + \ln\frac{2h}{t}\right) \quad (10)$$

For $W/h \leq 1/2\pi$

$$\frac{W_{eff}}{h} = \frac{W}{h} + \frac{t}{\pi h}\left(1 + \ln\frac{4hW}{t}\right)$$

Microstrip Transmission Line

For high frequencies quasi-TEM assump. is not valid and ε_{ff} and Z_0 are functions of freq.

v_p decreases with freq. increase $\Rightarrow$ ε_{ff} Increases. Also Z_0 increases with freq. while W_{eff} decreases.

•The freq below which the dispersion is neglected

$$f_0(GHz) = 0.3\sqrt{\frac{Z_0}{h(cm)\sqrt{\varepsilon_r - 1}}} \quad (11)$$

The effect of the dispersion on ε_{ff} :

$$\varepsilon_{ff}\left(f\left(GHz\right)\right) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{ff}}{1 + G(\frac{f}{f_p})^2} \qquad f_p = \frac{Z_0}{8\pi h(cm)} \quad G = 0.6 + 0.009Z_o \quad (12)$$

High impedance on thin substrates $\Rightarrow$ less dispersion.

Microstrip Transmission Line

Uses of microstrip tr. Lines:

- Series tr. Lines
- Short and open circuit stubs
- Microstrip + short/open circuited stubs can transform 50 Ohm resistor to any value of impedance
- Quarter-wave microstrip can transform 50 Ohm resistor to any value of resistance
- Quarter-wave transformer + one of the stubs = any value of impedance

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Smith Chart

•**Normalized Impedance:** it is the normalized load impedance to the characteristic impedance of the transmission line attached to the load

$$z = \frac{Z}{Z_o} = r + jx \qquad z = \frac{1+\Gamma}{1-\Gamma}$$

•Since the impedance is a complex number, the reflection coefficient will be a complex number

$$\Gamma = u + jv$$

$$r = \frac{1-u^2-v^2}{(1-u)^2+v^2} \qquad x = \frac{2v}{(1-u)^2+v^2}$$

Smith Chart

The impedance as a function of reflection coefficient can be re-written in the form:

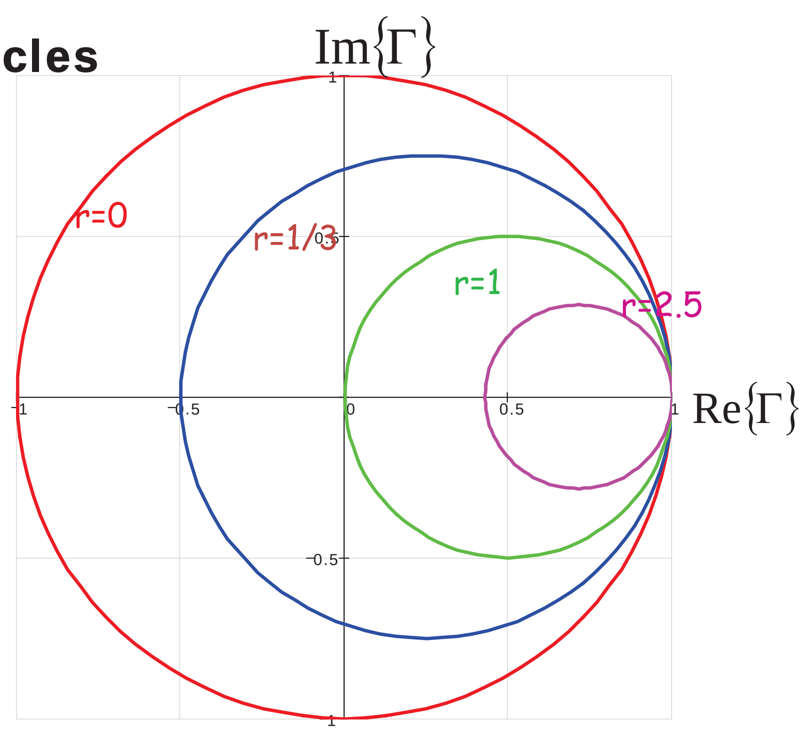
$$r = \frac{1-u^2-v^2}{(1-u)^2+v^2} \quad \Longrightarrow \quad \left(u-\frac{r}{1+r}\right)^2+v^2=\frac{1}{(1+r)^2}$$

$$x = \frac{2v}{(1-u)^2+v^2} \quad \Longrightarrow \quad (u-1)^2+\left(v-\frac{1}{x}\right)^2=\frac{1}{x^2}$$

These are equations for circles on the (u,v) plane

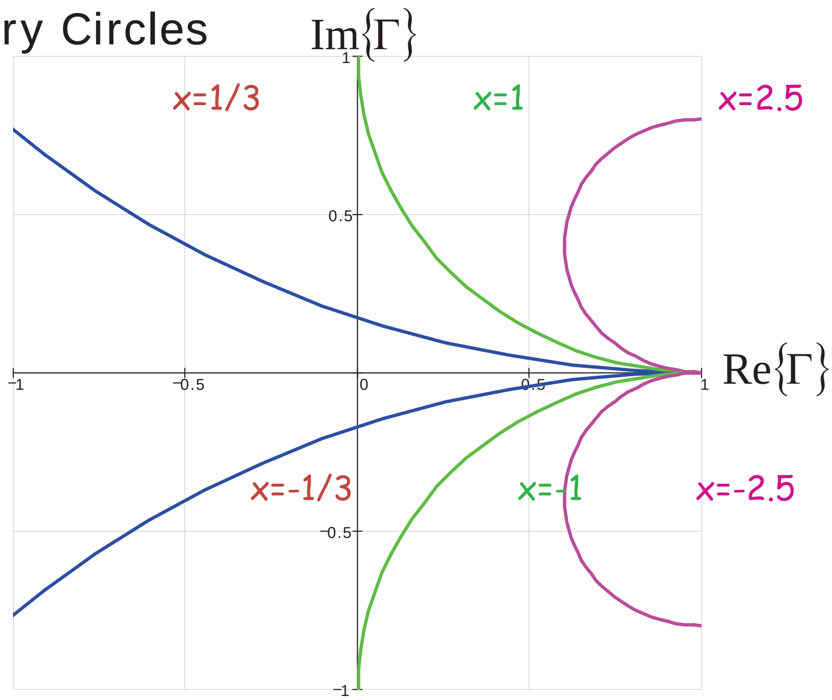
Smith Chart

Real Circles



Smith Chart

Imaginary Circles



Smith Chart

Impedance Smith Chart

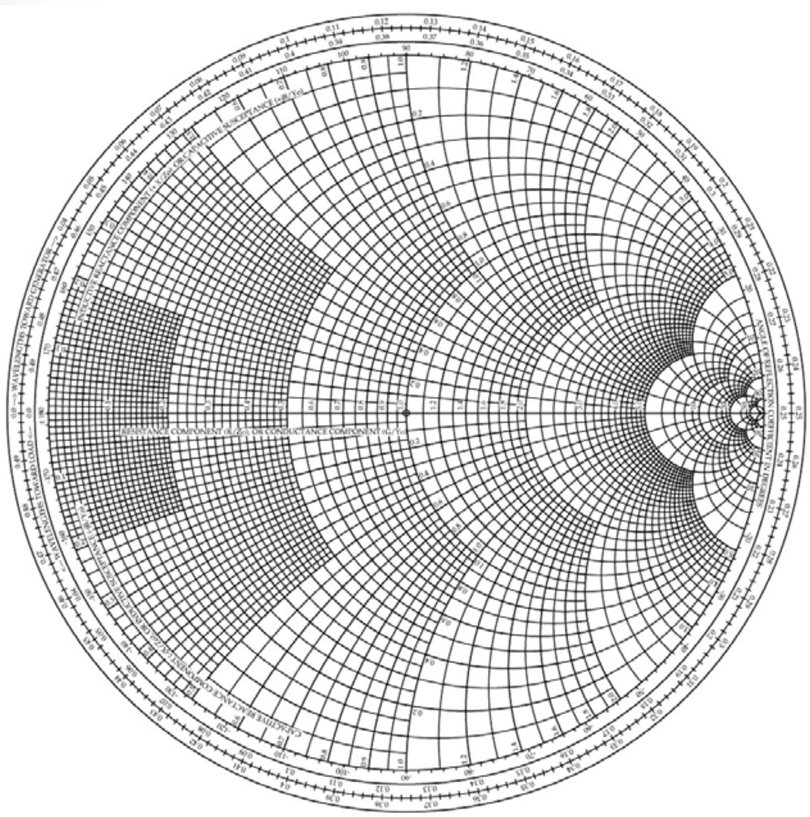


Figure 2.10

Smith Chart

Example 1

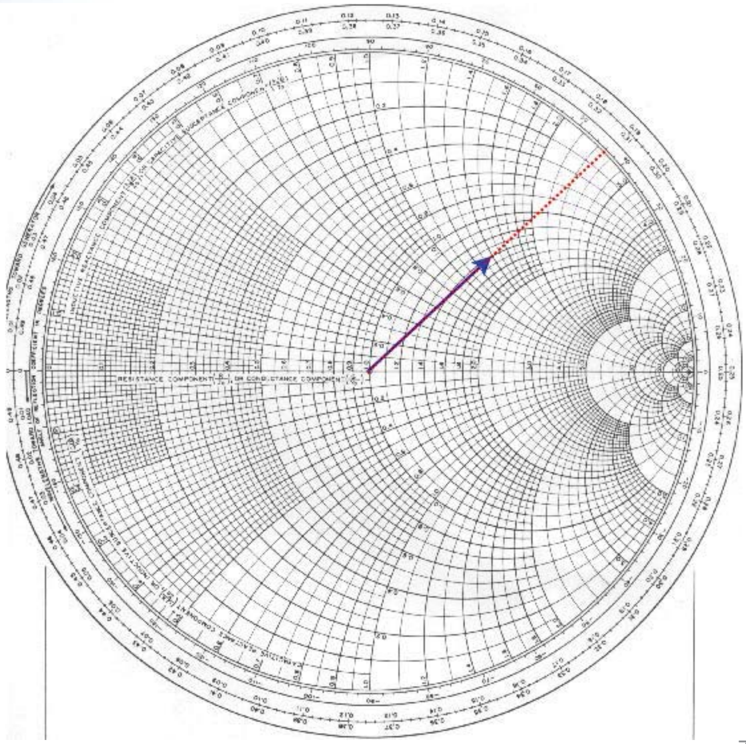
Given:

$$\Gamma_L = 0.5\angle 45°$$

$$Z_o = 50\Omega$$

What is Z_L?

$$Z_L = 50\Omega(1.35 + j1.35)$$
$$= 67.5\Omega + j67.5\Omega$$



Smith Chart

Example 2

Given:

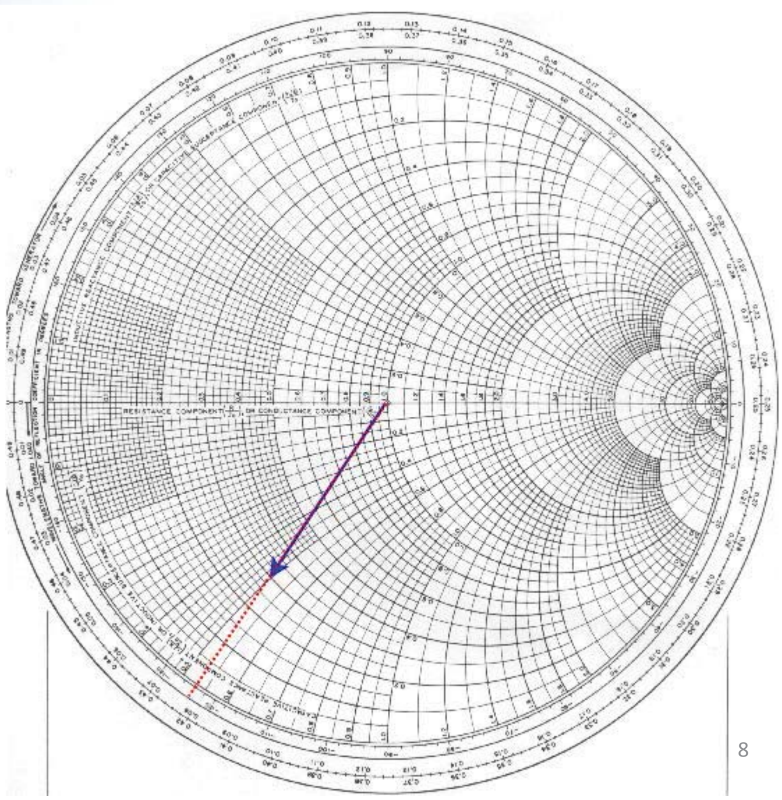
$$Z_L = 15\Omega - j25\Omega$$

$$Z_o = 50\Omega$$

What is Γ_L?

$$z_L = \frac{15\Omega - j25\Omega}{50\Omega}$$
$$= 0.3 - j0.5$$

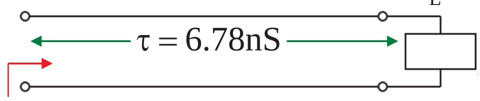
$$\Gamma_L = 0.618\angle -124°$$



Smith Chart

Example 3

$Z_L = 50\Omega + j50\Omega$



$Z_{in} = ?$ $\quad Z_o = 50\Omega$

What is Z_{in} at 50 MHz?

$$z_L = \frac{50\Omega + j50\Omega}{50\Omega}$$

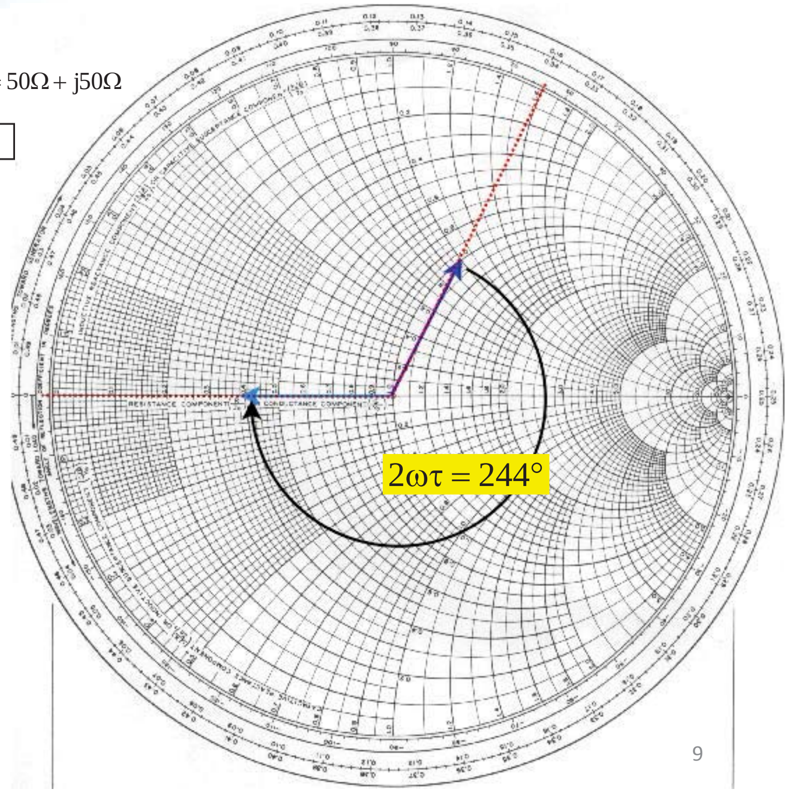
$$= 1.0 + j1.0 \quad \Rightarrow \Gamma_L = 0.445\angle 64°$$

$$\Gamma_{in} = \Gamma_L e^{-j2\beta d} = \Gamma_L e^{-j2\omega\tau}$$

$$2\omega\tau = 244°$$

$$\Gamma_{in} = 0.445\angle 180°$$

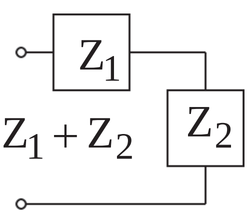
$$Z_L = 50\Omega(0.38 + j0.0) = 19\Omega$$



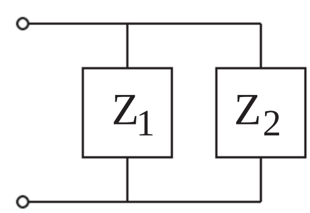
Admittance

- A matching network are combination of elements connected in series AND parallel.
- Impedance is well suited when working with series configurations.
- Impedance is NOT well suited when working with parallel configurations.

$$Z_L = Z_1 + Z_2$$

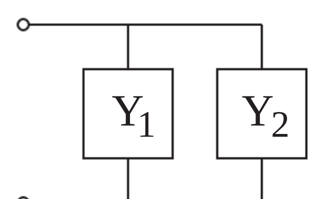


$$Z_L = \frac{Z_1 Z_2}{Z_1 + Z_2}$$



- For parallel loads it is better to work with admittance.

$$Y_L = Y_1 + Y_2$$



Smith Chart

Normalized Admittance

$$y = \frac{Y}{Y_o} = YZ_o = g + jb \qquad y = \frac{1-\Gamma}{1+\Gamma}$$

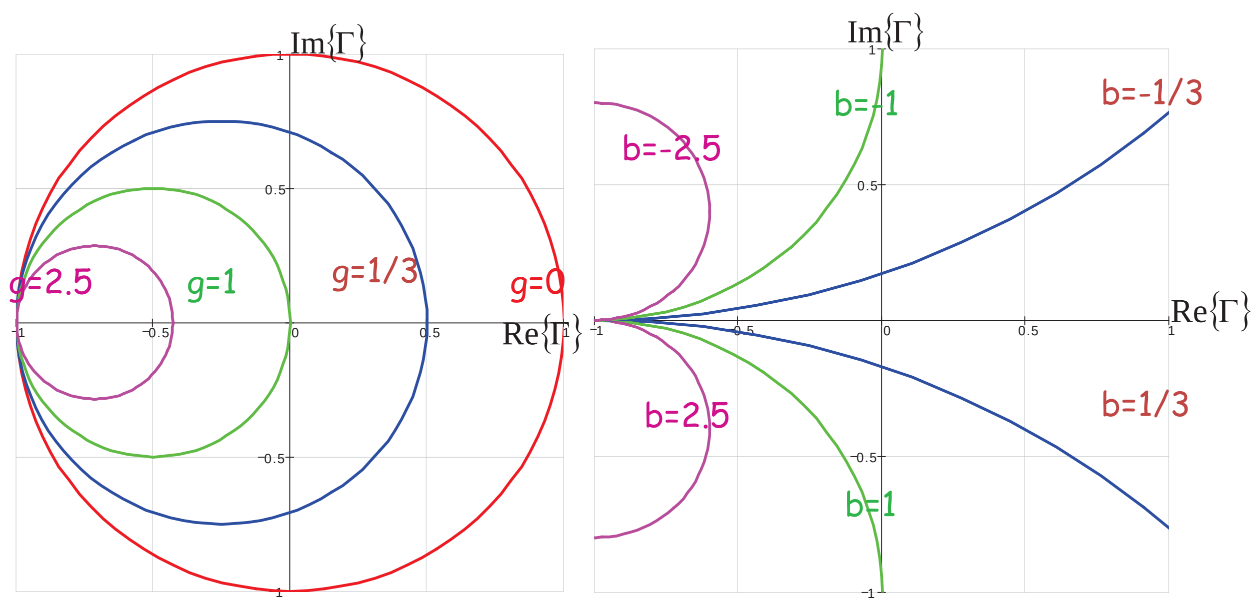
$$g = \frac{1-u^2-v^2}{(1+u)^2+v^2} \longrightarrow \left(u + \frac{g}{1+g}\right)^2 + v^2 = \frac{1}{(1+g)^2}$$

$$b = \frac{-2v}{(1+u)^2+v^2} \longrightarrow (u+1)^2 + \left(v + \frac{1}{b}\right)^2 = \frac{1}{b^2}$$

These are equations for circles on the (u,v) plane

Smith Chart

Admittance Smith Chart



Smith Charts

Impedance and Admittance Smith Charts

- For a matching network that contains elements connected in series and parallel, we will need two types of Smith charts
  - impedance Smith chart
  - admittance Smith Chart
- The admittance Smith chart is the impedance Smith chart rotated 180 degrees.
  - We could use one Smith chart and flip the reflection coefficient vector 180 degrees when switching between a series configuration to a parallel configuration.

Smith Chart

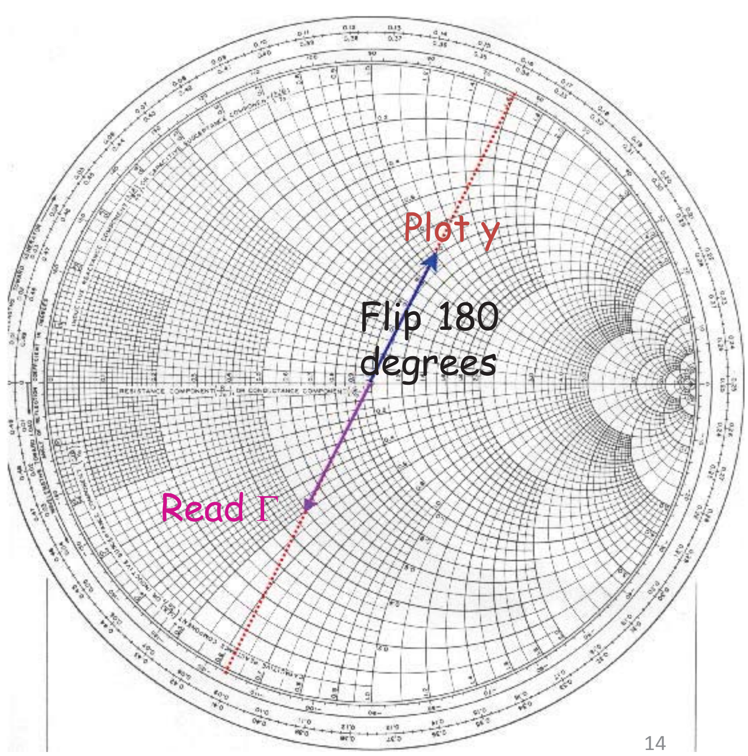
Example 1

$y = 1 + j1$

What is Γ?

- Procedure:
  - Plot 1+j1 on chart
    - vector $= 0.445\angle 64°$
  - Flip vector 180 degrees

$$\Gamma = 0.445\angle -116°$$



Smith Chart

Example 2

$\Gamma = 0.5\angle +45^\circ \quad Z_O = 50\Omega$

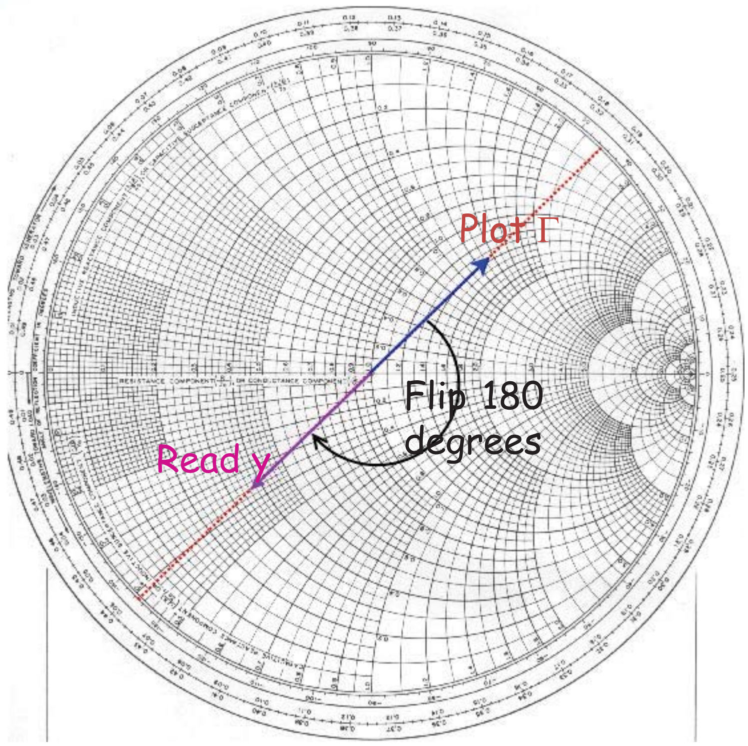
What is Y?

- Procedure:
  - Plot Γ
  - Flip vector by 180 degrees
  - Read coordinate

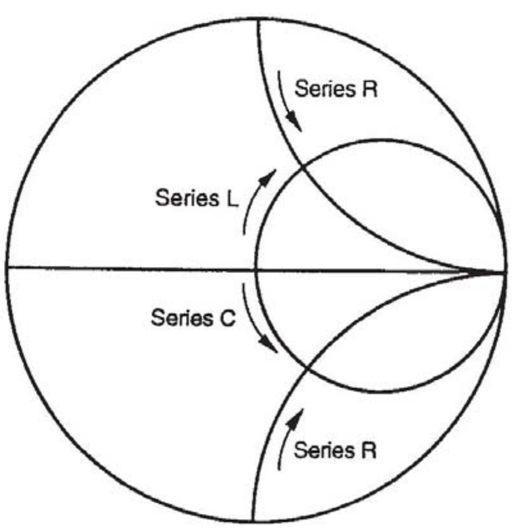
$$y = 0.38 - j0.36$$

$$Y = \frac{1}{50\Omega}(0.38 - j0.36)$$

$$Y = (7.6 - j7.2)x10^{-3}\text{mhos}$$



Smith Chart



Impedance Chart

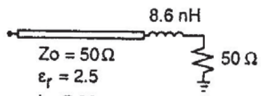
Adding a series L moves the impedance clockwise along a circle of constant resistance.

Adding a series C moves the impedance counter-clockwise along a circle of constant resistance.

Adding a series R moves the impedance along a line of constant reactance towards the open circuit point.

Smith Chart

What is the input impedance of the transmission line at 880 MHz?

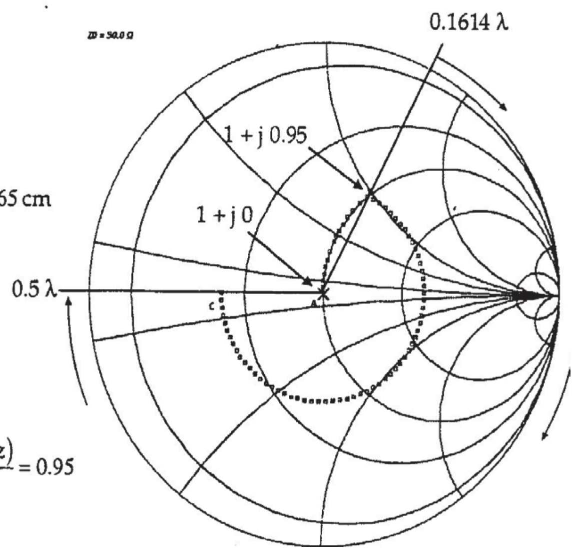


$$\lambda = \frac{c}{f\sqrt{\varepsilon_r}} = \frac{3(10^8)}{880(10^6)\sqrt{2.5}} = 21.65\text{ cm}$$

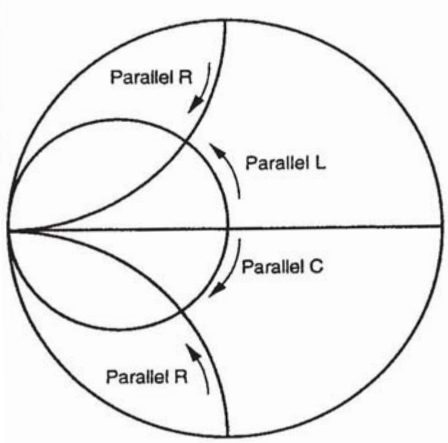
$$\ell = \left(\frac{7.33}{21.65}\right) = 0.3386\lambda$$

$$0.1614 + 0.3386 = 0.5$$

$$x_L = \frac{X_L}{50} = \frac{(8.6\text{nH})(2\pi 880\text{MHz})}{50} = 0.95$$



Smith Chart



Adding a parallel L moves the admittance counter-clockwise along a circle of constant conductance.

Adding a parallel C moves the admittance clockwise along a circle of consant conductance.

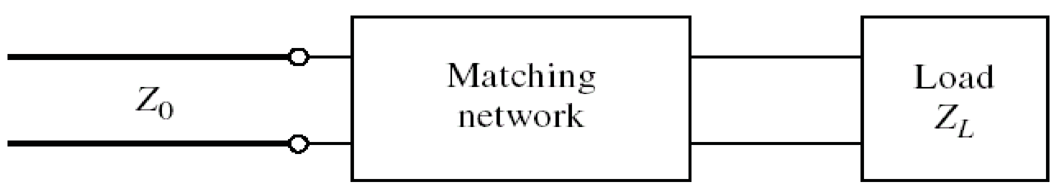
Adding a parallel R moves the admittance along a line of constant suseptance towards the short circuit point.

Admittance Chart

Matching Network

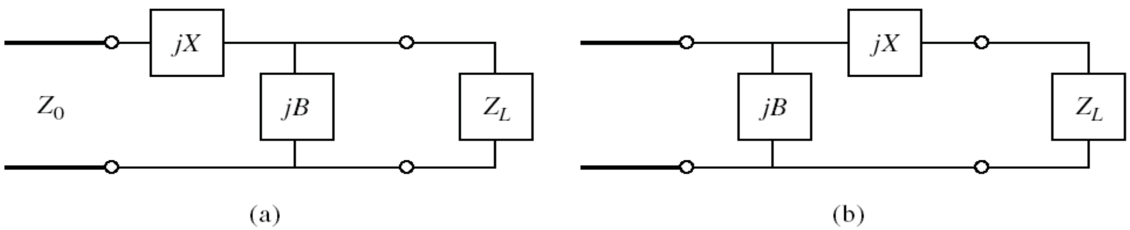
Matching Network

- A lossless network matching an arbitrary load impedance to a transmission line.



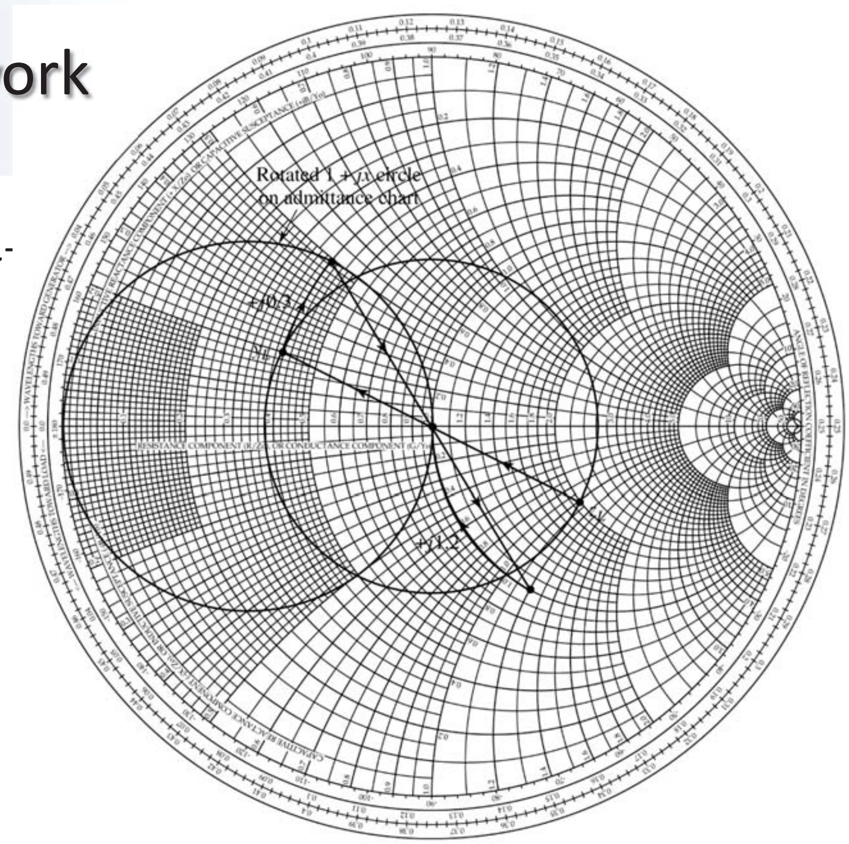
Matching Network

- L-section matching networks.
  - (a) Network for z_L inside the $1 + jx$ circle.
  - (b) Network for z_L outside the $1 + jx$ *circle*



Matching Network

- (*a*) Smith chart for the *L*-section matching networks.
- ZL=200-j100
- Zo=100Ohm
- Frequency=500MHz

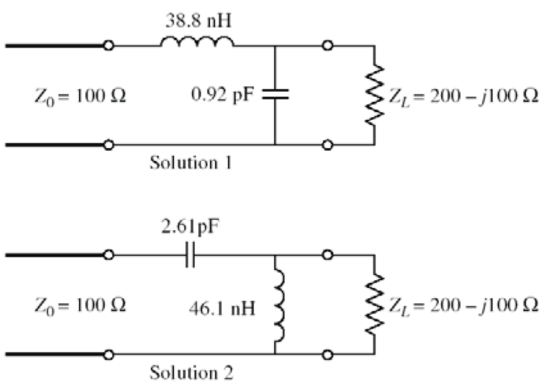


(a)

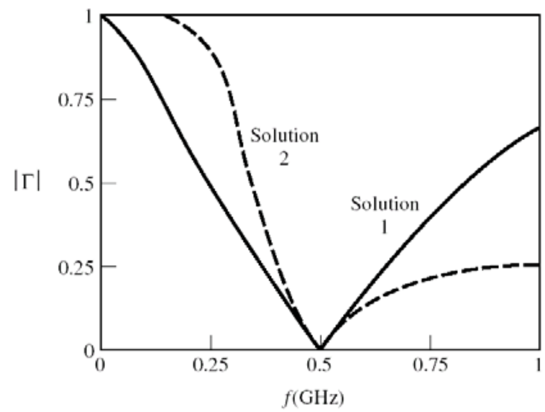
Figure 5.3a

Matching Network

- (*b*) The two possible *L*-section matching circuits. (*c*) Reflection coefficient magnitudes versus frequency for the matching circuits of (*b*).



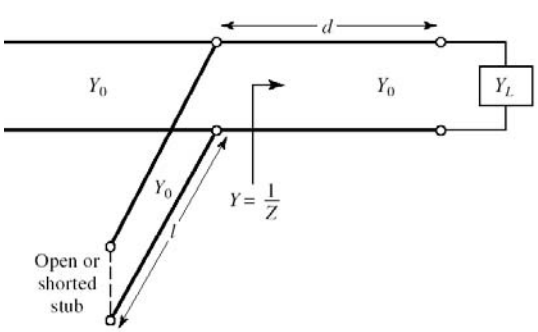
(b)



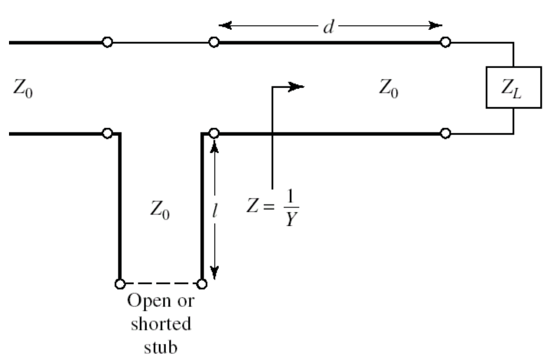
(c)

Matching Network

- Single-stub tuning circuits.
  (*a*) Shunt stub. (*b*) Series stub.



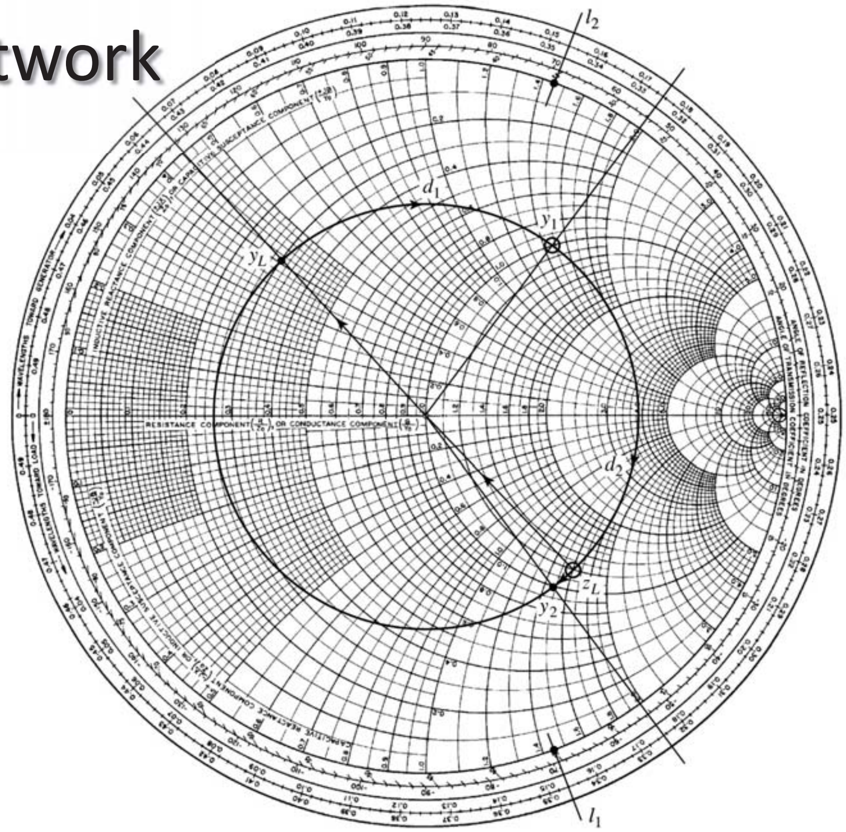
(a)



(b)

Matching Network

- (*a*) Smith chart for the shunt-stub tuners
- ZL=60-j80
- Zo=50Ohm
- Frequency=2GHz

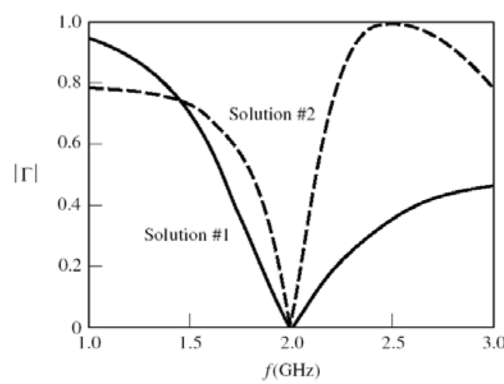


(a)

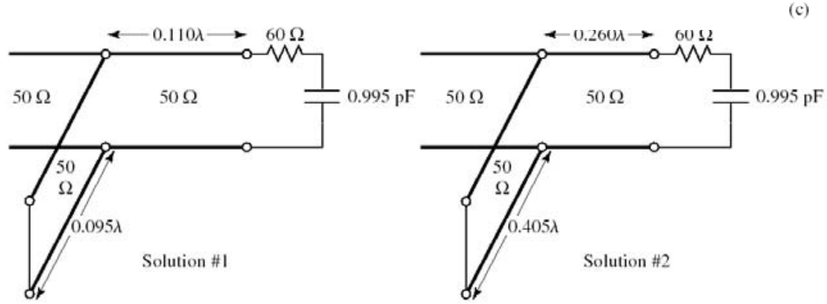
Figure 5.5a

Matching Network

- (*b*) The two shunt-stub tuning solutions. (*c*) Reflection coefficient magnitudes versus frequency for the tuning circuits of (*b*).



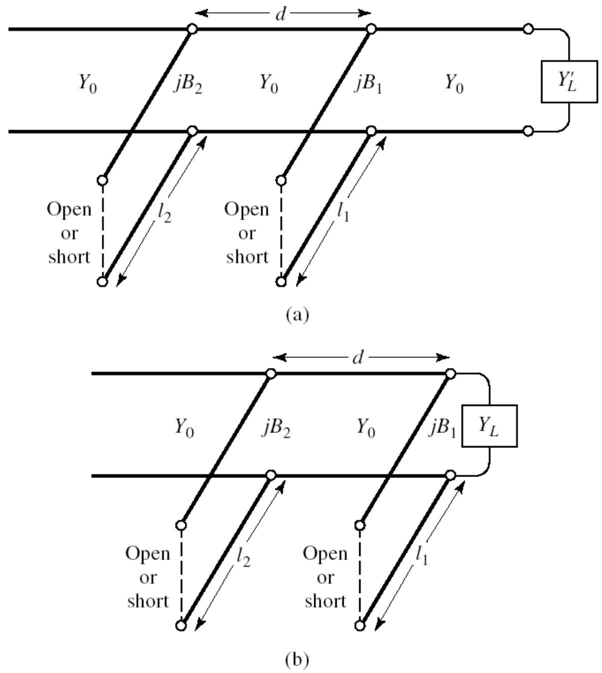
(c)



(b)

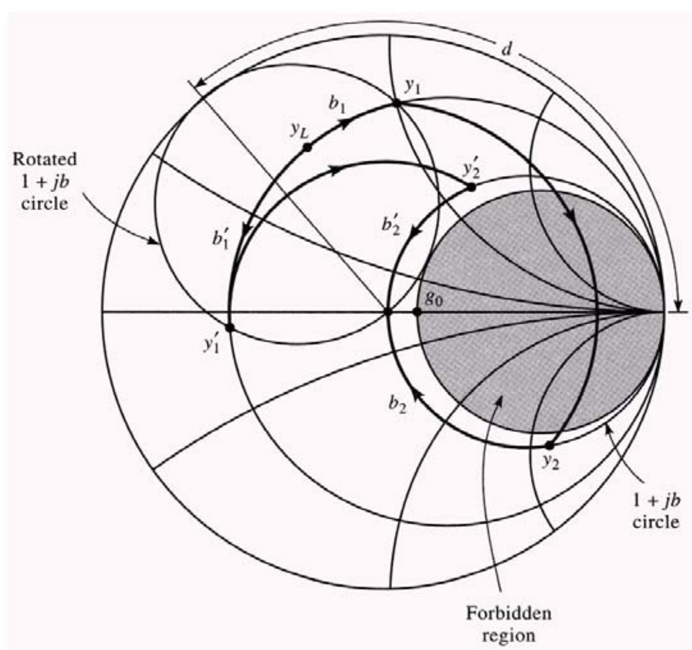
Matching Network

- Double-stub tuning.
  (*a*) Original circuit with the load an arbitrary distance from the first stub.
  (*b*) Equivalent-circuit with load at the first stub.



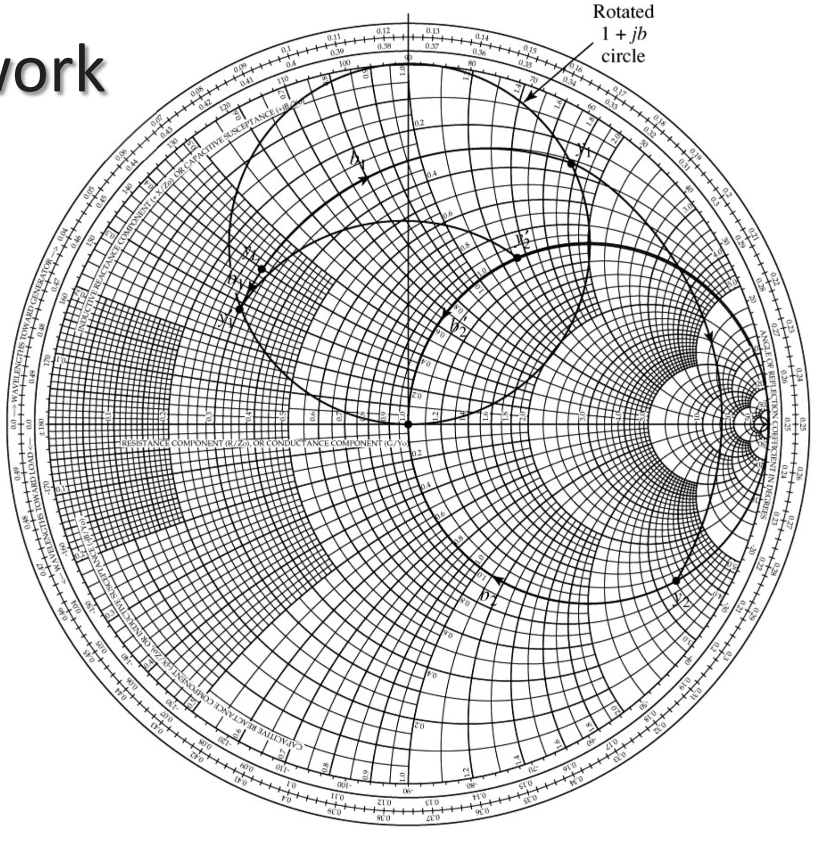
Matching Network

- Smith chart diagram for the operation of a double-stub tuner.



Matching Network

- (*a*) Smith chart for the double-stub tuners.
  ZL=60-j80
- Zo=50Ohm
- Frequency=2GHz
- d=λ/8

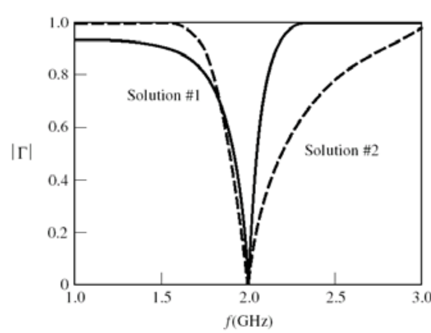


(a)

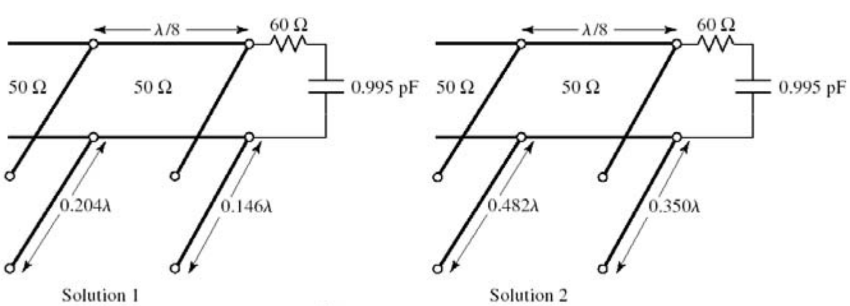
Figure 5.9a

Matching Network

- (*b*) The two double-stub tuning solutions. (*c*) Reflection coefficient magnitudes versus frequency for the tuning circuits of (*b*).



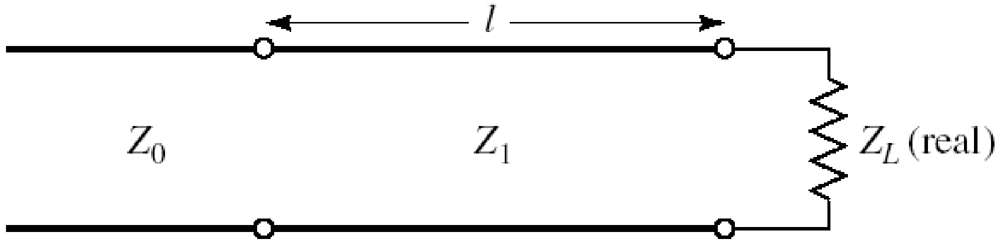
(c)



(b)

Matching Network

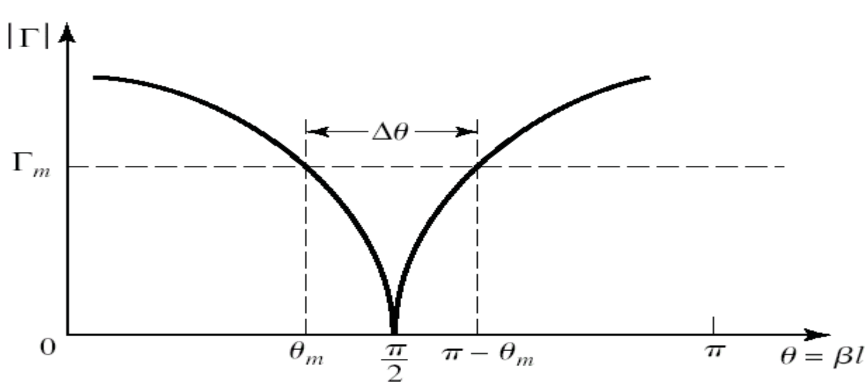
- A single-section quarter-wave matching transformer at the design frequency f_0.



$$\ell = \lambda_0 / 4$$

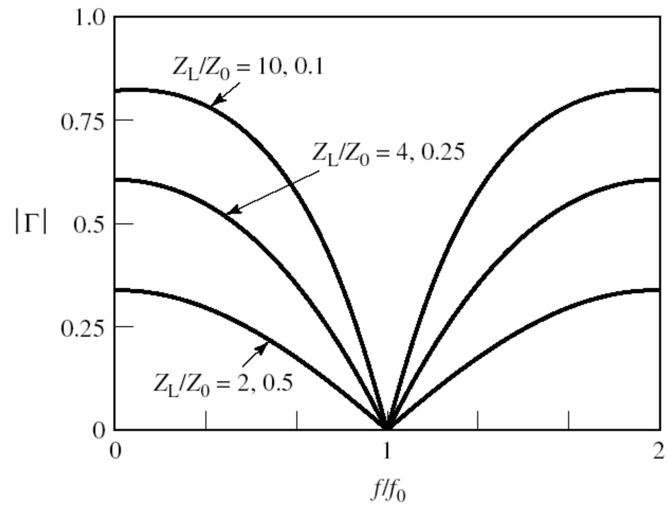
Matching Network

- Approximate behavior of the reflection coefficient magnitude for a single-section quarter-wave transformer operating near its design frequency.



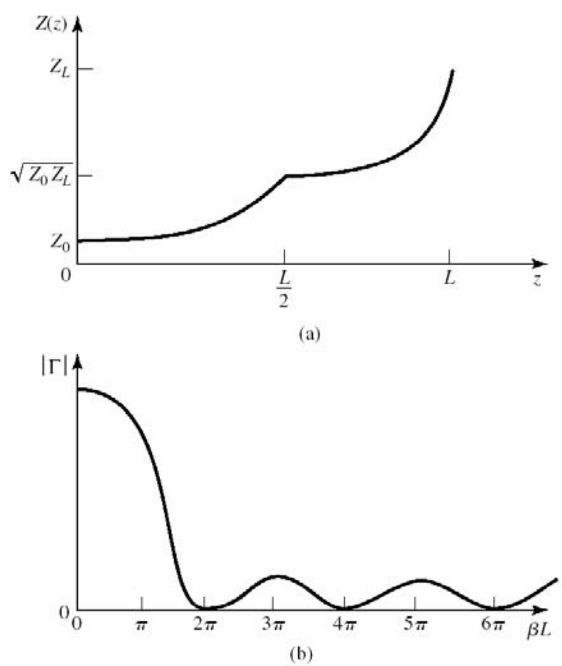
Matching Network

- Reflection coefficient magnitude versus frequency for a single-section quarter-wave matching transformer with various load mismatches.



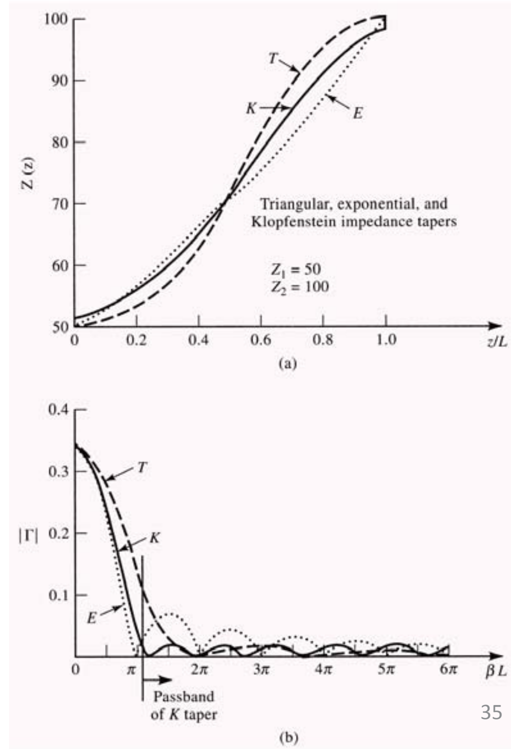
Matching Network

- A matching section with a triangular taper for $d(\ln Z/Z_0)/dz$.
  - (*a*) Variation of impedance.
  - (*b*) Resulting reflection coefficient magnitude response.



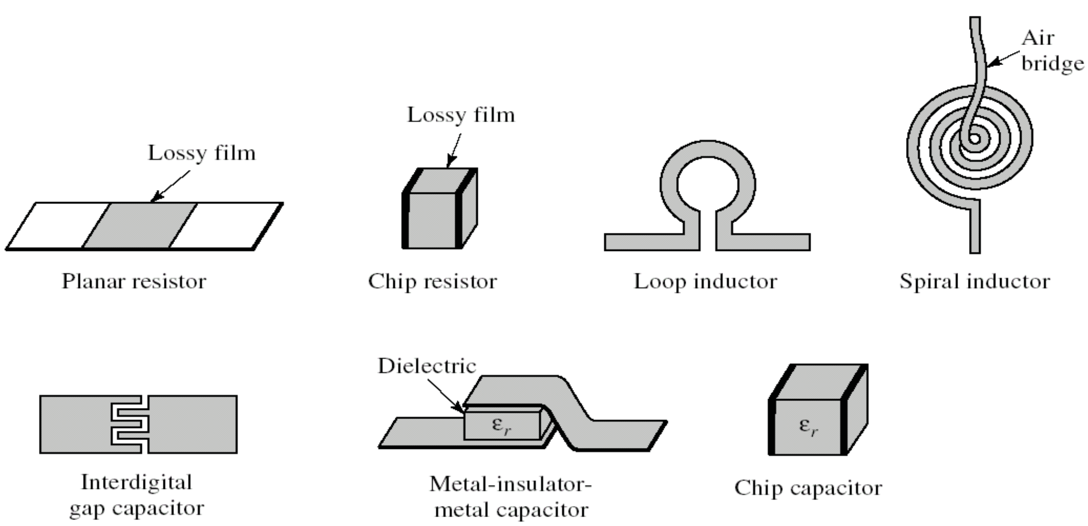
Matching Network

- (*a*) Impedance variations for the triangular, exponential, and Klopfenstein tapers.
- (*b*) Resulting reflection coefficient magnitude versus frequency for the tapers of (*a*).



Matching Network

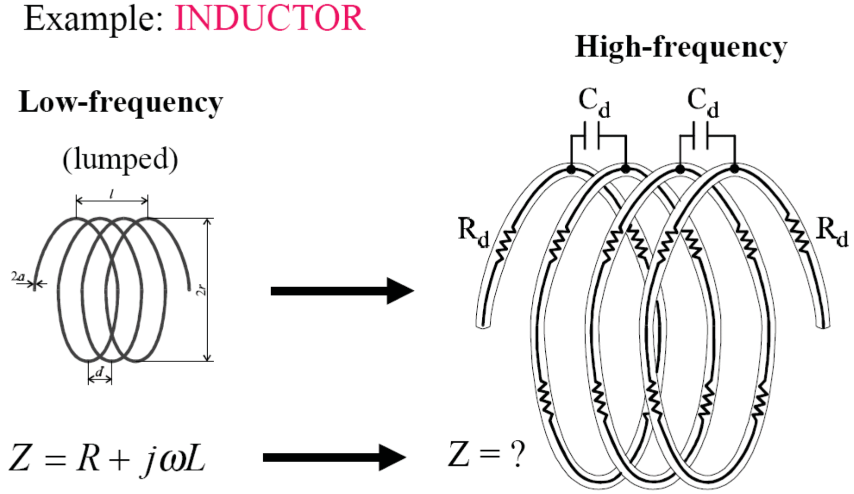
- Lumped Elements for Microwave Circuits



Microwave Components

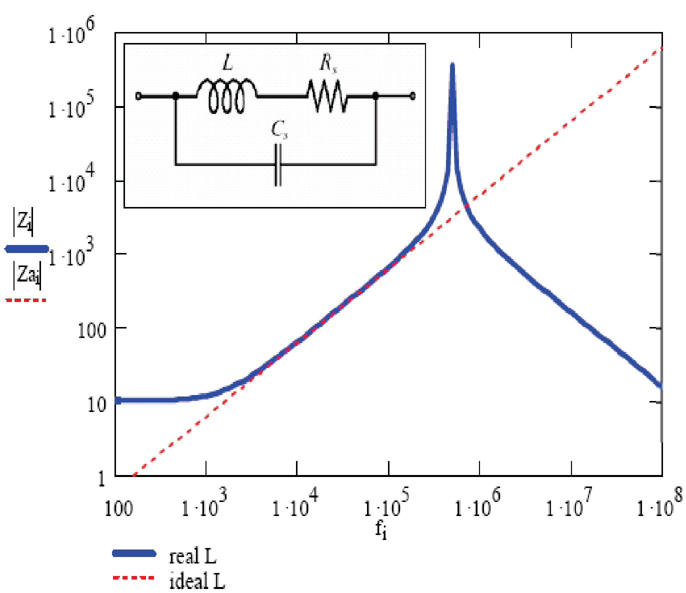
- Microwave technology involves predominantly distributed circuits analysis and design in contrast to lumped components

  - Example: INDUCTOR



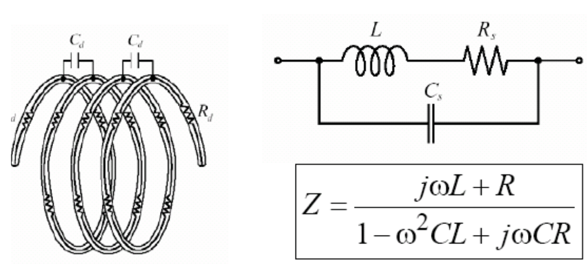
$Z = R + j\omega L \longrightarrow Z = ?$

Microwave Components



RF Inductor Model

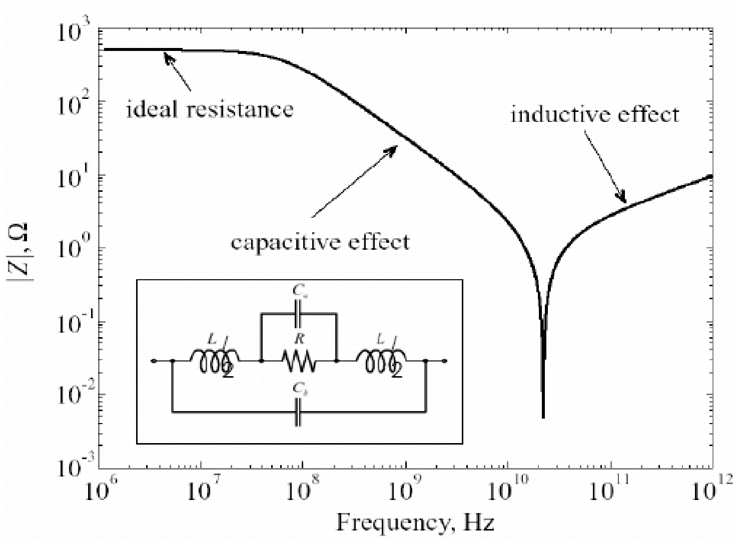
Parasitic capacitance and resistance



$$Z = \frac{j\omega L + R}{1 - \omega^2 CL + j\omega CR}$$

Microwave Components

RF Resistor Impedance



R. Ludwig and P. Bretchko, RF Circuit Design: Theory and Applications, Prentice Hall

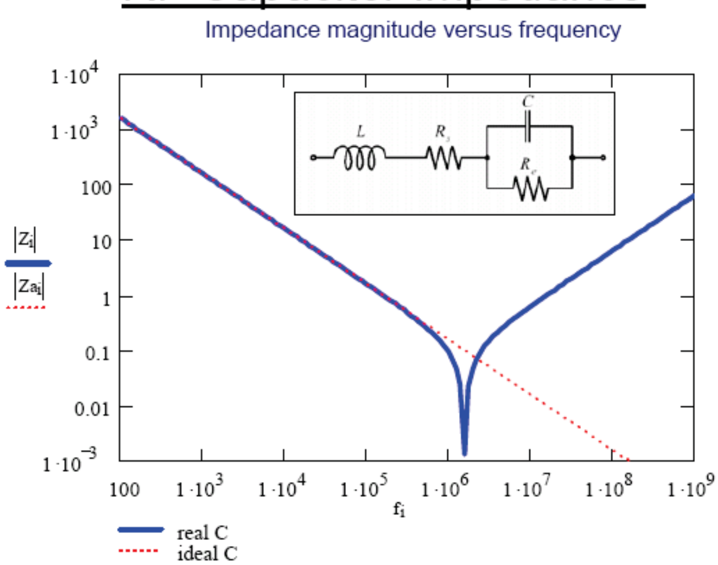
$$Z = j\omega L + \frac{R}{1 + j\omega CR}$$

$$C_b \approx 0$$

Parasitic inductance & capacitance

Microwave Components

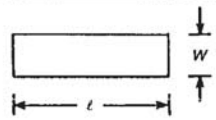
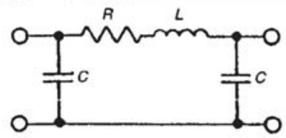
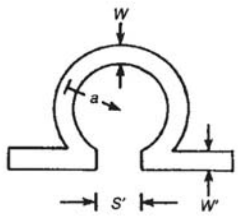
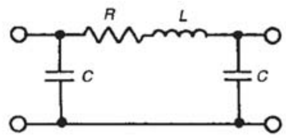
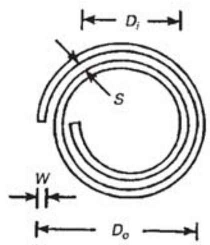
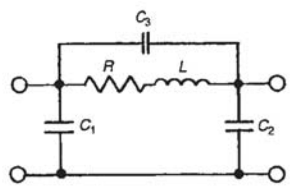
RF Capacitor Impedance



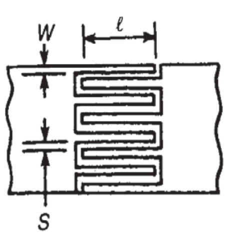
$$Z = j\omega L + \frac{1}{\omega C\left(j + \tan\Delta\right)}$$

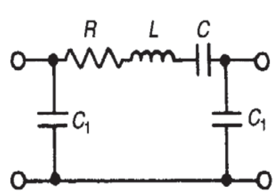
$$R_s \approx 0$$

Microwave Components

Inductors Strip		Equivalent Circuit	Expressions
Strip			$L\ (\text{nH}) = 2 \times 10^{-4} l \left[\ln\left(\frac{l}{W+t}\right) + 1.193 + 0.2235\frac{W+t}{l}\right] K_g$ $R\ (\Omega) = \frac{KR_s l}{2(W+t)}$
Loop			$L\ (\text{nH}) = 1.257 \times 10^{-3} a \left[\ln\left(\frac{a}{W+t}\right) + 0.078\right] K_g$ $R\ (\Omega) = \frac{KR_s}{W+t}\pi a$
Spiral			$L\ (\text{nH}) = 0.03937\frac{a^2 n^2}{8a+11c}K_g$ $a = \frac{D_o + D_i}{4}, \quad c = \frac{D_o - D_i}{2}$ $R\ (\Omega) = \frac{k\pi a n R_s}{W}$ $C_3\ (\text{pF}) = 3.5 \times 10^{-5} D_o + 0.06$

Microwave Components

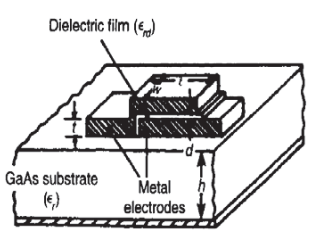


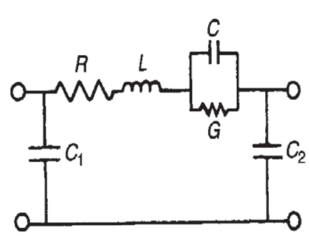


$$C\ (\text{pF}) = \frac{\epsilon_e 10^{-3}}{18\pi}\frac{K(k)}{K'(k)}(N-1)l$$

$$k = \tan^2\left(\frac{a\pi}{4b}\right), \quad a = \frac{W}{2}, \quad b = \frac{W+S}{2}$$

$$R\ (\Omega) = \frac{4}{3}\frac{R_s l}{WN}$$





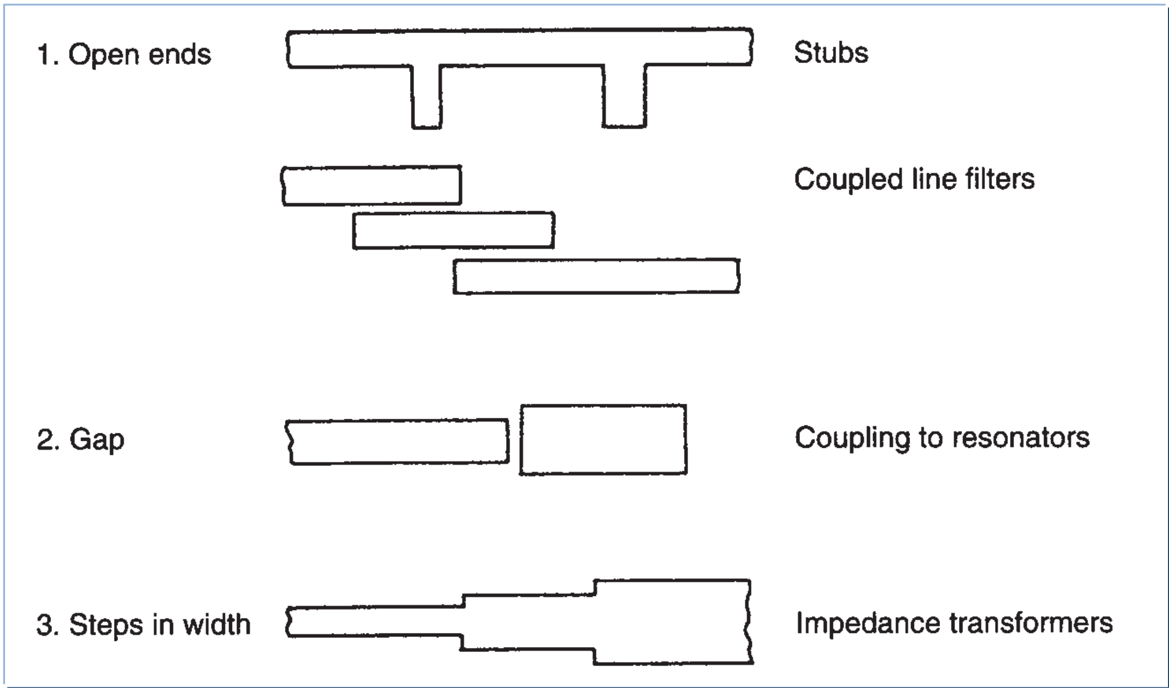
$$C\ (\text{pF}) = \frac{10^{-3}\epsilon_{rd} W l}{36\pi d}$$

$$R\ (\Omega) = \frac{KR_s l}{W+t}$$

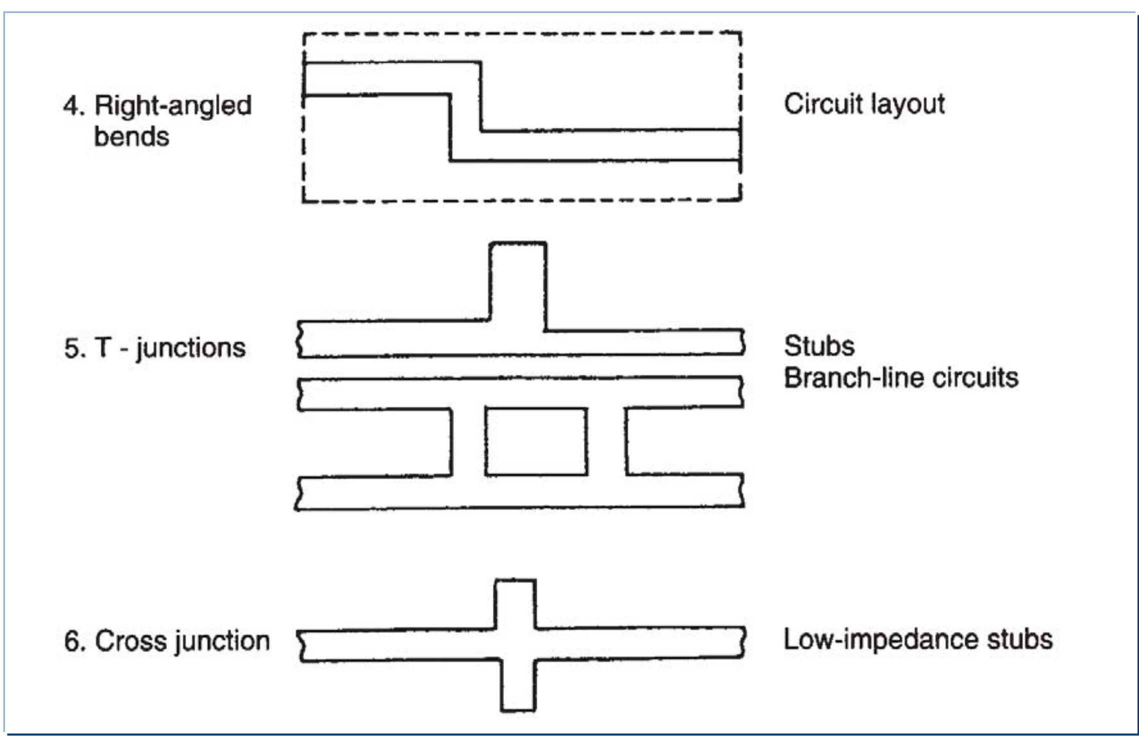
$$G\ (\Omega^{-1}) = \omega C \tan\delta$$

$$C_1 = C_2\ (\text{pF}) = 10^{-2}\left[\frac{\sqrt{\epsilon_e}}{Z_0(W,h,\epsilon_r)} - \frac{\epsilon_r W}{360\pi h}\right] l$$

Microstrip Microwave circuit



Microstrip Microwave circuit

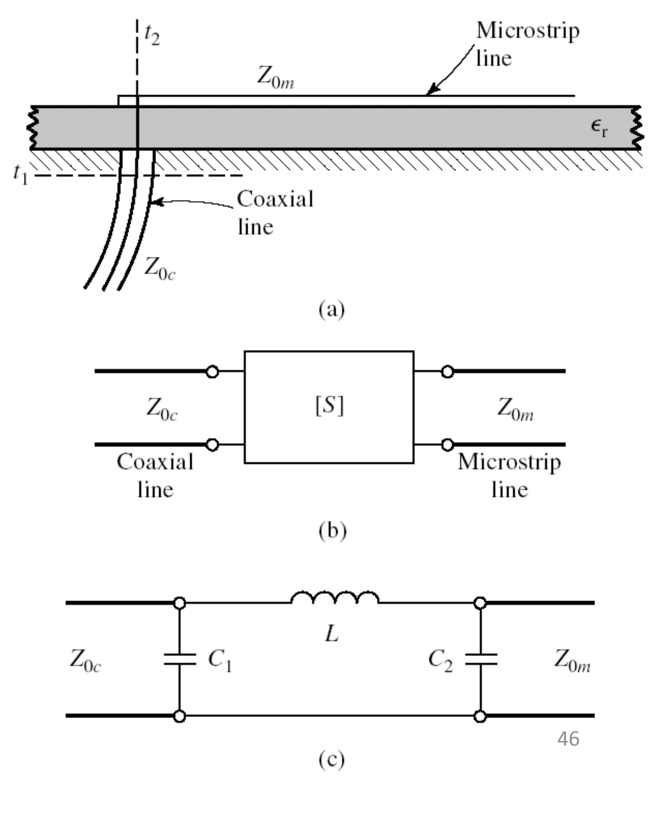


Microstrip Microwave circuit

- These discontinuities implies reactances (parasitic) that affects the circuit performance:
  - Frequency shift in narrow band circuits
  - Degradation in input and output VSWR
  - Higher ripple in gain flatness of broadband circuits
  - Interfacing problem in multifunction circuits
- These effects increases at higher freq
- They have either a T or Π equivalent circuit

Discontinuities effects

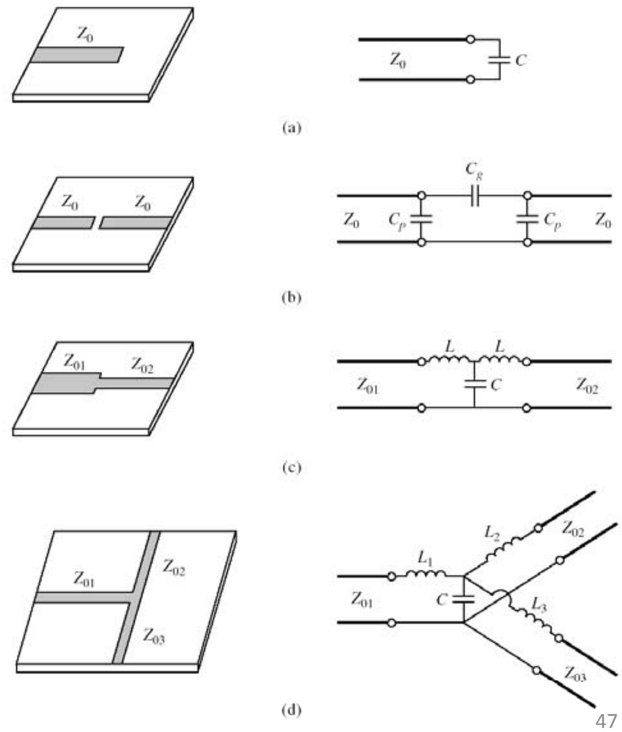
A coax-to-microstrip transition and equivalent circuit representations.
(*a*) Geometry of the transition.
(*b*) Representation of the transition by a “black box.”
(*c*) A possible equivalent circuit for the transition



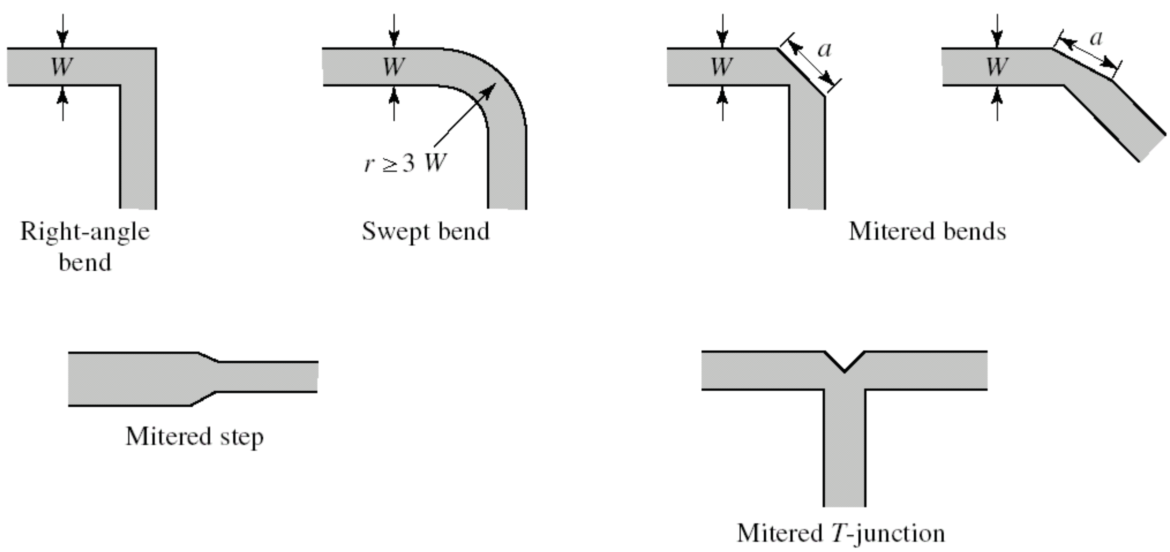
Discontinuities effects

Some common microstrip discontinuities.

(a) Open-ended microstrip.
(b) Gap in microstrip.
(c) Change in width.
(d) T-junction.



Microstrip Discontinuities effects



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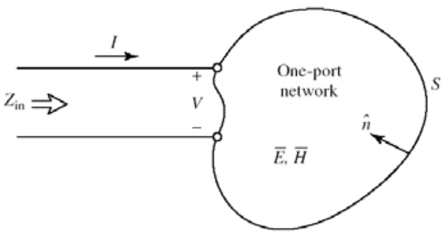
Linear microwave network parameter

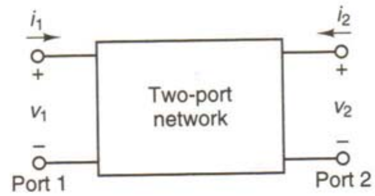
- Microwave Network Analysis is NOT….

Linear microwave network parameter

- Linear Microwave network can be fully described by the relations between the ports currents and voltages. The type of parameters used depends on the circuits interconnection

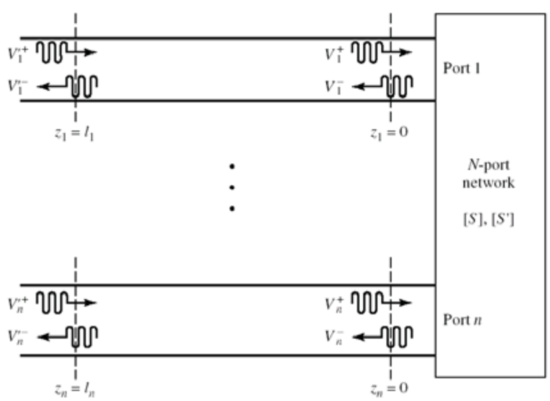


One-port network.

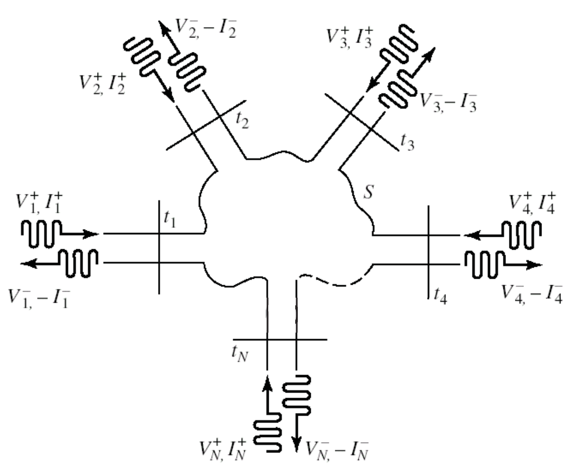


Two-port network.

Linear microwave network parameter



$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & & & \vdots \\ \vdots & & & \vdots \\ Z_{N1} & \cdots & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix},$$



N-port network.

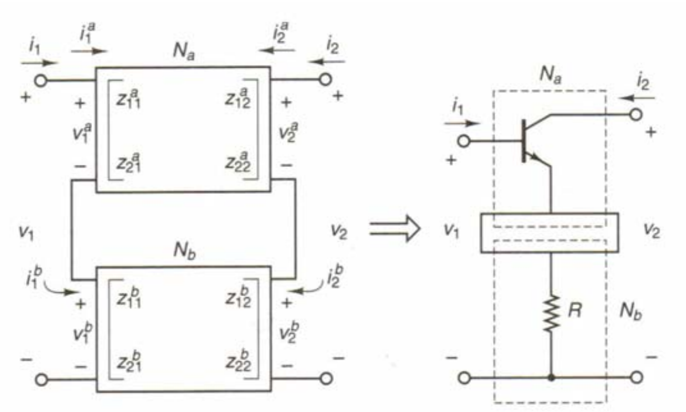
Linear microwave network parameter

- There are several sets of parameters used to express two independent variables from the 4 variables of the linear 2 port network
  - Z parameters
  - Y parameters
  - H parameters
  - ABCD parameters

Two Port Interconnection

- Two port networks could be interconnected according to different schemes such as

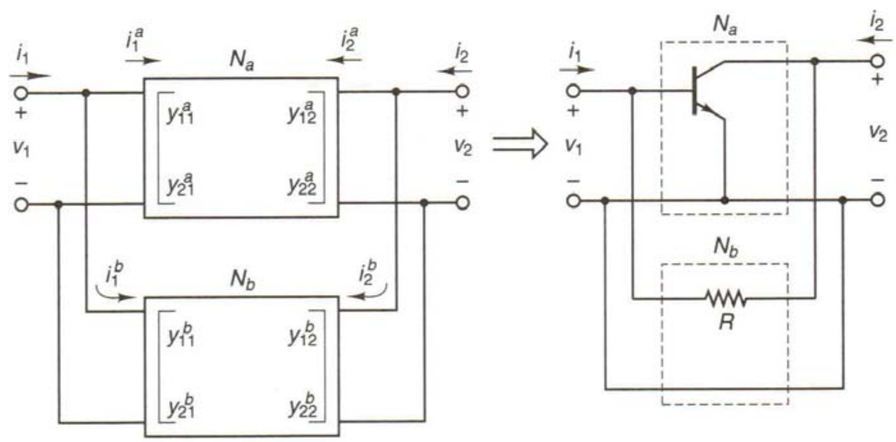
Series connection



Two Port Interconnection

- Two port networks could be interconnected according to different schemes such as

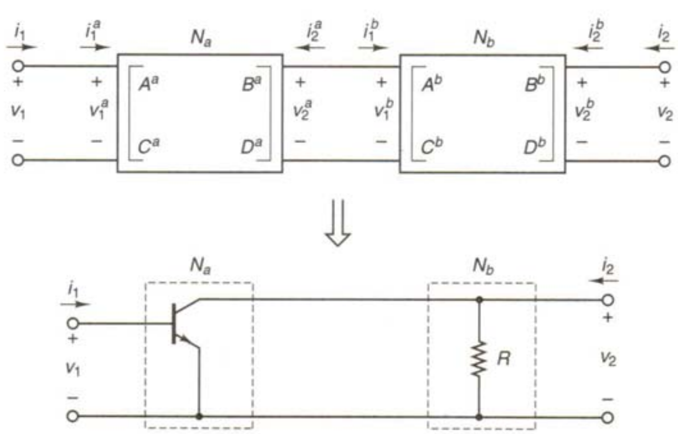
Parallel connection



Two Port Interconnection

- Two port networks could be interconnected according to different schemes such as

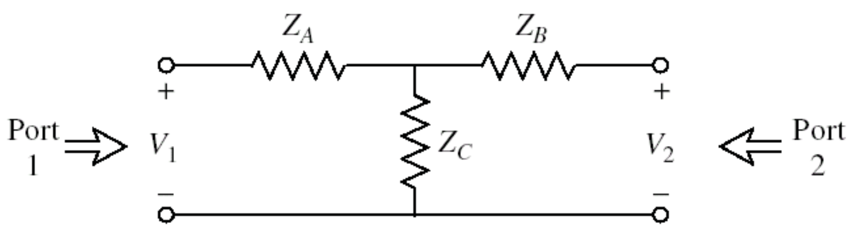
Cascade connection



Impedance Parameters

- Impedance parameters are very useful in designing impedance matching and power distribution system. The input and output terminal voltage can be expressed as:

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



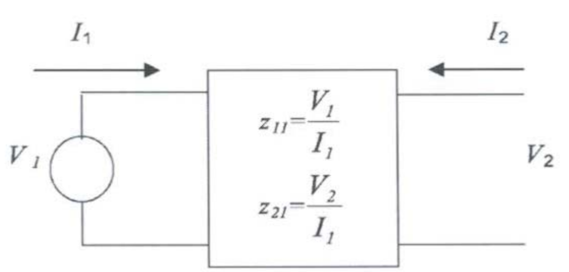
Impedance Parameters

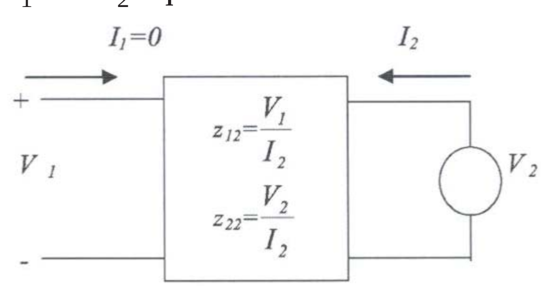
Rewrite the previous equation into matrix form as:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where impedance parameters of the system is Z = $\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$

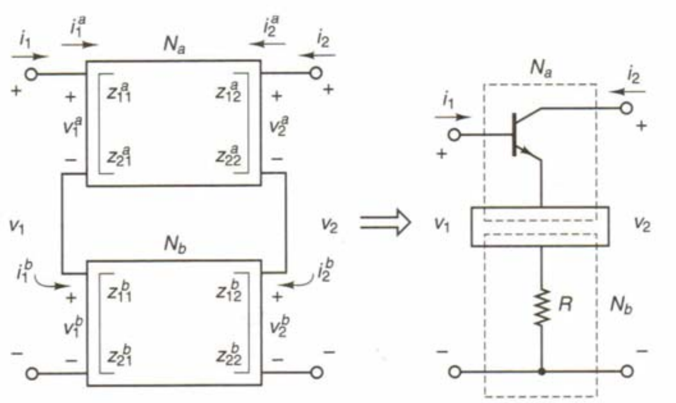
To find the parameters of the circuit, set I_1 and I_2 equal to zero





Impedance Parameters

- In the case of a series connection of two-port networks, the overall Z parameters can be found by adding the individual Z parameters



$$Z = Z^a + Z^b$$

Admittance Parameters

- Admittance parameters are very useful for describing the network when impedance parameters may not be determined.
- These parameters allow expressing the terminal current in term of the voltage.
- The input and output terminal current can be presented as follows:

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

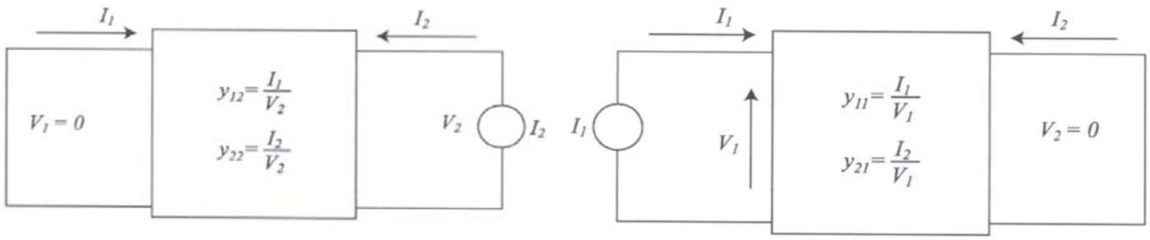
Rewrite into matrix form as:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where admittance parameters of the system is Y = $\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$

Admittance Parameters

To find the parameters of the circuit, set V1 and V2 equal to zero



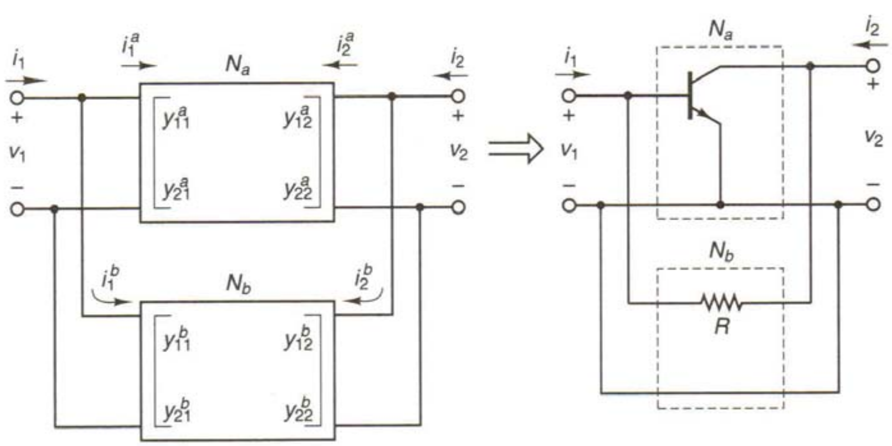
***y*$_{11}$:** short-circuit input impedance
***y*$_{12}$:** short-circuit transfer impedance from port 2 to 1
***y*$_{21}$:** short-circuit transfer impedance from port 1 to 2
***y*$_{22}$:** short-circuit output impedance

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Admittance Parameters

- The Y parameters of two 2-port networks connected in parallel is the sum of the two individual Y matrices



$$Y = Y^a + Y^b$$

Hybrid Parameters

- The third set of parameters is known as hybrid parameters or H-parameters.
- The input and output terminal current and voltage can be presented as follow:

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

- Matrix form $\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$ $H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$

Hybrid Parameters

- These parameters are very useful in transistor modeling
- The value of the parameters can be found based on the following equation.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Hybrid Parameters

- The H parameters are named specifically as follows:
  - *h*11 = short circuit input impedance
  - *h*12 = open circuit reverse voltage gain
  - *h*21 = short circuit forward current gain
  - *h*22 = open circuit output admittance

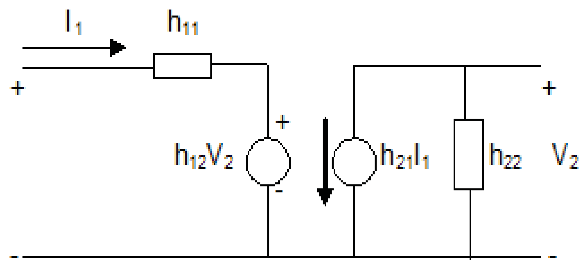
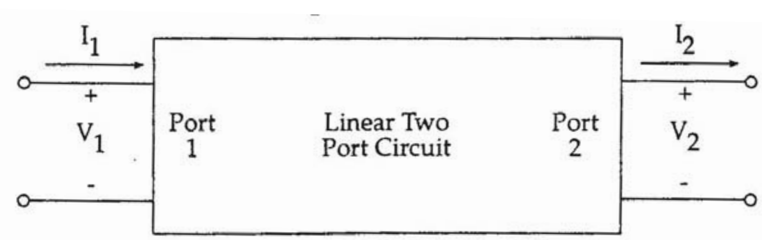


Figure show the H-parameter equivalent circuit for the two-port network.

Chain Parameter

- The chain parameters (ABCD) relate the input voltage and current at port1 to the voltage and current of the port 2



$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

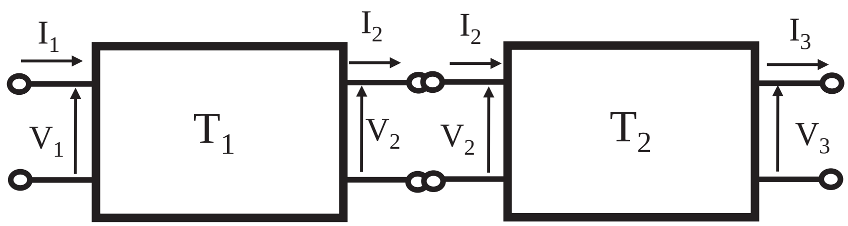
$\longleftrightarrow$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

Chain Parameters

- ABCD parameters are used to analyze a cascade connection of two port networks



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [T_1][T_2] \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

Scattering Parameters

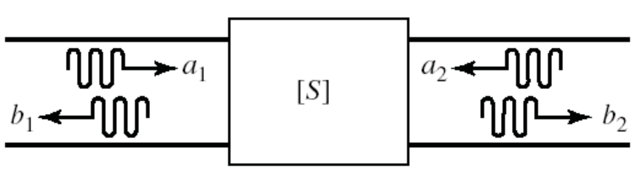
- There is a need to establish well-defined termination conditions in order to find the network descriptions for Z, Y, H, and ABCD networks
- Open and short voltage and current conditions are difficult to enforce. These conditions may lead to oscillation when a transistor is involved
- RF implies forward and backward traveling waves which can form standing waves destroying the elements

Solution S Parameters

- Input-output behavior of network is defined in terms of normalized power waves
- Ratio of the power waves are recorded in terms of so-called scattering parameters
- S-parameters are measured based on properly terminated transmission lines (and not open/short circuit conditions)

Solution S Parameters

- A fundamental two port model is developed adopting the convention that a is an input port and b is an output port.
- a is assigned to incident values while b indicates reflected values.

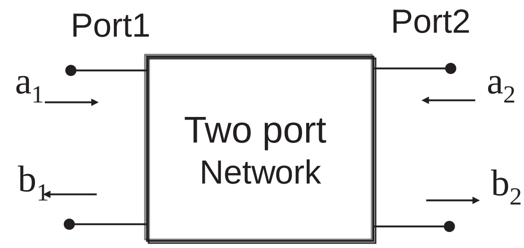


$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

Solution S Parameters

$$\Rightarrow b(x) = \Gamma(x)a(x)$$

$$\Leftrightarrow \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$



S_{11} = input reflection coefficient
S_{21} = forward transmission coefficient
$S_{21}{}^2$ = transducer power gain
S_{22} = output reflection coefficient
S_{12} = reverse transmission coefficient
$S_{12}{}^2$ = reverse transducer power gain

Scattering Parameters

Scattering matrix

Traveling waves: $V^+ = Ae^{-\gamma x}$, $V^- = Be^{\gamma x}$ so that the total voltage is $V(x) = V^+(x) + V^-(x)$.

Similarly for current: $I(x) = I^+(x) - I^-(x), = V^+/Z_0 - V^-/Z_0$

Refl. coeff.: $\Gamma(x) = V^-(x)/V^+(x)$

Introduce "normalized" variables: $v(x) = V(x)/\sqrt{Z_0}$, $i(x) = \sqrt{Z_0}I(x)$, $a(x) = V^+(x)/\sqrt{Z_0}$, $b(x) = V^-/\sqrt{Z_0}$, so that

$v(x) = a(x) + b(x) \quad i(x) = a(x) - b(x)$ and $b(x) = \Gamma(x)a(x)$

This defines a single port network. What about 2-port?

Scattering Parameters

2-port figure. Generalize eq. :

$$
\begin{aligned}
b_1 &= S_{11}a_1 + S_{12}a_2 \\
b_2 &= S_{21}a_1 + S_{22}a_2
\end{aligned}
\tag{10}
$$

or in matrix form.

Each reflected wave (b_1, b_2) has two contributions: one from the incident wave at the same port and another from the incident wave at the other port.

S-parameters, scattering parameters, scattering matrix. How to calculate them?

Scattering Parameters

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \quad \text{input refl. coeff. with output matched} \tag{11}$$

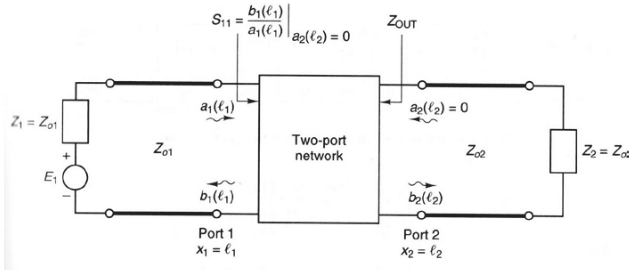
$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} \quad \text{reverse trans. coeff. with input matched} \tag{12}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \quad \text{transmission coeff. with output matched} \tag{13}$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} \quad \text{output refl. coeff. with input matched} \tag{14}$$

"matched" = termination equal to the tr. line ch. impedance.

Scattering Parameters



Note: Z_{out} (of 2-port) need not be matched to Z_{02}!!

Sufficient condition: $Z_L = Z_{02} \Rightarrow a_2 = 0$. Usually $Z_{01} = Z_{02}$. We measure "overall" S-parameters, i.e. including the tr. line on I/O.

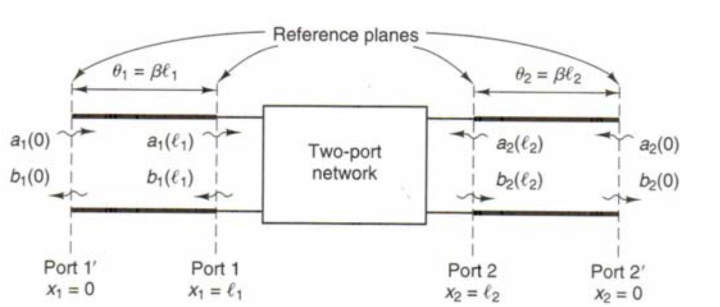
Advantage: using matched resistive terminations to measure S-matrix, transistor does not oscillate.

Occasional use for Chain scattering parameters or T-parameters — mainly for cascading networks.

Scattering Parameters

Shifting reference planes

why: to "de-embed" the 2-port from tr. lines on I/O. Reference planes: positions along tr. lines (usually beginning and end).



Scattering Parameters

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \text{ and } \begin{bmatrix} b_1' \\ b_2' \end{bmatrix} = \begin{bmatrix} S_{11}' & S_{12}' \\ S_{21}' & S_{22}' \end{bmatrix} \begin{bmatrix} a_1' \\ a_2' \end{bmatrix}$$

$\Theta = \beta l$ are electrical lengths of tr. lines. Signal is delayed by Θ as it travels from $x = 0$ to $x = l \Rightarrow$

$$\left.\begin{matrix} b_1' = b_1 e^{-j\Theta_1} & b_2' = b_2 e^{-j\Theta_2} \\ a_1 = a_1' e^{-j\Theta_1} & a_2 = a_2' e^{-j\Theta_2} \end{matrix}\right\} \Rightarrow \begin{matrix} b_1 = b_1' e^{j\Theta_1} & b_2 = b_2' e^{j\Theta_2} \\ a_1 = a_1' e^{-j\Theta_1} & a_2 = a_2' e^{-j\Theta_2} \end{matrix}$$

This is plugged into first part of eq. :

$$\begin{bmatrix} b_1' \\ b_2' \end{bmatrix} = \begin{bmatrix} S_{11} e^{-j2\Theta_1} & S_{12} e^{-j(\Theta_1+\Theta_2)} \\ S_{21} e^{-j(\Theta_1+\Theta_2)} & S_{22} e^{-j2\Theta_2} \end{bmatrix} \begin{bmatrix} a_1' \\ a_2' \end{bmatrix}$$

By inspection, first term on RHS must be $= [S']$ matrix

Reverse relations obtained by expressing primed variables in eq. and plugging into second part of eqs. $\Rightarrow$

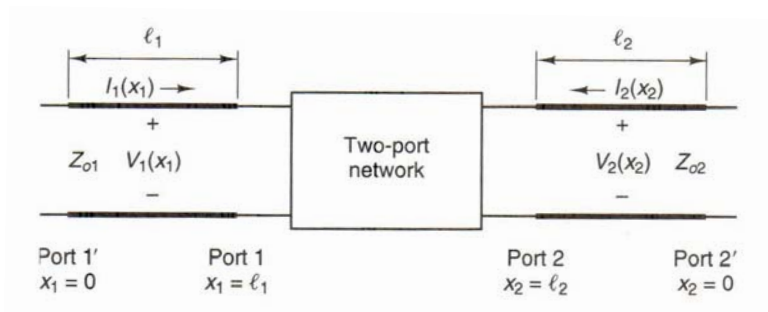
$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} S_{11}' e^{j2\Theta_1} & S_{12}' e^{j(\Theta_1+\Theta_2)} \\ S_{21}' e^{j(\Theta_1+\Theta_2)} & S_{22}' e^{j2\Theta_2} \end{bmatrix}$$

Scattering Parameters

Properties of S-parameters

Start with fig.below ; assume that tr. lines are lossless and Z_0 are real (usually 50 Ohm lines with 50 Ohm terminations).

At i-th port (here: 1 or 2):



Scattering Parameters

$$V_i(x_i) = V_i^+(x_i) + V_i^-(x_i) \quad (15)$$

$$I_i(x_i) = I_i^+(x_i) - I_i^-(x_i) = \frac{V_i^+(x_i)}{Z_{0i}} - \frac{V_i^-(x_i)}{Z_{0i}} \quad (16)$$

$$\begin{aligned}
\text{define } a_i(x_i) &= \frac{V_i^+(x_i)}{\sqrt{Z_{0i}}} = \sqrt{Z_{0i}} I_i^+(x_i) \\
&= \frac{1}{2\sqrt{Z_{0i}}}[V_i(x_i) + Z_{0i} I_i(x_i)] && (17) \\
\text{and similarly } b_i(x_i) &= \frac{V_i^-(x_i)}{\sqrt{Z_{0i}}} = \sqrt{Z_{0i}} I_i^-(x_i) \\
&= \frac{1}{2\sqrt{Z_{0i}}}[V_i(x_i) - Z_{0i} I_i(x_i)] && (18)
\end{aligned}$$

V-s and I-s are peak values. For sin() signal $\Rightarrow$ divide by $\sqrt{2}$.

Scattering Parameters

Average power associated with incident wave

$$\begin{aligned}
P_i^+(x=0) &= \frac{1}{2}\text{Re}[V_i^+(0)(I^+(0))^*] \\
&= \frac{|V_i^+(0)|^2}{2Z_{0i}} = \frac{a_i(0)a_i^*(0)}{2} = \frac{|a_i(0)|^2}{2} && (19)
\end{aligned}$$

and for reflected power

$$P_i^-(x=0) = \frac{|V_i^-(0)|^2}{2Z_{0i}} = \frac{b_i(0)b_i^*(0)}{2} = \frac{|b_i(0)|^2}{2} \quad (20)$$

Lines are lossless $\Rightarrow P_i^+(0) = P_i^+(l)$ and $P_i^-(0) = P_i^-(l)$

$\Rightarrow |a_i(x)|^2$ and $|b_i(x)|^2$ represent power associated with incident and reflected waves anywhere on tr. lines.

Scattering Parameters

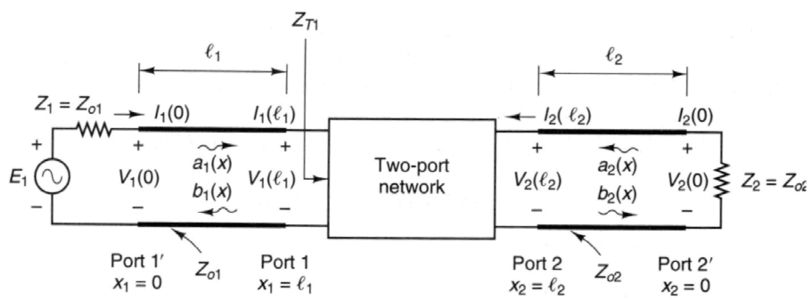


Fig. 1.6.2

Scattering Parameters

Consider Fig. 1.6.2: (port $1'$ excited by generator $E_1\angle 0^\circ$ and ports match terminated.

$$\text{At } x_2 = 0 \ : \ V_2(x_2 = 0) = -I_2(0)Z_{02}, \ a_2(x_2 = 0) = 0 \quad (21)$$

$$\text{At } x_1 = 0 \ : \ V_1(x_1 = 0) = E_1 - Z_{01}I_1(0) \quad (22)$$

Resulting in (use eq. 17)

$$a_1(x_1) = \frac{E_1}{2\sqrt{Z_{01}}} \quad \text{or} \quad |a_1|^2 = \frac{|E_1|^2}{4Z_{01}} \Rightarrow P_1^+(0) = \frac{|E_1|^2}{8Z_{01}} \quad (23)$$

$\Rightarrow |a_1(x_1 = 0)|^2$ represents the power available from the source at port $1'$ and since tr. line is lossless it is also power available at port 1. Called P_{avs}. Independent of the input impedance of the 2-port network.

Scattering Parameters

If $Z_1 \neq Z_{01}$, after simple derivation (see book):

$$\frac{1}{2}(|a_1(0)|^2 - |b_1(0)|^2) = \frac{1}{2}\text{Re}[I_i(0)V_1^*(0)] \tag{24}$$

which represents power delivered to port 1 (or $1'$). Call it P_1 so that $|b_1(0)|^2 = P_{avs} - P_1$ is reflected power from port 1 to port $1'$.

Similarly, $P_L' = P_L = |b_2|^2/2 = Z_{02}|I_2(0)|^2$ represents the power delivered to the load Z_{02}

Summary: Generator send available power $|a_1(0)|^2$ toward input port 1. This power is independent of input impedance Z_1. If $Z_1 = Z_{01}$ (input matched to tr. line) then reflected power $=0$. Otherwise, some is reflected back to generator ($= |b_1(0)|^2$). Net power delivered to port 1 is $|a_1(0)|^2 - |b_1(0)|^2$.

Scattering Parameters

How to use this to calculate S-parameters? See fig. 1.6.3 and use definition of S_{11}

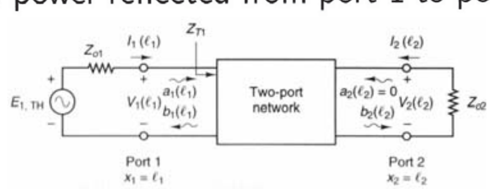
$$S_{11} = \left.\frac{b_1(l_1)}{a_1(l_1)}\right|_{a_2(l_2)=0} = \left.\frac{V_1^-(l_1)}{V_1^+(l_1)}\right|_{V_2^+(l_2)=0} = \frac{Z_{T1} - Z_{01}}{Z_{T1} + Z_{01}} \tag{25}$$

S_{11} **is reflection coefficient of port 1 when port 2 is match terminated.**

Also, note that

$$|S_{11}|^2 = \left|\frac{b_1(l_1)}{a_1(l_1)}\right|^2_{a_2(l_2)=0} = \frac{P_{avs} - P_1}{P_{avs}} \tag{26}$$

i.e. it is $=$ ratio of power reflected from port 1 to power available at port 1.



Scattering Parameters

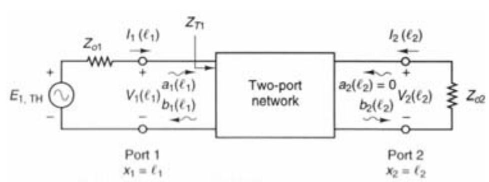
Completely analogous situation for S_{22}, just change indices (see fig. 1.6.4)

What about S_{21} (and S_{12})? Take the definition:

$$S_{21} = \left.\frac{b_2(l_2)}{a_1(l_1)}\right|_{a_2(l_2)=0} = \left.\frac{\sqrt{Z_{02}}I_2^-(l_2)}{\sqrt{Z_{01}}I_1^+(l_1}\right|_{I_2^+(l_2)=0} \tag{27}$$

The total current at l_2 consists only of I_2^- (matching condition ensures that), i.e. $I_2(l_2) = -I_2^-(l_2)$.

$$S_{21} = \left.\frac{-\sqrt{Z_{02}}I_2(l_2)}{\sqrt{Z_{01}}I_1^+(l_1)}\right|_{I_2^+(l_2)=0} \tag{28}$$



Scattering Parameters

Furthermore, $I_1^+(l_1) = E_{1,TH}/(2Z_{01})$

(why 2? $a_1(0) = E_1/(2\sqrt{Z_{01}})$
$\Rightarrow a_1(l_1) = E_1/(2\sqrt{Z_{01}})\exp(-j\Theta_1)$.

$Z_{TH} = Z_{01}$ (matched line), $E_{TH} = |E_1|\angle - \Theta_1$. $\Rightarrow a_1(l_1) = \sqrt{Z_{01}}I_1^+(l_1) = E_{TH}/(2\sqrt{Z_{01}}) \Rightarrow I_1^+(l_1) = E_{TH}/(2Z_{01})$)

Also: $V_2(l_2) = -Z_{02}I_2(l_2)$. $\Rightarrow$

$$S_{21} = \frac{\sqrt{Z_{02}}V_2(l_2)/Z_{02}}{\sqrt{Z_{01}}E_{1,TH}/(2Z_{01})} = 2\sqrt{\frac{Z_{01}}{Z_{02}}}\frac{V_2(l_2)}{E_{1,TH}} \tag{29}$$

$\Rightarrow S_{21}$ is **forward voltage transmission coefficient from port 1 to port 2**.

Scattering Parameters

Also:

$$|S_{21}|^2 = \frac{\frac{1}{2}|V_2(l_2)|^2/Z_{02}}{|E_{1,TH}|^2/(8Z_{01})} = \frac{\text{P delivered to load}}{P_{avs}} \tag{30}$$

i.e. $|S_{21}|^2$ is **transducer power gain** (note the loading conditions!).

S_{12} analogous — change the indices.

Scattering Parameters

$S_{11} = b_1(l_1)/a_1(l_1)$ $(a_2(l_2) = 0)$ is input reflection coeff. with output match terminated (tr. line matching condition)

$|S_{11}|^2 = (P_{avs} - P_1)/P_{avs}$ = ratio of power reflected from port 1 to power available from source

$S_{21} = 2\sqrt{Z_{01}/Z_{02}}V_2(l_2)/E_{1,TH}$ is forward transmission coeff. from port1 to port2 with output matched

$|S_{21}|^2 = P_L/P_{avs}$ is ratio of power delivered to load Z_{02} to available power = transducer power gain

For S_{22} and S_{12} just interchange ports (terminals).

Properties of S matrix- Reciprocal

- if transmission from port 1 to port 2 is the same as transmission in the reverse direction, the device is reciprocal.
- Most circuits are, except for active devices, and devices that use gyrotropic (and other complex anisotropic) materials.
- Typical non-reciprocal devices are amplifiers, and devices with magnetized ferrites such as circulators and isolators.
- Phenomena in magnetized plasma (as in ionosphere) are also non-reciprocal
- if the device is reciprocal, than S21=S12, or in general

$$[S] = [S]^T$$

Properties of S matrix- Symmetry

if there is symmetry in the device, then the ports can be easily interchanged than

$$S_{11}=S_{22}=S_{ii}$$
$$S_{21}=S_{12},\ S_{nm}=S_{mn}$$

$$[S] = \begin{bmatrix} S_{11} & S_{21} \\ S_{21} & S_{11} \end{bmatrix}$$

$$[S] = \begin{bmatrix} S_{11} & S_{21} & \cdots & \cdots & S_{n1} \\ S_{21} & \ddots & & S_{n-1,2} & S_{n2} \\ \vdots & & \ddots & & \\ \vdots & S_{n-1,2} & & \ddots & \\ S_{n1} & S_{n2} & \cdots & \cdots & S_{11} \end{bmatrix}$$

Properties of S matrix- Losslessness

- If there is no losses in the device then the incident and reflected powers are equal.

$$[a]^T [a]^* = [b]^T [b]^* \qquad [b] = [S][a]$$

$$[a]^T [a]^* = [[S][a]]^T [S]^* [a]^*$$

$$[a]^T [a]^* = [a]^T [S]^T [S]^* [a]^*$$

$$\Downarrow$$

$$[S]^T [S]^* = [U]$$

Properties of S matrix- Losslessness

$$[U] = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

$$[S]^T [S]^* = [U] \qquad [S]^{T*} [S] = [U]$$

$$[S]^* = \left\{[S]^T\right\}^{-1} \qquad [S]^{-1} = [S]^{T*}$$

Properties of S matrix- Losslessness

Losslessness   no losses in the device

$$[S]^T[S]^* = [U]$$

$$\begin{bmatrix} S_{11} & S_{21} & \cdots & \cdots & S_{n1} \\ S_{12} & \ddots & & S_{n-1,2} & S_{n2} \\ \vdots & & \ddots & & \\ \vdots & S_{2,n-1} & & \ddots & \\ S_{1n} & S_{2n} & \cdots & \cdots & S_{nn} \end{bmatrix} \cdot \begin{bmatrix} S_{11}^* & S_{12}^* & \cdots & \cdots & S_{1n}^* \\ S_{21}^* & \ddots & & S_{2,n-1}^* & S_{2n}^* \\ \vdots & & \ddots & & \\ \vdots & S_{n-1,2}^* & & \ddots & \\ S_{n1}^* & S_{n2}^* & \cdots & \cdots & S_{nn}^* \end{bmatrix} = \begin{bmatrix} 1 & & \\ & \ddots & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$$

Properties of S matrix- Losslessness

$$\begin{bmatrix} S_{11} & S_{21} & \cdots & \cdots & S_{n1} \\ S_{12} & \ddots & & S_{n-1,2} & S_{n2} \\ \vdots & & \ddots & & \\ \vdots & S_{2,n-1} & & \ddots & \\ S_{1n} & S_{2n} & \cdots & \cdots & S_{nn} \end{bmatrix} \cdot \begin{bmatrix} S_{11}^* & S_{12}^* & \cdots & \cdots & S_{1n}^* \\ S_{21}^* & \ddots & & S_{2,n-1}^* & S_{2n}^* \\ \vdots & & \ddots & & \\ \vdots & S_{n-1,2}^* & & \ddots & \\ S_{n1}^* & S_{n2}^* & \cdots & \cdots & S_{nn}^* \end{bmatrix} = \begin{bmatrix} 1 & & \\ & \ddots & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$$

$$\sum_{k=1}^{N} S_{ki} S_{ki}^* = 1$$

Properties of S matrix- Losslessness

$$
\begin{bmatrix}
S_{11} & S_{21} & \cdots & \cdots & S_{n1} \\
S_{12} & \ddots & & S_{n-1,2} & S_{n2} \\
\vdots & & \ddots & & \\
\vdots & S_{2,n-1} & & \ddots & \\
S_{1n} & S_{2n} & \cdots & \cdots & S_{nn}
\end{bmatrix}
\cdot
\begin{bmatrix}
S_{11}^{*} & S_{12}^{*} & \cdots & \cdots & S_{1n}^{*} \\
S_{21}^{*} & \ddots & & S_{2,n-1}^{*} & S_{2n}^{*} \\
\vdots & & \ddots & & \\
\vdots & S_{n-1,2}^{*} & & \ddots & \\
S_{n1}^{*} & S_{n2}^{*} & \cdots & \cdots & S_{nn}^{*}
\end{bmatrix}
=
\begin{bmatrix}
1 & 0 & & \\
& \ddots & & \\
& & \ddots & \\
& & & 1
\end{bmatrix}
$$

$$\sum_{k=1}^{N} S_{ki} S_{kj}^{*} = 0 \quad i \neq j$$

For reciprocal devices, same can be shown for rows

Parameters Conversion Chart

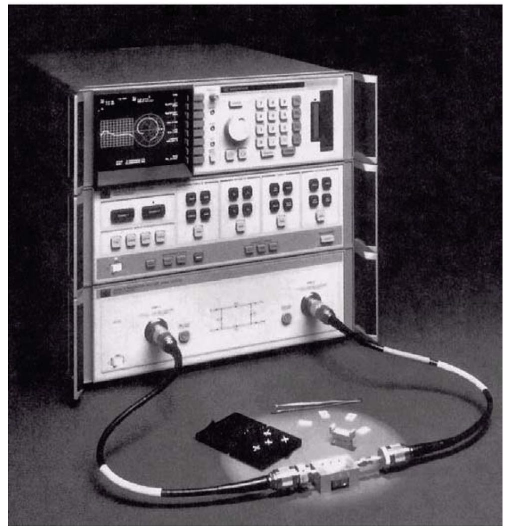
	S	Z	Y	$ABCD$
S_{11}	S_{11}	$\frac{(Z_{11}-Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0-Y_{11})(Y_0+Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D}$
S_{12}	S_{12}	$\frac{2Z_{12}Z_0}{\Delta Z}$	$\frac{-2Y_{12}Y_0}{\Delta Y}$	$\frac{2(AD-BC)}{A+B/Z_0+CZ_0+D}$
S_{21}	S_{21}	$\frac{2Z_{21}Z_0}{\Delta Z}$	$\frac{-2Y_{21}Y_0}{\Delta Y}$	$\frac{2}{A+B/Z_0+CZ_0+D}$
S_{22}	S_{22}	$\frac{(Z_{11}+Z_0)(Z_{22}-Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0+Y_{11})(Y_0-Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D}$
Z_{11}	$Z_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{11}	$\frac{Y_{22}}{\|Y\|}$	$\frac{A}{C}$
Z_{12}	$Z_0\frac{2S_{12}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{12}	$\frac{-Y_{12}}{\|Y\|}$	$\frac{AD-BC}{C}$
Z_{21}	$Z_0\frac{2S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{21}	$\frac{-Y_{21}}{\|Y\|}$	$\frac{1}{C}$
Z_{22}	$Z_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{22}	$\frac{Y_{11}}{\|Y\|}$	$\frac{D}{C}$
Y_{11}	$Y_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{22}}{\|Z\|}$	Y_{11}	$\frac{D}{B}$
Y_{12}	$Y_0\frac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{12}}{\|Z\|}$	Y_{12}	$\frac{BC-AD}{B}$
Y_{21}	$Y_0\frac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{21}}{\|Z\|}$	Y_{21}	$\frac{-1}{B}$
Y_{22}	$Y_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{11}}{\|Z\|}$	Y_{22}	$\frac{A}{B}$
A	$\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{11}}{Z_{21}}$	$\frac{-Y_{22}}{Y_{21}}$	A
B	$Z_0\frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{\|Z\|}{Z_{21}}$	$\frac{-1}{Y_{21}}$	B
C	$\frac{1}{Z_0}\frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{1}{Z_{21}}$	$\frac{-\|Y\|}{Y_{21}}$	C
D	$\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{22}}{Z_{21}}$	$\frac{-Y_{11}}{Y_{21}}$	D

$|Z| = Z_{11}Z_{22} - Z_{12}Z_{21}$; $|Y| = Y_{11}Y_{22} - Y_{12}Y_{21}$; $\Delta Y = (Y_{11}+Y_0)(Y_{22}+Y_0) - Y_{12}Y_{21}$; $\Delta Z = (Z_{11}+Z_0)(Z_{22}+Z_0) - Z_{12}Z_{21}$; $Y_0 = 1/Z_0$

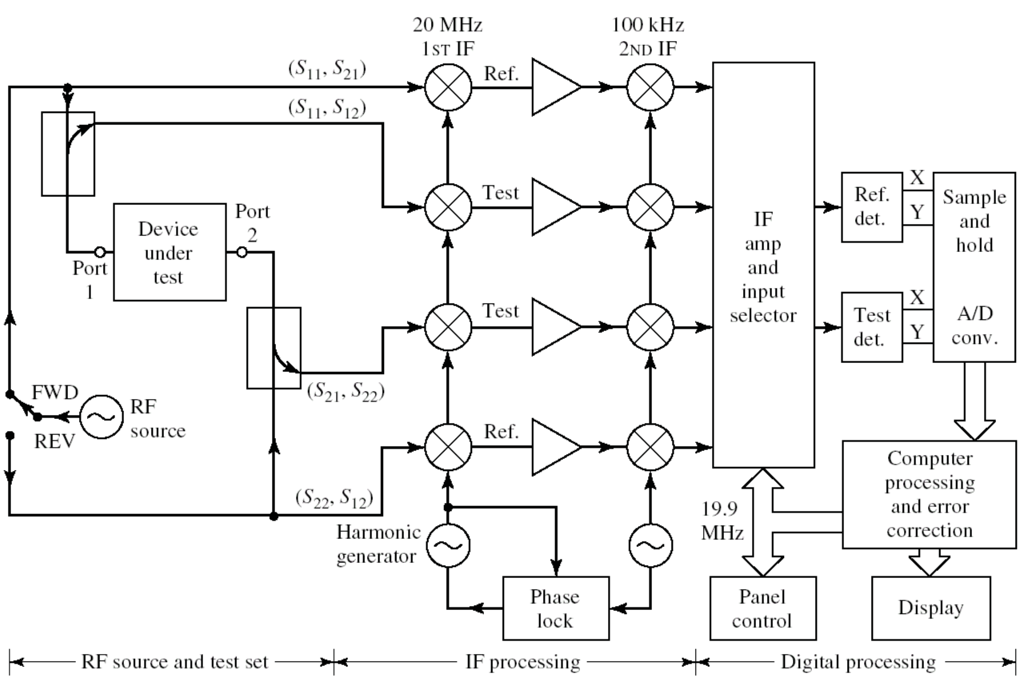
1

S Parameters Measurement

- A photograph of the Hewlett-Packard HP8510B Network Analyzer.
- This test instrument is used to measure the scattering parameters (magnitude and phase) of a one- or two-port microwave network from 0.05 GHz to 26.5 GHz.
- Built-in microprocessors provide
  - error correction
  - a high degree of accuracy
  - and a wide choice of display formats.
- This analyzer can also perform a fast Fourier transform of the frequency domain data to provide a time domain response of the network under test.

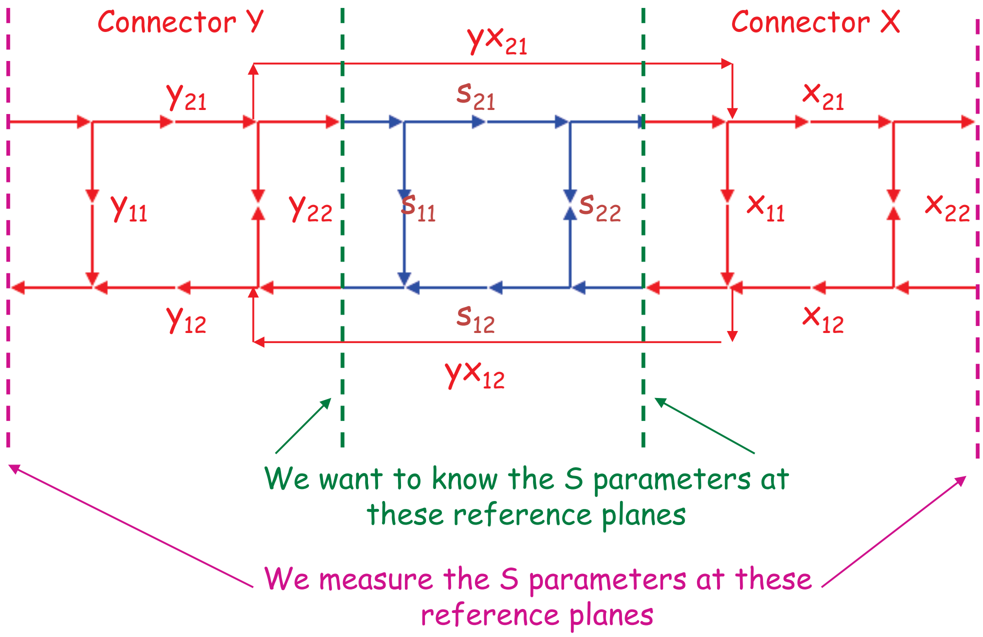


S Parameters Measurement



Network Analyzer Calibration

- To measure the pure S parameters of a device, we need to eliminate the effects of cables, connectors, etc. attaching the device to the network analyzer



Network Analyzer Calibration

- There are 10 unknowns in the connectors
- We need 10 independent measurements to eliminate these unknowns
- Develop calibration standards
- Place the standards in place of the Device Under Test (DUT) and measure the S- parameters of the standards and the connectors
- Because the S parameters of the calibration standards are known (theoretically), the S parameters of the connectors can be determined and can be mathematically eliminated once the DUT is placed back in the measuring fixtures.

Network Analyzer Calibration

- Since we measure four S parameters for each calibration standard, we need at least three independent standards.
- Different set of standards exist
  - SOL: Short, Open, Load
  - TRL: Thru, Reflect, Load
  - TRM: Thru, Reflect, Match

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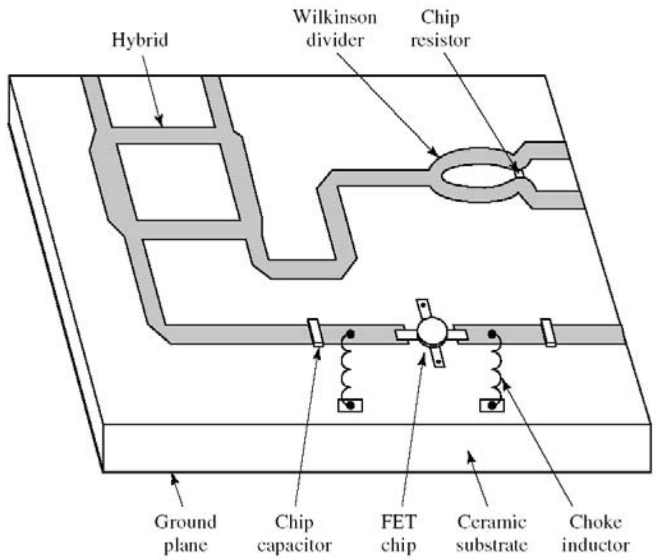
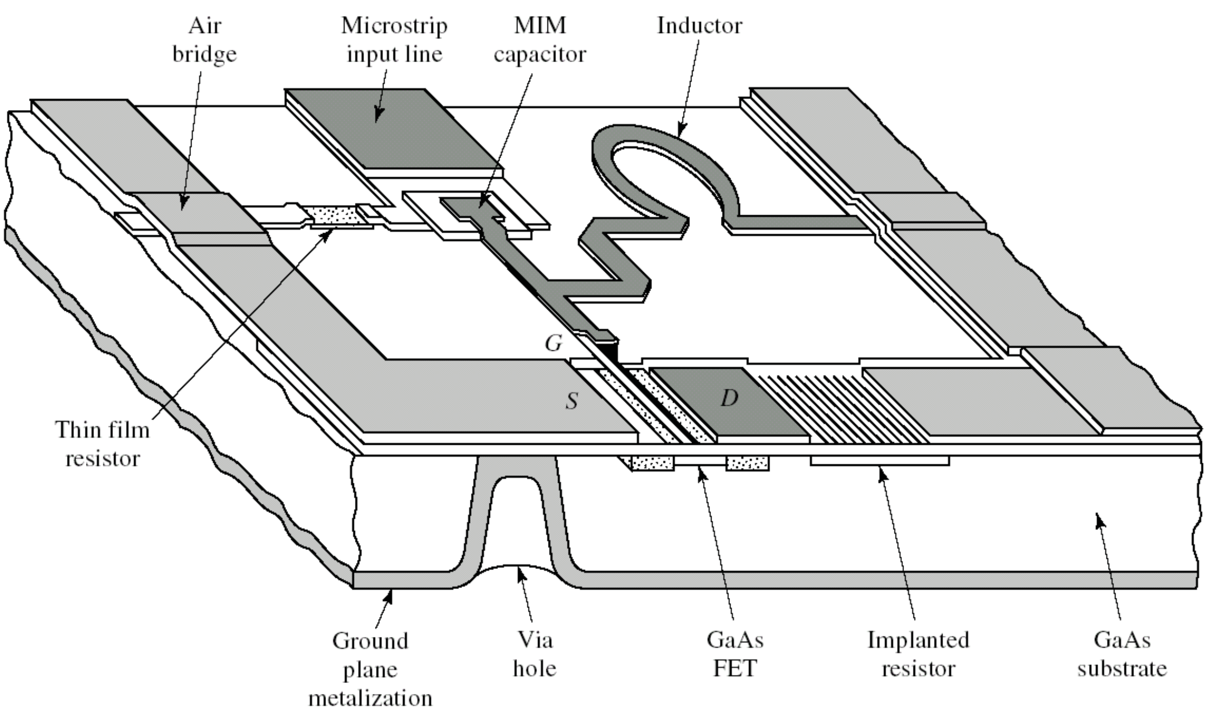


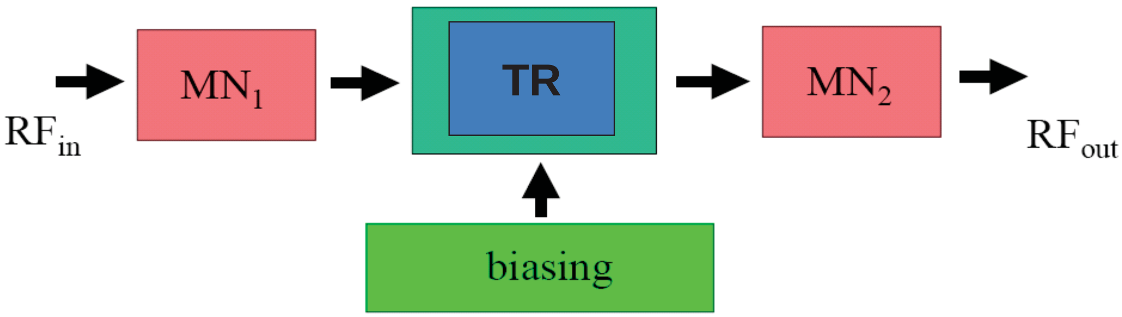
Layout of a hybrid microwave integrated circuit



Layout of a monolithic microwave integrated circuit

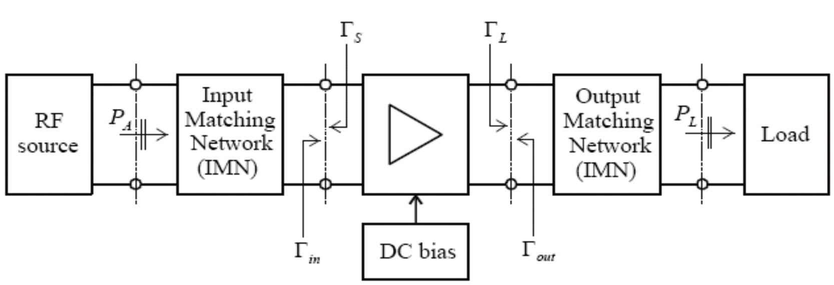
RF Amplifier

- Objective: Design a complete single-stage RF amplifier operated at f GHz which includes biasing, matching networks, and RF/DC isolation



Strategy

- Design DC biasing conditions
- Select S-parameters for given bias and operating frequency
- Build input and output matching networks for desired frequency response. For power considerations, matching networks are assumed lossless
- Include RF/DC isolation
- Simulate amplifier performance on the computer aided sign software (ADS)

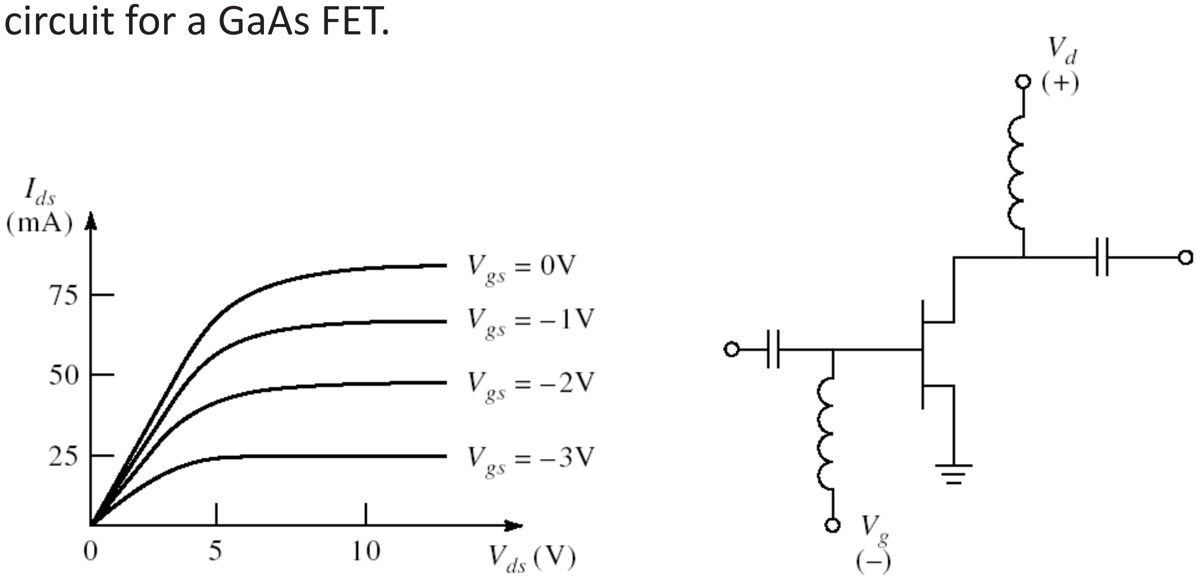


Biasing networks -Reading-

- Biasing networks are needed to set appropriate operating conditions for active devices
- There are two types:
  - Passive biasing (or self-biasing)
    - resistive networks
    - drawback: poor temperature stability
  - Active biasing
    - additional active components (thermally coupled)
    - drawback: complexity, added power consumption

Biasing networks -Reading-

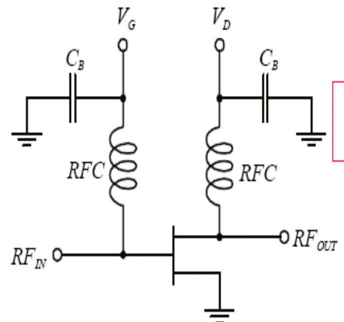
(*a*) DC characteristics of a GaAs FET; (*b*) biasing and decoupling circuit for a GaAs FET.



(a) (b)

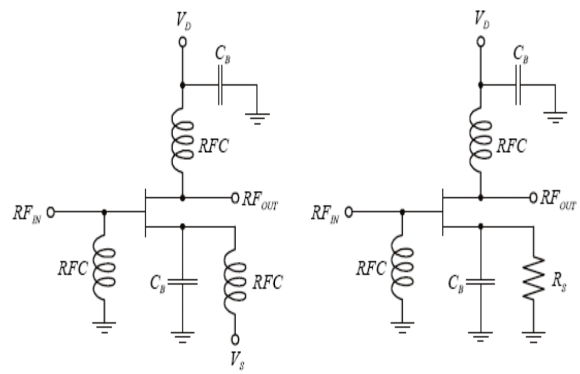
FET biasing -Reading-

Bi-polar power supply



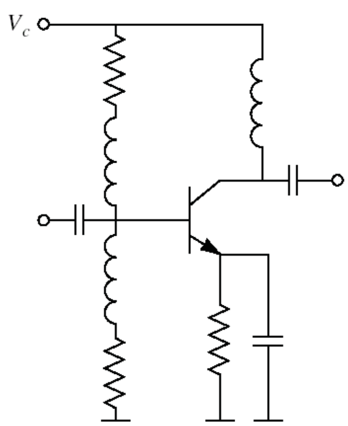
$V_G<0$ and $V_D>0$

Uni-polar power supply

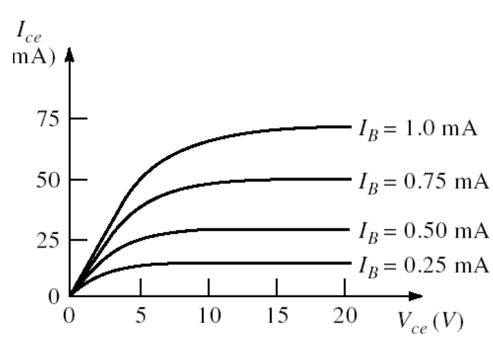


Biasing networks -Reading-

(*a*) DC characteristics of a silicon bipolar transistor; (*b*) biasing and decoupling circuit for a bipolar transistor.



(b)

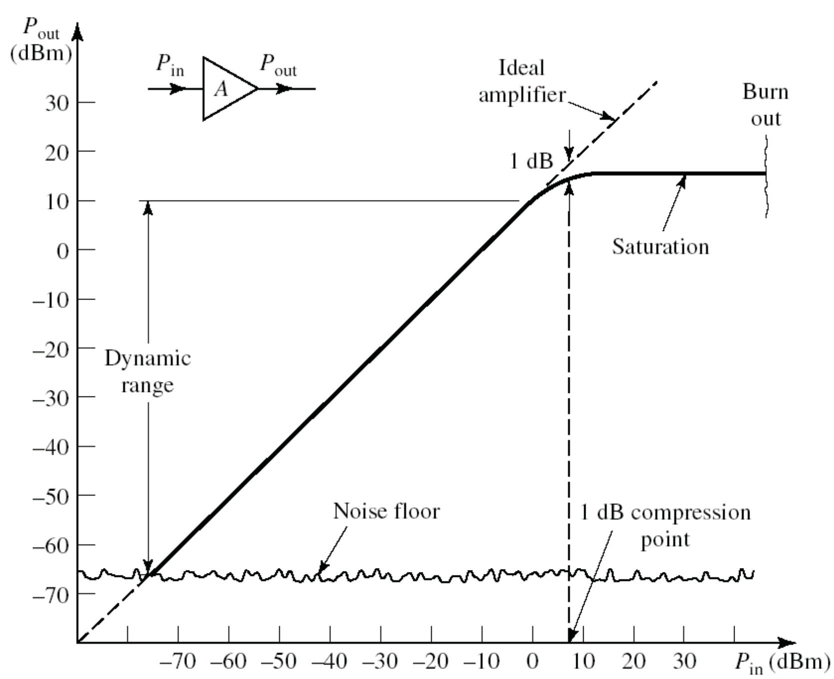


(a)

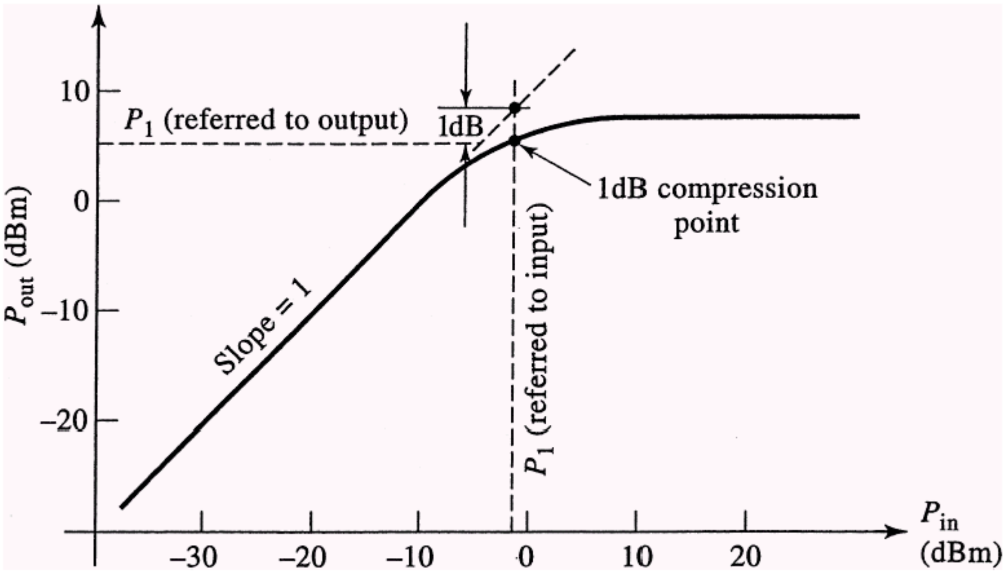
RF Transistors -Reading-

- BJT: low noise, linear power amplification, power applications
- GaAs FET: very low noise, low power
- HEMT (High electron mobility transistor): very high frequency (f > 20 GHz)
- Major issue when dealing with RF transistors:
  - Noise
    - shot noise in emitter-base
    - shot noise in collector system
    - thermal noise in base resistance
  - How to reduce noise
    - minimize current flow across pn-junction
    - minimize resistance
  - Solution
    - Finger structure of base, emitter configuration

Linearity Specification -Reading-



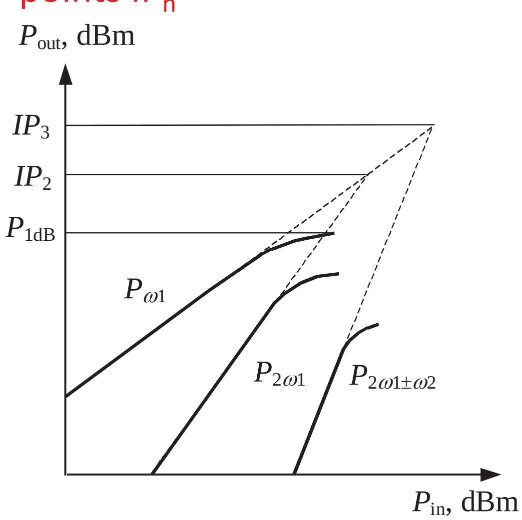
Linearity Specification



Definition of the 1 dB compression point for a nonlinear amplifier.

Linearity Specification -Reading-

- Output powers of linear, second- or third-order component show straight-line behavior and vary by 1 dB, 2 dB and 3 dB, respectively, with 1-dB variation of input power
- These straight lines intersect at some points which are called intercept points IP_n



$$\boxed{P_{IM_n} = nP_{\omega_1} - (n-1)\, IP_n \text{ (dBm)}}$$

Second harmonic component

$$P_{2\omega_1} = 2P_{\omega_1} - IP_2 \text{ (dBm)}$$

Third-order intermodulation component

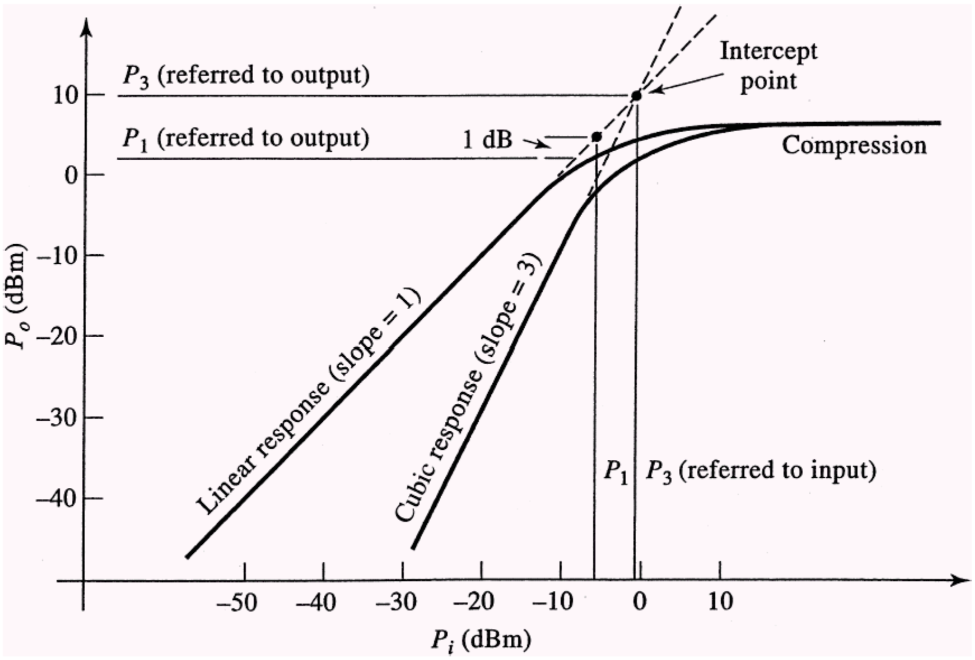
$$P_{2\omega_1-\omega_2} = 3P_{\omega_1} - 2IP_3 \text{ (dBm)}$$

P 1-dB gain compression point

$$P_{1dB} = IP_3 - 10 \text{ (dBm)}$$

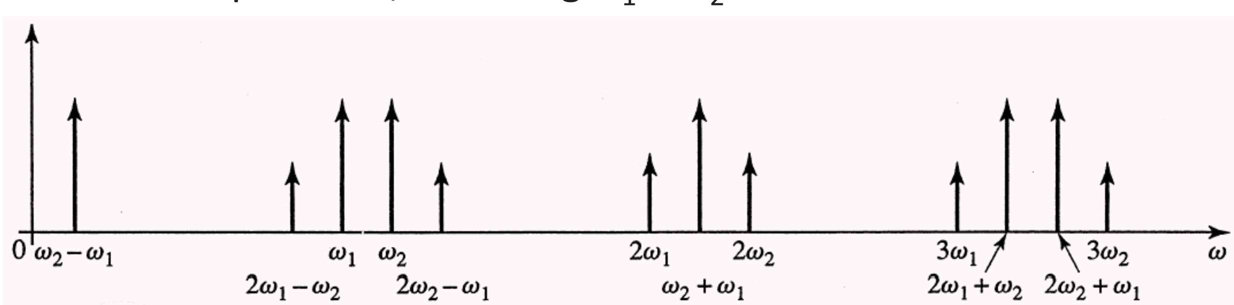
Linearity Specification

Third-order intercept diagram for a nonlinear component.



Linearity Specification -Reading-

Output spectrum of second- and third-order two-tone inter-modulation products, assuming $\omega_1 < \omega_2$.



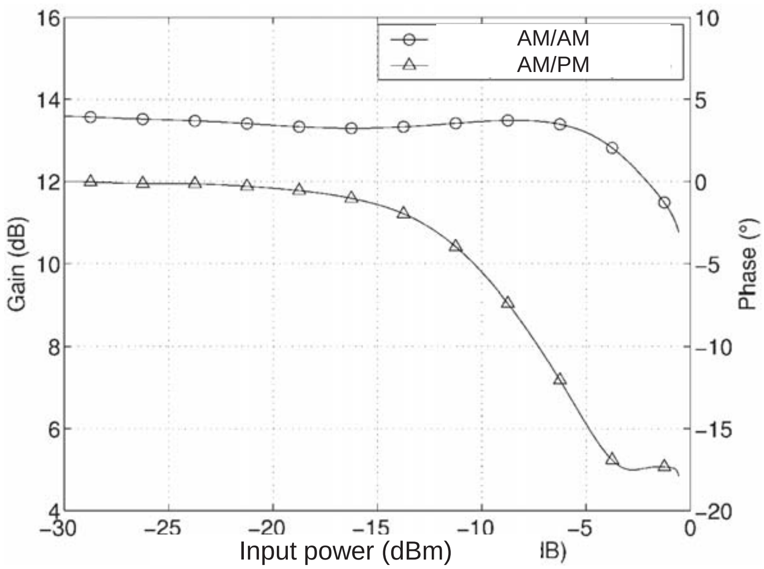
$$IM_3 = 10\log_{10}\left(P_{2\omega_1-\omega_2}/P\right) = P_{2\omega_1-\omega_2} - P \ (\text{dBc})$$ - third-order intermodulation product

$$IM_5 = 10\log_{10}\left(P_{3\omega_1-2\omega_2}/P\right) = P_{3\omega_1-2\omega_2} - P \ (\text{dBc})$$ - fifth-order intermodulation product

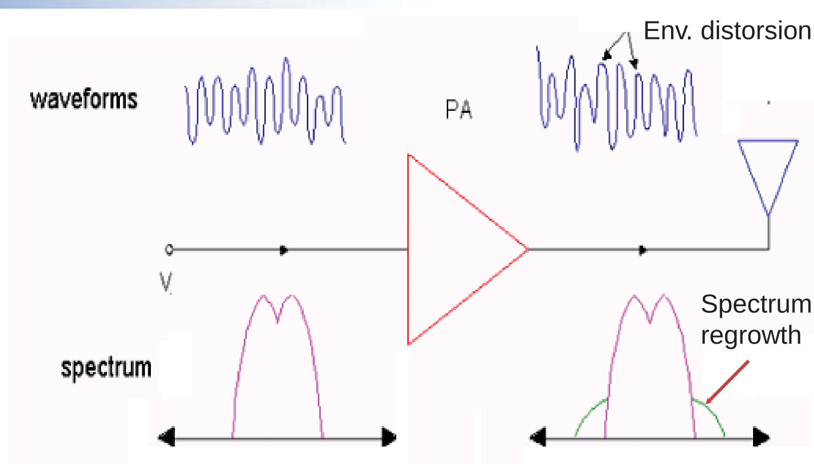
where $P = P_{\omega_1} = P_{\omega_2}$

Linearity Specification -Reading-

- AM/AM and AM/PM Characteristics



Linearity Specification -Reading-



Amplifier non-linearities introduce:

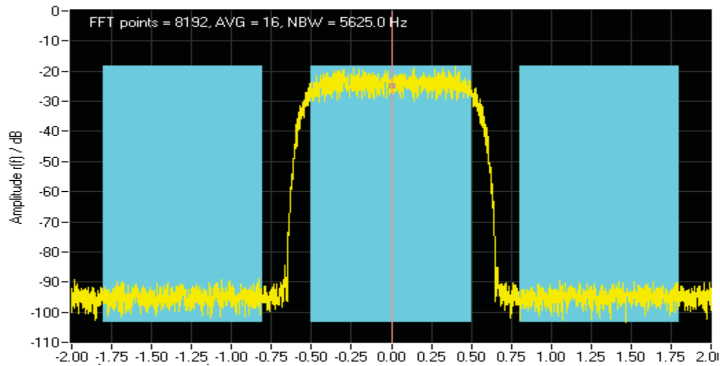
- Spectrum regrowth (out of band distortion) which deteriorates the APCR $\Rightarrow$ the PA fails to pass the mask specifications.
- Information distortion (in band distortion) $\Rightarrow$ bad EVM.

Linearity Specification -Reading-

•Adjacent Channel Power Ration (ACPR)

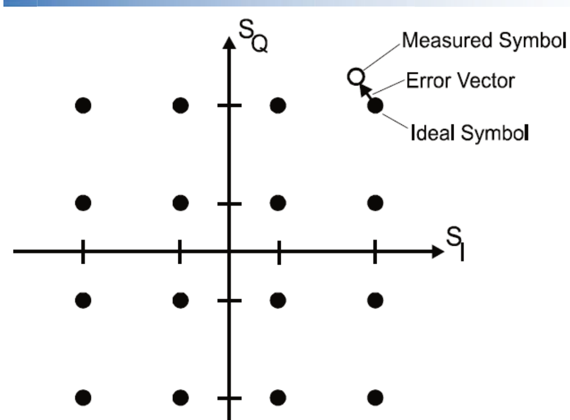
the ratio between the out-of-band power spectral density at the specified offset channels and the in-band spectral density.

$$ACPR(dB) = P_{offset}(dB) - P_{mean}(dB)$$

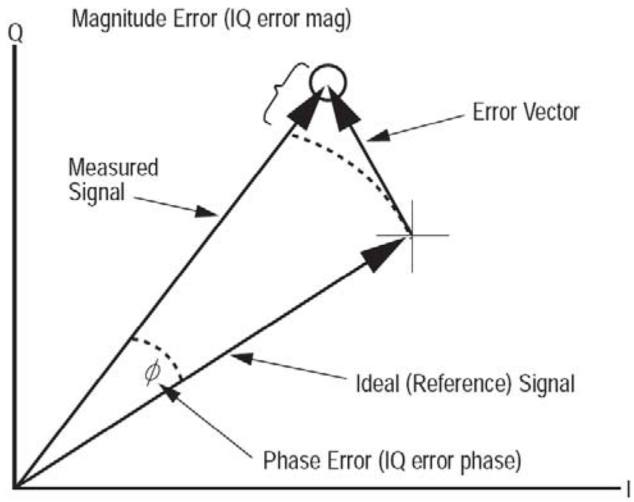


W-CDMA

Linearity Specification -Reading-



$$EVM = \left[\frac{\frac{1}{N}\sum_{r=1}^{N} | S_{\text{ideal},r} - S_{\text{meas},r} |^2}{\frac{1}{N}\sum_{r=1}^{N} | S_{\text{ideal},r} |^2} \right]^{\frac{1}{2}}$$



EVM reveals the vector difference between the measured and ideal signals.

Error Vector Magnitude (EVM)

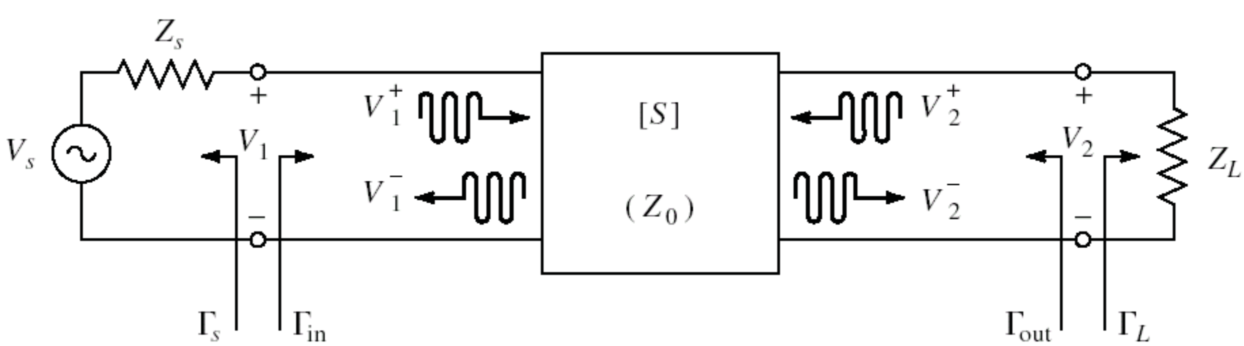
** N is the number of unique symbols in the constellation.*

Design of Linear Amplifier

- Consider: Stability, power gain, bandwidth, noise, dc bias.
- Start with a set of specifications and select proper transistor.
- Determine transistor loading (source and load reflection coeff.) for particular stability and gain criteria.

Stability

Stability = resistance to "spontaneous" oscillations. Possible if either input or output port has negative resistance, i.e. $|\Gamma_{IN}| > 1$ or $|\Gamma_{OUT}| > 1$. For unilateral device that means $|S_{11}| > 1$ and $|S_{22}| > 1$.



A two-port network with general source and load impedances

Stability

•**Unconditionally stable** at freq f if real parts of Z_{IN} and Z_{OUT} are greater than zero for all passive load and source impedances.

•If a 2-port network is **not unconditionally stable** it is **potentially unstable**, i.e. some passive load and source terminations result in input and output impedances with Re(Z) < 0.

Stability

Conditions for unconditional stability:

$$|\Gamma_S| < 1, \quad |\Gamma_L| < 1 \quad (9)$$

$$|\Gamma_{IN}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1 \quad (10)$$

$$|\Gamma_{OUT}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{22}\Gamma_S} \right| < 1 \quad (11)$$

From this determine where $|\Gamma_{IN}| = |\Gamma_{OUT}| = 1$

$$\left| \Gamma_L - \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \right| = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \quad (12)$$

Stability

$$\left|\Gamma_S - \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2}\right| = \left|\frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2}\right| \quad (13)$$

With $\Delta = S_{11}S_{22} - S_{12}S_{21}$ is S-matrix determinant.

Geometrical interpretation: in polar coordinates these are two circles displaced by some amount from origin.

- For $|\Gamma_{IN}| = 1$, Γ_L values satisfying the conditions for unconditional stability will be on one side of the circle which has:

center:

$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}$$

radius:

$$r_L = \left|\frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2}\right|$$

This is **output stability circle**

Stability

Figs. 3.3.2 and 3.3.3. Which side of the circle is unconditionally stable (and which one is not)?

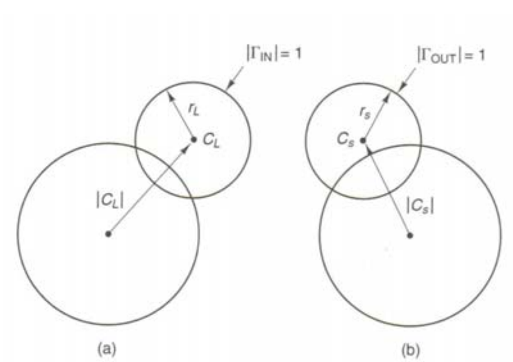


Fig 3.3.2

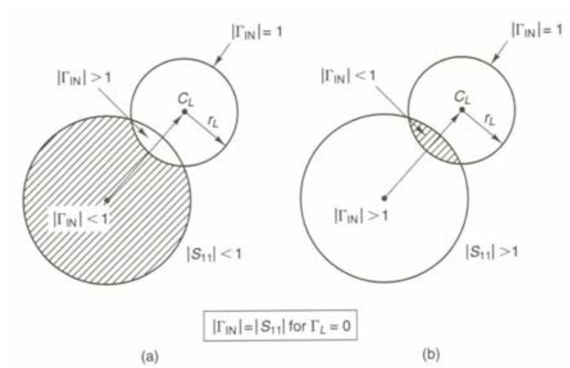


Fig 3.3.3

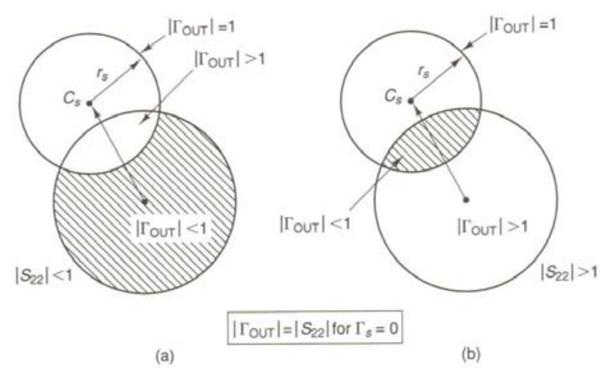
Stability

Observe: for $\Gamma_L = 0$, $Z_L = Z_0 \rightarrow |\Gamma_{IN}| = |S_{11}|$. For $|S_{11}| < 1 \rightarrow |\Gamma_{IN}| < 1$ when $Z_L = Z_0$ (origin).

→ Based on value of S_{11} , origin is either in stable or not. For $|S_{11}| < 1$ it is part of the stable region. Conversely, if $|S_{11}| > 1$ then origin is part of the potentially unstable region. (fig. 3.3.3).

Note that $|\Gamma_L| < 1$ is required in the above.

Similar analysis for **input stability circle** (fig. 3.3.4).



$$r_S = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2} \right|$$

$$C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2}$$

Stability

•For unconditional stability circles do not cross S-chart → for no value of Z_L or Z_S will the circuit oscillate. For $|S_{11}| < 1$ we have $||C_L - r_L| > 1$ and for $|S_{22}| < 1$ we have $||C_S| - r_S| > 1$.

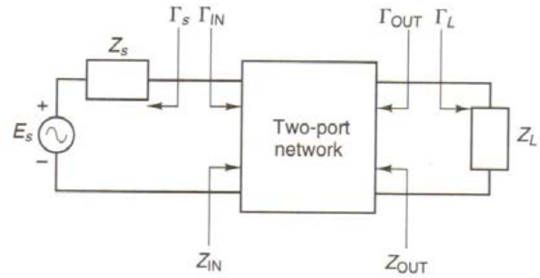
•Note that $|S_{11}| > 1$ and $|S_{22}| > 1$ are automatically excluded from unconditional stability since $\Gamma_S = 0$ produces $|\Gamma_{OUT}| > 1$ and $\Gamma_L = 0$ produces $|\Gamma_{IN}| > 1$

•Define stability factor K

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1 \quad (14)$$

$$1 - |S_{11}|^2 > |S_{12}S_{21}| \quad 1 - |S_{22}|^2 > |S_{12}S_{21}| \quad (15)$$

Stability



Written differently:

$$K > 1 \quad and \quad B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 > 0 \quad (16)$$

Or even simpler

$$K > 1 \quad and \quad |\Delta| < 1 \quad (\Delta = S_{11}S_{22} - S_{12}S_{21}) \quad (17)$$

Unilateral amplifier: K → ∞ and for unconditionally stability we only need $|S_{11}| < 1$ and $|S_{22}| < 1$.

It is always possible to make the circuit stable if the total input and output loop resistances are positive

$$\mathrm{Re}(Z_S + Z_{IN}) > 0 \quad \mathrm{Re}(Z_L + Z_{OUT}) > 0 \quad (18)$$

Stability

Accomplished by:

- Resistively loading the transistor
  - By adding resistances we change Γ_S and Γ_L, but loose power too and affect the noise performances. Added resistance can be absorbed into transistor S-matrix (how?)
- Adding negative feedback
  - Negative feedback could be used to make $S_{12} = 0$ to stabilize transistor; not commonly done.
- Both not recommended for narrowband design but popular for broadband amplifiers

Stability

Example: Determine the stability and show how resistive loading can stabilize the transistor

$$[S]_{800MHz} = \begin{bmatrix} 0.65/-95 & 0.035/40 \\ 5/115 & 0.8/-35 \end{bmatrix}$$

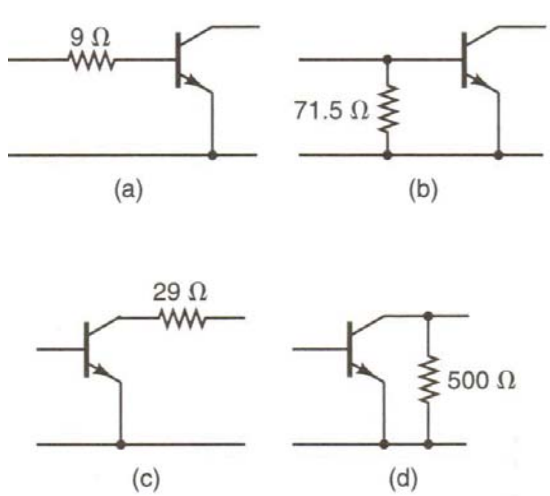
$K = 0.547; \Delta = 0.504\angle 249.6 \Rightarrow$ Potentially unstable

input and output stability circles

$$\begin{cases} Cs = 1.79\angle 122 \\ rs = 1.04 \end{cases}$$

$$\begin{cases} Cl = 1.3\angle 48 \\ rl = 0.45 \end{cases}$$

Stability



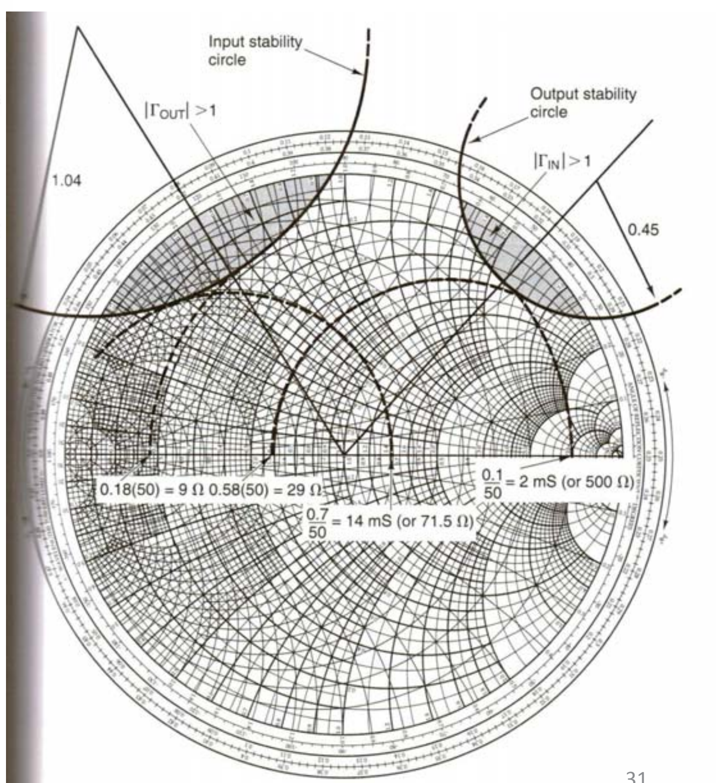
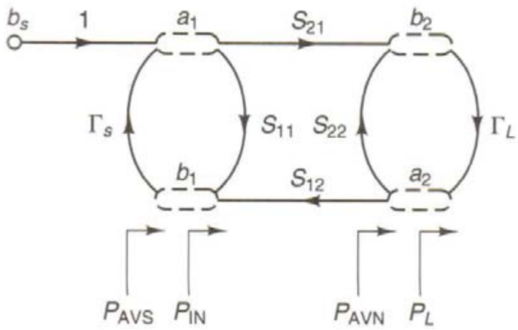


Figure 3.3.7 Input and output stability circles.

Power gain equations

Power gain definitions:



$$P_{IN} = \frac{1}{2}|a_1|^2 - \frac{1}{2}|b_1|^2$$

$$P_L = \frac{1}{2}|b_2|^2 - \frac{1}{2}|a_2|^2$$

$$P_{AVS} = P_{IN}\big|_{\Gamma_{IN} = \Gamma_s^*}$$

$$P_{AVN} = P_L\big|_{\Gamma_L = \Gamma_{OUT}^*}$$

$$G_T = \frac{P_L}{P_{avs}} = \frac{\text{power delivered to load}}{\text{power available to source}} = \text{transducer} \quad (1)$$

$$G_P = \frac{P_L}{P_{in}} = \frac{\text{power delivered to load}}{\text{power input to network}} = \text{operating} \quad (2)$$

$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{\text{power available from network}}{\text{power available from source}} = \text{available} \quad (3)$$

Power gain equations

$$G_T = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{IN}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \quad (4)$$

$$G_T = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_{OUT}\Gamma_L|^2} \quad (5)$$

$$G_P = \frac{1}{1-|\Gamma_{IN}|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = f(\Gamma_L,[S]) \quad (6)$$

$$G_A = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}|S_{21}|^2\frac{1}{1-|\Gamma_{out}|^2} = f(\Gamma_s,[S]) \quad (7)$$

$$\Gamma_{IN} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} \qquad \Gamma_{OUT} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S} \quad (8)$$

Power gain equations

Unilateral network: $S_{12} = 0 \rightarrow \Gamma_{IN} = S_{11}$, $\Gamma_{OUT} = S_{22}$. In that case: Unilateral transducer power gain:

$$G_{TU} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \quad (19)$$

Think of it as a product of three components:

$G_{TU} = G_S \cdot G_0 \cdot G_L$. See fig. 3.2.2.

G_S affects the degree of mismatch between Γ_S and S_{11} .

It is passive, but can have gain > 1. Since there is an "intrinsic" mismatch loss between Z_s and S_{11} (i.e. between Γ_S and S_{11}) $\rightarrow$ decreasing that loss provides gain.

Same for G_L.

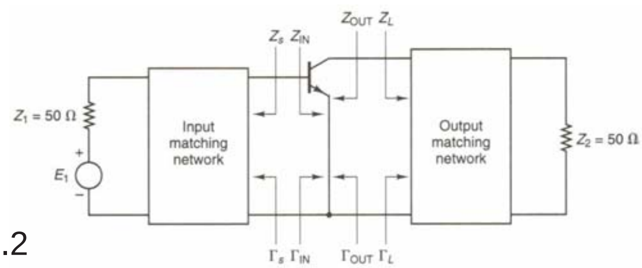


Fig 3.2.2

Power gain equations

In dB-s: $G_{TU}(dB) = G_S(dB) + G_0(dB) + G_L(dB)$.

Optimize (match) Γ_S and Γ_L for max. G_S and G_L

$\rightarrow \Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$ ($|S_{11}| < 1$, $|S_{22}| < 1$).

$$G_{S,\max} = \frac{1}{1-|S_{11}|^2} \qquad G_{L,\max} = \frac{1}{1-|S_{22}|^2} \quad (20)$$

$$G_{TU,\max} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2} \quad (21)$$

Constant gain circles: unilateral case

Constant gain circles: unilateral case

$$G_{TU} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \quad (22)$$

Or $G_{TU} = G_S \cdot G_0 \cdot G_L$. For general analysis, write G_S,G_L as

$$G_i = \frac{1-|\Gamma_i|^2}{|1-S_{ii}\Gamma_i|^2} \quad (23)$$

Two cases:

1. Unconditionally stable: $|S_{ii}| < 1$
2. Potentially unstable: $|S_{ii}| > 1$

Constant gain circles: unilateral case

Case 1. Max. value of $G_{i,max}$ → match Γ_i and S_{ii} , i.e. $\Gamma_i = S_{ii}^*$

$$G_{i,\max} = \frac{1}{1-|S_{ii}|^2} \quad (24)$$

Terminations that produce $G_{i,max}$ are called optimum terminations.

From definition of G_i , if $|\Gamma_i| = 1$ → $G_i = 0$. Other values of $|\Gamma_i|$ produce values $0 \le G_i \le G_{i,max}$

Analysis of G_i via Smith chart leads to constant gain circles.

Introduce normalized gain factor:

$$g_i = \frac{G_i}{G_{i,\max}} = G_i(1-|S_{ii}|^2), \quad and \quad 0 \le g_i \le 1 \quad (25)$$

Constant gain circles: unilateral case

Also define $\Gamma_i = U_i + jV_i$, and $S_{ii} = A_{ii} + jB_{ii}$ and plug it into eq. 25. Many manipulations later…

$$\left[U_i - \frac{g_i A_{ii}}{1-|S_{ii}|^2(1-g_i)}\right]^2 + \left[V_i - \frac{g_i B_{ii}}{1-|S_{ii}|^2(1-g_i)}\right] = \left[\frac{\sqrt{1-g_i}\,(1-|S_{ii}|^2)}{1-|S_{ii}|^2(1-g_i)}\right]^2$$

These are circles with g_i as parameter.

Centers are at:

$$U_c = \frac{g_i A_{ii}}{1-|S_{ii}|^2(1-g_i)}, \quad V_c = -\frac{g_i B_{ii}}{1-|S_{ii}|^2(1-g_i)} \qquad (26)$$

With radius

$$R_i = \frac{\sqrt{1-g_i}\,(1-|S_{ii}|^2)}{1-|S_{ii}|^2(1-g_i)} \qquad (27)$$

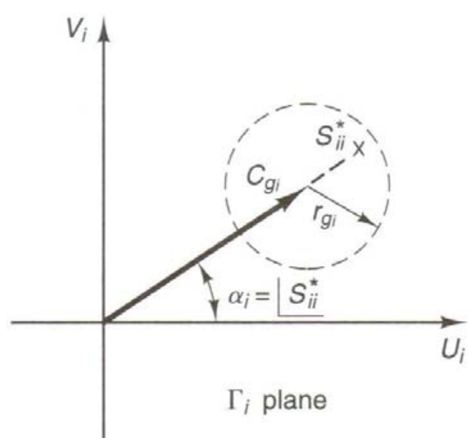
Constant gain circles: unilateral case

Writing differently, distance from origin

$$d_i = \sqrt{U_c^2 + V_c^2} = \frac{g_i |S_{ii}|}{1-|S_{ii}|^2(1-g_i)} \qquad (28)$$

And angle

$$\tan\alpha = \frac{V_c}{U_c} \Rightarrow \alpha_i = \tan^{-1}\frac{-B_{ii}}{A_{ii}} \qquad (29)$$



Const. gain circles located at distance d_i along the line drawn from origin to point S_{ii}^*(= A_{ii} - jB_{ii})

Constant gain circles: unilateral case

Procedure for const. gain circle (s) construction:

- Locate S_{11}^* and draw line from origin to it. At S_{ii}^* gain is $G_{i,max} = 1/(1 - |S_{ii}|^2)$, and the radius is $R_{i,max} = 0$ (point).
- Find values for $0 \leq G_i \leq G_{i,max}$ (using $G_i = (1- |\Gamma_i|^2)/|1-S_{ii}\Gamma_i|^2$), and calculate $g_i = G_i / G_{i,max}$
- Determine d_i, R_i for each g_i (eqs. 28 and 27)
- 0 dB circle ($G_i = 1$) always goes through origin, or for $G_i = 1 \rightarrow \Gamma_i = 0$ so that

$$g_{i,0db} = 1 - |S_{ii}|^2 \quad and \quad R_{i,0db} = d_{i,0db} = \frac{|S_{ii}|}{1+|S_{ii}|^2} \qquad (30)$$

Constant gain circles: unilateral case

Example $$[S]_{800MHz} = \begin{bmatrix} 0.73/175 & 0.0 \\ 4.45/65 & 0.21/-80 \end{bmatrix}$$

a) Optimum termination→ $\Gamma_s = S_{11}^* = 0.73\angle -175$ and $\Gamma_L = S_{22}^* = 0.21\angle 80$

b) Find $G_{s,max} = 1/(1 - |S_{11}|^2) = 2.141$ (= 3.31 dB)

Find $G_{L,max} = 1/(1 - |S_{22}|^2) = 1.046$ (= 0.19 dB)

Find $G_0 = |S_{21}|^2 = 19.8$ (= 12.97 dB); for total sum them up.

→ $G_{TU,max} = G_{s,max} + G_{L,max} + G_0$ =16.47dB

c) Draw the Gs gain circles

Construct table of g_s, d_s, R_s for various G_i-s (fig. 3.4.4)

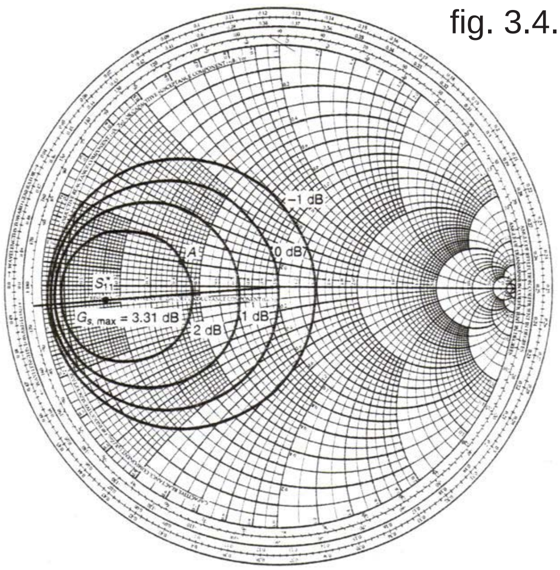
(why no matching for output?)

Construct matching network for given gain (fig. 3.4.5)

G_{TU}=0.19+12.97+2.0=15.16dB

Constant gain circles: unilateral case

fig. 3.4.4

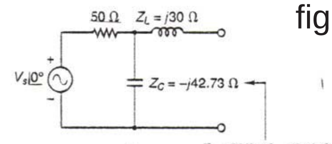


(a)

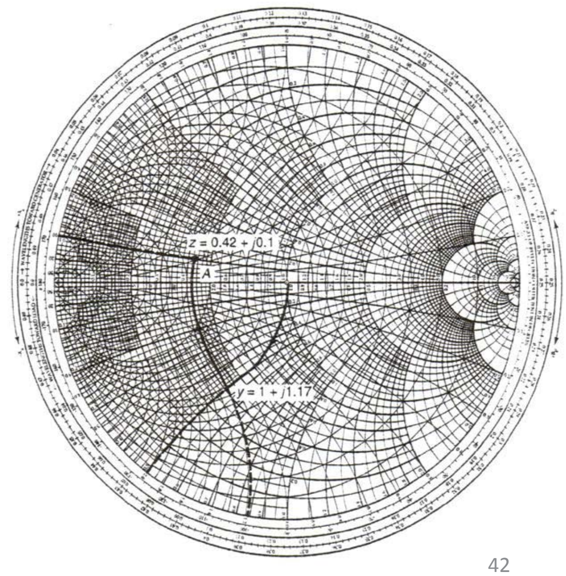
G_s (dB):	2	1	0	−1
G_s	1.59	1.26	1	0.79
g_s	0.743	0.588	0.467	0.369
$\|C_{g_s}\|$	0.629	0.55	0.476	0.406
r_{g_s}	0.274	0.384	0.476	0.559

(b)

fig. 3.4.5



(a)



(b)

Constant gain circles: unilateral case – Reading-

<u>Potentially unstable case (case 2)</u>

Unilateral network: $|S_{ii}| > 1$ (i.e. negative input resistance).

Based on (23) for $\Gamma_i = 1/S_{ii}$ then $G_i \to \infty$.

The critical value is $\Gamma_{i,c} = 1/S_{ii}$

Under "normal" circumstances ($|S_{ii}| < 1$) above condition cannot be satisfied by passive terminations

For $\Gamma_{i,c} = 1/S_{ii}$ real parts of impedances associated with $\Gamma_{i,c}$ and S_{ii} cancel → no resistance in the circuit!! Oscillations follow.

Remember that for negative resistances we plot the inverse, complex conjugate of the original value; here we plot $1/S_{ii}^*$ on S-chart and read resistance circles as negative and reactance as given.

Constant gain circles: unilateral case – Reading-

Normalized gain $g_i = G_i\,(1 - |S_{ii}|^2)$ can be negative! However, the same formulas for R_i and d_i hold (see eqs. 27, 28), except the centers are located along the origin - $1/S_{ii}$ line.

Need to determine stable region: use conditions:

$$\mathrm{Re}(Z_S) > |\mathrm{Re}(Z_{IN})| \quad and \quad \mathrm{Re}(Z_L) > |\mathrm{Re}(Z_{OUT})| \quad (31)$$

More reading, see book "Microwave Transistor amplifier analysis and design" page 234

Unilateral figure of merit

Unilateral figure of merit

When is it permissible to assume $S_{12} = 0$ (unilateral device)?

$$\frac{G_T}{G_{TU}} = \frac{1}{|1-X|^2},\; X = \frac{S_{12}S_{21}\Gamma_S\Gamma_L}{(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)} \quad (32)$$

Bounded by
$$\frac{1}{(1+|X|)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-|X|)^2} \quad (33)$$

G_{TU} max when conj. Matching provided: $\Gamma_s = S_{11}^*$ etc. giving max error introduced by $S_{12} = 0$.

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2} \quad (34)$$

Unilateral figure of merit
$$U = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)} \quad (35)$$

Simultaneous conjugate match

Simultaneous conjugate match: Bilateral case

What to do if $S_{12} \neq 0$ (or, U is not small)?

Most general expressions for Γ_{IN} and Γ_{OUT} still hold:

$$\Gamma_{IN} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} (= \Gamma_S^*) \quad (36)$$

$$\Gamma_{OUT} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} (= \Gamma_L^*) \quad (37)$$

For max. power transfer $\Gamma_{IN} = \Gamma_S^*$ and $\Gamma_{OUT} = \Gamma_L^*$ fig. to illustrate.

Solve two eqs. With two unknowns: Γ_L ,Γ_S. Call the solution Γ_{MS} and Γ_{ML} (for simultaneously matched cases).

Simultaneous conjugate match

Solution:

$$\Gamma_{MS} = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \qquad \Gamma_{ML} = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} \quad (38)$$

$$C_1 = S_{11} - \Delta S_{22}^* \qquad B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \quad (39)$$

$$C_2 = S_{22} - \Delta S_{11}^* \qquad B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \quad (40)$$

Which sign to use above?

First, $|\Gamma_{MS}|$, $|\Gamma_{ML}| < 1$ is necessary for unconditional stability.

Further analysis shows the necessary condition to be K > 1 where K is stability factor! From before we know the additional requirement: $|\Delta| < 1 \rightarrow B_1 > 0$ and $B_2 > 0$

Simultaneous conjugate match

All of the negative signs (-) should be used in Γ_{MS}, Γ_{ML} when calculating the simultaneous con. match for unconditionally stable two-port network.

Potentially unstable case: analysis done in terms of G_P, G_A

What is the max. G_T associated with Γ_{MS}, Γ_{ML} ?

$$G_{T,\max} = \frac{(1-|\Gamma_{MS}|^2)|S_{21}|^2(1-|\Gamma_{ML}|^2)}{|(1-S_{11}\Gamma_{MS})(1-S_{22}\Gamma_{ML})-S_{12}S_{21}\Gamma_{MS}\Gamma_{ML}|^2} \quad (41)$$

$$= \frac{1}{1-|\Gamma_{MS}|^2}|S_{21}|^2\frac{1-|\Gamma_{ML}|^2}{|1-S_{22}\Gamma_{ML}|^2}$$

For K<-1 or K<1 Simultaneous marching does not exist

Simultaneous conjugate match

Further manipulation:

$$G_{T,\max} = \frac{|S_{21}|}{|S_{12}|}(K-\sqrt{K^2-1}) \quad (42)$$

- Note: $G_{T,max}$ depends only on transistor parameters (S-matrix).
- Γ_{MS}, Γ_{ML} don't enter the picture because they are determined by conjugate matching requirement.
- If we had some control of [S], what would max. $G_{T,max}$ be?
- Since $K-\sqrt{(K^2-1)}$ falls monotonically to 0, max value is for K=1.
- The value of $G_{T,max}$ for K=1 is called maximum stable gain (MSG)
- $G_{MSG}=|S_{21}|/|S_{12}|$. Condition K=1 can be achieved by resistive loading or by feedback.

Simultaneous conjugate match

Example

Design a GaAs FET amplifier at 6 GHz with $G_{T,max}$.

$$[S]_{6GHz} = \begin{bmatrix} 0.641 / -171.3 & 0.057 / 16.3 \\ 2.058 / 28.5 & 0.572 / -95.7 \end{bmatrix}$$

- Calc. K and |Δ| for stability: K = 1.504, Δ = 0.301∟109° (is it unconditionally stable?).
- Can we assume the FET unilateral? Calculate U = 0.1085 →

$$-0.89dB \le \frac{G_T}{G_{TU}} < 1dB \Rightarrow S_{12} \ne 0 \quad (52)$$

Simultaneous conjugate match

- Calc. simultaneous conjugate match (need B_1,B_2 ,C_1,C_2)

B1=0.9928;B2=0.8255;C1=0.4786 ∟-177.3;C2=0.3911 ∟-103.9

→ Γ_{MS} = 0.76∟177°, Γ_{ML} = 0.71∟103°

- From K, S_{21} ,S_{12} → $G_{T,max}$ = 11.3 dB.
- Given Γ_{MS}, Γ_{ML} find matching networks.

From S-chart: Y_{MS} = 7.2 – j1.23 = 0.144 – j0.0246 S,

and Y_{ML} = 0.414 – j1.19 = 0.0083 – j0.0238 S.

Simultaneous conjugate match

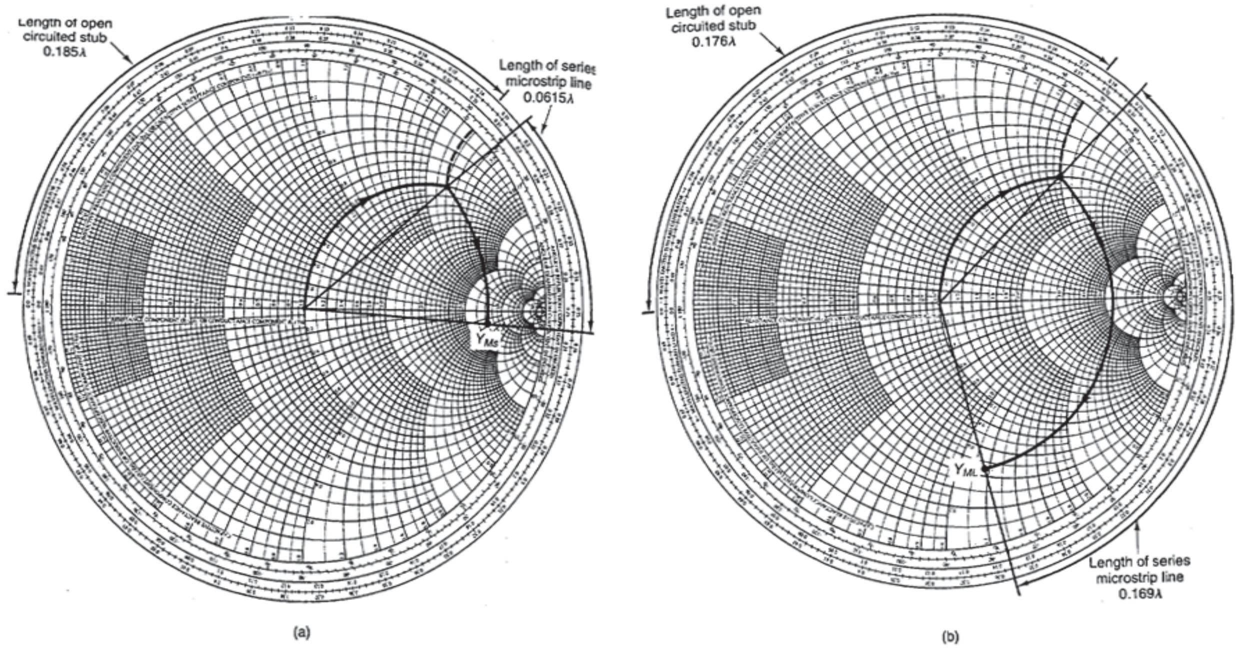


Figure 3.6.3 (a) Design of the input matching network; (b) design of the output matching network.

Simultaneous conjugate match

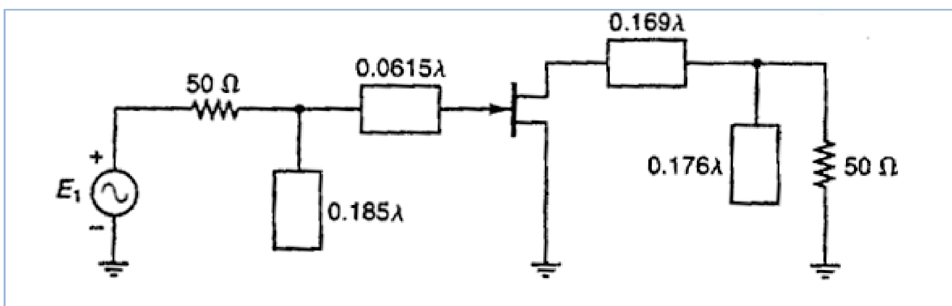


Figure 3.6.4 The ac schematic of a GaAs FET microwave amplifier. All microstrip lines have a characteristic impedance of 50 Ω.

Constant gain circles

Constant gain circles: bilateral case

Conditions: K > 1 and |Δ| < 1.

Design may call for $G_T \neq G_{Tmax}$ (produced by conjugate matching); what to do then? Need to choose Γ_S, Γ_L differently.

Start with:

$$G_T = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{IN}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_{OUT}\Gamma_L|^2} = G_S^{'}G_0G_L \quad (43)$$

$$G_S^{'} = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{IN}\Gamma_S|^2} \quad G_0 = |S_{21}|^2 \quad G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \quad (44)$$

Constant gain circles

Compare this with expressions in unilateral case

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} \quad G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \quad (45)$$

→ Γ_{IN} replaced S_{11}

General procedure follows unilateral case, but it is iterative because Γ_{IN} depends on choice of Γ_L

Constant gain circles

Procedure:

• Pick Γ_L for given G_L gain (see eq. above). Const. gain circles can be drawn using unilateral gain formulas

$$g_L = G_L / G_{L,\max} = G_L(1-|S_{22}|^2) \quad (46)$$

$$R_L = \frac{\sqrt{1-g_L}\,(1-|S_{22}|^2)}{1-|S_{22}|^2(1-g_L)} \quad (47)$$

$$d_L = \frac{g_L|S_{22}|}{1-|S_{22}|^2(1-g_L)} \quad (48)$$

Angle determined by S_{22}^*: line connecting the origin with S_{22}^* will have centers of circles on it.

• Calculate Γ_{IN} from $\Gamma_{IN} = S_{11} + S_{12}S_{21}\Gamma_L/(1 - S_{22}\Gamma_L)$

• G_S' const. gain circles drawn using the same eqs. (46, 47, 48) but Γ_{IN} goes where S_{11} would normally go:

Constant gain circles

$$g_S = G_S' / G_{S,\max} = G_S'(1-|\Gamma_{IN}|^2) \quad (49)$$

$$R_S = \frac{\sqrt{1-g_S}\,(1-|\Gamma_{IN}|^2)}{1-|\Gamma_{IN}|^2(1-g_S)} \quad (50)$$

$$d_S = \frac{g_S|\Gamma_{IN}|}{1-|\Gamma_{IN}|^2(1-g_S)} \quad (51)$$

Angle determined by Γ_{IN}^* (instead of S_{11}^*)

Select the desired Γ_S for a given G_S' gain. The available values of G_S' may not be satisfactory, e.g. $G_{S,\max} = 1/(1 - |\Gamma_{IN}|^2)$ may be too small, or values of possible Γ_S may not be good etc → go back to start and choose different Γ_L and repeat the procedure

• Once Γ_S and Γ_L are picked up, design matching networks as before

Constant gain circles

How about potentially unstable bilateral case (K < 1 or |Δ| > 1)?

Design procedures based on G_T is not good and is better done based on operating power gain equation.

Conclusion on bilateral case: anything but design for $G_{T,max}$ is tedious; if transistor is unconditionally stable simultaneously conjugate match can be found resulting in $G_{T,max}$.

Operating and available power gain circles

Operating power gain

$$G_P = \frac{P_L}{P_{IN}} = \frac{\text{power delivered to load}}{\text{power input to network}} \qquad (1)$$

it is independent of source impedance.

Design procedure is simple for both unconditionally stable and potentially unstable transistors.

Unconditionally stable bilateral case:

$$G_P = \frac{1}{1-|\Gamma_{IN}|}|S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \qquad (2)$$

Operating and available power gain circles

$$G_P = |S_{21}|^2 \frac{\left(1-|\Gamma_L|^2\right)}{\left(1-\left|\frac{S_{11}-\Delta\Gamma_L}{1-S_{22}\Gamma_L}\right|^2\right)|1-S_{22}\Gamma_L|^2} = |S_{21}|^2 g_P \qquad (3)$$

$$g_P = \frac{1-|\Gamma_L|^2}{1-|S_{11}|^2+|\Gamma_L|^2\left(|S_{22}|^2-|\Delta|^2\right)-2\,\mathrm{Re}\left(\Gamma_L C_2\right)} \qquad (4)$$

$$C_2 = S_{22}-\Delta S_{11}^* \qquad (5)$$

G_P, g_P functions only of $[S], \Gamma_L$. Procedure for obtaining const. gain circles as before. Final result:

– **Circle radius:**

$$R_P = \frac{\left[1-2K|S_{12}S_{21}|g_P+|S_{12}S_{21}|^2 g_P^2\right]^{1/2}}{\left|1+g_P\left(|S_{22}|^2-|\Delta|^2\right)\right|} \qquad (6)$$

Operating and available power gain circles

- **distance from origin:**

$$d_P = \frac{g_p|C_2^*|}{\left|1+g_P\left(|S_{22}|^2-|\Delta|^2\right)\right|} \qquad (7)$$

- Angle: g_P is a real number. Looking at expr. for center of the circle (eqs. not given), the direction (angle) of the centers will be determined by C_2^*.

$$C_P = \frac{g_P C_2^*}{1+g_P\left(|S_{22}|^2-|\Delta|^2\right)} \qquad (8)$$

$R_P = 0 \Rightarrow$

g_P have its max. for

$$g_{p,\max}^2 |S_{12}S_{21}|^2 - 2K|S_{12}S_{21}|g_{p,\max}+1=0 \qquad (9)$$

$$g_{p,\max} = \frac{1}{|S_{12}S_{21}|}\left(K-\sqrt{K^2-1}\right) \qquad (10)$$

Operating and available power gain circles

$$\Rightarrow G_{P,\max} = \frac{|S_{21}|}{|S_{12}|}\left(K - \sqrt{K^2 - 1}\right) \qquad (11)$$

Given G_P we select Γ_L from constant operating power gain circles. In order to get max. G_P, Γ_L is selected at distance where $g_{p,\max} = G_{P,\max} \big/ |S_{21}|^2$ i.e. calculate $G_{P,\max}$ from eq. 11, find $g_{P,\max} = G_{P,\max} \big/ |S_{21}|^2$, and plug it into eq. for d_P (eq. 7). Intersection of that with direction of C_2^* gives Γ_L for $G_{P,\max}$.

- Max. output power obtained if input is conj. matched, i.e. $\Gamma_S = \Gamma_{IN}^*$

in this case the input power is equal to max. available power from source $\Rightarrow G_{T,\max} = G_{P,\max}$. Note that values of Γ_S, Γ_L that give $G_{P,\max}$ are identical to Γ_{Ms}, Γ_{ML} .

Operating and available power gain circles

Procedure

- Specify G_P; radius, center and angle are given in eqs. above.
- Select the desired Γ_L
- For given Γ_L max. output power is obtained by conjugate matching on input, i.e. with

$$\Gamma_S = \Gamma_{IN}^* = \left(S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}\right)^* \qquad (12)$$

This value of Γ_S produces the transducer gain $G_T = G_P$ (if $G_P = G_{P,\max}$ is specified at the beginning, G_T will be equal to $G_{P,\max}$.)

Operating and available power gain circles

Example Using the same previous transistor, design for $G_P = 9dB$

– $|S_{21}|^2 = 4.235$ or 6.27 dB. $\Rightarrow$ $g_P = G_P / |S_{21}|^2 = 1.875$

$K = 1.504, |\Delta| = 0.3014, C_2 = 0.3911\angle -103.9$

– Calculate $R_P = 0.431$ and $C_P = 0.508\angle 103.9°$

– Select some Γ_L point, say pt. A where $\Gamma_L = 0.36\angle 47.5°$

– Calculate Γ_S from known $\Gamma_L \Rightarrow \Gamma_S = 0.629\angle 175°$

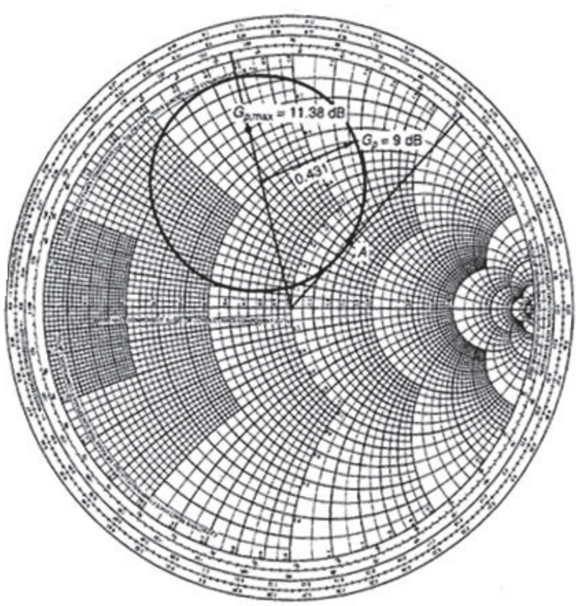
Same procedure can be used to find $G_{P,\max}$ (for simultaneous conjugate matching) => different circle:

$g_{p,\max} = 3.24,\ R_{p,\max} = 0,\ C_{p,\max} = 0.718\angle 103.9°$

This leads to values for $\Gamma_{L,\max} = 0.718\angle 103.9°$. Use it to find

$\Gamma_{s,\max} = 0.762\angle 177° \Rightarrow$ same as Γ_{Ms}, Γ_{ML} from before.

Operating and available power gain circles



(a)

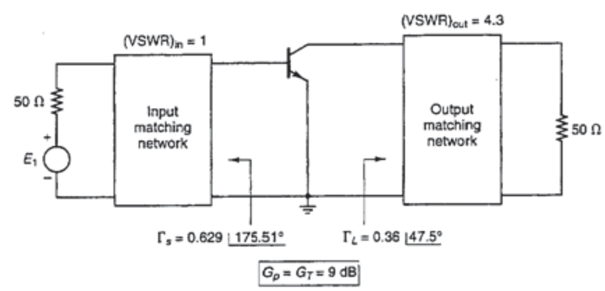


Figure 3.7.1 (a) Operating power-gain circle for $G_p = 9$ dB and location of $G_{p,\max} = 11.38$ dB; (b) the block diagram of the amplifier.

Operating and available power gain circles

Example

Design a GaAs FET amplifier at 8 GHz with G_p=10dB.

$$S = \begin{bmatrix} 0.5\angle -180 & 0.08\angle -30 \\ 2.5\angle 70 & 0.8\angle -100 \end{bmatrix}$$

- Calc. K and |Δ| for stability: K = 0.4, Δ = 0.223∟62.12°

→ The transistor is potentially unstable.

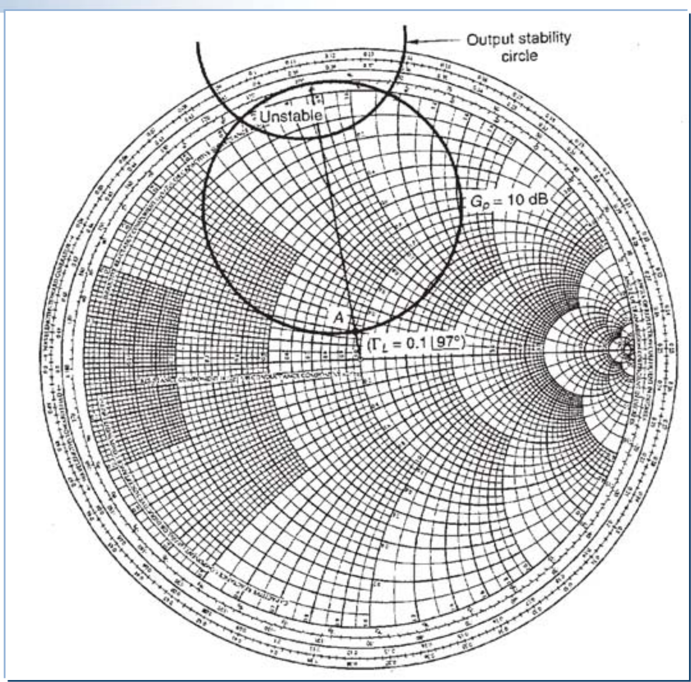
- G_{MSG}=2.5/0.08=31.25=14.9dB
- 10dB power gain circle, using (6) and (8):

$$C_p = 0.572\angle 97.2, \quad r_p = 0.473$$

- Output Stability Circle

$$C_L = 1.18\angle 97.2, \quad r_L = 0.34$$

Operating and available power gain circles



Operating and available power gain circles

– At point A: $\Gamma_L = 0.1\angle 97, \quad Z_L = 50(0.96 + j0.19)\Omega$

– Set Calculate $\Gamma_s = \Gamma_{in}^* = S_{11} + \dfrac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.52\angle 179.32$

– Check if Calculate Γ_s is in the stable region

– Input stability circle $Cs = 1.67\angle 171, \quad r_s = 1.0$

Operating and available power gain circles

Available power gain

$$G_A = \frac{P_{AVN}}{P_{AVS}} = \frac{\text{P avail. from network}}{\text{P avail. from source}} \qquad (13)$$

$$G_A = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1}{1 - |\Gamma_{OUT}|^2} \qquad (14)$$

Derivation analogous to operating power gain case:

$$g_a = \frac{G_A}{|S_{21}|^2} \qquad (15)$$

$$C_1 = S_{11} - \Delta S_{22}^*$$

Operating and available power gain circles

$$R_a = \frac{\left[1 - 2K\left|S_{12}S_{21}\right| g_a + \left|S_{12}S_{21}\right|^2 g_a^2\right]^{1/2}}{\left|1 + g_a\left(\left|S_{11}\right|^2 - \left|\Delta\right|^2\right)\right|} \qquad (16)$$

$$C_a = \frac{g_a C_1^*}{1 + g_a\left(\left|S_{11}\right|^2 - \left|\Delta\right|^2\right)} \qquad (17)$$

By plotting const. available gain circles a Γ_S can be picked up to produce desired G_A. With Γ_S known, max. power on output is obtained for conjugate matched load so that

$$\Gamma_L = \Gamma_{OUT}^* = \left(S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}\right)^* \qquad (18)$$

for

$$\Gamma_L = \Gamma_{OUT}^* \Rightarrow G_T = G_A$$

Operating and available power gain circles

Potentially unstable bilateral case

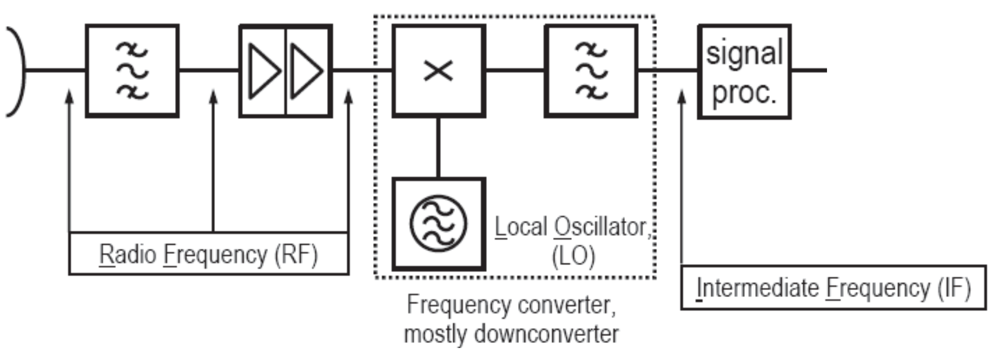
We have to worry about stability! Procedure is still the same:

- Specify G_P and draw const. operating gain circles. (calc. R_P, C_P)
- Draw output stability circle (calc. r_L, C_L ; remember that they give Γ_L-s for which $\left|\Gamma_{IN}\right| = 1$)
- Choose Γ_L in stable region (stay away from circle)
- Draw input stability circles (calc. r_s, C_s)
- Calculate Γ_{IN} and see if $\Gamma_s = \Gamma_{IN}^*$ is in the stable region
- If Γ_s is not in stable region, or is too close to stability circle:
  a) select Γ_s arbitrary, b) select new G_p, c) try different Γ_L

Operating and available power gain circles

Γ_s, Γ_L should be away from the stability circles to avoid oscillations during tuning or due to variation of parameters.

Noise Optimization of the Receiver RF front end

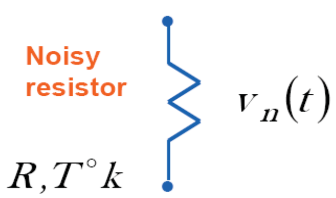


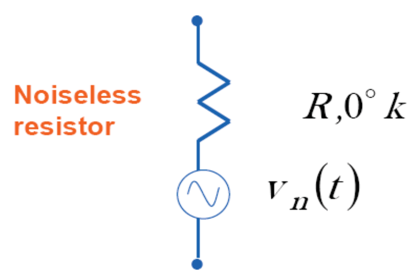
Linear Amplifier Design

- Why we need Low Noise Amplifier?
  - In receivers, a preamplifier with a low noise figure is needed
- How we design a microwave amplifier for low noise?
  - A trade-off exists between noise figure and gain of an amplifier. So, to obtain both minimum noise figure and maximum gain a compromise must be made.

Noise Review

- Noise power (due to thermal noise)





- Planck's black body radiation law, rms noise voltage across a resistor R is

$$v_n = \sqrt{4kT \int \frac{hf/kT}{e^{hf/kT} - 1} R(f)\, df}$$

- hf << kT at microwave frequencies

$$\rightarrow \quad e^{hf/kT} - 1 \cong \frac{hf}{kT} \rightarrow v_n = \sqrt{4kTBR}$$

h: Planck constant

k: Boltzmann constant

Noise Review

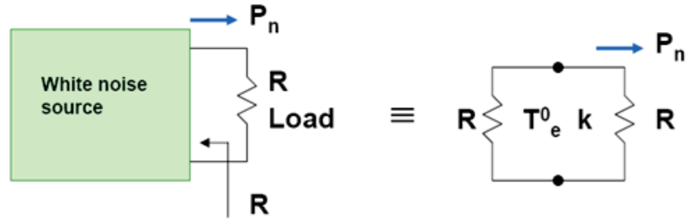
- The maximum power delivered from the noisy resistor is P_n is

$$P_n = \left(\frac{v_n}{2R}\right)R = \frac{v_n^2}{4R} = kTB$$

- It is considered equally across an entire microwave band.
- A resistor temperature at 300K, noise power for a 10kHz bandwidth receiver → $P_n = 4.14 \times 10^{-17}$ W = -176dBW= -146dBm
- At the standard temperature of 290K, the noise power density available from a lossy passive network in a 1Hz bandwidth is -174dBm/Hz.

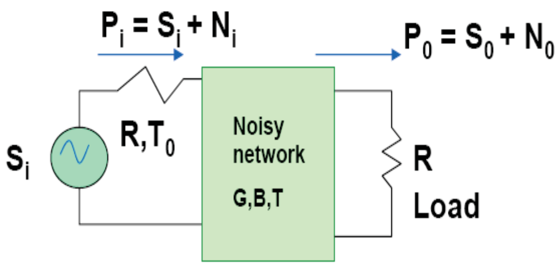
Noise Review

- Equivalent noise temperature: the absolute temperature to generate the same noise power, not the physical temperature of the device equivalent noise temperature $T_e \equiv P_n / kB$



Noise Figure

- Noise figure is a figure of merit to measure the degradation of SNR of a system



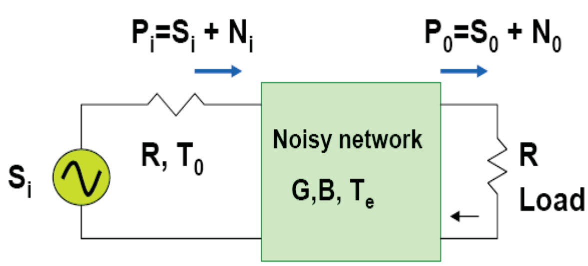
$$NF = \frac{(S/N)_i}{(S/N)_o} \geq 1$$

$$NF_{dB} = 10\log NF \geq 0dB$$

- For a passive device with G=1/L and the temperature T, N_0 = kTB = N_i , S_o =GS_i ,

$$NF = \frac{(S/N)_i}{(S/N)_o} = \frac{S_i}{S_o}\frac{N_o}{N_i} = L$$

Noise Figure



$$NF = \frac{(S/N)_i}{(S/N)_o}$$

$$= \frac{S_i}{kT_0B}\frac{Gk(T_0+T_e)B}{GS_i}$$

$$= 1 + \frac{T_e}{T_0} \geq 1$$

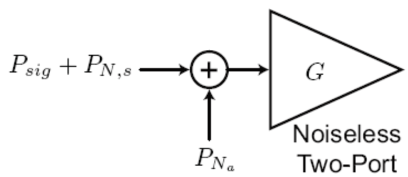
$$\rightarrow T_e = (NF-1)T_0$$

Noise Figure

- While the noise of a two-port originates from *inside* the two-port, it's suitable to act as if the two-port is noiseless and to imagine that the noise added to either the input or output of the amplifier.
- The input referred noise is more commonly used.
- From circuit theory, any two-port can be simplified to a noiseless two-port with an input referred voltage and current.
- The total input noise can be derived from the definition of F

$$F - 1 = \frac{P_{N_a}}{P_{N_s}}$$

$$P_{N_a} = P_{N_s}(F - 1)$$



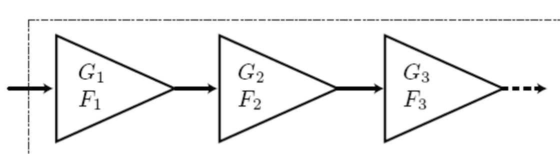
Noise Figure

- The overall noise figure of several amplifier stages or two-ports connected in cascade?
  - We can use the concept of a noiseless two-port cascade and simply input refer all the noise sources.
- The noise from the second stage can be input refereed if the power gain is known and the stages are matched

$$P_{N_{ia,2}} = \frac{P_{N_s}(F_2 - 1)}{G_1}$$

- The noise from the k^{th} stage can be similarly input referred

$$P_{N_{ia,k}} = \frac{P_{N_s}(F_k - 1)}{G_1 \cdot G_2 \cdots G_{k-1}}$$



Noise Figure

- Therefore the total noise figure is $$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots$$
- The first stage of the cascade is very important.
- The noise figure of the cascade is bounded by the noise of the first stage whereas the noise figure of other stages is divided by the gain preceding the stage.
- The first stage amplifier, or the low noise amplifier (LNA), is one of the most important blocks in a receiver.
- The noise of the first this stage must be as low as possible and the gain as high as possible in order to reject the subsequent noise in the system, especially the mixer (which tends to have a high noise figure).
- Any attenuation before the LNA must be minimized (cables, mismatch, loss in filters, switches, diplexers).

Noise Figure

- Two stage amplifier noise figure
- If G_A is available power gain at each stage; P_{N1}, P_{N2} are noise powers caused by internal amplitude noise, then, the total noise power is

$$P_{N0,tot} = G_{A2}(G_{A1}P_{Ni} + P_{N1}) + P_{N2}$$

- The corresponding noise figure is

$$F = \frac{P_{N0,tot}}{P_{Ni}G_{A2}G_{A1}} = 1 + \frac{P_{N1}}{G_{A1}P_{Ni}} + \frac{P_{N2}}{G_{A2}G_{A1}P_{Ni}} = F_1 + \frac{F_2 - 1}{G_{A1}}$$

where $F_1 = 1 + \frac{P_{N1}}{P_{Ni}G_{A1}}, \quad F_2 = 1 + \frac{P_{N2}}{P_{Ni}G_{A2}}$

and F_1, F_2 are noise figures for each amplifier stage.

- Noise figure of the 2nd stage reduced by G_{A1}

Noise Figure

- Design trade-off: It may not always be the best to minimize 1st stage noise, if this results in large reduction of power gain since that may increase the contribution from the second part.
- General conclusion: There is always a compromise between gain and noise power.
- Is it better to put the Amplifier 1 first or the Amplifier 2? The overall noise figure F for the two cases is

$$F_{12} = F_1 + \frac{F_2 - 1}{G_{A1}}, F_{21} = F_2 + \frac{F_1 - 1}{G_{A2}}$$

Noise Figure

- We $F_{12} < F_{21}$ require (i.e. the 1st amp should go first)

$$F_1 - 1 + \frac{F_2 - 1}{G_{A1}} < F_2 - 1 + \frac{F_1 - 1}{G_{A2}}$$

$$\frac{F_1 - 1}{1 - 1/G_{A1}} (= M_1) < \frac{F_2 - 1}{1 - 1/G_{A2}} (= M_2)$$

- Where M1, M2 are **noise measures** for the two amplifiers respectively. If then Amplifier 1 goes first.

Noise Figure

- This noise figure can be expressed as

$$F = F_{\min} + \frac{R_n}{G_s}\left[\left(G_s - G_{opt}\right)^2 + \left(B_s - B_{opt}\right)^2\right]$$

  where

  - $Y_s = G_s + jB_s$
  - $Y_{opt} = G_{opt} + jB_{opt}$
  - F_{min} = lowest possible noise factor
  - Y_{opt} = optimum source admittance for minimum noise
- Normalizing everything to Z_o leads to:

$$F = F_{\min} + \frac{r_n}{g_s}\left|y_s - y_{opt}\right|^2$$

Noise Figure

- Using reflection coefficients:

$$F = F_{\min} + \frac{4r_n\left|\Gamma_S - \Gamma_{opt}\right|^2}{\left(1 - \left|\Gamma_S\right|^2\right)\left|1 + \Gamma_{opt}\right|^2} \qquad y_S = \frac{1 - \Gamma_S}{1 + \Gamma_S} \qquad y_o = \frac{1 - \Gamma_{opt}}{1 + \Gamma_{opt}}$$

- $F_{\min}$, r_n, Γ_{opt} are noise parameters given by manufacturer or can be measured.
- There is a minimum noise factor possible for a device, F_{min}, that is achieved only when a particular reflection coefficient, Γ_{opt} is presented to the input.
- So, $\Gamma_s = \Gamma_{opt}$ leads to the minimum noise figure for the amplifier built with this transistor.

Noise Figure

- Noise figure is often dependent on the bias point selection for the transistor.

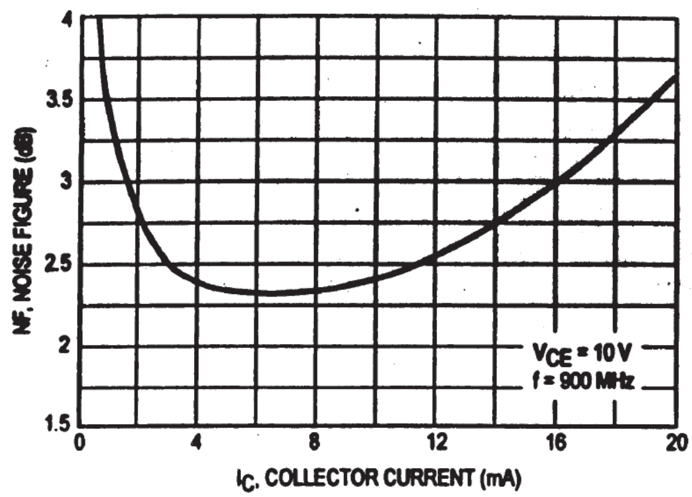
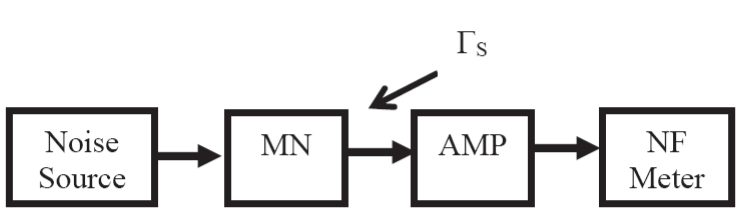


Figure 12. Noise Figure versus Collector Current

Two Port noise parameters measurement

- To measure the 2 port noise parameters:
  - 1. Change Γs until min. F is observed. read F from NF meter, use the network analyzer to measure Γopt = Γs
  - 2. Set Γs=0 (term. in Zo) and Measure F to get r_n

$$r_n = \left(F_{\Gamma_S=0} - F_{\min}\right)\frac{\left|1+\Gamma_{opt}\right|^2}{4\left|\Gamma_{opt}\right|^2}$$



Noise Circles

- We can also use the equation to predict noise figure vs. Γ_S. The equation can be reconfigured:

$$\frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{1-\left|\Gamma_S\right|^2} = \underbrace{\frac{F_i - F_{\min}}{4r_n}\left|1+\Gamma_{opt}\right|^2}_{\text{constant for each } F_i} \equiv N_i$$

- Then circles can be drawn on the source Γs Smith chart that correspond to a particular noise figure.
- When these noise circles are plotted with available gain circles, we can show the tradeoff between min. noise and gain.

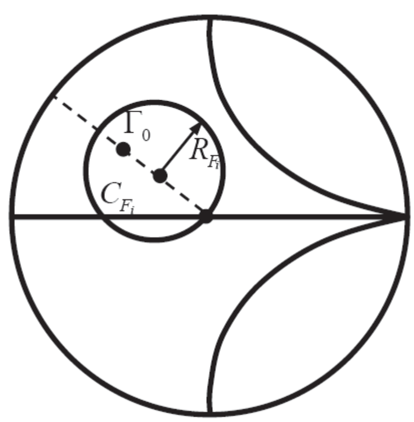
Noise Circles

- To calculate noise circles:

$$N_i = \frac{F_i - F_{\min}}{4r_n}\left|1+\Gamma_{opt}\right|^2 = \textit{noise figure parameter}$$

$$C_{Fi} = \frac{\Gamma_{opt}}{1+N_i}, \qquad \text{(center)}$$

$$R_{Fi} = \frac{1}{1+N_i}\sqrt{N_i^2 + N_i\left(1-\left|\Gamma_{opt}\right|^2\right)} \qquad \text{(radius)}$$



Noise Circles

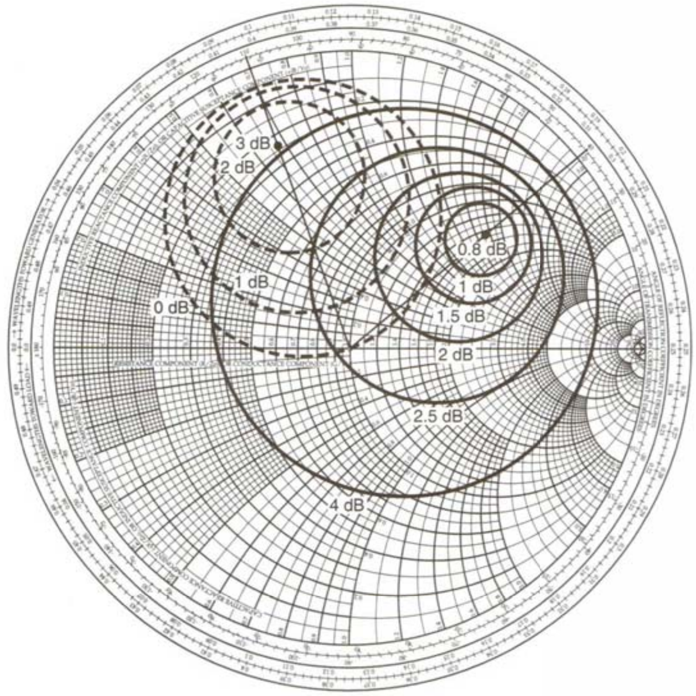


Figure 4.3.3 Noise figure circles (solid curves) and G_s constant-gain circles (dashed curves). The transistor is a GaAs FET with $V_{DS} = 4$ V, $I_{DS} = 12$ mA, and $f = 6$ GHz.

Noise Circles

Additional comment about gain circles

- Device data sheets often plot G_T with the output matched on the Γ_s plane. This is the available power gain = G_A . $$G_A = \frac{P_{AVN}}{P_{AVS}}$$
- Since output is always matched, G_A is independent of Γ_L

$$G_A = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} |S_{21}|^2 \frac{1}{1-|\Gamma_{OUT}|^2}$$

(since $\Gamma_L = \Gamma_{OUT}^*$)

Depends upon input match because actual power absorbed in the input is not necessarily the same as Γ_{MS} (unless conjugate matched at input).

Noise Circles

- If input is also conjugate matched, then we get the maximum available gain:

$$G_A = G_{A.\max} = MAG = G_{T,\max}$$

$$= \frac{1}{\left(1-|S_{11}|^2\right)} |S_{21}|^2 \frac{1}{1-|\Gamma_{OUT}|^2}$$

$$\frac{1}{1-|S_{22}|^2} \text{ (if unilateral)}$$

Example (1)

The scattering and noise parameters of a BJT measured at a bias point for low-noise operation ($V_{CE} = 10$ V, $I_C = 4$ mA) at $f = 4$ GHz are

$$S_{11} = 0.552\angle 169^\circ$$

$$S_{12} = 0.049\angle 23^\circ$$

$$S_{21} = 1.681\angle 26^\circ$$

$$S_{22} = 0.839\angle -67^\circ$$

and

$$F_{\min} = 2.5 \text{ dB}$$

$$\Gamma_{\text{opt}} = 0.475\angle 166^\circ$$

$$R_n = 3.5\ \Omega$$

Design a microwave transistor amplifier to have a minimum noise figure. (This example is based on a design from Hewlett-Packard Application Note 967 [4.2].)

Example (1)

Solution. For this transistor, it follows that $K = 1.012$ and $|\Delta| = 0.419\underline{|111.04^\circ}$. Therefore, the transistor is unconditionally stable at 4 GHz. Using (3.6.10), (3.6.5), and (3.6.6), it also follows that $G_{T,\max} = G_{A,\max} = 14.7$ dB, $\Gamma_{Ms} = 0.941\underline{|-154^\circ}$, and $\Gamma_{ML} = 0.979\underline{|70^\circ}$.

A minimum noise figure of 2.5 dB is obtained with $\Gamma_s = \Gamma_{\text{opt}} = 0.475\underline{|166^\circ}$. The constant noise figure circles in Fig. 4.3.4 for $F_i = 2.5$ to 3 dB were calculated using (4.3.6), (4.3.7), and (4.3.8). For example, the $F_i = 2.8$-dB circle was obtained as follows:

$$N_i = \frac{1.905 - 1.778}{4(3.5/50)}|1 + 0.475\underline{|166^\circ}|^2 = 0.1378$$

$$C_{F_i} = \frac{0.475\underline{|166^\circ}}{1 + 0.1378} = 0.417\underline{|166^\circ}$$

and

$$r_{F_i} = \frac{1}{1 + 0.1378}\sqrt{(0.1378)^2 + 0.1378[1 - (0.475)^2]} = 0.312$$

Example (1)

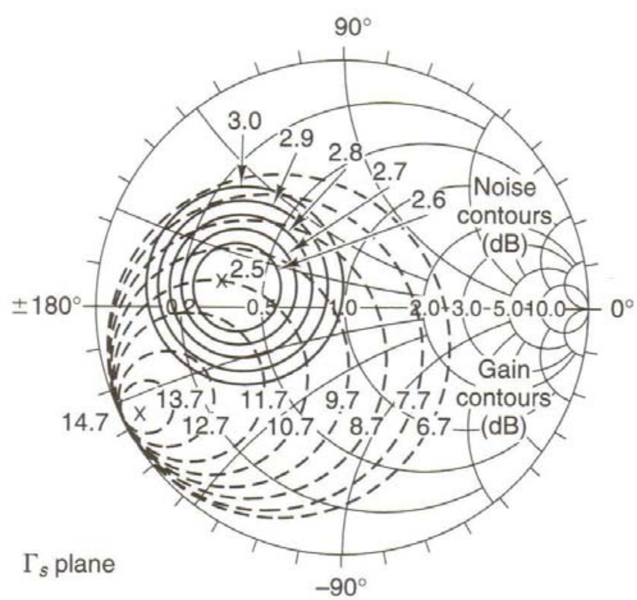


Figure 4.3.4 Constant noise figure circles and available power gain circles. (From Ref. [4.2]; courtesy of Hewlett-Packard.)

Example (1)

With $\Gamma_s = \Gamma_{opt}$, the resulting available gain is $G_A = 11$ dB. Therefore, a sacrifice in gain was needed to obtain optimum noise performance. The load reflection coefficient is selected to provide maximum power transfer to the load (i.e., $\Gamma_L = \Gamma^*_{OUT}$). With $\Gamma_s = \Gamma_{opt}$, the value of Γ_L is

$$\Gamma_L = \left(S_{22} + \frac{S_{12}S_{21}\Gamma_{opt}}{1 - S_{11}\Gamma_{opt}}\right)^* = 0.844\underline{|70.4^\circ}$$

Since $\Gamma_L = \Gamma^*_{OUT}$, the VSWR at the output is 1, and the resulting gains are $G_T = G_A = 11$ dB and $G_p = 12.7$ dB. Also, $\Gamma_{IN} = 0.744\underline{|157^\circ}$ and, using (3.8.1) and (3.8.2), the input VSWR is 4.26.

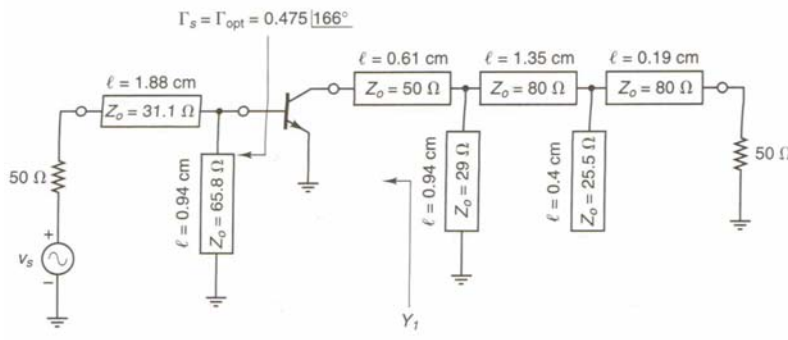


Figure 4.3.5 Amplifier schematic. The microstrip lengths are given for $\epsilon_{ff} = 1$ at $f = 4$ GHz. (From Ref. [4.2]; courtesy of Hewlett-Packard.)

Example (2)

Example 4.3.2

The S parameters and the noise parameters of the AT-41470 (a low-noise silicon BJT) at 4 GHz, $V_{CE} = 8$ V, and $I_C = 10$ mA are

$$S_{11} = 0.6\underline{|146^\circ} \qquad F_{min} = 3 \text{ dB}$$
$$S_{12} = 0.085\underline{|62^\circ} \qquad \Gamma_{opt} = 0.45\underline{|-150^\circ}$$
$$S_{21} = 1.97\underline{|32^\circ} \qquad r_n = 0.2$$
$$S_{22} = 0.52\underline{|-63^\circ}$$

Design a low-noise amplifier with $(\text{VSWR})_{in} < 1.8$.

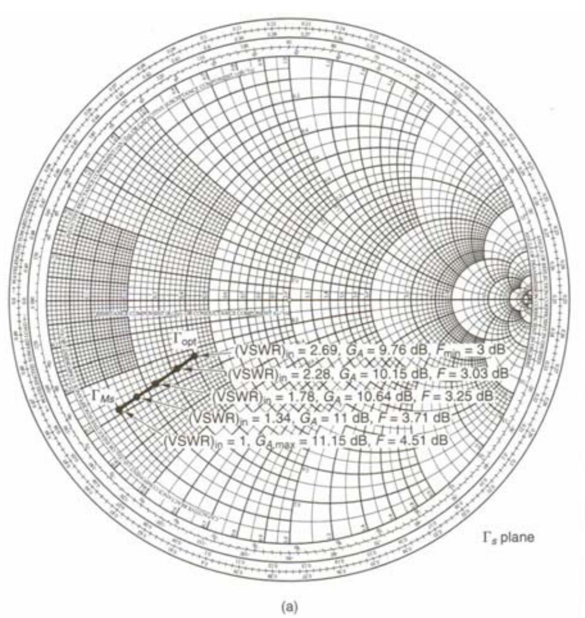
Example (2)

Solution. This transistor is unconditionally stable at 4 GHz since $K = 1.172$ and $|\Delta| = 0.151$. From (3.6.5) and (3.6.6), the simultaneous conjugate match terminations are $\Gamma_{Ms} = 0.824\angle -147.81^\circ$ and $\Gamma_{ML} = 0.791\angle 60.45^\circ$; and $G_{A,\max} = 11.15$ dB.

In Fig. 4.3.8a, the values of Γ_{opt} and Γ_{Ms} are plotted, as well as three other values of Γ_s located on a straight line drawn from Γ_{opt} to Γ_{Ms}. In Fig. 4.3.8b, we have tabulated the resulting values of the noise, $(VSWR)_{in}$, and G_A at the values of Γ_s indicated in Fig. 4.3.8a.

An examination of Fig. 4.3.8b shows that the design target of low noise with $(VSWR)_{in} < 1.8$ can be obtained with $\Gamma_s = 0.64\angle -148.2^\circ$. In fact, with this value of Γ_s the noise figure is only 0.25 dB higher than F_{min}, the gain $G_A = 10.64$ dB is only 0.51 dB lower than $G_{A,\max}$, and the value of $(VSWR)_{in} = 1.78$ represents an improvement of 33.83% over the $(VSWR)_{in}$ value of 2.69 at Γ_{opt}. The block diagram of the low-noise amplifier is shown in Fig. 4.3.8c.

Example (2)



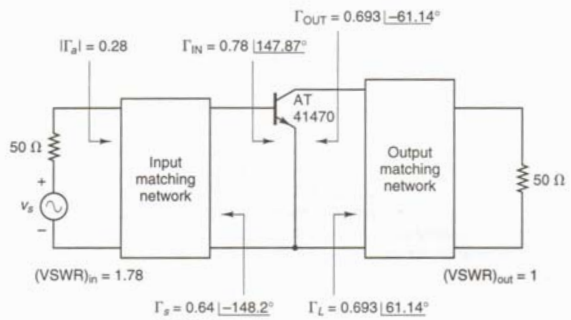
(a)

Figure 4.3.8 (a) Trade-offs between noise figure, available gain, and $(VSWR)_{in}$; (b) calculations of F, G_A, and $(VSWR)_{in}$ at the values of Γ_s indicated in (a); (c) the block diagram of the low-noise amplifier.

Example (2)

Γ_s	$\Gamma_L = \Gamma_{OUT}^*$	Γ_{IN}	$\lvert\Gamma_a\rvert$	$(VSWR)_{in}$	G_A (dB)	F (dB)
$0.45\angle{-150^\circ}$	$0.623\angle{62.09^\circ}$	$0.753\angle{147.97^\circ}$	0.458	2.69	9.76	3
$0.53\angle{-149^\circ}$	$0.649\angle{61.68^\circ}$	$0.762\angle{147.87^\circ}$	0.39	2.28	10.15	3.03
$0.64\angle{-148.2^\circ}$	$0.693\angle{61.14^\circ}$	$0.78\angle{147.87^\circ}$	0.28	1.78	10.64	3.25
$0.74\angle{-148^\circ}$	$0.741\angle{60.78^\circ}$	$0.8\angle{147.86^\circ}$	0.147	1.34	11	3.71
$0.824\angle{-147.81^\circ}$	$0.791\angle{60.45^\circ}$	$0.824\angle{147.81^\circ}$	0	1	11.15	4.51

(b)



$F = 3.25$ dB
$G_A = 10.64$ dB

(c)

Figure 4.3.8 Continued

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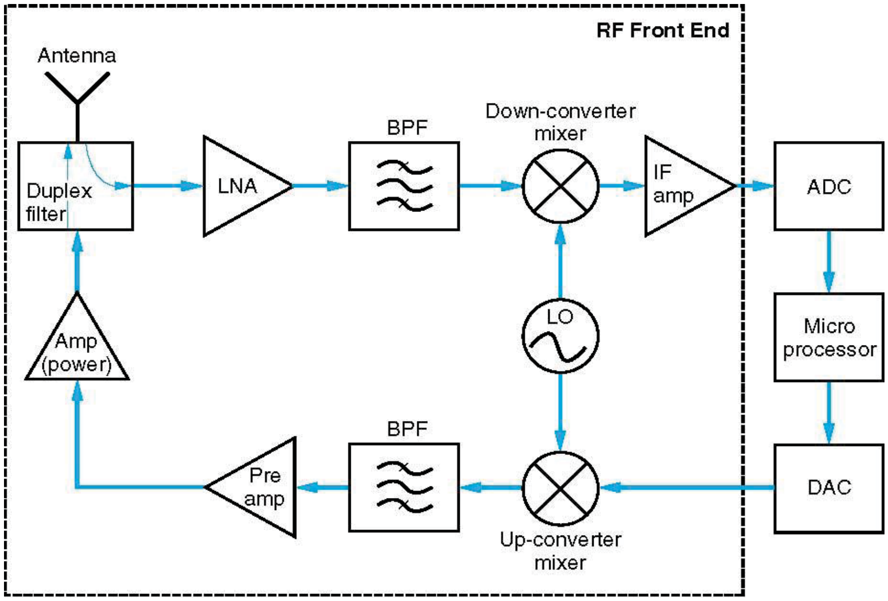
Why we need filter

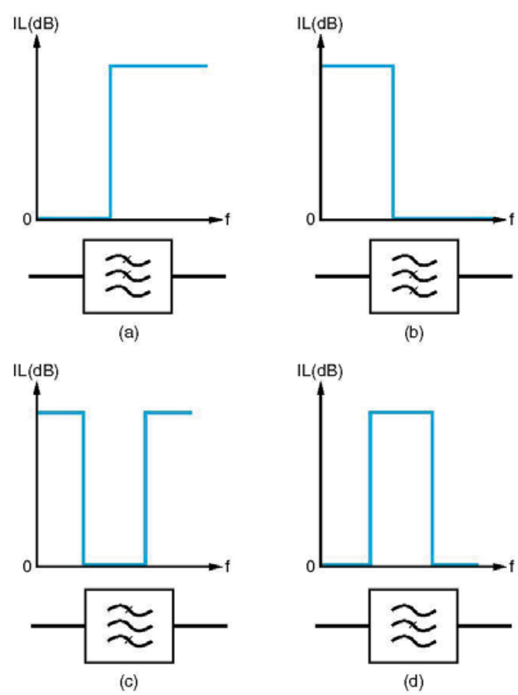


A typical transceiver RF front end configuration.

Filters types

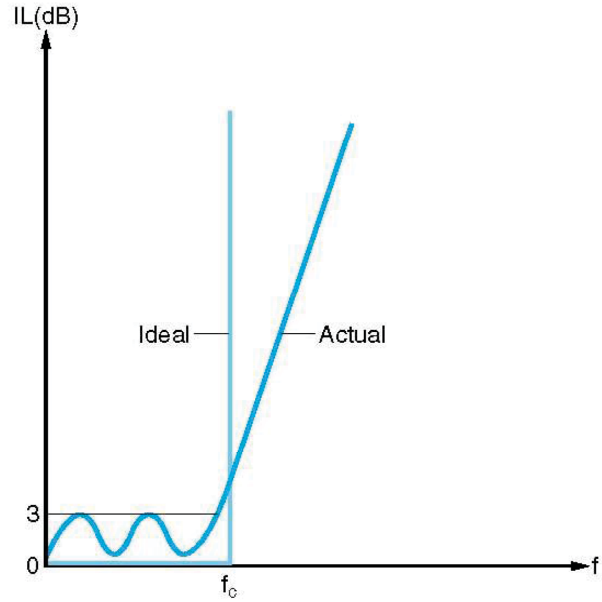
Filters are two-port networks that achieve a certain desired frequency response.

Ideal performance of the four filter types, along with their circuit symbols:
(a) low-pass,
(b) high-pass,
(c) bandpass, and
(d) bandstop.



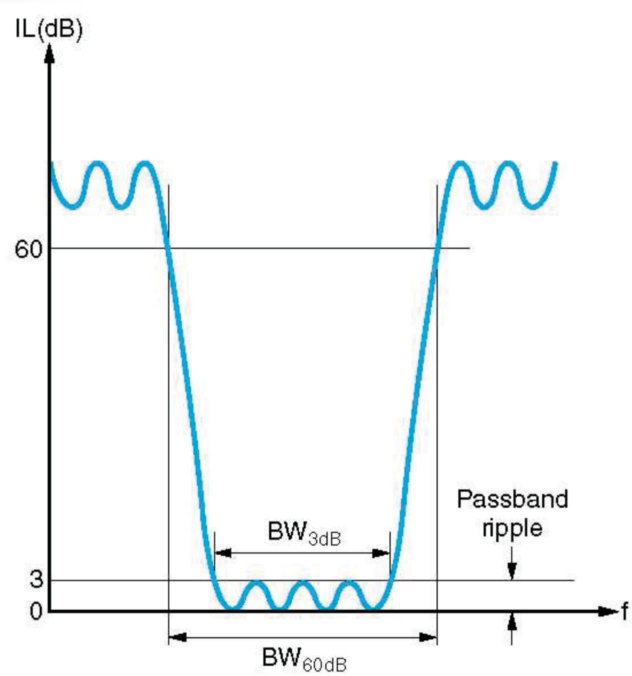
Filters performances

Typical performance for a low-pass filter compared with an ideal one.



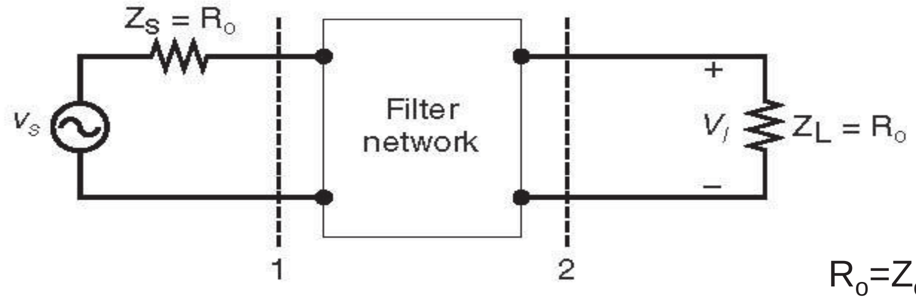
Filters performances

Typical filter performance for a band pass filter



Microwave Filters

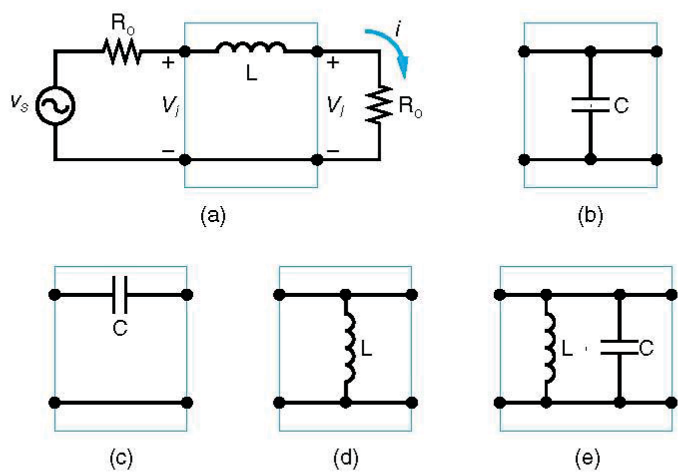
Filter network inserted in an Z_o impedance system.



$R_o = Z_o$

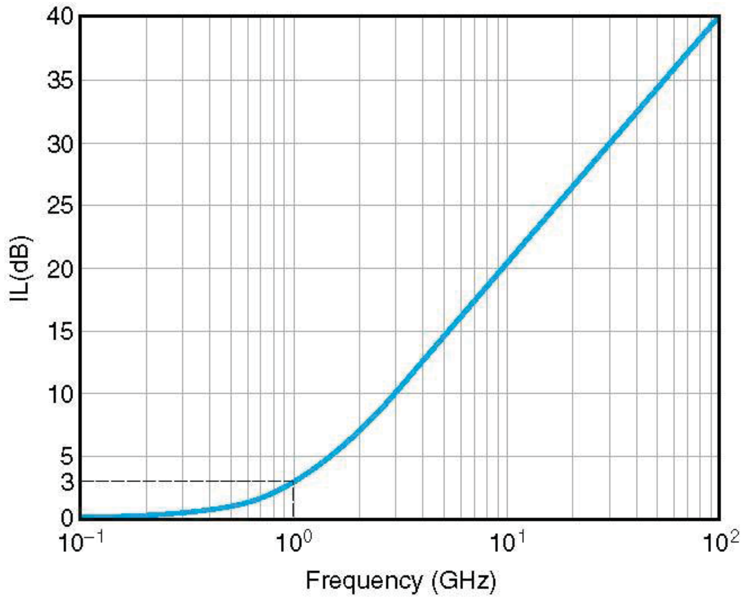
Microwave Filters

Simple lumped-element filters;
(a) and (b) low-pass filters,
(c) and (d) high pass filters,
(e) a bandpass filter (a *tank* circuit).



Microwave Filters

Characteristics for the simple low-pass filter



Microwave filters

$$P_{LR} = \frac{\text{Power available a source}}{\text{Power delivered to load}} = \frac{P_{inc}}{P_{Load}} = \frac{1}{1-|\Gamma(\omega)|^2}$$

Γ is an even function of ω, it can therefore be expressed as a polynomial in ω^2.

$$|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2)+N(\omega^2)} \quad \Rightarrow \quad P_{LR} = 1+\frac{M(\omega^2)}{N(\omega^2)}$$

Maximally flat:
(Butterworth)

$$P_{LR} = 1+k^2\left(\frac{\omega}{\omega_c}\right)^{2N}$$

N is a filter order

Equal ripple:
(Chebyshev)

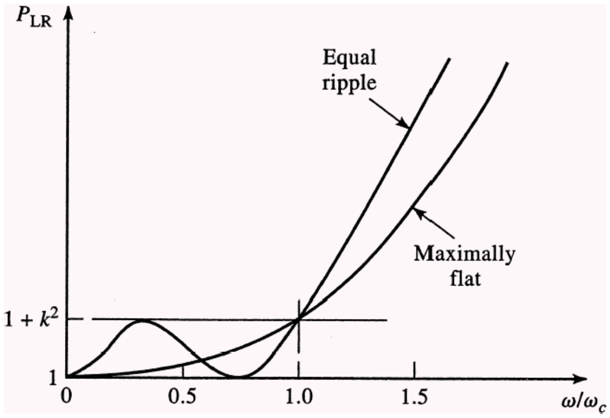
$$P_{LR} = 1+k^2T_N^2\left(\frac{\omega}{\omega_c}\right)$$

$T_N(x)$ Oscillates between ± 1

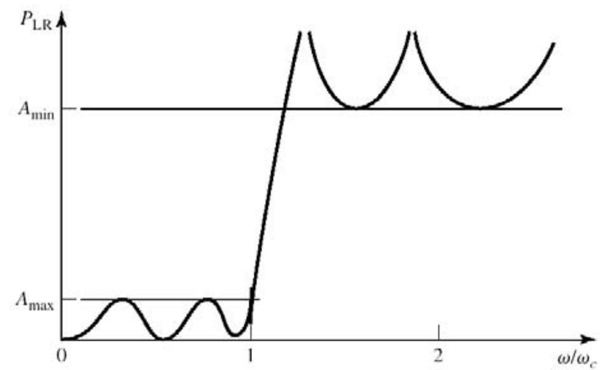
$$\text{large } x \Rightarrow T_N(x) \approx \frac{1}{2}(2x)^N \Rightarrow P_{LR} \approx \frac{k^2}{4}\left(\frac{2\omega}{\omega_c}\right)^{2N}$$

IL is $\left(2^{2N}\right)/4$ greater than Butterworth case

Microwave filters



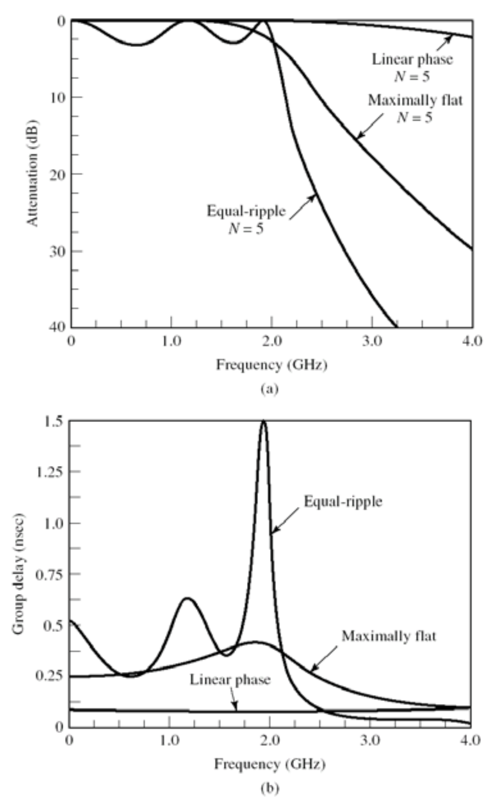
Maximally flat and equal-ripple low-pass filter responses (*N* = 3).



Elliptic function low-pass filter response.

Microwave filters

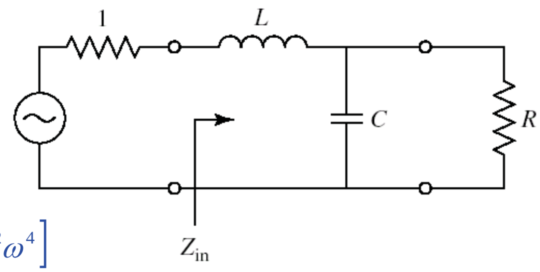
Frequency response of the filter design.
(*a*) Amplitude response.
(*b*) Group delay response.



Microwave filters

Maximally flat low pass filter prototype

Consider 2 elements filter prototype. For prototype we assume source impedance 1Ω and cutoff frequency (don't confuse with waveguide cutoff) of ω_c =1. We also assume 3dB attenuation at ω_c, hence k=1



$$P_{LR} = 1 + k^2\left(\frac{\omega}{\omega_c}\right)^{2*2} = 1 + \omega^4$$

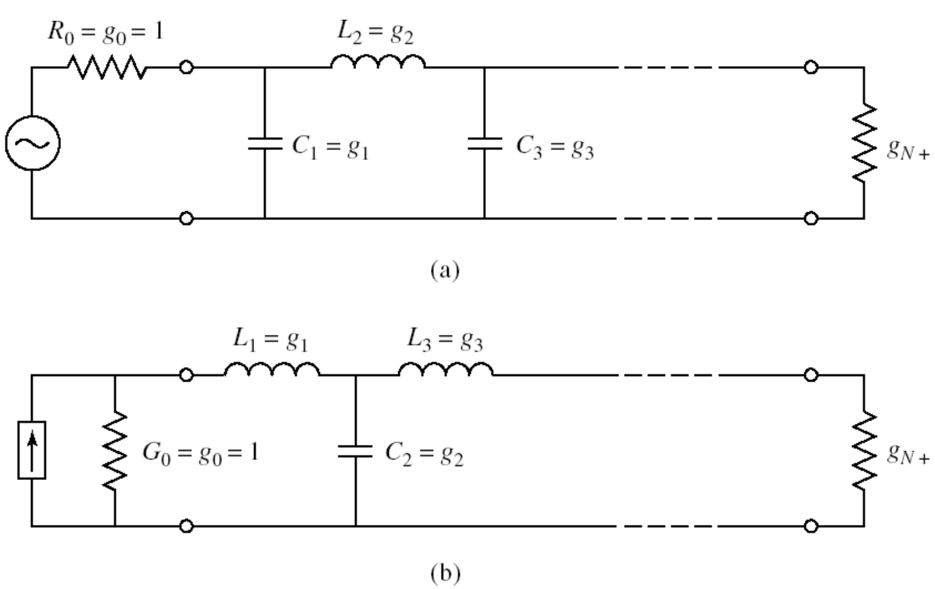
$$P_{LR} = 1 + \frac{1}{4R}\left[(1-R)^2 + \left(R^2C^2 + L^2 - 2LCR^2\right)\omega^2 + R^2C^2L^2\omega^4\right]$$

$$R = 1$$

$$C^2 + L^2 - 2LC = 0 \Rightarrow L = C$$

$$P_{LR} = 1 + \frac{1}{4}\left[C^2L^2\right]\omega^4 \Rightarrow L = C = \sqrt{2}$$

Microwave Filters



Ladder circuits for low-pass filter prototypes and their element definitions. (*a*) Prototype beginning with a shunt element. (*b*) Prototype beginning with a series element.

Microwave Filters

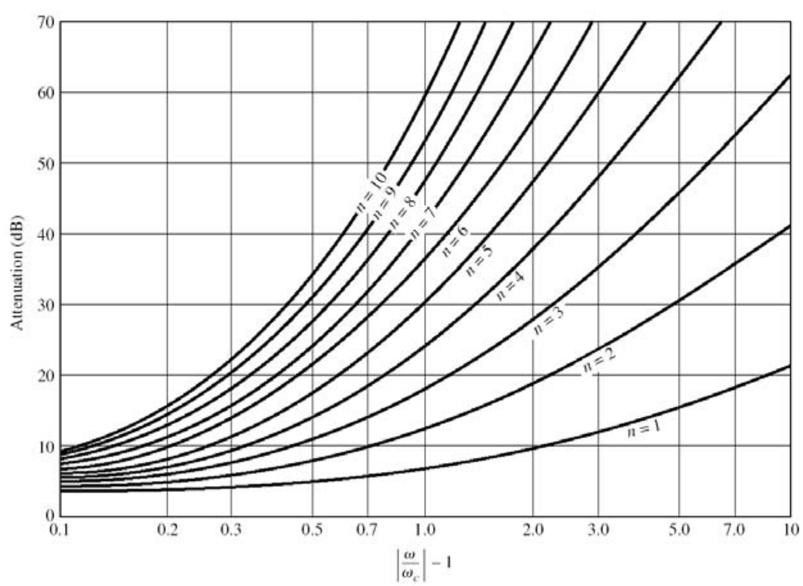
Properties of the Butterworth filter

- Attenuation at cut-off=3dB
- Order of the filter N controls the cut-off rate
- Slope of cut-off k_d =-20N dB/decade or k_o =-6N dB/octave

Low-Pass Butterworth Filter Coefficients

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Microwave Filters



Attenuation versus normalized frequency for maximally flat filter prototypes.

Microwave Filters

Properties of the Chebysev filter

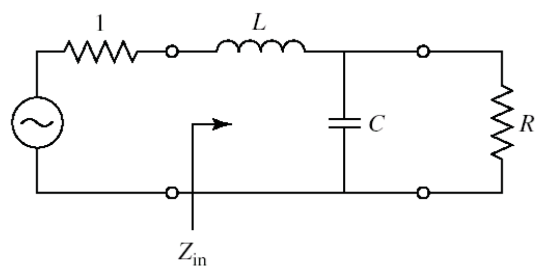
$$\text{if } \omega_c = 1 \Rightarrow P_{LR} = 1 + k^2 T_N^2(\omega)$$

$$T_N(0) = \begin{cases} 0 \text{ for N odd} \\ 1 \text{ for N Even} \end{cases}$$

Consider 2 elements filter prototype. impedance 1Ω and cutoff frequency of $\omega_c = 1$

$$P_{LR} = 1 + k^2\left(4\omega^4 - 4\omega^2 + 1\right) = 1 + \frac{1}{4R}\left[(1-R)^2 + \left(R^2C^2 + L^2 - 2LCR^2\right)\omega^2 + R^2C^2L^2\omega^4\right]$$

$$\begin{cases} k^2 = \dfrac{(1-R)^2}{4R} \\ 4k^2 = \dfrac{L^2C^2R^2}{4R} \\ -4k^2 = \dfrac{\left(C^2R^2 + L^2 - 2LCR^2\right)}{4R} \end{cases}$$



In General

$$\text{Ripple} = -10\log\left(1 + k^2\right) dB$$

$$\text{Insertion loss in the stopband} = 20\log(k) + 6(N-1) + 20N\log(\omega) dB$$

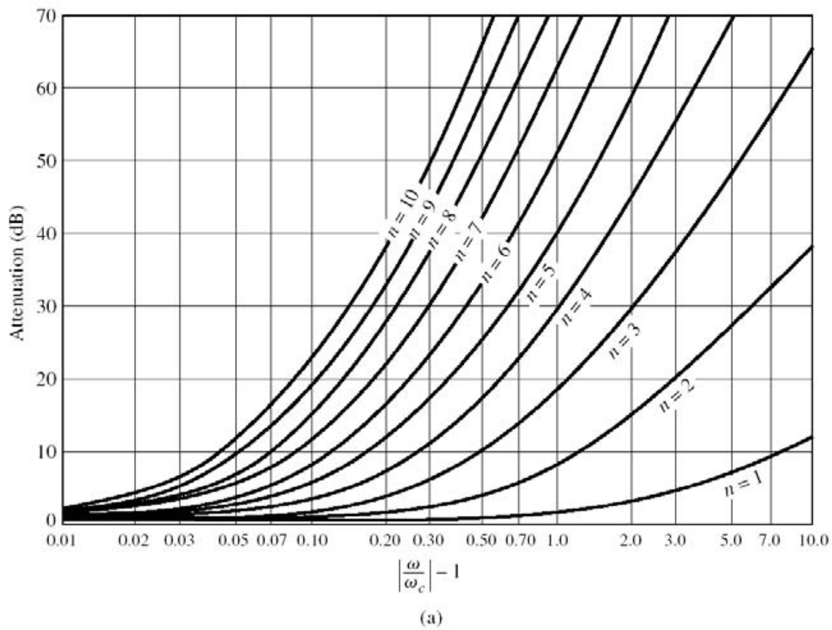
$$\text{at much higher freq than } \omega_c = 20N\log(\omega)\ dB$$

Microwave Filters

Low-Pass Chebysev Filter Coefficients – 0.5 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7939	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

Microwave Filters



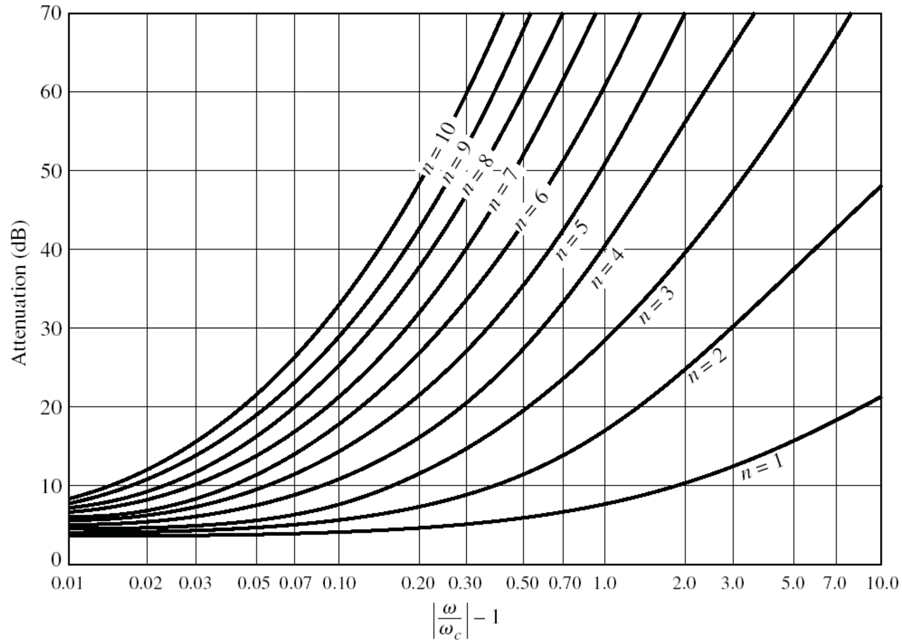
Attenuation versus normalized frequency for equal-ripple filter prototypes. (*a*) 0.5 dB ripple level.

Microwave Filters

Low-Pass Chebysev Filter Coefficients – 3 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Microwave Filters



(b)

Attenuation versus normalized frequency for equal-ripple filter prototypes. (*b*) 3.0 dB ripple level.

Microwave Filters

- The design procedures for any type of response are similar and follow broadly the same steps.

Filter specifications → Low-pass prototype → Scaling and conversion → Implementation

A typical filter design procedure (insertion method)

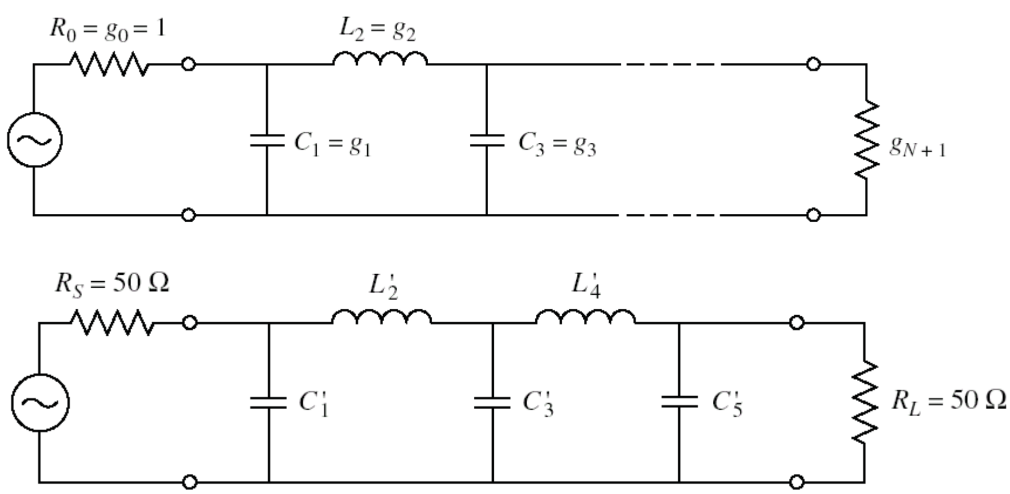
- The first step in filter design is to determine the element values of the 'Prototype Filter' from which all other filters are derived.

Microwave Filters

- The prototype filter is a low-pass filter with a cut-off frequency of 1 rad/sec and 1Ω terminations at both input and output .
- Impedance Transformation
  - To transform the filter for terminations other than 1Ω we multiply the resistances and the inductances by the required factor and divide the capacitors by the same factor.
  - This leaves the frequency response unchanged.

Microwave Filters

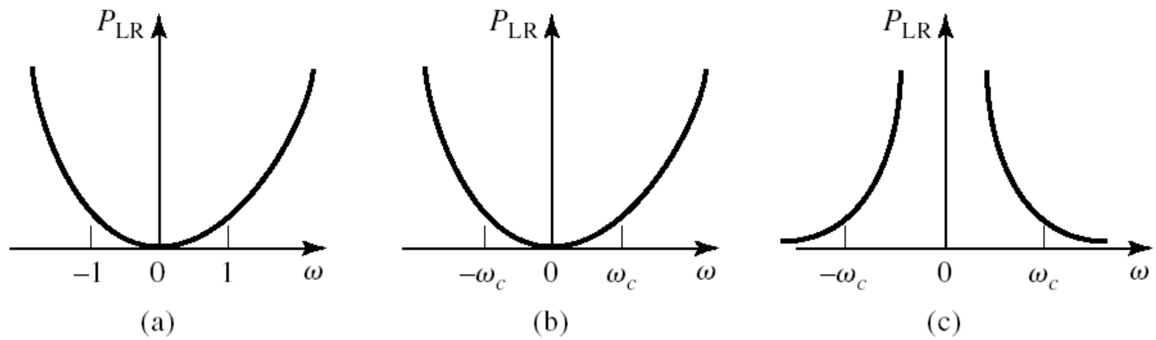
- **Impedance transformation**



Low-pass maximally flat filter circuit for Example 8.3.

Microwave Filters

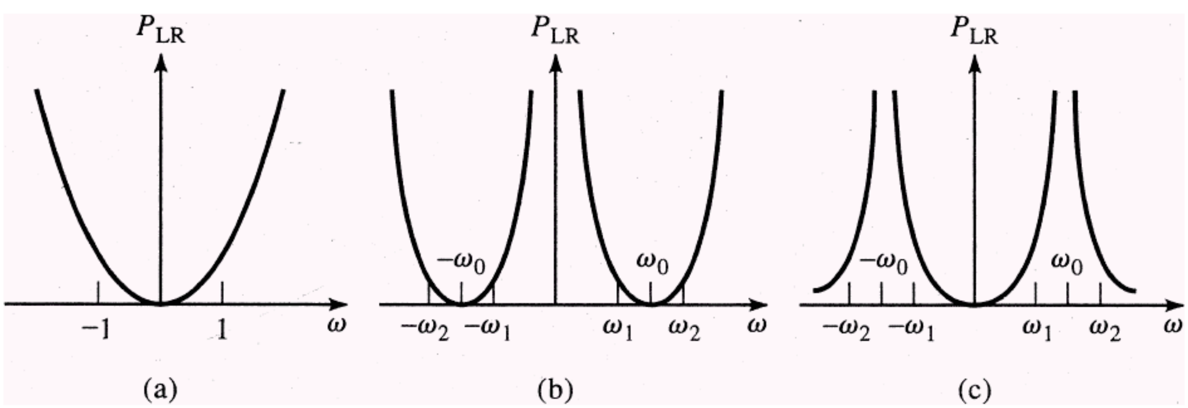
Frequency Transformation



Frequency scaling for low-pass filters and transformation to a high-pass response.

(*a*) Low-pass filter prototype response for $\omega_c = 1$.
(*b*) Frequency scaling for low-pass response.
(*c*) Transformation to high-pass response.

Microwave Filters



Bandpass and bandstop frequency transformation.

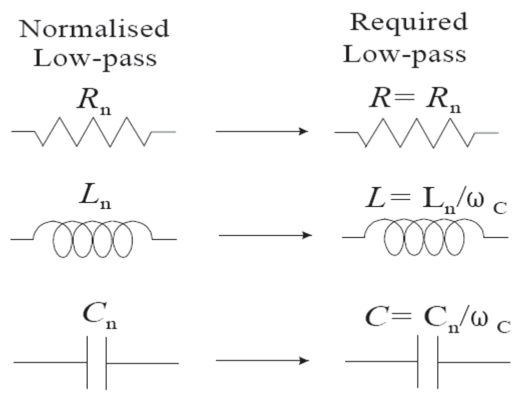
(*a*) Low-pass filter prototype response for $\omega_c = 1$.
(*b*) Transformation to bandpass response.
(*c*) Transformation to bandstop response.

Microwave Filters

Low-pass to Low-pass Transformation

- To change the cut-off frequency of the prototype filter from 1 rad/sec to ω_c rad/sec, we divide the values of all the inductors and capacitors by ω_c .
- This leaves the response as a low-pass and only changes the cut-off frequency.

$$\omega \leftarrow \frac{\omega}{\omega_c}$$

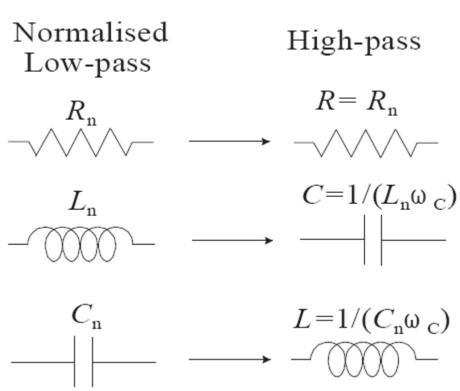


Microwave Filters

Low-pass to High-pass Transformation

- In this case we want to change the prototype response from a low-pass with a cut-off frequency of 1 rad/sec to a high-pass with a cut-off frequency of ω_C .
- Every inductor is replaced by a capacitor and every capacitor is replaced by an inductor.

$$\omega \leftarrow -\frac{\omega}{\omega_c}$$



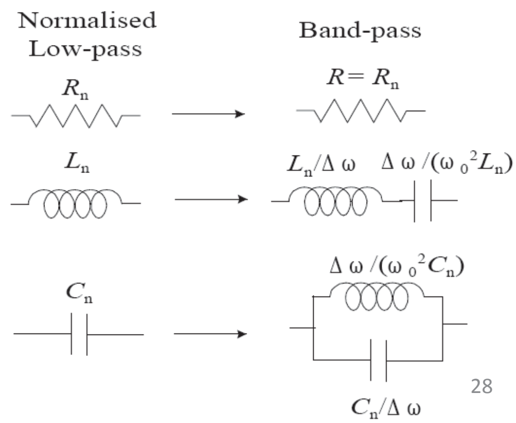
Microwave Filters

Low-pass to Band-pass Transformation

$$\omega \leftarrow \frac{\omega_o}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)$$

- In this case we want to change the prototype response from a low-pass with a cut-off frequency of 1 rad/sec to a band-pass with a band between ω_1 and ω_2 .
- Every inductor is replaced by a series resonance circuit and every capacitor is replaced by a parallel resonance circuit.

Where ω1 and ω2 are the band edges, ωo=sqrt(ω1ω2) and Δω= ω2 - ω1 In other words ωo is the centre frequency and Δω the pass bandwidth.



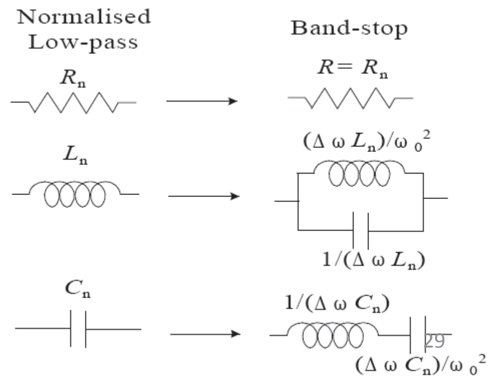
Microwave Filters

$$\omega \leftarrow -\frac{1}{\frac{\omega_o}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}$$

Low-pass to Band-stop Transformation

- In this case we want to change the prototype response from a low-pass with a cut-off frequency of 1 rad/sec to a band-stop between ω_1 and ω_2 .
- Every inductor is replaced by a parallel resonance circuit and every capacitor is replaced by a series resonance circuit.

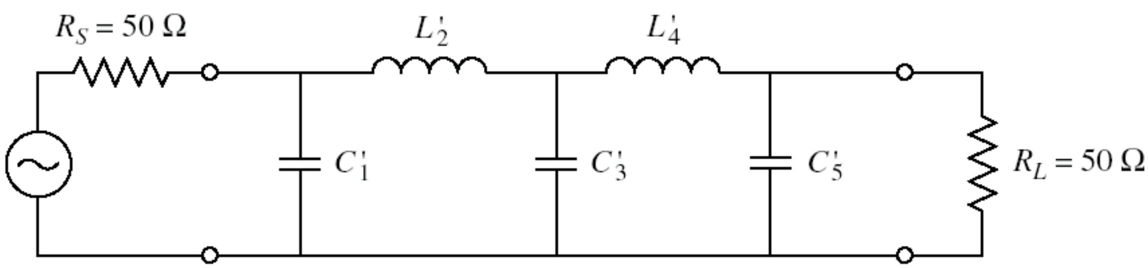
Where ω1 and ω2 are the band edges, ωo=sqrt(ω1ω2) and Δω= ω2-ω1 In other words ωo is the centre frequency and Δω the pass bandwidth.



Microwave Filters

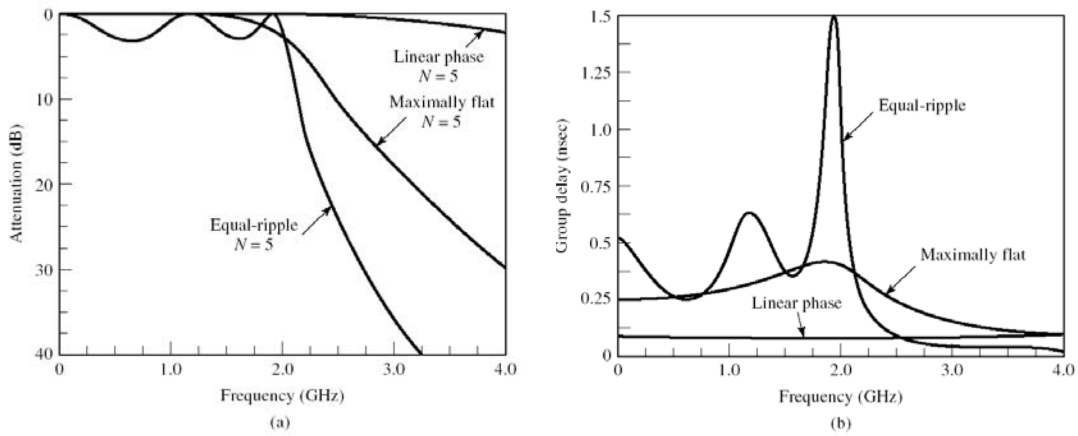
Design maximally flat low pass filter with a cutoff freg 2GHz, impedance 50ohm, at least 15dB insertion loss at 3GHz $\left|\frac{\omega}{\omega_c}\right| - 1 = 0.5$

N	g1	g2	g3	g4	g5	g6
1	2.0000	1.0000				
2	1.4142	1.4142	1.0000			
3	1.0000	2.0000	1.0000	1.0000		
4	0.7654	1.8478	1.8478	0.7654	1.0000	
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000



Low-pass maximally flat filter circuit for Example 8.3.

Microwave Filters



Frequency response of the filter design of Example 8.3.
(*a*) Amplitude response.
(*b*) Group delay response.

Microwave Filters

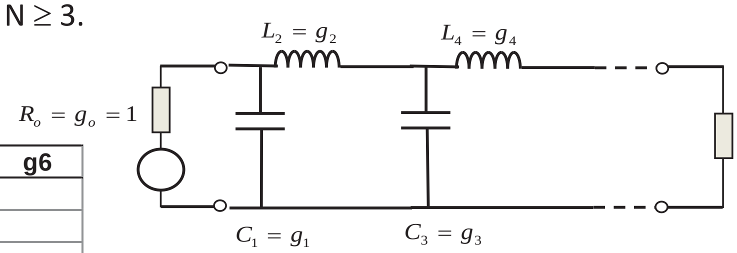
Maximally flat filter Example

Filter specifications → Low-pass prototype → Scaling and conversion → Implementation

Design a maximally flat low pass filter with cutoff frequency 8GHz and a minimum attenuation of 10dB at 11GHz. How many filter elements are required?

$$\frac{\omega}{2\pi} = 11GHz \qquad \frac{\omega_c}{2\pi} = 8GHz \qquad \left|\frac{\omega}{\omega_c}\right| - 1 = \frac{11}{8} - 1 = 0.375$$

From attenuation figure we read that N ≥ 3.



N	g1	g2	g3	g4	g5	g6
1	2.0000	1.0000				
2	1.4142	1.4142	1.0000			
3	1.0000	2.0000	1.0000	1.0000		
4	0.7654	1.8478	1.8478	0.7654	1.0000	
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000

Microwave Filters

Maximally flat filter Example

Filter Transformations

Filter specifications → Low-pass prototype → Scaling and conversion → Implementation

Impedance scaling

$$L' = R_o L$$

$$C' = \frac{C}{R_o}$$

$$R_S' = R_o$$

$$R_L' = R_0 R_L$$

Frequency scaling

Low pass

$$\omega \leftarrow \frac{\omega}{\omega_c}$$

$$L_k' = R_o \frac{L_k}{\omega_c}$$

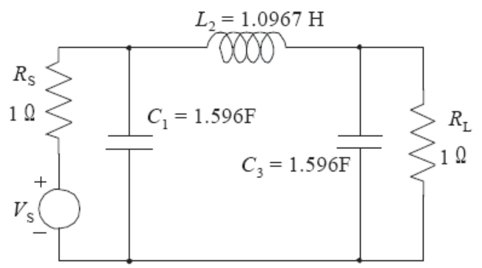
$$C_k' = \frac{C}{R_o \omega_c}$$

Microwave Filters

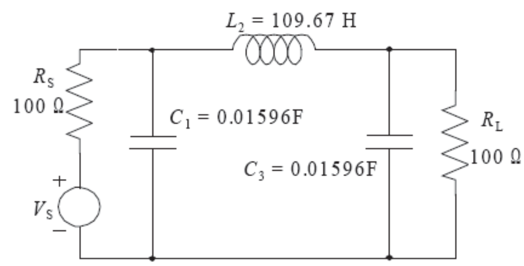
Equal ripple filter Example

Specification for a band-pass Chebyshev filter
Terminations $R_1 = R_2 = 100\ \Omega$, Maximum ripple in pass-band = 0.5 dB
Cut-off frequencies = 1 GHz and 1.5 GHz, Minimum slope of cut-off = -50dB/decade

Design Steps
First we determine the order n of the filter from $20n = 50$ or $n = 2.5$. We take $n = 3$.
From tables or by applying the formula we get the values of the prototype filter.



Prototype Filter

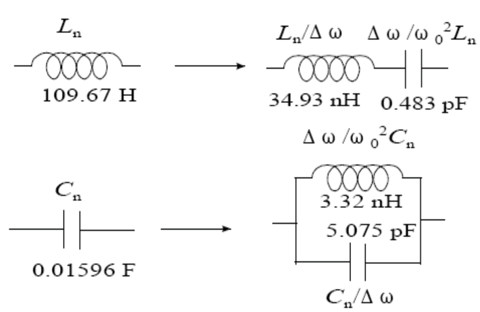


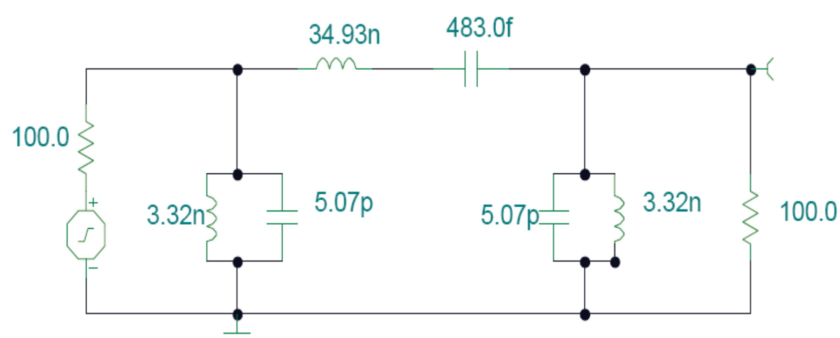
After impedance transformation

Next we apply the impedance transformation by multiplying the resistors and inductors by 100 and dividing the capacitors by 100.

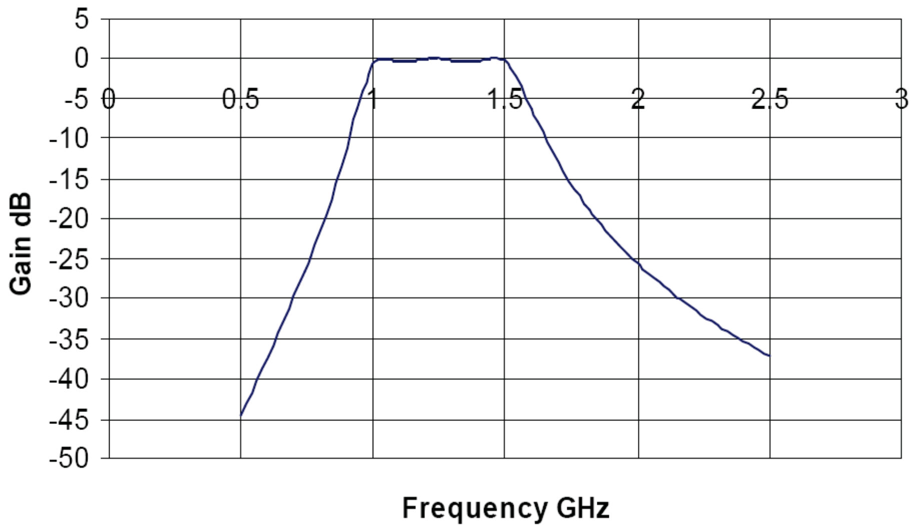
Microwave Filters

Now we apply the low-pass to band-pass transformation
$\omega_0^2 = \omega_1\,\omega_2 = 59.2 \times 10^{18}$ and $\Delta\omega = \omega_2 - \omega_1 = 3.14 \times 10^9$
The resistors remain unchanged but we transform the capacitors and inductors.





Microwave Filters



Equal ripple filter Example

Filters Implementation

Filters Implementation

Microwave Filters

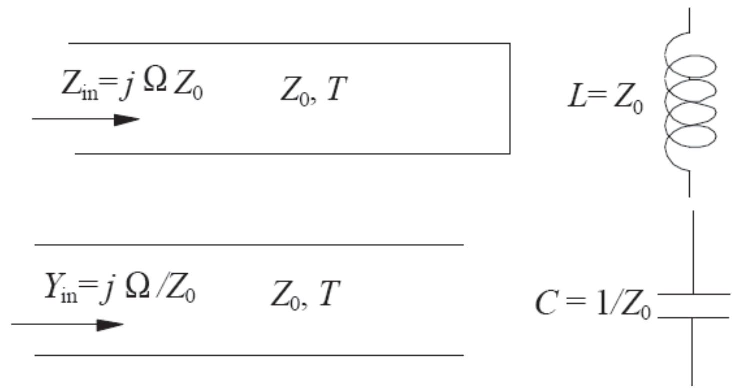
(Richard transformation)

The Short and Open Stub Filter

The inductors and capacitors of the lumped equivalent filter can be transformed to distributed elements using the relations we derived earlier. This is done after performing the impedance and frequency transformations on the prototype filter.

Short Circuited Stub
Frequency Ω not ω

Open Circuited Stub
Frequency Ω not ω

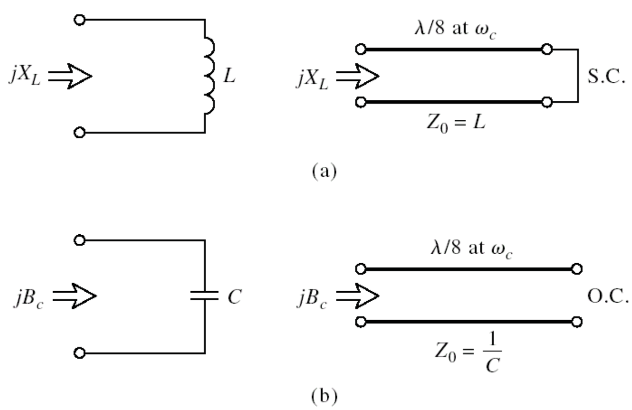


$$\omega \to \Omega \Rightarrow \begin{cases} \text{the reactance of the inductor } jX_L = j\omega L = j\Omega L \to \text{can be obtained SC stub} \\ \text{the susceptance of the capacitor } jB_C = j\omega C = j\Omega L \to \text{can be obtained OC stub} \end{cases}$$

$$\Omega = \tan\beta l = \tan\left(\frac{\omega l}{v_p}\right) \to \Omega = 1 \text{ if } l = \lambda/8$$

Microwave Filters

Richard's transformation. (*a*) For an inductor to a short-circuited stub. (*b*) For a capacitor to an open-circuited stub.

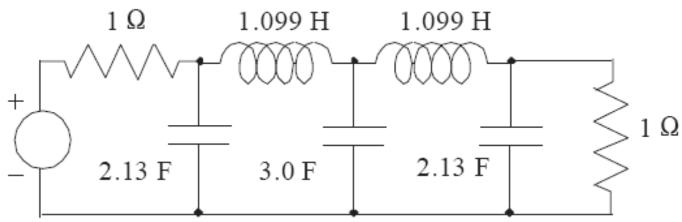


Microwave Filters

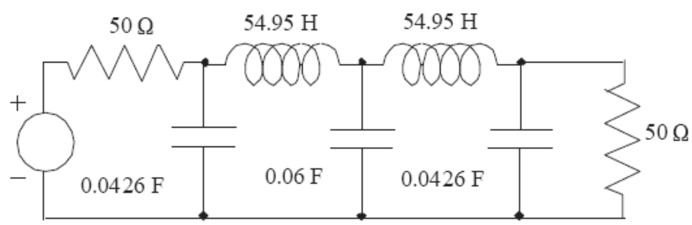
Example

As an example, we want to design a low-pass 5th order filter with a cut-off frequency of 5 GHz, a pass-band ripple of 1 dB and 50 Ω terminations.

We use the tables to get element values of the prototype

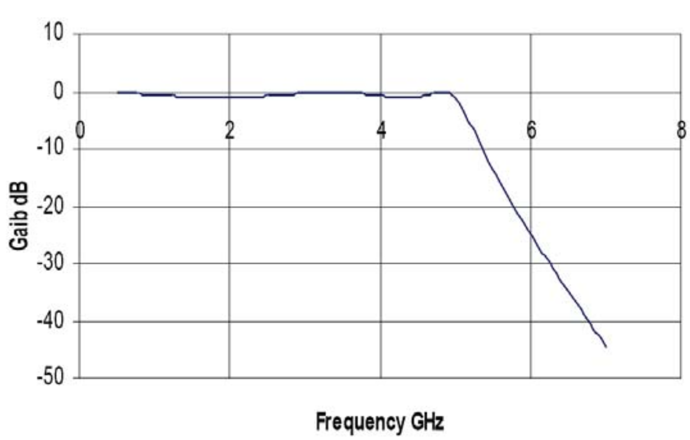


Next we perform the impedance transformation to achieve 50 Ω terminations



Microwave Filters

Next we derive the microwave filter with open-circuited and short-circuited stubs. All stubs will have a delay of 25 ps. Each 0.0426 F capacitor is replaced by an open-circuited stub with $Z_0 = 1/C = 23.47\ \Omega$. The 0.06 F capacitor is replaced by an open- circuited stub with $Z_0 = 1/C = 16.66\ \Omega$ and each 54.95 H inductor is replaced by a Short circuited stub with $Z_0 = L = 54.95\ \Omega$.



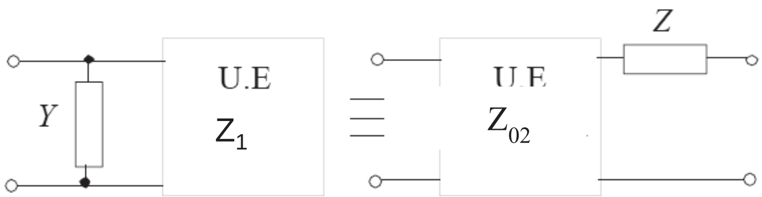
Microwave Filters

Kuroda Identity

The last example will be difficult to implement in practice for the following reasons:

- All the stubs are effectively connected to the same physical point,
- Practical distributed elements, such as microstrip, must have one earth terminal.
- impractical characteristic impedances into more realizable ones

These problems are overcome by employing the Kuroda identities to transform the filter into a practical realisation. In most microwave filters, it is customary to use transmission lines with the same delay *T*. Such circuits are called *commensurate* circuits and each transmission line is called a *unit element* (UE). The TL length is λ/8



(a) (b)

The Kuroda identities prove that the two circuits shown are identical.
The ABCD matrix of the circuit on the left-hand side is given by

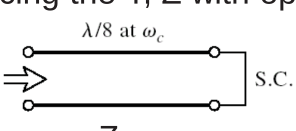
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} 1 & j\Omega Z_1 \\ \dfrac{j\Omega}{Z_1} & 1 \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 & j\Omega Z_1 \\ Y + \dfrac{j\Omega}{Z_1} & 1 + j\Omega Z_1 \end{bmatrix} \quad \Omega = \tan \beta l$$

Microwave Filters

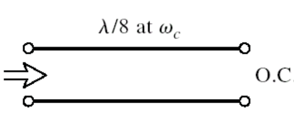
And for the circuit in the right-hand side

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 & j\Omega Z_{02} \\ \dfrac{j\Omega}{Z_{02}} & 1 \end{bmatrix} \times \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} = \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 & j\Omega Z_{02} + Z \\ \dfrac{j\Omega}{Z_{02}} & \dfrac{j\Omega Z}{Z_{02}} + 1 \end{bmatrix}$$

Replacing the Y, Z with open-circuited and short-circuited stubs respectively

$Z = j\Omega Z_{01} \Rightarrow$ 

Z_{01}

$Y = j\Omega / Z_2 \Rightarrow$ 

Z_2

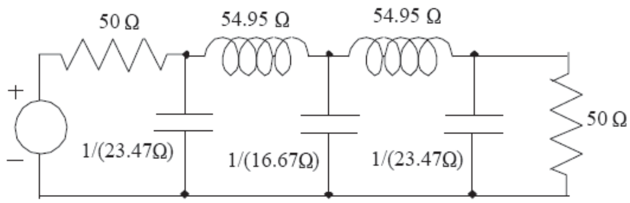
$$\Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix}_L = \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 & j\Omega Z_1 \\ \dfrac{j\Omega}{Z_1} + \dfrac{j\Omega}{Z_2} & 1 - \Omega^2 \dfrac{Z_1}{Z_2} \end{bmatrix} \quad \Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix}_R = \frac{1}{\sqrt{1+\Omega^2}} \begin{bmatrix} 1 & j\Omega (Z_{01} + Z_{02}) \\ \dfrac{j\Omega}{Z_{02}} & 1 - \Omega^2 \dfrac{Z_{01}}{Z_{02}} \end{bmatrix}$$

The two circuits are identical if $\begin{bmatrix} A & B \\ C & D \end{bmatrix}_L = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_R \Rightarrow \begin{cases} Z_{02} = Z_2 / n^2 \\ Z_{01} = Z_1 / n^2 \end{cases}$

Where $n^2 = 1 + \dfrac{Z_2}{Z_1}$

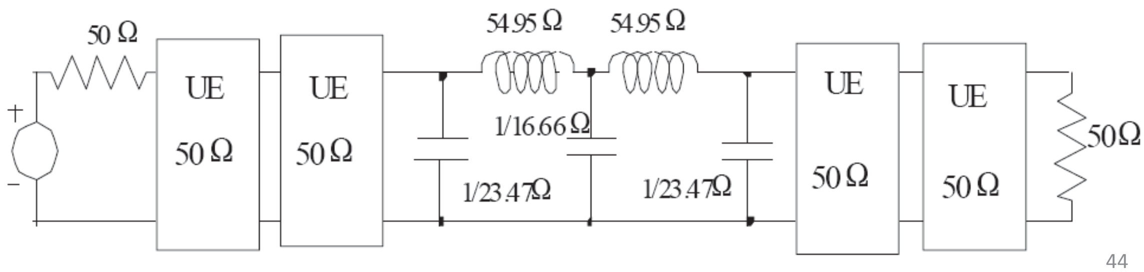
Microwave Filters

Example



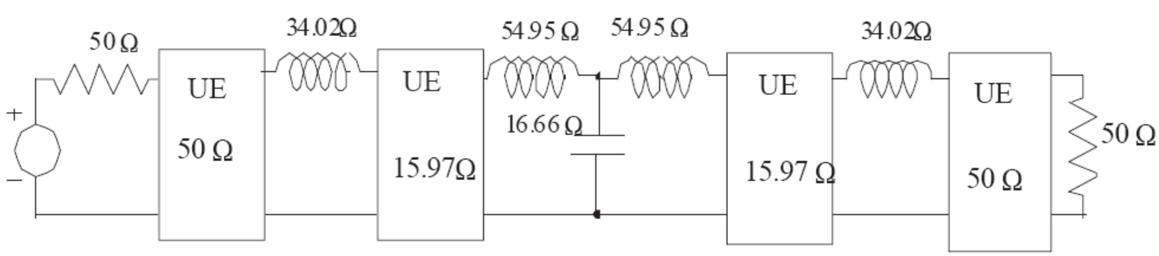
We apply the Kuroda identities to the last example. The filter is redrawn and for convenience we have drawn short circuited stubs as inductors and open circuited stubs as capacitors. The values are the characteristic impedances of the stubs.

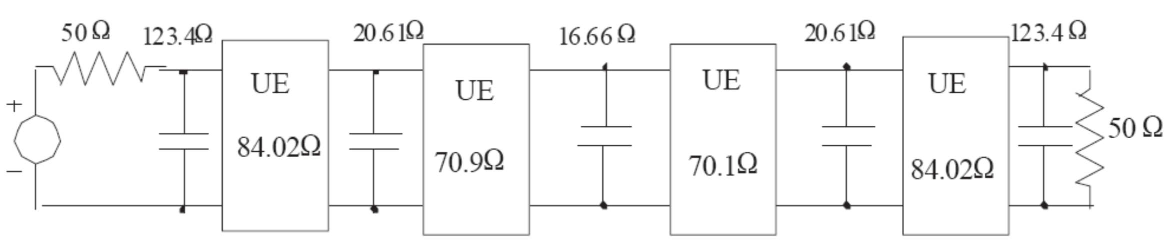
Next we add unit elements at both the input and the output all with a characteristic impedance of 50 Ω. These unit elements will not alter the response of the filter but will only introduce an extra delay or phase shift between input and output.



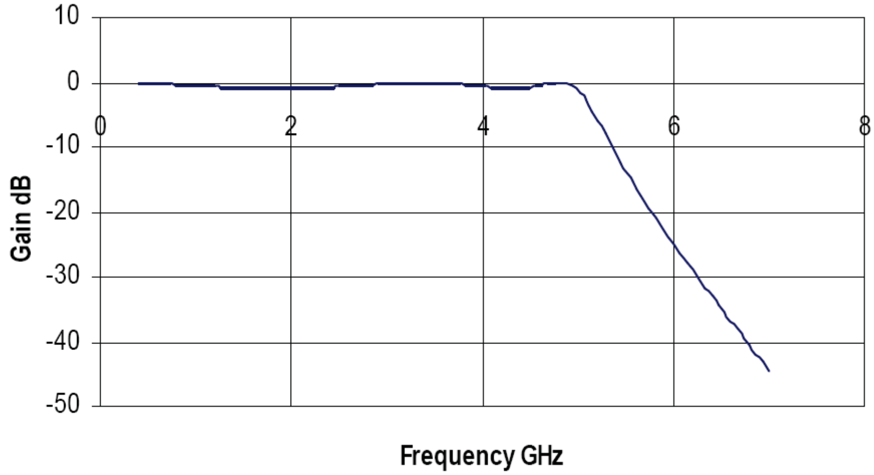
Microwave Filters

Next we apply the Kuroda identities to transform a circuit consisting only of short-circuited stubs and unit elements. The Kuroda identities enable us to calculate the element values.





Microwave Filters



Microwave Filters

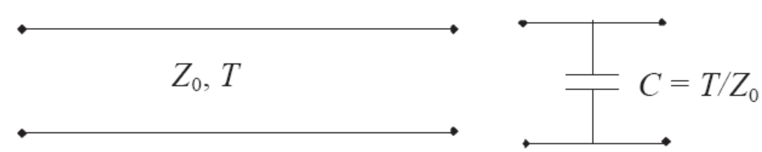
Stepped Impedance LPF

Microwave filters using distributed elements (transmission lines, waveguides ..etc) are also derived from prototype filters. There are several possible topologies and design methods. We shall consider some of these methods.

Low-Pass Filters using Cascaded lines

We shall make use of the equivalence between short cascade transmission lines and lumped elements

Cascade line
$l<<\lambda$ and
Zo<<1



Cascade line
$l<<\lambda$ and
Zo>>1

The design procedure starts by finding the prototype filter, then applying the impedance and frequency transformations and finally deriving the equivalent transmission lines.

Microwave Filters

Short section of high impedance line → Inductor

Short section of high impedance line → Capacitor

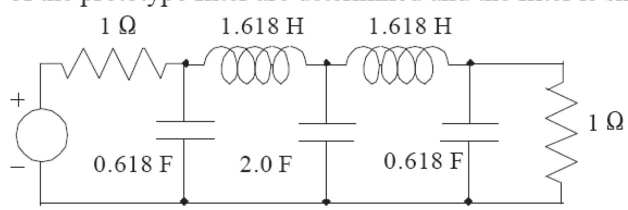
Microwave Filters

Example

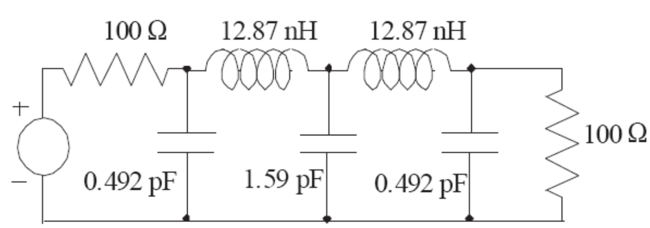
We are required to design a low-pass Butterworth fiter with a cut-off frequency of 2 GHz. The slope of the cut-off should be greater than 90 dB/decade and the terminations are 100 Ω.

We first determine the order of the filter from $k_d = -20\ n = 90$, which gives $n = 4.5$. We take $n = 5$.

The elements of the prototype filter are determined and the filter is shown below.



We need to increase the impedances by a factor n_z of 100 and the frequency by a factor $n_f = 2\pi \times 2 \times 10^9$. The resistors are multiplied by n_z , the inductors by n_z / n_f and the capacitors by $1/(n_z n_f)$. The denormalised filter is shown below.



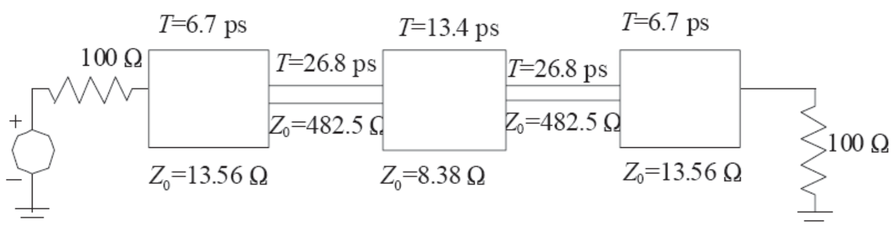
Microwave Filters

Next we derive the transmission line filter. The wavelength at 2 GHz is 15 cm and since The transmission line lengths should be less than $\lambda/10$, we take the lengths as 0.2 cm. This gives a delay $T = 0.2/(3\times10^{10}) = 6.67$ ps. We assumed that the wave travels with the free space velocity of 3×10^{10} cm/sec. We can now calculate the characteristic impedances of the equivalent transmission lines.

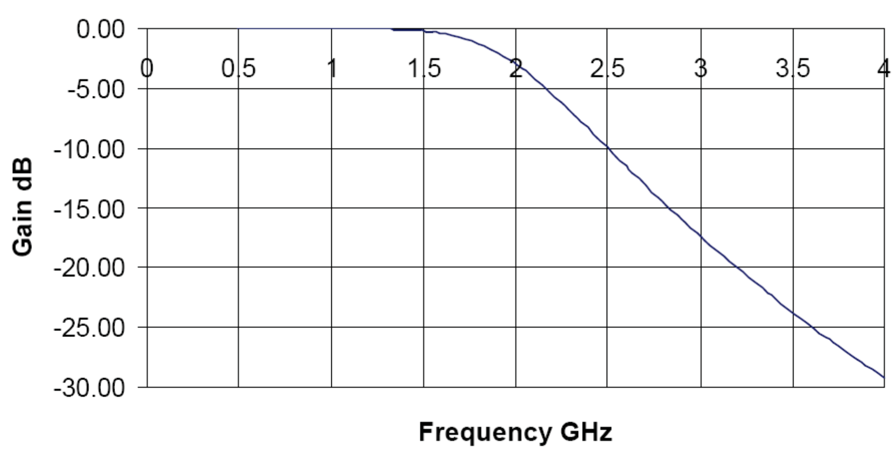
For the capacitor of 0.492 pF $Z_0 = T/C = 13.56\ \Omega$
For the capacitor of 1.59 pF $Z_0 = T/C = 4.12\ \Omega$
For the inductor of 12.87 nH $Z_0 = L/T = 1930\ \Omega$

The values of 1930 and 4.12 are not practical. We then change the delay T for these elements
For the capacitor of 1.59 pF we take T = 13.4 ps and $Z_0 = T/C = 8.24\ \Omega$
For the inductor of 12.87 nH we take T = 26.8 ps and $Z_0 = L/T = 482.5\ \Omega$

The derived transmission line filter is shown below.



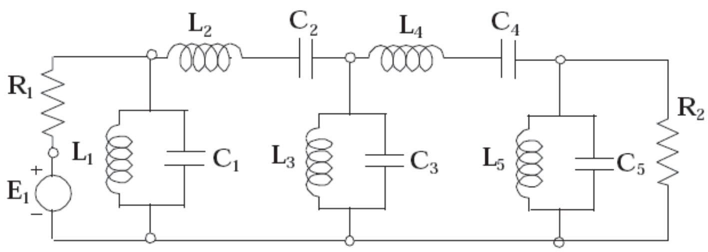
Microwave Filters



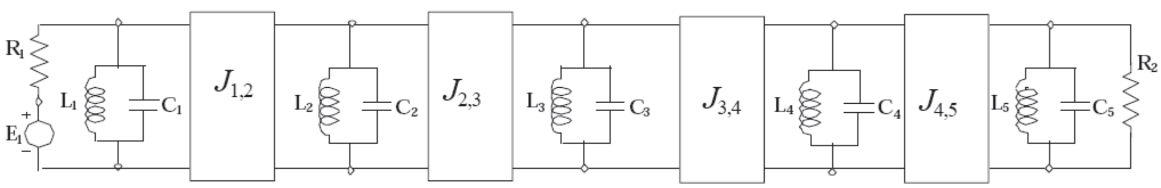
Microwave Filters

Couples Line Filters

So far the microwave filters we designed were low-pass filters. We want to consider band-pass and band-stop microwave filters. The lumped band-pass filter has the following topology

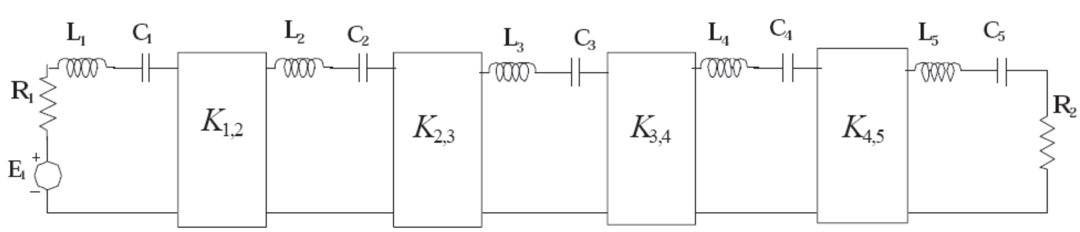


In applying the Kuroda identities we saw how the unit element (U.E) acted as an impedance inverter by converting a series impedance to a parallel admittance and vice versa. If we assume that we can apply admittance inverters to the above circuit then we can represent all the series resonators by parallel resonators.



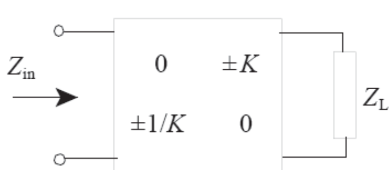
Microwave Filters

We can also use impedance inverters to represent the parallel admittances by series impedances.

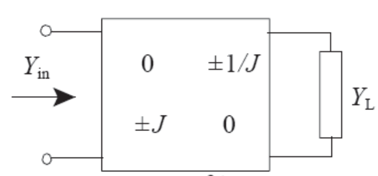


The ABCD matrices for the impedance and admittance inverters are given by

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & \pm K \\ \pm \dfrac{1}{K} & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & \pm \dfrac{1}{J} \\ \pm J & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$



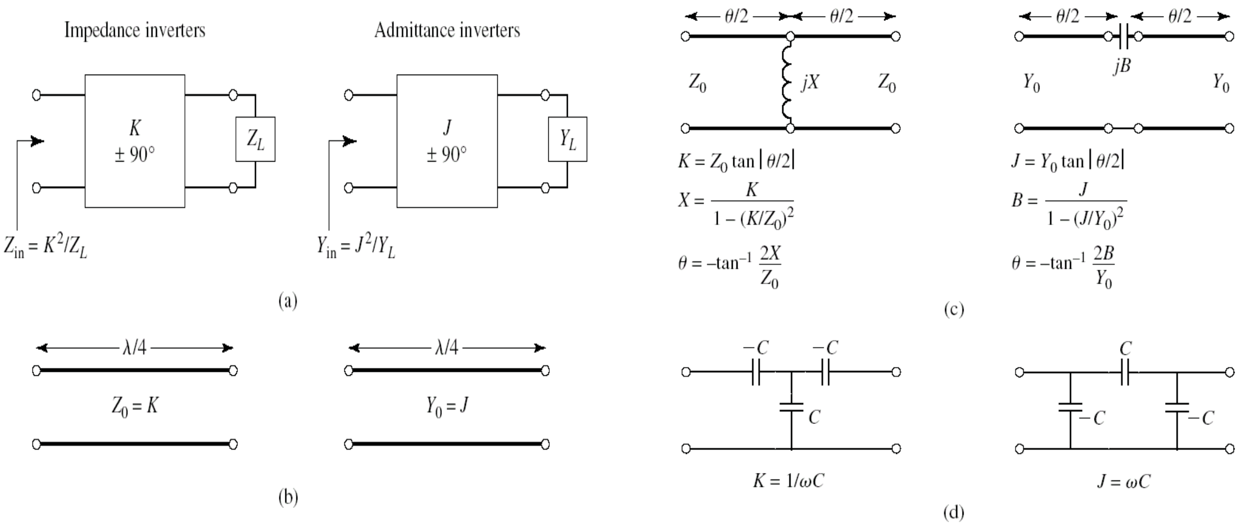
$Z_{in} = K^2/Z_L$



$Y_{in} = J^2/Y_L$

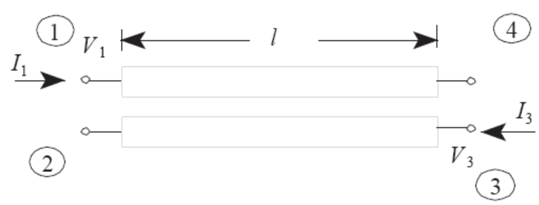
The are various forms of implementing impedance and admittance converters. We shall consider the edge coupled transmission line as an admittance converter for use in filter design.

Microwave Filters



Impedance and admittance inverters. (*a*) Operation of impedance and admittance inverters. (*b*) Implementation as quarter-wave transformers. . (c) Implementation using transmission lines and reactive elements. (d) Implementation using capacitor networks.

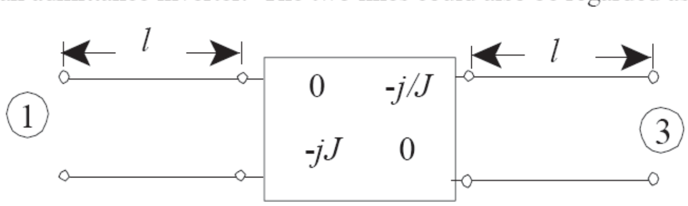
Microwave Filters



When ports 2 and 4 are left open-circuited, The structure will consist of two transmission lines in cascade with a coupling gap between them. The ABCD matrix between ports 1 and 4 is given by

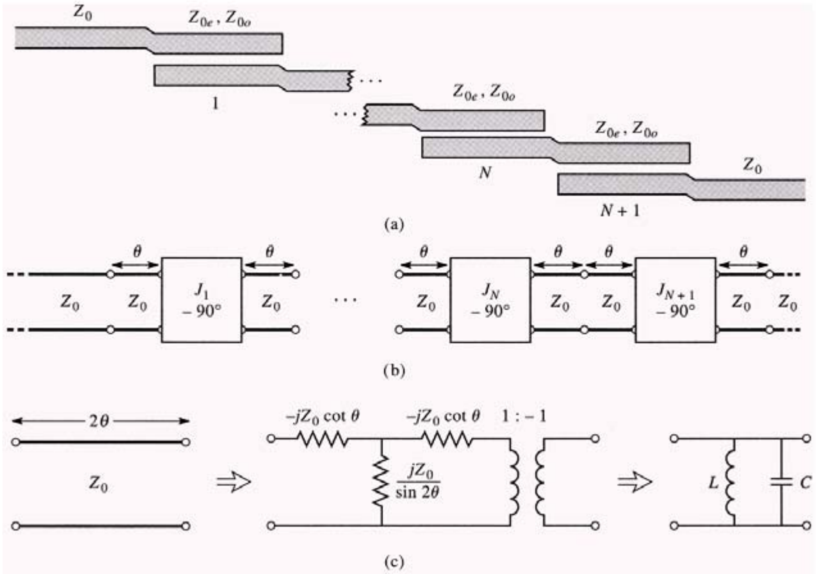
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos(\theta) & jZ_0\sin(\theta) \\ \dfrac{j\sin(\theta)}{Z_0} & \cos(\theta) \end{bmatrix} \begin{bmatrix} 0 & -j/J \\ -jJ & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta) & jZ_0\sin(\theta) \\ \dfrac{j\sin(\theta)}{Z_0} & \cos(\theta) \end{bmatrix}$$

An equivalent circuit of the structure is shown below. It consists of two transmission lines and an admittance inverter. The two lines could also be regarded as two resonators.



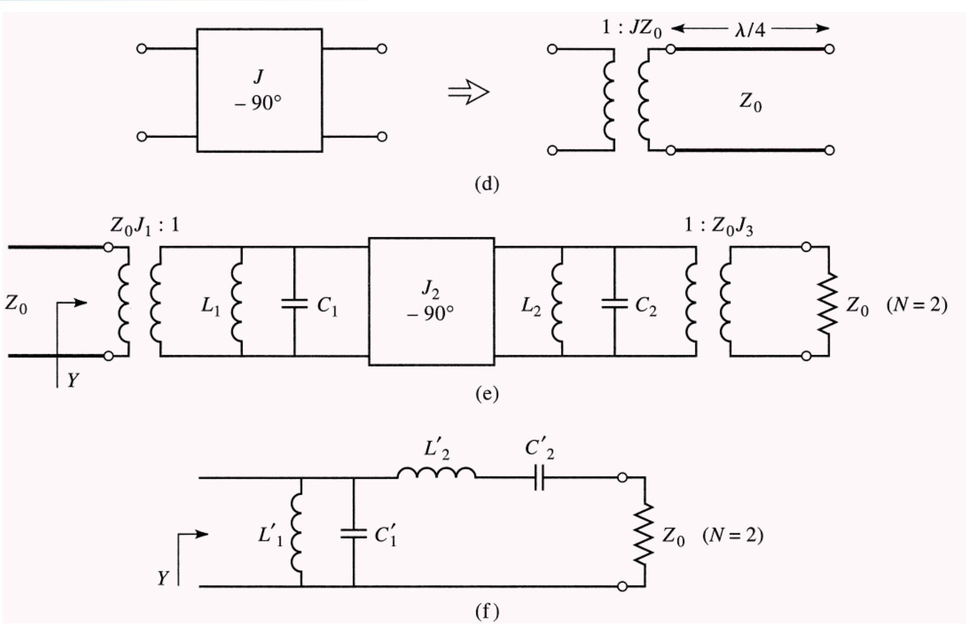
This equivalent circuit is ideal for coupled resonator filters.

Microwave Filters



Development of an equivalent circuit for derivation of design equations for a coupled line bandass filter. (*a*) Layout of an N + 1 section coupled line bandpass filter.
(*b*) Using equivalent circuit of Figure 8.44 for each coupled line section.
(*c*) Equivalent circuit for transmission lines of length 2θ.

Microwave Filters



Development of an equivalent circuit for derivation of design equations for a coupled line bandass filter. (*d*) Equivalent circuit for the admittance inverters. (*e*) Using results of (*c*) and (*d*) for the N = 2 case. (*f*) Lumped-element circuit for a bandpass filter for N = 2.

Microwave Filters

Design Steps for Coupled-Line Filters

1. Determine the order of the filter and obtain the prototype values.
2. Determine the required fractional bandwidth from

$$\Delta\omega = \frac{\omega_2 - \omega_1}{\omega_0} \quad \text{where} \quad \omega_0 = \frac{\omega_1 + \omega_2}{2}$$

3. Determine the coefficients of the admittance inverters from

$$J_{0,1} = \frac{1}{Z_0}\sqrt{\frac{\pi\Delta\omega}{2g_0 g_1}} \qquad J_{i,i+1} = \frac{1}{Z_0}\frac{\pi\Delta\omega}{\sqrt{4g_i g_{i+1}}} \qquad J_{N,N+1} = \frac{1}{Z_0}\sqrt{\frac{\pi\Delta\omega}{2g_N g_{N+1}}}$$

4. Determine the odd and even characteristic impedances of the coupled sections from

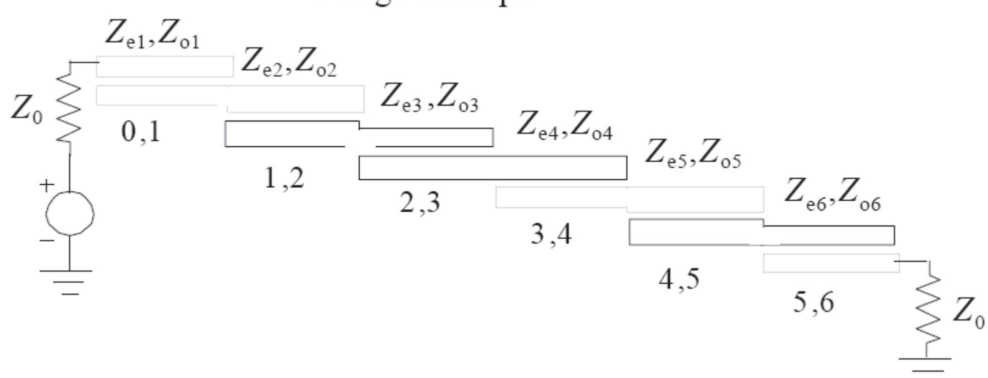
$$Z_{0o}\big|_{i,i+1} = Z_0\left[1 - Z_0 J_{i,i+1} + \left(Z_0 J_{i,i+1}\right)^2\right]$$

$$Z_{0e}\big|_{i,i+1} = Z_0\left[1 + Z_0 J_{i,i+1} + \left(Z_0 J_{i,i+1}\right)^2\right]$$

Note the length of each section is $\lambda/4$ at the centre frequency.

Microwave Filters

Design Example



We want to design a fifth order Chebyshev coupled-line filter in a 50 Ω system for a pass-band between 9.5 and 10.5 GHz.
The lengths of the sections are all $\lambda/4$ and at the centre frequency of 10 GHz this corresponds to a delay of 25 ps.

$Z_{e1} = Z_{e5} = 75.35\ \Omega$, $Z_{o1} = Z_{o5} = 38.34\ \Omega$ $\quad$ $Z_{e2} = Z_{e5} = 57.05\ \Omega$, $Z_{o2} = Z_{o5} = 44.52\ \Omega$
$Z_{e3} = Z_{e5} = 55.23\ \Omega$, $Z_{o3} =\ Z_{o4}\ \ 45.68\ \Omega$
The terminations $Z_0 = 50$.
Note that the number of sections exceeds the order of the filter by 1.

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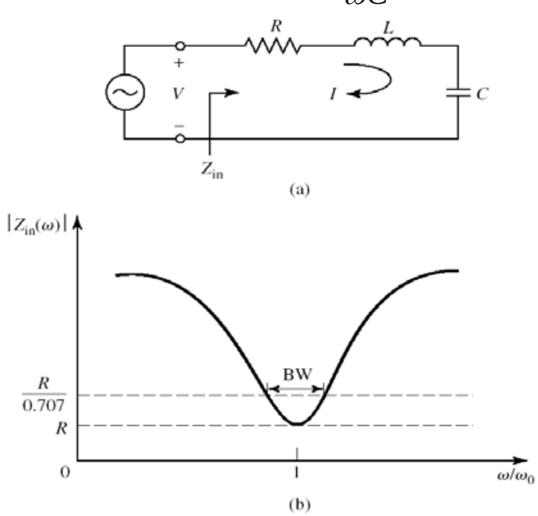
Series Resonators

$$Z_{in} = R + j\omega L - j\frac{1}{\omega C}$$



A series *RLC* resonator and its response. (*a*) The series RLC circuit. (*b*) The input impedance magnitude versus frequency.

Resonance occurs when the average stored magnetic and electric energies are equal and Z_{in} is purely real

$$Q = \omega\frac{\text{Time average energy stored}}{\text{Energy lost per second}} = \omega\frac{W_m + W_e}{P_l}$$

$$Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC} \qquad \omega_o = \frac{1}{\sqrt{LC}}$$

Near the resonance $\omega = \omega_o + \Delta\omega$

$$Z_{in} = R + j\omega L\left(1 - \frac{1}{\omega^2 LC}\right) = R + j\omega L\left(\frac{\omega^2 - \omega_o{}^2}{\omega^2}\right)$$

$$= R + j\frac{2RQ\Delta\omega}{\omega_o} \text{ where } \omega^2 - \omega_o^2 \approx 2\omega\Delta\omega \text{ for small } \Delta\omega$$

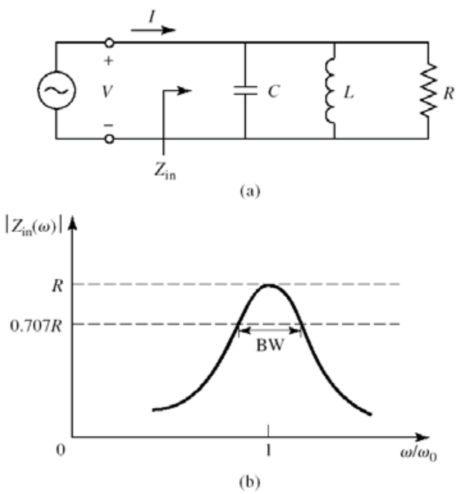
BW=resonator’s half power fractional bandwidth

$$|Z_{in}|^2 = 2R^2$$

$$\frac{BW}{2} = \Delta\omega / \omega_o \Rightarrow BW = 1/Q$$

Parallel Resonators

$$Z_{in} = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C \right)^{-1}$$



A parallel RLC resonator and its response. (*a*) The parallel RLC circuit. (*b*) The input impedance magnitude versus frequency.

Resonance occurs when the average stored magnetic and electric energies are equal and Z_{in} is purely real

$$\omega_o = \frac{1}{\sqrt{LC}} \qquad Q = \frac{R}{\omega_o L} = \omega_o RC$$

$$Q = \omega \frac{\text{Time average energy stored}}{\text{Energy lost per second}}$$

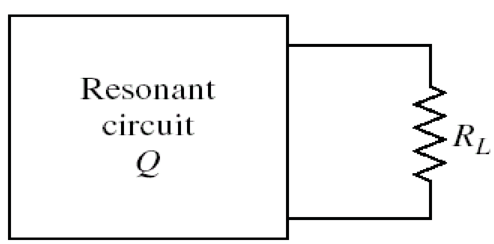
$$BW = 1/Q$$

Near the resonance $\omega = \omega_o + \Delta\omega$

$$Z_{in} \approx \left(\frac{1}{R} + 2j\Delta\omega C \right)^{-1} = \frac{R}{1 + 2jQ\Delta\omega/\omega_o}$$

$$R \to \infty \qquad Z_{in} = \frac{1}{j2C(\omega - \omega_o)}$$

Loaded Q Factor



A resonant circuit connected to an external load, R_L.

$$Q_e = \begin{cases} \dfrac{\omega_o L}{R_L} & \text{for series connection} \Rightarrow \text{effective Resistance} = R_L + R \\ \dfrac{R_L}{\omega_o L} & \text{for parallel connection} \Rightarrow \text{effective Resistance} = \dfrac{R_L R}{R_L + R} \end{cases}$$

$$\frac{1}{Q_L} = \frac{1}{Q} + \frac{1}{Q_e}$$

Microwave Resonators

TABLE 6.1 Summary of Results for Series and Parallel Resonators

Quantity	Series Resonator	Parallel Resonator
Input Impedance/admittance	$Z_{in} = R + j\omega L - j\frac{1}{\omega C}$	$Y_{in} = \frac{1}{R} + j\omega C - j\frac{1}{\omega L}$
	$\simeq R + j\frac{2RQ\Delta\omega}{\omega_0}$	$\simeq \frac{1}{R} + j\frac{2Q\Delta\omega}{R\omega_0}$
Power loss	$P_{loss} = \frac{1}{2}\|I\|^2 R$	$P_{loss} = \frac{1}{2}\frac{\|V\|^2}{R}$
Stored magnetic energy	$W_m = \frac{1}{4}\|I\|^2 L$	$W_m = \frac{1}{4}\|V\|^2\frac{1}{\omega^2 L}$
Stored electric energy	$W_e = \frac{1}{4}\|I\|^2\frac{1}{\omega^2 C}$	$W_e = \frac{1}{4}\|V\|^2 C$
Resonant frequency	$\omega_0 = \frac{1}{\sqrt{LC}}$	$\omega_0 = \frac{1}{\sqrt{LC}}$
Unloaded Q	$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$	$Q = \omega_0 RC = \frac{R}{\omega_0 L}$
External Q	$Q_e = \frac{\omega_0 L}{R_L}$	$Q_e = \frac{R_L}{\omega_0 L}$

Transmission Line Resonators

$$Z_{in} = jZ_o \tanh(\alpha + j\beta)l = Z_o \frac{\tanh \alpha l + j\tan\beta l}{1 + j\tanh\alpha l \tan\beta l}$$

if we assume that $\alpha l \ll 1$ so that $\tanh \alpha l \approx \alpha l = 0$

$$\tan\beta l = \tan(\frac{\omega l}{v_p}) = \tan(\frac{\omega_o l}{v_p} + \frac{\Delta\omega l}{v_p}) = \tan(\pi + \frac{\Delta\omega\pi}{\omega_o}) \approx \frac{\Delta\omega\pi}{\omega_o}$$

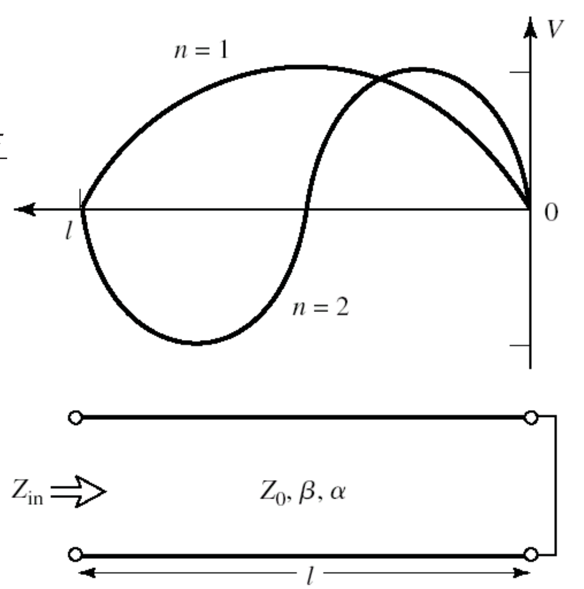
$$\Rightarrow Z_{in} \approx Z_o \frac{\alpha l + j\left(\frac{\Delta\omega\pi}{\omega_o}\right)}{1 + j\alpha l\left(\frac{\Delta\omega\pi}{\omega_o}\right)} \approx Z_o\left(\alpha l + j\left(\frac{\Delta\omega\pi}{\omega_o}\right)\right)$$

since $\left(\frac{\Delta\omega\alpha l}{\omega_o}\right) \ll 1$

$Z_{in} \approx R + 2jL\Delta\omega$

series RLC resonator equivalent circuit

$$R = Z_o\alpha l;\quad L = \frac{Z_o\pi}{2\omega_o}; C = \frac{1}{\omega_o^2 L} \qquad Q = \frac{\omega_o L}{R} = \frac{\beta}{2\alpha}$$



A short-circuited length of lossy transmission line, and the voltage distributions resonators.

for n=1 ($\ell = \lambda/2$) and n=2 ($\ell = \lambda$)

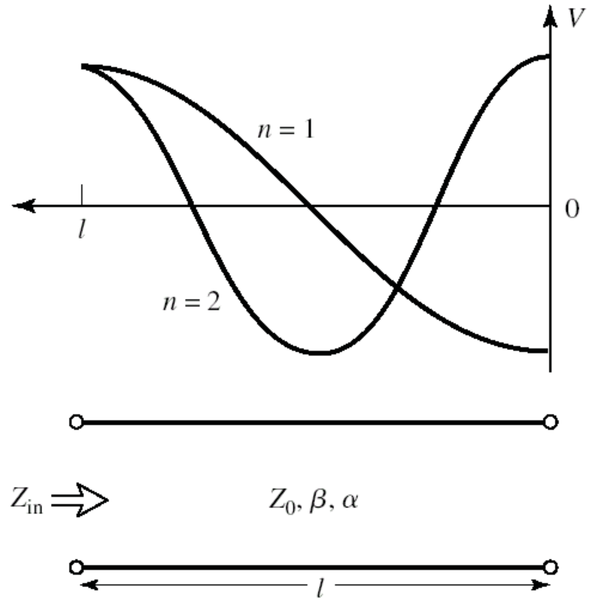
Transmission Line Resonators

$$Z_{in} \approx \frac{Z_o}{\alpha l + j(\Delta\omega\pi / \omega_o)}$$

Parallel RLC resonator equivalent circuit

$$R = Z_o / \alpha l; \quad L = \frac{1}{\omega_o^2 C}; C = \frac{\pi}{2 Z_o \omega_o}$$

$$Q = \frac{\beta}{2\alpha}$$



An open-circuited length of lossy transmission line, and the voltage distributions for n = 1 resonators.

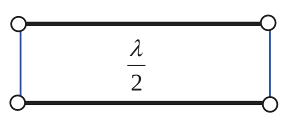
Rectangular Waveguide Cavity Resonator

We can look at them as short circuit section of transmission line

m,n th TE or TM mode propagation constant

$$\beta_{mn} = \sqrt{\omega^2 \mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$E_t(x,y,z) = e(x,y)\left[A^+ e^{-j\beta_{mn} z} + A^- e^{j\beta_{mn} z}\right]$$



Boundry conditions enforced

$$E_x = E_y = 0 \text{ for } z = 0, d$$

$$\Rightarrow \beta_{mn} d = l\pi \qquad \rightarrow \beta_{mn} = \frac{l\pi}{d} \qquad l = 1,2,3...$$

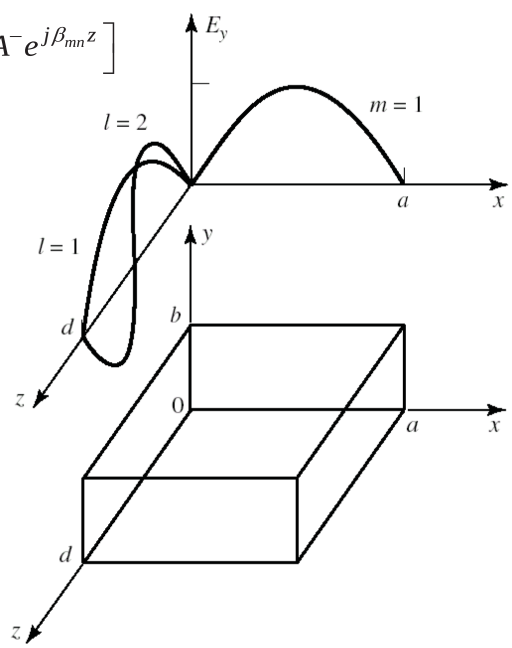
$$\Rightarrow \left(\frac{l\pi}{d}\right)^2 = \omega^2 \mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

$$\frac{2\pi f_{mn,l}}{c} = \sqrt{\left(\frac{l\pi}{d}\right)^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

then the resonance frequency

$$f_{mn,l} = \frac{c}{2\pi}\sqrt{\left(\frac{l}{d}\right)^2 + \left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

if $b < a < d$ the lowest and dominant resonant TE (resp TM) mode will be TE101 (resp. TM110)



A rectangular resonant cavity, and the electric field distributions for the TE_{101} and TE_{102} resonant modes.

Rectangular waveguide cavity resonator Q factor for TE10l

$$Q = \left(\frac{1}{Q_c} + \frac{1}{Q_d} \right)^{-1}$$

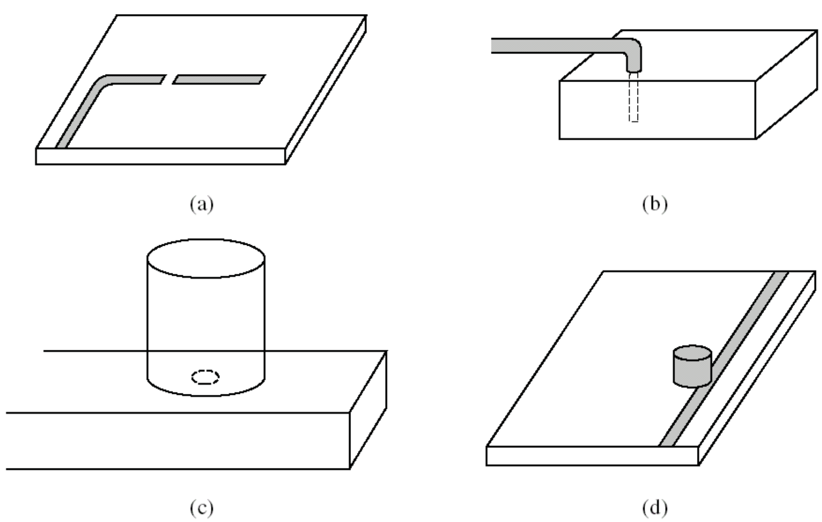
$$Q_d = \frac{1}{\tan \delta}$$

$$Q_c = \frac{(2\pi ad/\lambda)^3 \eta}{2\pi^2 R_s} \frac{1}{\left(2l^2a^3b + 2bd^3 + l^2a^3d + ad^3\right)}$$

$$R_s = \sqrt{\omega\mu_o/2\sigma}$$

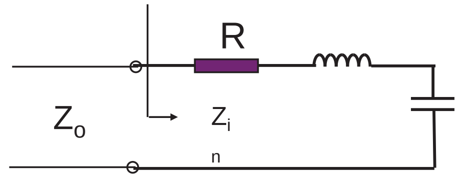
$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

Microwave Resonators Coupling



Coupling to microwave resonators. (*a*) A microstrip transmission line resonator gap coupled to a microstrip feedline. (*b*) A rectangular cavity resonator fed by a coaxial probe. (*c*) A circular cavity resonator aperture coupled to a rectangular waveguide. (*d*) A dielectric resonator coupled to a microstrip feedline.

Resonators - coupling

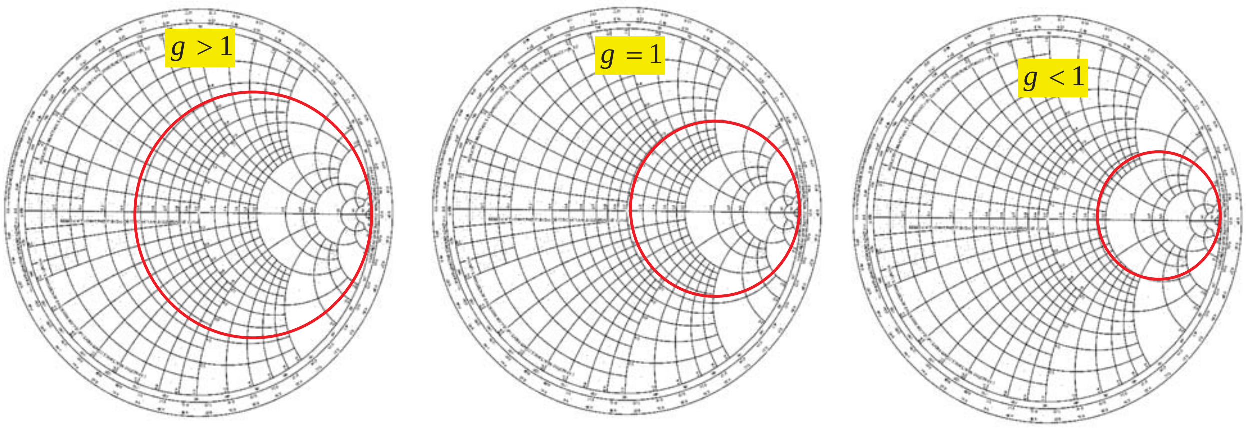


Critical coupling occurs when $2Q_L = Q_e = Q$

If we define coupling coefficient $g = \frac{Q_u}{Q_e}$ then we have

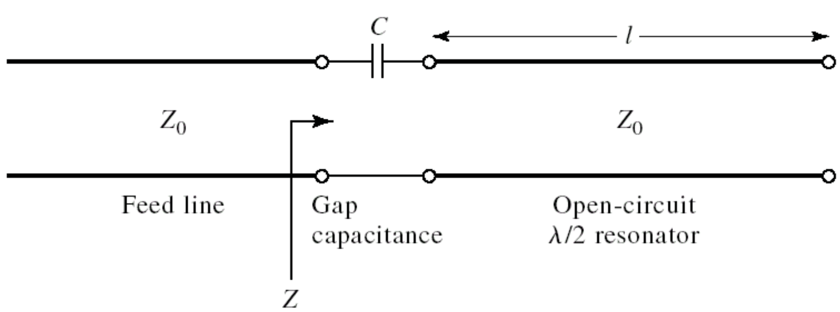
series RLC $g = \frac{Z_0}{R}$ parallel RLC $g = \frac{R}{Z_0}$

- under coupled resonator if g<1
- critically coupled resonator if g=1 (resonator matched to the feed line)
- over coupled resonator if g>1



Series RLC circuit

Gap coupled microstrip resonator resonators



$$b_c = Z_o \omega C$$

$$Z(\omega) = Z_o \left(\frac{\pi}{2Qb_c^2} + j\frac{\pi(\omega - \omega_o)}{\omega_o b_c^2} \right)$$

$$Series\ resonator \Rightarrow R = Z_o \left(\frac{\pi}{2Qb_c^2} \right) \quad b_c = \sqrt{\left(\frac{\pi}{2Q}\right)}$$

the coupling coeticient $g = \frac{Z_o}{R} = \frac{\pi}{2Q_u b^2{}_c}$

for critical coupling $Z_o = R \quad \Rightarrow b_c = \sqrt{\left(\frac{\pi}{2Q}\right)}$

for undercoupled resonator $\Rightarrow b_c < \sqrt{\left(\frac{\pi}{2Q}\right)}$

for overcoupled resonator $\Rightarrow b_c > \sqrt{\left(\frac{\pi}{2Q}\right)}$

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ECE671
Microwave and RF Engineering

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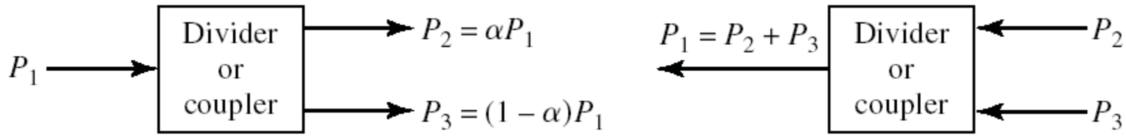
Microwave Couplers and Dividers

Power division and combining:
(*a*) Power division.
(*b*) Power combining.



(a) (b)

Microwave Couplers and Dividers

$$S=\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \xrightarrow{\text{Reciprocal, matched}} \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

Lossless networks→ at least two of the (S_{12} , S_{13} , S_{23}) must be zero

⇓

Three port networks can not be lossless, reciprocal, matched at all ports

⇓

One of these conditions should be relaxed for a realizable device

Microwave Couplers and Dividers

- Three port network matched at all port

$$S=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

- If lossless

$$\begin{matrix} S_{12}=S_{23}=S_{31}=0 \\ or \\ S_{21}=S_{32}=S_{13}=0 \end{matrix} \quad and \quad \begin{matrix} |S_{21}|=|S_{32}|=|S_{13}|=1 \\ |S_{12}|=|S_{23}|=|S_{31}|=1 \end{matrix}$$

⇓

Matched at all ports, lossless three port network is nonreciprocal

Microwave Couplers and Dividers

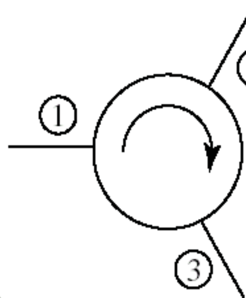
•**Microwave circulator**: three port network, lossless, matched but non reciprocal. It uses generally ferrite (anisotropic) material.

•The two types of circulators and their [S] matrices.

a) Clockwise circulation.

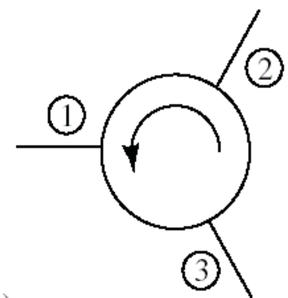
b) Counterclockwise circulation.

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



(a)

$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



(b)

Microwave Couplers and Dividers

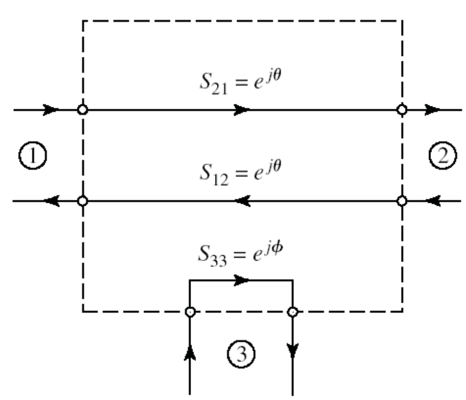
Three-port network can be reciprocal and lossless if only two ports are matched.

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

Lossless, Unitary condition ⇒

$$S_{13} = S_{23} = 0$$
$$|S_{12}| = |S_{21}| = 1$$
$$|S_{33}| = 1$$

$$[S] = \begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\phi} \end{bmatrix}$$



Microwave Couplers and Dividers

- Four Port Networks, reciprocal and matched at all ports

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

- Lossless→ Unitary condition
  - Multiplications of (row 1, row 2) and (row 3, row 4)

$$S_{14}^{*}\left(|S_{13}|^2 - |S_{24}|^2\right) = 0$$

  - Multiplication (Row 1 and row 3) and (Row 4 and row 2)

$$S_{23}\left(|S_{12}|^2 - |S_{34}|^2\right) = 0$$

Microwave Couplers and Dividers

- These conditions can be satisfied if

$$S_{14} = S_{23} = 0 \Rightarrow \text{Directional coupler}$$
$$\Downarrow$$
$$|S_{13}| = |S_{24}| \text{and } |S_{12}| = |S_{34}|$$

- In addition the following phase relationship should be maintained

$$\theta + \phi = \pi \pm 2\pi$$

$$where\ S_{12} = S_{34} = \alpha, S_{13} = \beta e^{j\theta} \text{ and } S_{24} = \beta e^{j\phi}$$

$$\alpha^2 + \beta^2 = 1$$

Microwave Couplers and Dividers

- Two directional couplers cases are possible:
  - Symmetrical coupler: $\theta = \phi = \pi/2$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

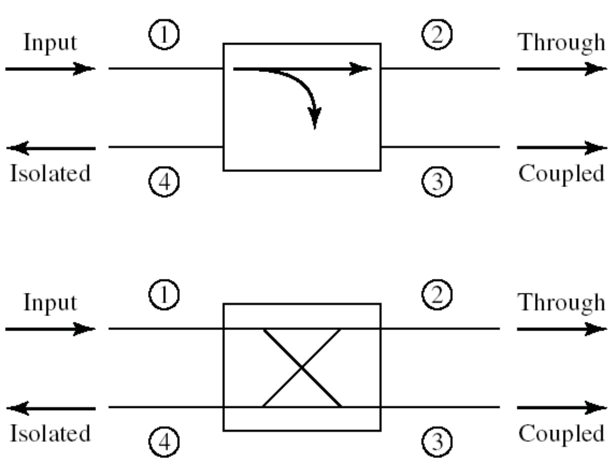
  - Ant Symmetrical coupler: $\theta = 0; \phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

•Coupling factor between Port 1 and Port 3
$|S_{13}|^2 = \alpha^2$
•Transmission coefficient between Port1 and port 2
$|S_{12}|^2 = \beta^2 = 1 - \alpha^2$
•No power to port 4

Microwave Couplers and Dividers

Two commonly used symbols for directional couplers, and power flow conventions.



$$Coupling = C = 10\log\frac{P_1}{P_3} = -20\log\beta \text{ dB}$$

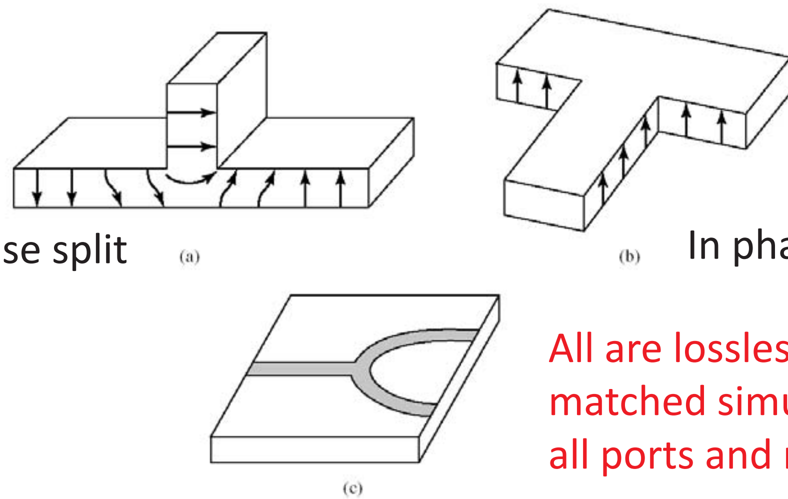
$$Directivity = D = 10\log\frac{P_3}{P_4} = -20\log\frac{\beta}{|S_{14}|} \text{ dB}$$

$$Isolation = I = 10\log\frac{P_1}{P_4} = -20\log|S_{14}| \text{ dB}$$

I=C+D

Microwave Couplers and Dividers

Various T-junction power dividers:
(a) *E* plane waveguide T.
(b) *H* plane waveguide T.
(c) Microstrip T-junction.



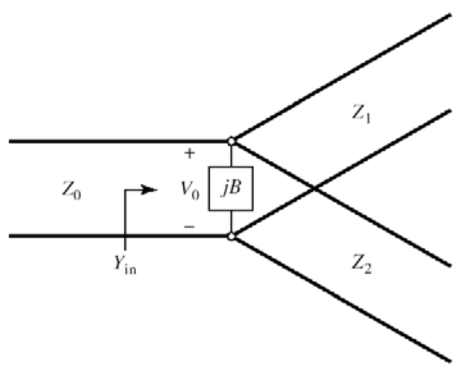
Out of phase split

In phase split

All are lossless → can't be matched simultaneously at all ports and reciprocal

Microwave Couplers and Dividers

Transmission line model of a lossless T-junction.



B: models the stored energy in the discontinuity at the junction

$$Y_{in} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_o}$$

Z1,Z2 are selected to provide a given division ratio

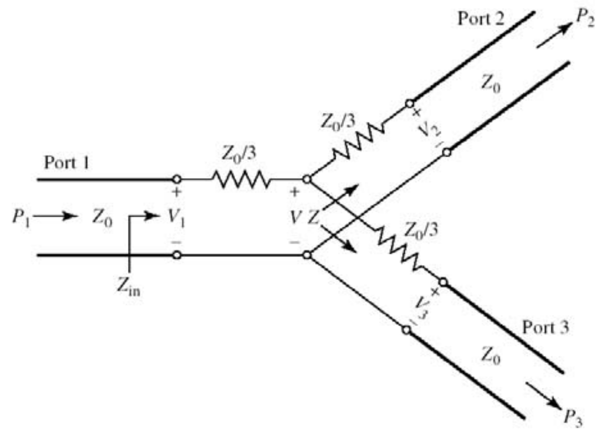
Example: Z1=Z2=100ohm→3dB division

Note: no isolation between the two output ports, the two output ports are mismatched

Microwave Couplers and Dividers

Equal Resistive power divider.

$Z_{in} = Z_o$ ⇨



$$S_{11} = 0 \text{ and similarly } S_{22} = S_{33} = 0$$

$$V_2 = V_3 = V \frac{Z_0}{Z_0/3 + Z_0} = \frac{1}{2} V_1$$

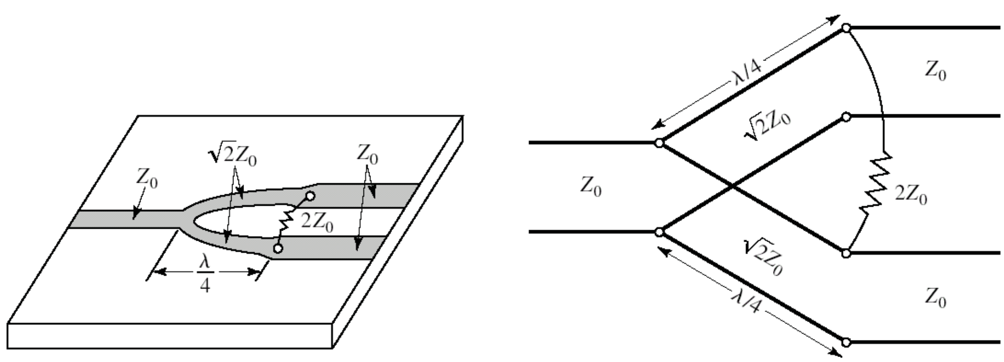
$$\rightarrow S_{21} = S_{31} = \frac{1}{2} = \text{-6dB}$$

$$S = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

- Reciprocal, all matched but lossy divider
- Half of the power is dissipated in the resistances

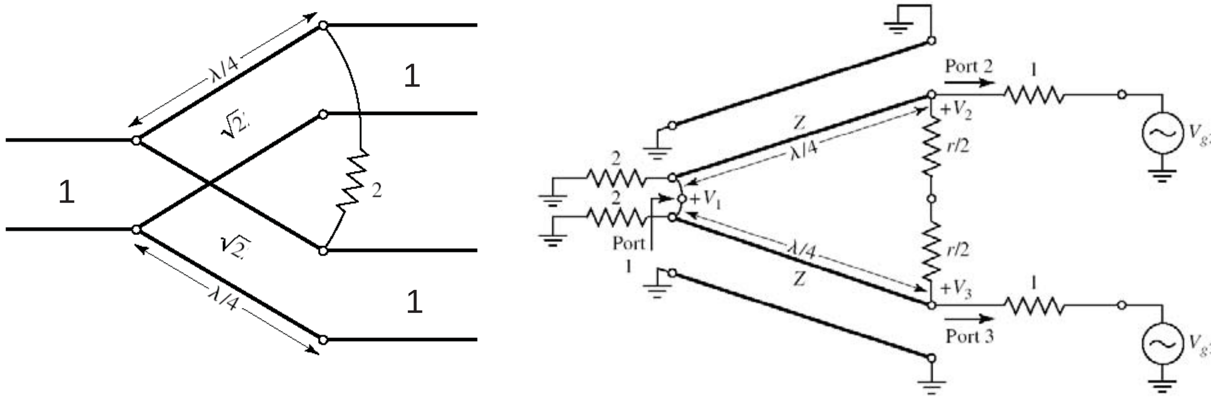
Microwave Couplers and Dividers

The Wilkinson power divider.
(*a*) An equal-split Wilkinson power divider in micro-strip form.
(*b*) Equivalent transmission line circuit.



(a) (b)

Microwave Couplers and Dividers



Wilkinson power divider circuit in normalized and symmetric form.

$Z = \sqrt{2}$

$r = 2$

•If voltages V_{g2} and V_{g3} are in phase, we have even mode excitation, and open circuit can be placed between the two arms
•When V_{g2}= - V_{g3} we have odd mode excitation, short circuit can be placed in the plane of the symmetry

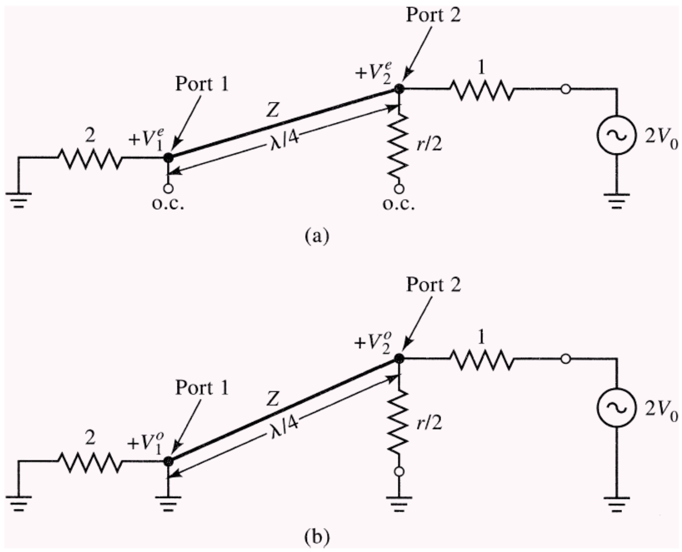
Microwave Couplers and Dividers

Plan:
• consider odd and even excitations
• compute voltages at different ports
• compute reflection coefficient at ports
• create S-matrix

We will consider odd and even excitation of ports 2 & 3
We know already that any other excitation can be built from that

Then we will excite port 1

Microwave Couplers and Dividers



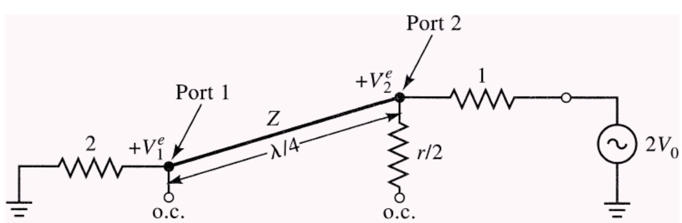
Bisection of the circuit

(a) Even-mode excitation.

(b) Odd-mode excitation.

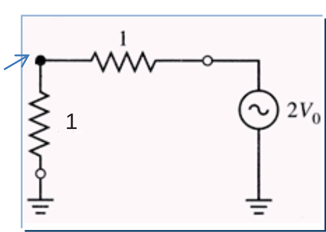
Microwave Couplers and Dividers

Even mode

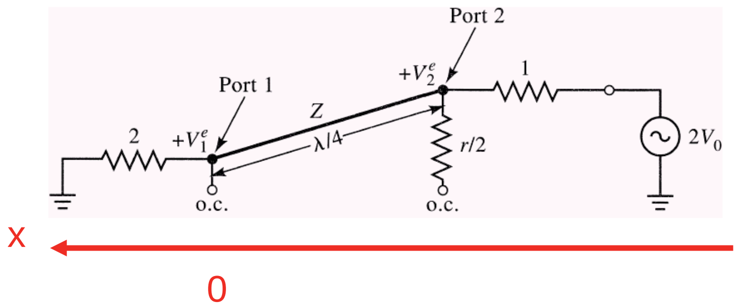


Match!

$V_2^e = V_o$



Microwave Couplers and Dividers



$$V(x) = V^+(e^{-j\beta x} + \Gamma e^{j\beta x})$$

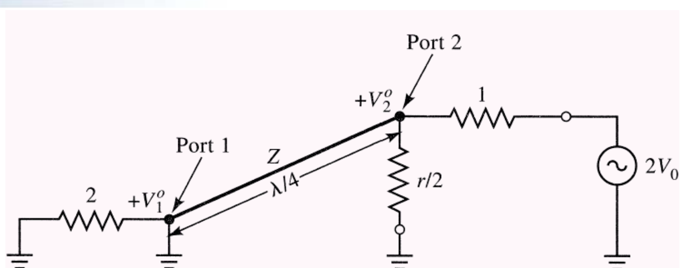
x=0 $\quad V_1^e = V(0) = jV^+(1+\Gamma) = jV_o \dfrac{\Gamma+1}{\Gamma-1}$

x=-λ/4 $\quad V_2^e = V(-\lambda/4) = jV^+(1-\Gamma) = V_o \qquad e^{-j\beta\frac{\lambda}{4}} = e^{-j\frac{\pi}{2}} = -j$

$$V_1^e = jV_o \frac{\Gamma+1}{\Gamma-1} \qquad \Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}} \qquad V_1^e = jV_o \frac{\frac{2-\sqrt{2}}{2+\sqrt{2}}+1}{\frac{2-\sqrt{2}}{2+\sqrt{2}}-1} = -jV_o \frac{4}{2\sqrt{2}} \qquad \Rightarrow V_1^e = -jV_o\sqrt{2}$$

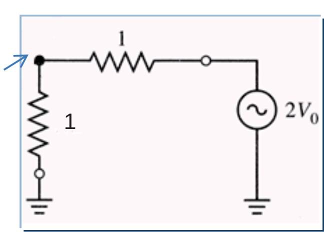
Microwave Couplers and Dividers

Odd mode



$V_1^e = 0$

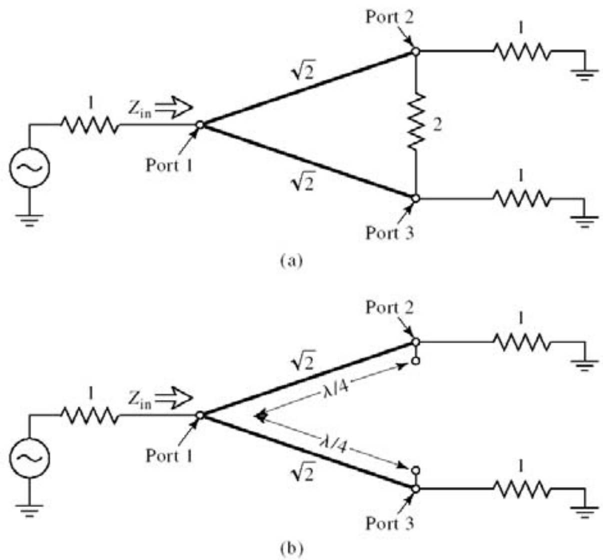
$V_2^o = V_o$



Matched, S_{22}=0

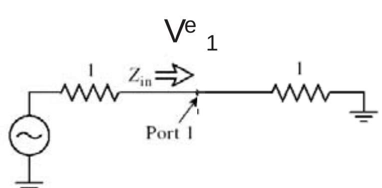
Similarly, Matched, S_{33}=0

Microwave Couplers and Dividers



Analysis of the Wilkinson divider to find S_{11}.
(a) The terminated Wilkinson divider.
(b) Bisection of the circuit in (*a*).

Microwave Couplers and Dividers



$$Z_{in} = \frac{1}{2}\left(\sqrt{2}\right)^2 = 1$$

Matched, $S_{11}=0$

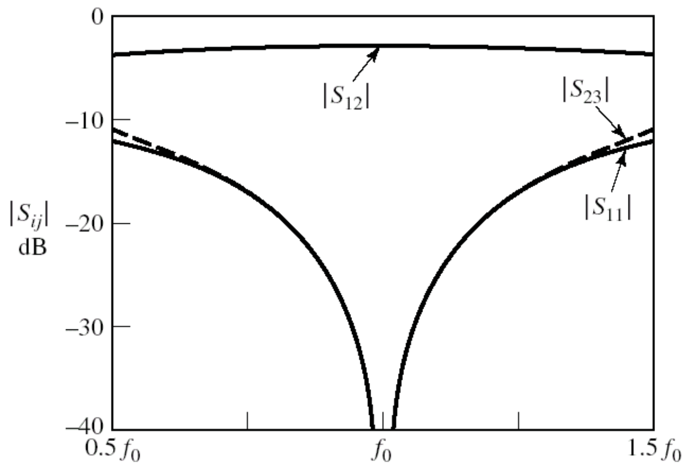
Ports 2 and 3 are separated either by Short or Open circuit. No power goes between. → $S_{23}=S_{32}=0$

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{V_1^+=V_3^+=0} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = \frac{-j\sqrt{2}\mathrm{V} + 0}{1\mathrm{V} + 1\mathrm{V}} = -j\frac{1}{\sqrt{2}}$$

$$S = \begin{bmatrix} 0 & \frac{-j}{\sqrt{2}} & \frac{-j}{\sqrt{2}} \\ \frac{-j}{\sqrt{2}} & 0 & 0 \\ \frac{-j}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

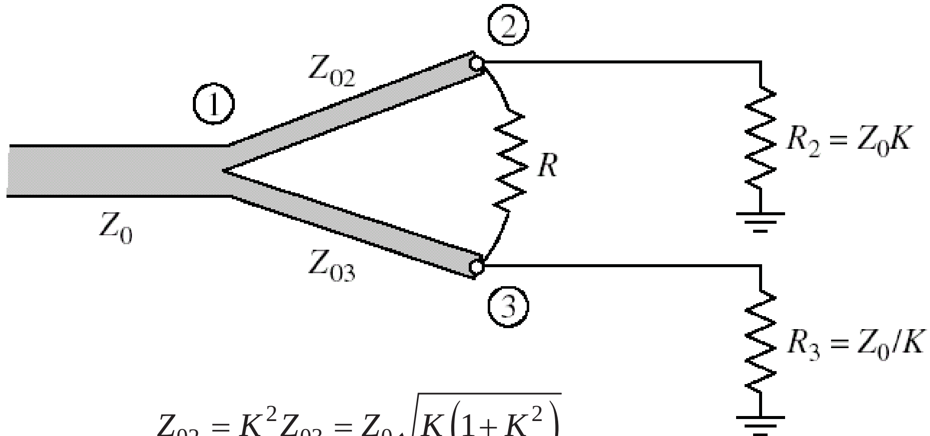
Microwave Couplers and Dividers

Frequency response of an equal-split Wilkinson power divider. Port 1 is the input port; ports 2 and 3 are the output ports.



Microwave Couplers and Dividers

A Wilkinson power divider in microstrip form having unequal power division.



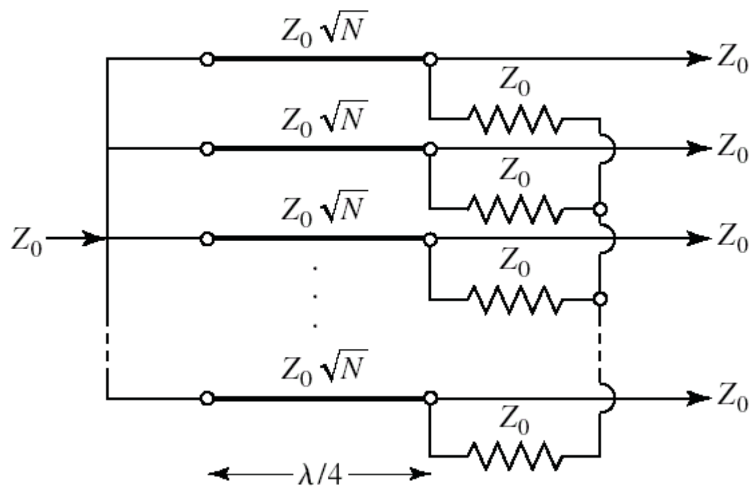
$$K^2 = \frac{P_3}{P_2}$$

$$Z_{02} = K^2 Z_{03} = Z_0\sqrt{K\left(1+K^2\right)}$$

$$Z_{03} = Z_0\sqrt{\frac{1+K^2}{K^3}}$$

$$R = Z_0\left(K + \frac{1}{K}\right)$$

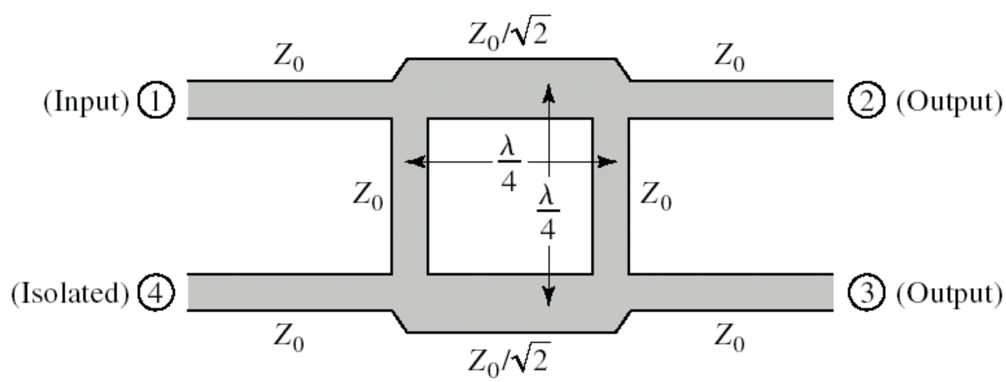
Microwave Couplers and Dividers



An *N*-way, equal-split Wilkinson power divider.

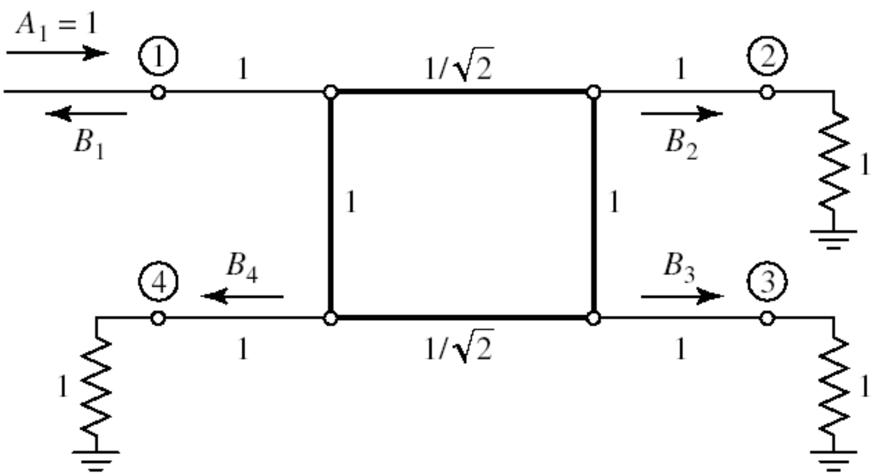
Microwave Couplers and Dividers

Quadrature hybrid (or branch-line) coupler.



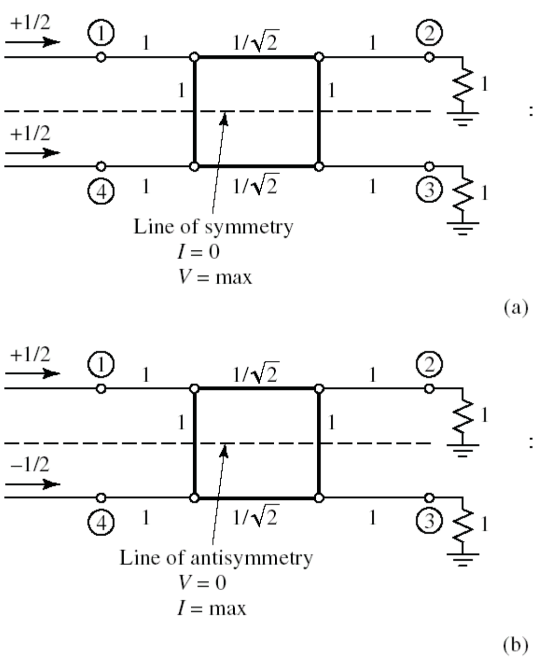
$$[S] = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

Microwave Couplers and Dividers



Circuit of the branch-line hybrid coupler in a normalized form.

Microwave Couplers and Dividers



$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o$$

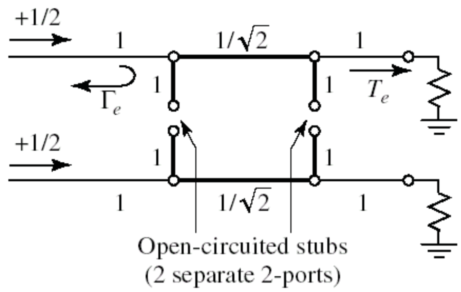
$$B_2 = \frac{1}{2}T_e + \frac{1}{2}T_o$$

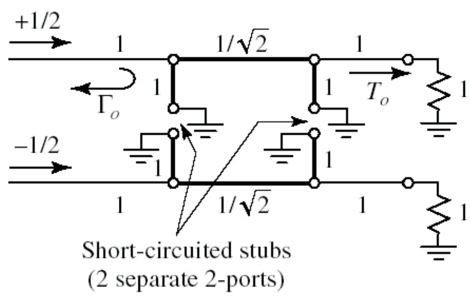
$$B_3 = \frac{1}{2}T_e - \frac{1}{2}T_o$$

$$B_3 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

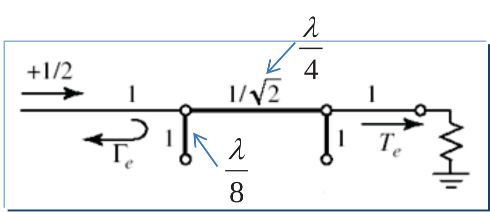
Decomposition of the branch-line coupler into even- and odd-mode excitations. (*a*) Even mode (*e*). (*b*) Odd mode (*o*).

Microwave Couplers and Dividers



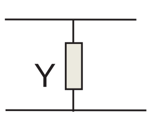


Microwave Couplers and Dividers



We have 3 two ports: shunt, line section and shunt. We'll use ABCD matrices.

Shunt:



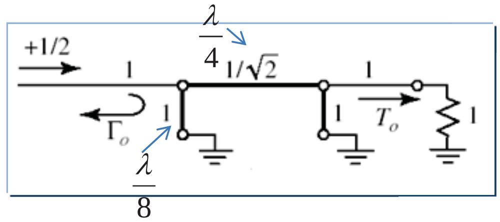
$$Y = j\tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{8}\right) = j\tan\left(\frac{\pi}{4}\right) = j \qquad \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix}$$

TL Section:

$$\begin{bmatrix} \cos(\beta l) & Z_o j\sin(\beta l) \\ \frac{j}{Z_o}\sin(\beta l) & \cos(\beta l) \end{bmatrix} = \begin{bmatrix} \cos(\frac{\pi}{2}) & \frac{1}{\sqrt{2}} j\sin(\frac{\pi}{2}) \\ j\sqrt{2}\sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) \end{bmatrix} = \begin{bmatrix} 0 & \frac{j}{\sqrt{2}} \\ j\sqrt{2} & 0 \end{bmatrix}$$

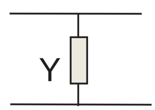
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} \begin{bmatrix} 0 & \frac{j}{\sqrt{2}} \\ j\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & j \\ j & -1 \end{bmatrix}$$

Microwave Couplers and Dividers



We have 3 two ports: shunt, line section and shunt. We'll use ABCD matrices.

Shunt:



$$Y = j\tan\left(\frac{2\pi}{\lambda}\left(\frac{\lambda}{8}+\frac{\lambda}{4}\right)\right) = j\tan\left(\frac{3\pi}{4}\right) = -j$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} \begin{bmatrix} 0 & \frac{j}{\sqrt{2}} \\ j\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j & 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$$

Microwave Couplers and Dividers

$$\Gamma = S_{11} = \frac{b_1}{a_1} = \frac{A+B-C-D}{A+B+C+D} \qquad T = S_{21} = \frac{b_2}{a_1} = \frac{2}{A+B+C+D}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & j \\ j & -1 \end{bmatrix} \Rightarrow$$

$$\Gamma_e = \frac{A+B-C-D}{A+B+C+D} = \frac{-1+j-j+1}{A+B+C+D} = 0$$

$$T_e = 2\sqrt{2}(-1+j+j-1)^{-1} = -\frac{1}{\sqrt{2}}(1+j)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix} \Rightarrow$$

$$\Gamma_o = \frac{1+j-j-1}{A+B+C+D} = 0$$

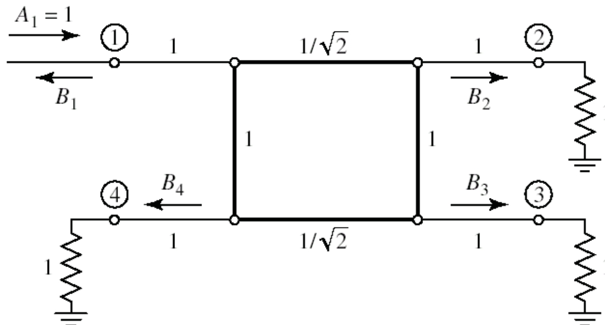
$$T_o = 2\sqrt{2}(1+j+j+1)^{-1} = \frac{1}{\sqrt{2}}(1-j)$$

Microwave Couplers and Dividers

$$\Gamma_e = \Gamma_o = 0$$

$$T_e = -\frac{1}{\sqrt{2}}(1+j)$$

$$T_o = \frac{1}{\sqrt{2}}(1-j)$$



$$b_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \quad\Rightarrow\quad b_1 = 0$$

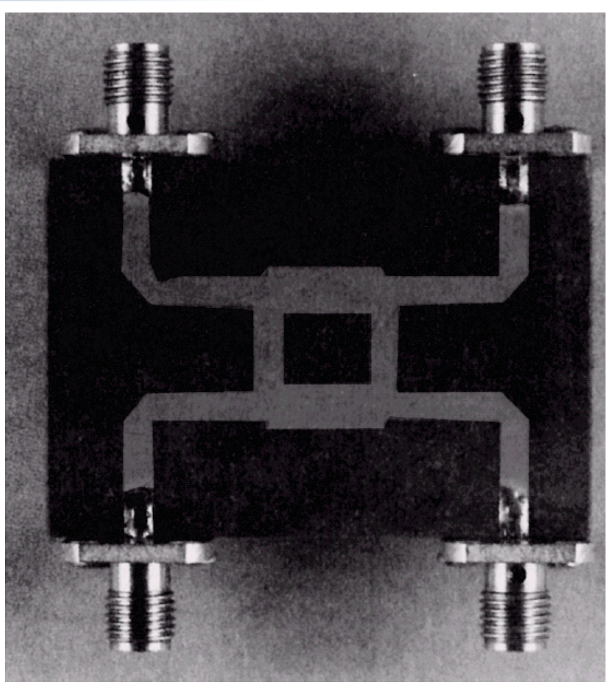
$$b_2 = \frac{1}{2}T_e + \frac{1}{2}T_o \quad\Rightarrow\quad b_2 = -\frac{j}{\sqrt{2}}$$

$$b_3 = \frac{1}{2}T_e - \frac{1}{2}T_o \quad\Rightarrow\quad b_3 = -\frac{1}{\sqrt{2}}$$

$$b_4 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o \quad\Rightarrow\quad b_4 = 0$$

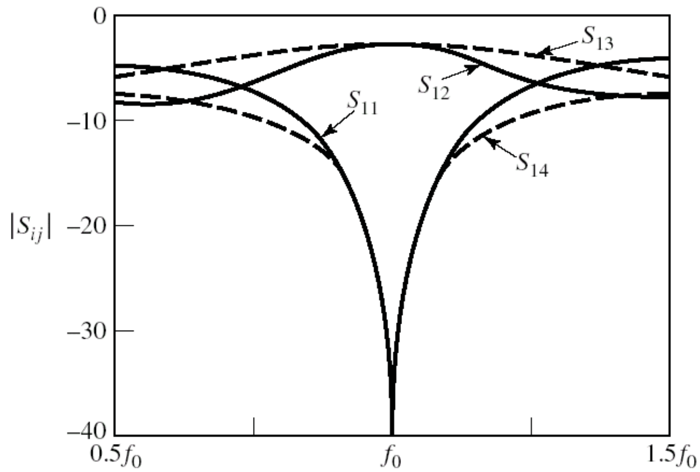
$$[S] = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 & . & . & . \\ j & . & . & . \\ 1 & . & . & . \\ 0 & . & . & . \end{bmatrix}$$

Microwave Couplers and Dividers



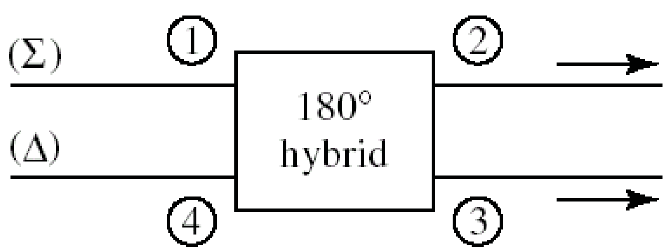
Microwave Couplers and Dividers

S parameter magnitudes versus frequency for the branch-line coupler



Microwave Couplers and Dividers

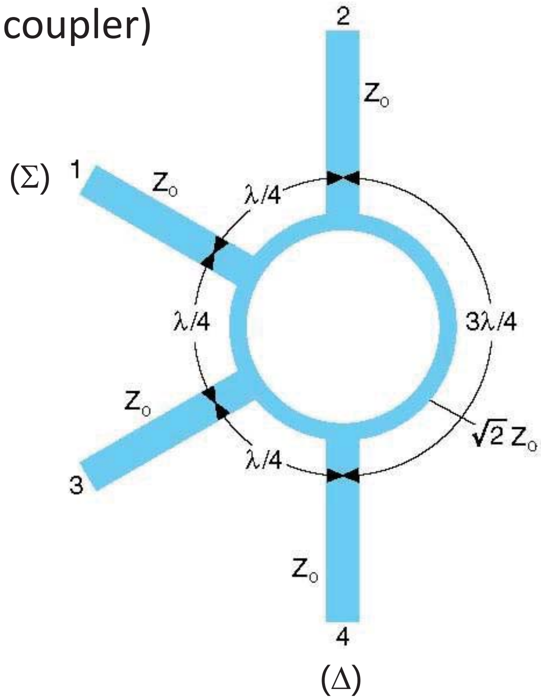
Symbol for a 180° hybrid junction (rat-trace coupler).



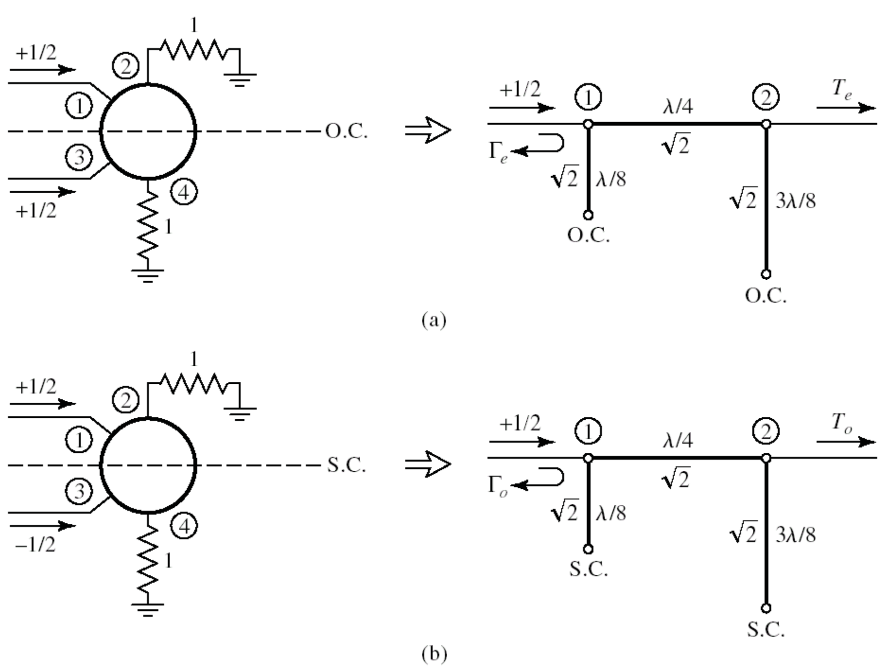
Microwave Couplers and Dividers

Ring hybrid (or rat-race coupler)

$$[S]=\frac{-j}{\sqrt{2}}\begin{bmatrix}0 & 1 & 1 & 0\\ 1 & 0 & 0 & -1\\ 1 & 0 & 0 & 1\\ 0 & -1 & 1 & 0\end{bmatrix}$$



Microwave Couplers and Dividers



Even- and odd-mode decomposition of the ring hybrid when port 1 is excited with a unit amplitude incident wave. (*a*) Even mode. (*b*) Odd mode.

Microwave Couplers and Dividers

$$B_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \qquad \Gamma_e = \frac{-j}{\sqrt{2}} \qquad B_1 = 0$$

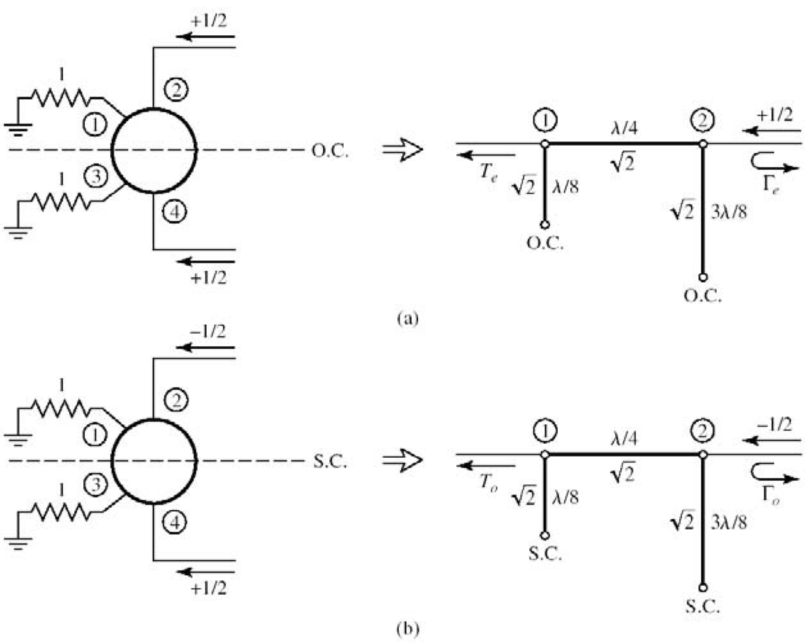
$$B_2 = \frac{1}{2}T_e + \frac{1}{2}T_o \qquad T_e = \frac{-j}{\sqrt{2}} \qquad B_2 = \frac{-j}{\sqrt{2}}$$

$$\Rightarrow$$

$$B_3 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o \qquad \Gamma_o = \frac{j}{\sqrt{2}} \qquad B_3 = \frac{-j}{\sqrt{2}}$$

$$B_3 = \frac{1}{2}T_e - \frac{1}{2}T_o \qquad T_o = \frac{-j}{\sqrt{2}} \qquad B_4 = 0$$

Microwave Couplers and Dividers



Even- and odd-mode decomposition of the ring hybrid when port 4 is excited with a unit amplitude incident wave. (*a*) Even mode. (*b*) Odd mode.

Microwave Couplers and Dividers

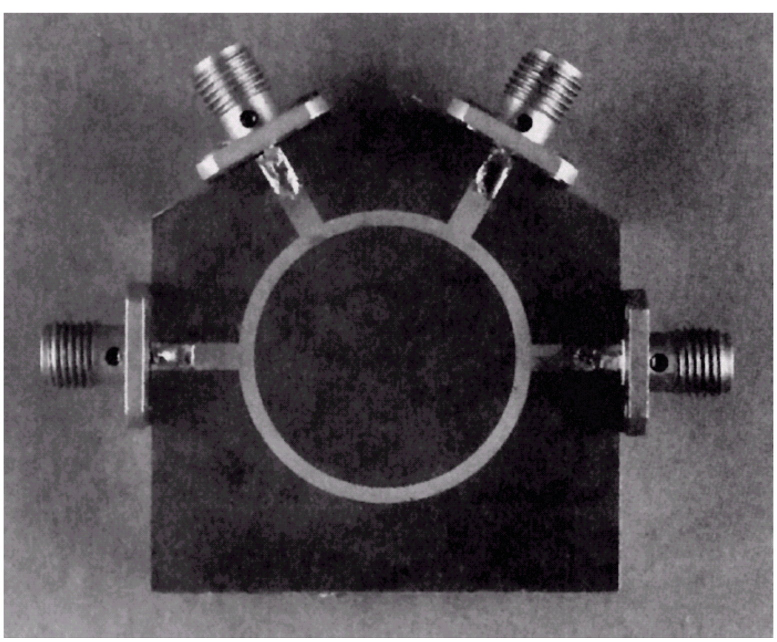
$$B_1 = \frac{1}{2}T_e - \frac{1}{2}T_o \qquad \Gamma_e = \frac{j}{\sqrt{2}} \qquad B_1 = 0$$

$$B_2 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o \qquad T_e = \frac{-j}{\sqrt{2}} \qquad B_2 = \frac{j}{\sqrt{2}}$$

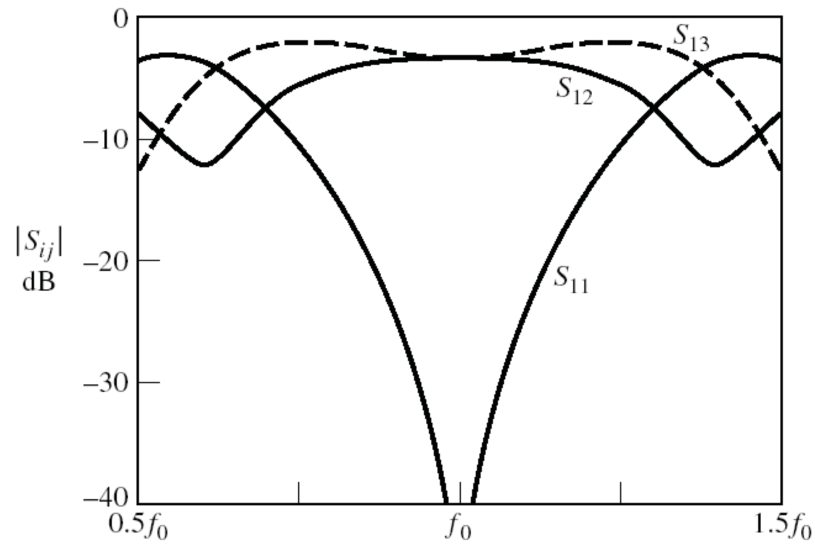
$$B_3 = \frac{1}{2}T_e + \frac{1}{2}T_o \qquad \Gamma_o = \frac{-j}{\sqrt{2}} \qquad \Rightarrow \qquad B_3 = \frac{-j}{\sqrt{2}}$$

$$B_3 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \qquad T_o = \frac{-j}{\sqrt{2}} \qquad B_4 = 0$$

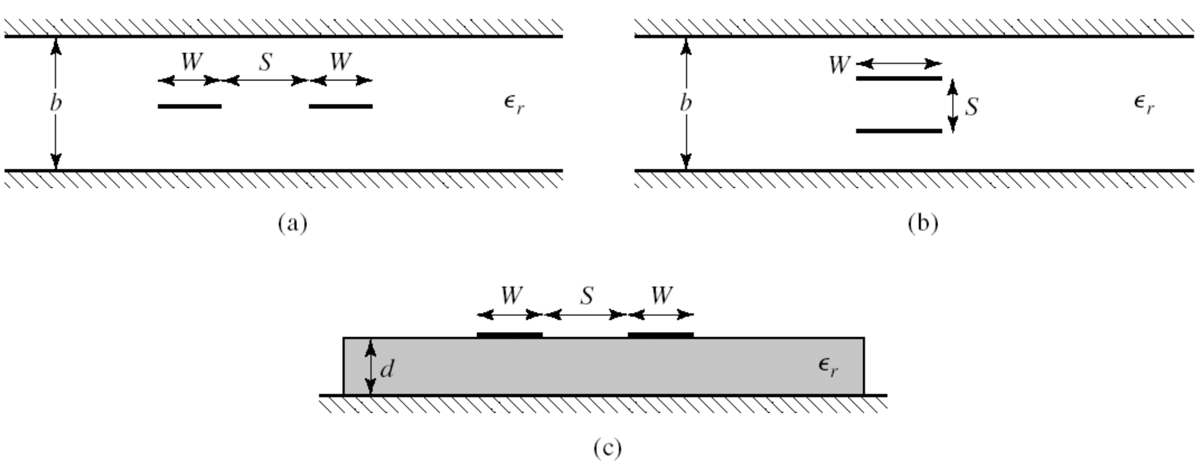
Microwave Couplers and Dividers



Microwave Couplers and Dividers

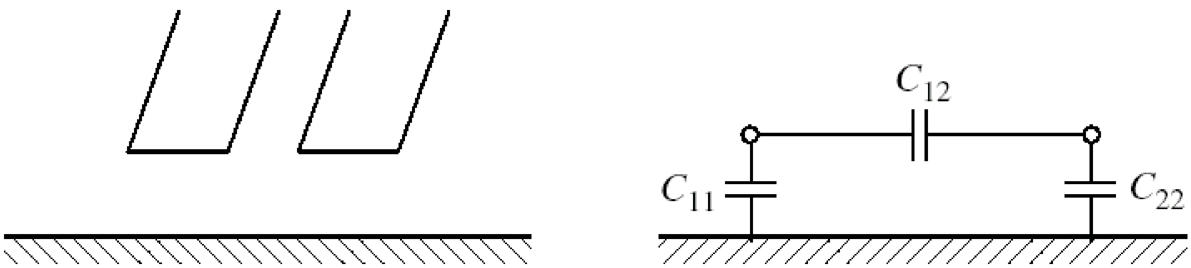


Microwave Couplers and Dividers



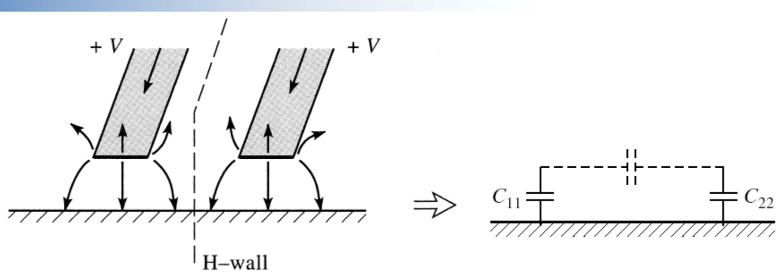
Various coupled transmission line geometries. (*a*) Coupled stripline (planar, or edge-coupled). (*b*) Coupled stripline (stacked, or broadside-coupled). (*c*) Coupled microstrip.

Microwave Couplers and Dividers



A three-wire coupled transmission line and its equivalent capacitance network.

Microwave Couplers and Dividers

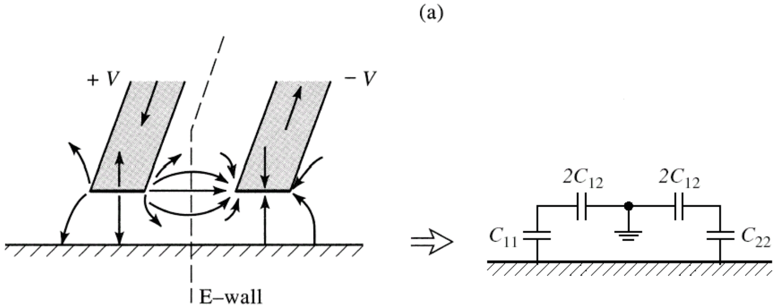


(a)

$$Z_{0e} = \sqrt{\frac{L}{C_e}} = \frac{\sqrt{LC_e}}{C_e} = \frac{1}{v_{pe}C_e}; C_e = C_1$$

$$v_{pe} = \frac{1}{\sqrt{LC_e}}$$

$$C_1 = C_{22} = C_{11}$$



(b)

$$Z_{0o} = \sqrt{\frac{L}{C_o}} = \frac{\sqrt{LC_o}}{C_o} = \frac{1}{v_{po}C_o}; C_o = C_1 + 2C_{12}$$

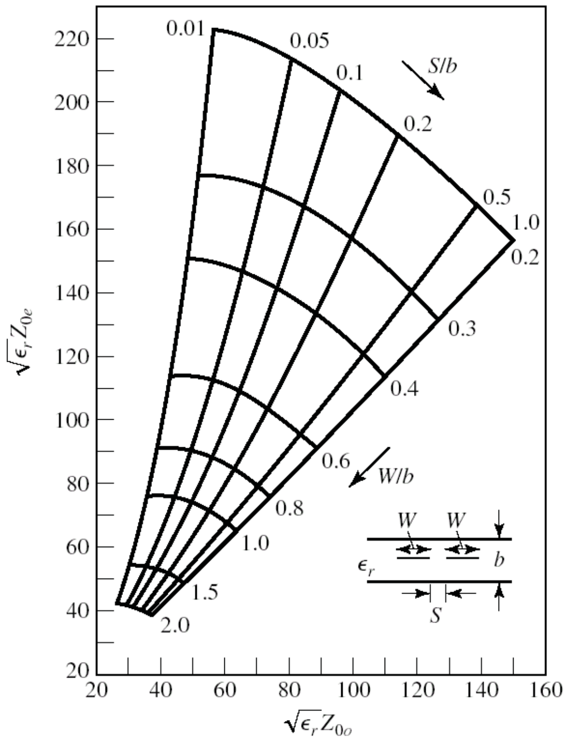
$$v_{po} = \frac{1}{\sqrt{LC_o}}$$

Even- and odd-mode excitations for a coupled line, and the resulting equivalent capacitance networks. (*a*) Even-mode excitation. (*b*) Odd-mode excitation.

Assumption: identical fringing effect and symmetrical structure

Microwave Couplers and Dividers

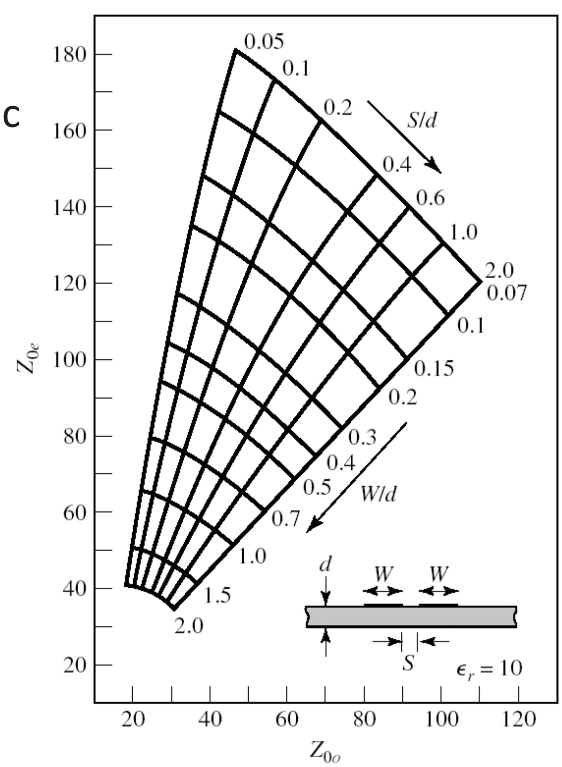
•Analytical techniques can be used to determine the characteristic impedances in case of pure TEM



Normalized even-and odd-mode characteristic impedance design data for edge-coupled striplines.

Microwave Couplers and Dividers

• Numerical approximations can be used to determine the characteristic impedances in case of Quasi-TEM

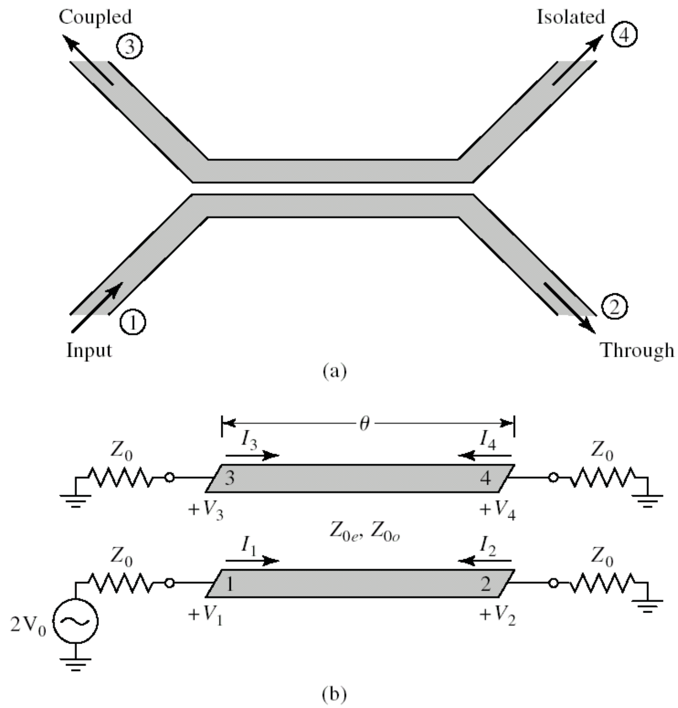


Even- and odd-mode characteristic impedance design data for coupled microstrip lines on a substrate with ε_r = 10.

Microwave Couplers and Dividers

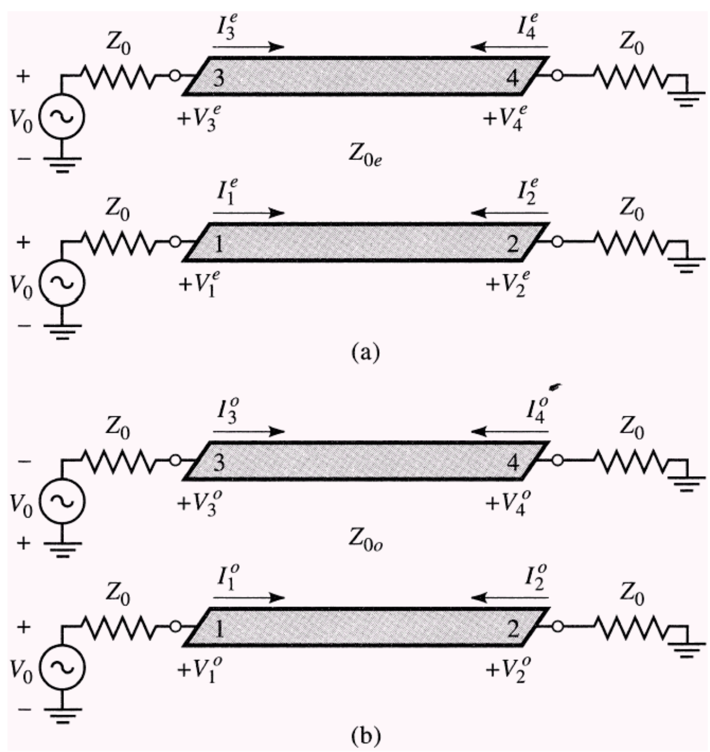
Coupled Line Directional Couplers

A single-section coupled line coupler.
(*a*) Geometry and port designations.
(*b*) The schematic circuit.



Microwave Couplers and Dividers

Decomposition of the coupled line coupler circuit into even- and odd-mode excitation.
(*a*) Even mode.
(*b*) Odd mode.



Microwave Couplers and Dividers

$$Z_{in} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^e}{I_1^e + I_1^o} \text{ where}$$

$$V_1^e = V_0 \frac{Z_{in}^e}{Z_{in}^e + Z_0}; I_1^e = V_0 \frac{1}{Z_{in}^e + Z_0}$$

$$V_1^o = V_0 \frac{Z_{in}^o}{Z_{in}^o + Z_0}; I_1^o = V_0 \frac{1}{Z_{in}^o + Z_0}$$

$$Z_{in}^e = Z_{0e} \frac{Z_0 + jZ_{0e} \tan\theta}{Z_{0e} + jZ_0 \tan\theta}$$

$$Z_{in}^o = Z_{0o} \frac{Z_0 + jZ_{0o} \tan\theta}{Z_{0o} + jZ_0 \tan\theta}$$

$$\Rightarrow Z_{in} = Z_0 + \frac{2\left(Z_{in}^e Z_{in}^o - Z_0^2\right)}{Z_{in}^e + Z_{in}^o + 2Z_0}$$

if we force $Z_{in}^e Z_{in}^o = Z_0^2 \Rightarrow Z_{in} = Z_0$

Microwave Couplers and Dividers

$$\Rightarrow V_1 = V_0$$

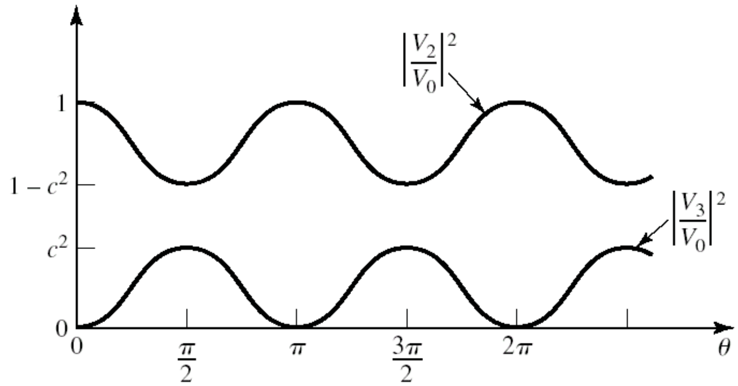
$$V_3 = V_3^e + V_3^o = V_1^e - V_1^o = V_0 \frac{Z_{in}^e}{Z_{in}^e + Z_0} - V_0 \frac{Z_{in}^o}{Z_{in}^o + Z_0} = V_0 \frac{j\left(Z_{0e} + Z_{0o}\right)\tan\theta}{2Z_0 + j\left(Z_{0e} + Z_{0o}\right)\tan\theta}$$

$$\text{let } C = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}} \Rightarrow V_0 \frac{jC\tan\theta}{\sqrt{1-C^2} + j\tan\theta}$$

$$V_4 = V_4^e + V_4^o = V_2^e - V_2^o$$

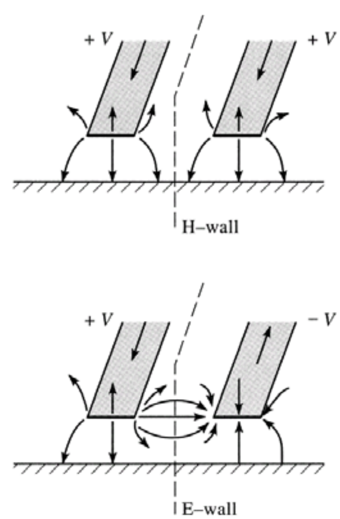
$$V_2 = V_2^e + V_2^o = V_0 \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta}$$

Microwave Couplers and Dividers



Coupled and through port voltages (squared) versus frequency for the coupled line coupler

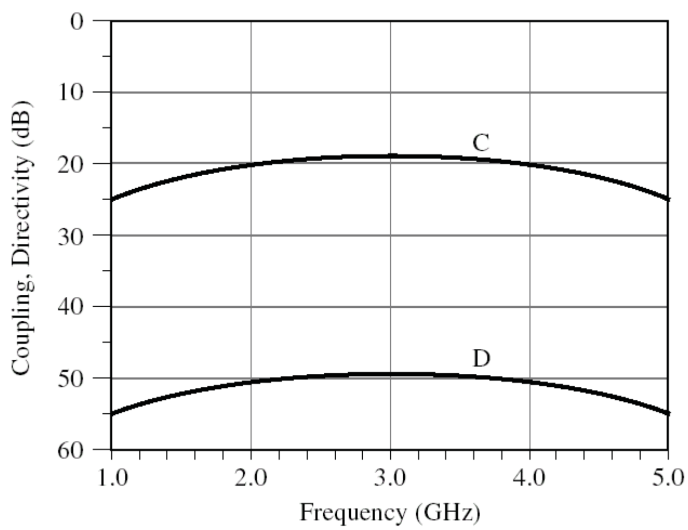
Microwave Couplers and Dividers



If we assume that phase velocities for odd and even modes are the same (true for TEM, but generally not true for microstrip) we have:

$$Z_{oe} = Z_o\sqrt{\frac{1+C}{1-C}} \qquad Z_{oo} = Z_o\sqrt{\frac{1-C}{1+C}}$$

Microwave Couplers and Dividers

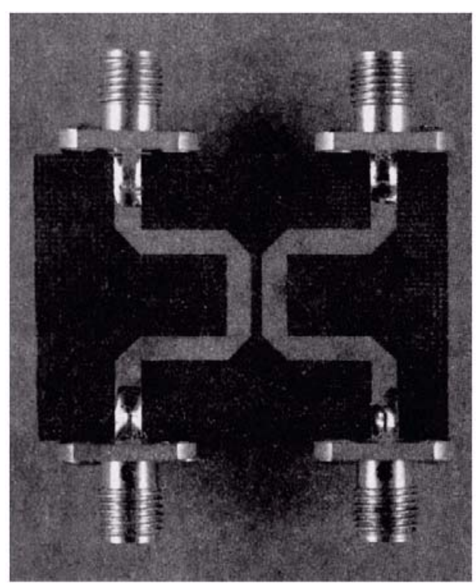


Coupling versus frequency for the single-section coupler

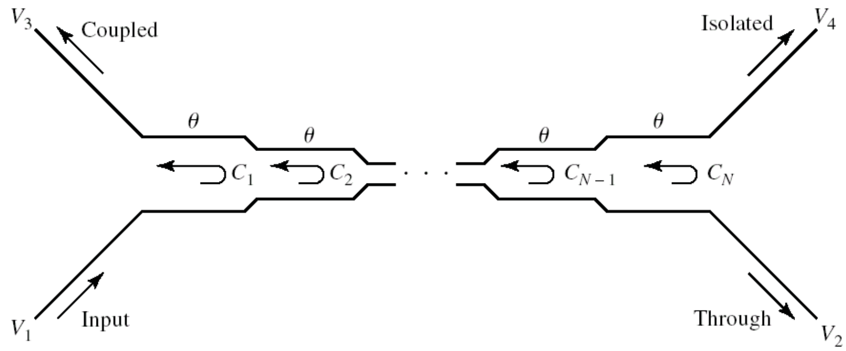
Microwave Couplers and Dividers

Photograph of a single-section microstrip coupled line coupler.

Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory.



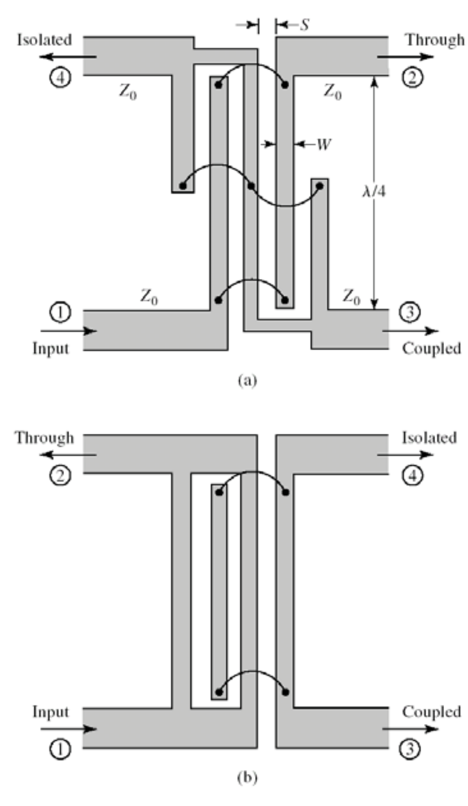
Microwave Couplers and Dividers



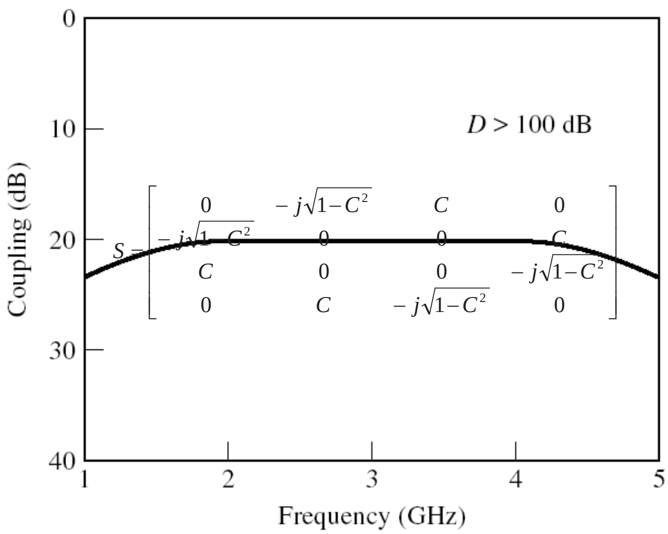
An *N*-section coupled line coupler.

Microwave Couplers and Dividers

The Lange coupler.
(*a*) Layout in microstrip form.
(*b*) The unfolded Lange coupler.



Microwave Couplers and Dividers



$$S = \begin{bmatrix} 0 & -j\sqrt{1-C^2} & C & 0 \\ -j\sqrt{1-C^2} & 0 & 0 & C \\ C & 0 & 0 & -j\sqrt{1-C^2} \\ 0 & C & -j\sqrt{1-C^2} & 0 \end{bmatrix}$$

Coupling versus frequency for the three-section binomial coupler

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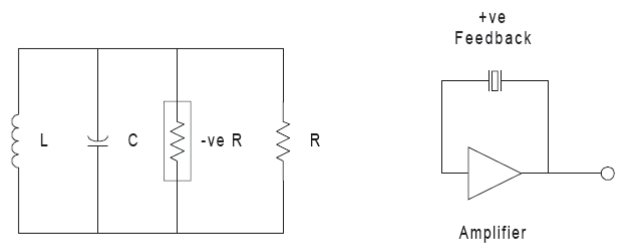
Oscillator Design

- Oscillators can be categorized as either amplifiers with positive feedback, or as negative resistance circuits.



(a) Negative Resistance Oscillator (d) Positive Feedback Oscillator

- At RF and Microwave frequencies the negative resistance design technique is generally favored.

Oscillator Design

$$a_L = \frac{a_n \Gamma_{in}(j\omega)}{1 - \Gamma_L(j\omega)\Gamma_{in}(j\omega)} \qquad \Gamma_{in}(j\omega) = \frac{Z_{in} - Z_o}{Z_{in} + Z_o} \qquad \Gamma_L(j\omega) = \frac{Z_L - Z_o}{Z_L + Z_o}$$

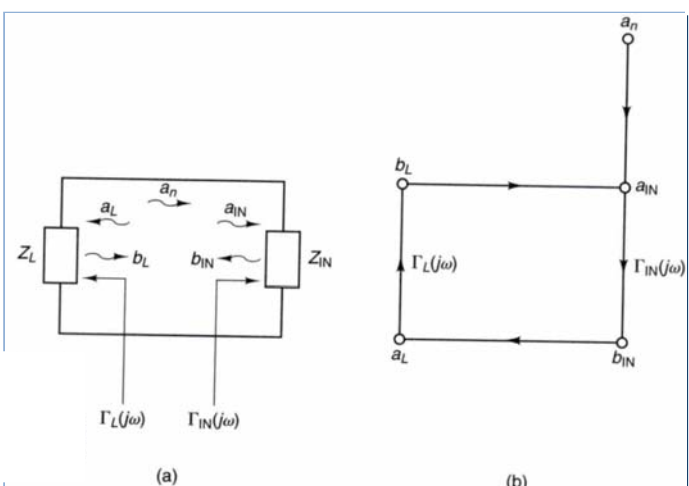


Figure 5.2.2 (a) A microwave circuit; (b) the flow graph.

- Oscillation occur when $\Gamma_L(j\omega)\Gamma_{in}(j\omega) = 1$

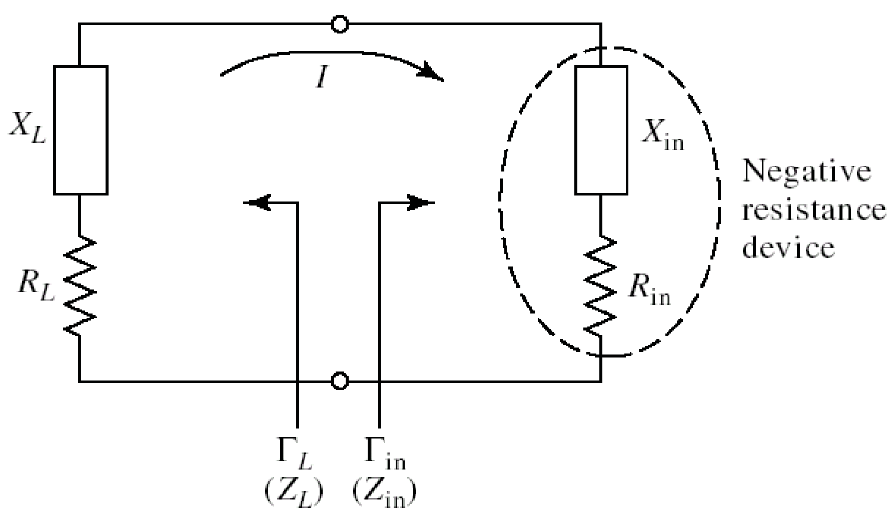
Oscillator Design

- The procedure is to design an active negative resistance circuit which, under large-signal steady-state conditions, exactly cancels out the load and any other positive resistance in the closed loop circuit.
- This leaves the equivalent circuit represented by a single L and C in either parallel (as illustrated before) or series configuration.
- At a frequency the reactance will be equal and opposite, and this resonant frequency is given by the standard formula

$$f = \frac{1}{2\pi\sqrt{LC}}$$

- In the presence of excess negative resistance in the small-signal state, any small perturbation caused, for example by noise, will rapidly build up into a large signal steady-state resonance.

One Port Oscillator



Circuit for a one-port negative-resistance oscillator.

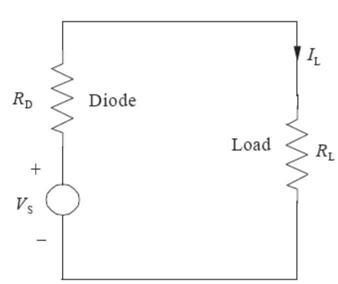
One Port Oscillator

- One port oscillators consist of a negative resistance diode and a load.
- At microwave frequencies IMPATT and Gunn diodes are used. The diode is represented by an impedance $Z_D = R_D + jX_D$. R_D is negative.

  R_L and X_L such that the maximum power is delivered to the load?
- At the required frequency of oscillation, we choose the reactance X_L to tune out the diode reactance X_D. This means that $X_L = -X_D$.
- This leaves only the resistive parts of both impedances.

One Port Oscillator

- To obtain the appropriate value of R_L we assume that the diode includes an a.c. voltage generator V_S



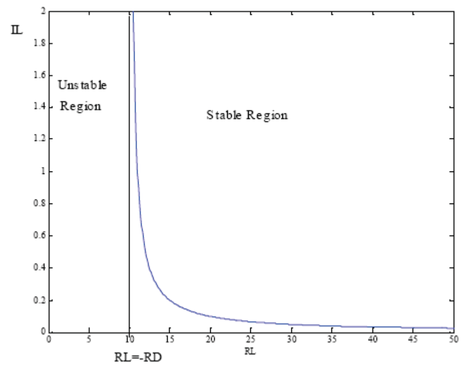
- We want to obtain the condition that the signal can sustain itself without the fictitious generator V_S

One Port Oscillator

- The load current I_L is given by $I_L = \dfrac{V_S}{R_D + R_L}$
- The oscillation condition, we need to adjust the load so

$$Z_L = R_L + jX_L = -R_D - jX_D = -Z_D$$

- The condition $X_L = -X_D$ ensures that the circuit is tuned at the required frequency
- The condition for the oscillation to build up, in small signal: $R_L \leq -R_D$.



One Port Oscillator

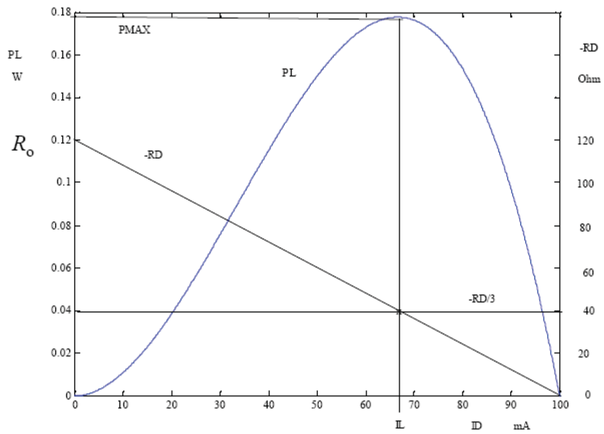
- when the diode oscillates the current and the power in the load will be infinite If the above analysis is true.
- The power does not reach infinity since the diode R_D is a non-linear function of the current.
- We assume that R_D varies linearly with current

$$R_D = -R_0 + aI_D$$

  R_0 is the small signal resistance and a is the slope
- When the diode starts to oscillates the signal builds up, the value of the negative resistance will drop until it is equal to the load resistance. This determines the operating point and the load current I_L.
- The power in the load is given by $P_L = I_L^2 R_L = I_D^2 R_L$

One Port Oscillator

- The resistance function and the variation of output power with I_D



- From the plot, we see that the maximum power occurs when the load is $R_o/3$.

One Port Oscillator

- To ensure the required frequency we need $X_L = -X_D$
- To sustain the oscillations we need $R_L \leq -R_o$
- For maximum power output we need $R_L = -R_o/3$

In other words

- The start oscillation condition is

$$Z_L = R_L + jX_L = -Z_D$$

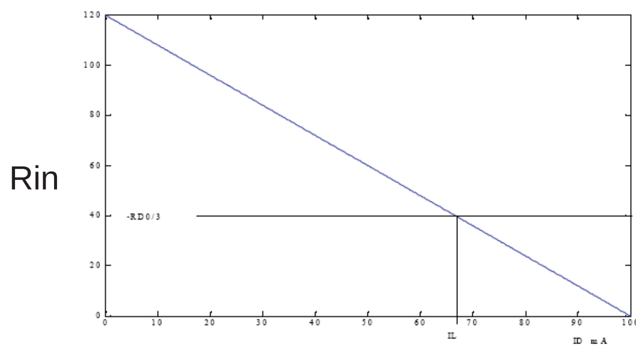
- The maximum power condition is

$$Z_L = R_L + X_L = -R_D/3 - jX_D$$

One Port Oscillator

- The impedance of a diode (Zin) is equivalent to a negative resistor and a capacitor. At 10 GHz and small signal level is R_{in}=-120Ω and C_{in}= 2pF. When the signal current was 50 mA, the parameters were R_{in} = -60 Ω and C_{in} was unchanged. Assuming that the negative resistance varies linearly with the signal current and that the capacitor is unaffected by the signal level.

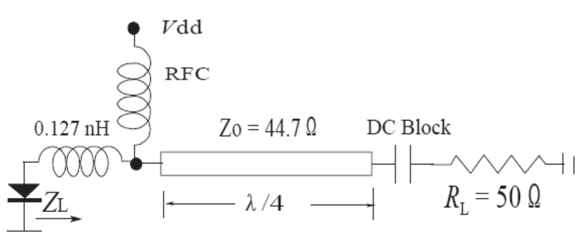
  Design an oscillator using the above diode that deliver maximum power at 10 GHz to a 50 Ω load and calculate that power.



One Port Oscillator

- The diode reactance *X*in = 1/*j*wC_{in} = -*j* 7.96 Ω. When R_{in} = -40 Ω the power is maximum and the diode impedance is given by *Z*in = -40 – *j* 7.96 Ω. Hence the required load impedance is Z_L = 40 + *j* 7.96Ω.

  An inductance of 0.127 nH will tune the diode and a quarter wave transformer of characteristic impedance 44.7Ω will transform the 50Ω load to 40Ω.



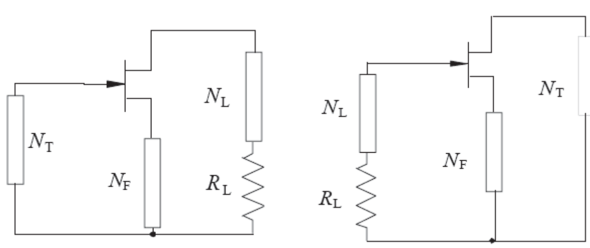
One Port Oscillator

- The value of the DC blocking capacitor is not critical as long as its impedance is low at 10 GHz with respect to the 50 Ω load. A capacitor of 20 pF will have a reactance of 0.796 Ω
- Since the current at the operating point is 66.7 mA. The power P_L in the load can be calculated from

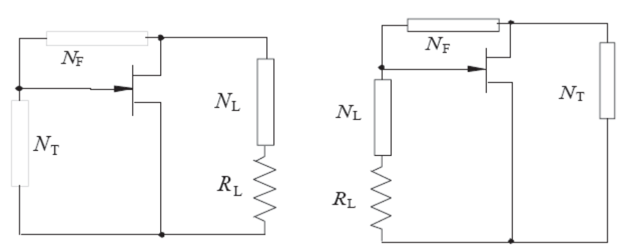
$$P_L = I_L^2 \frac{R_D}{3} = \left(66.7 \times 10^{-3}\right)^2 \times 40 = 177.8 \quad \text{mW}$$

Two Port Oscillator

- Two-port oscillators use bipolar junction transistors and FETs as the active elements



Oscillator with series feedback



Oscillator with parallel feedback

Two Port Oscillator

- Negative resistors are easily designed taking a three terminal active device and applying the correct amount of feedback to a common port, such that the magnitude of the input reflection coefficient becomes greater than one.
- The input of the 2-port negative resistance circuit can simply be terminated in the opposite sign reactance to complete the oscillator circuit.
- Several RF oscillator configurations have become standard: the Colpitts, Hartly and Clapp circuits
- Alternatively high-Q series or parallel resonator circuits can be used to generate higher quality and therefore lower phase noise oscillators.

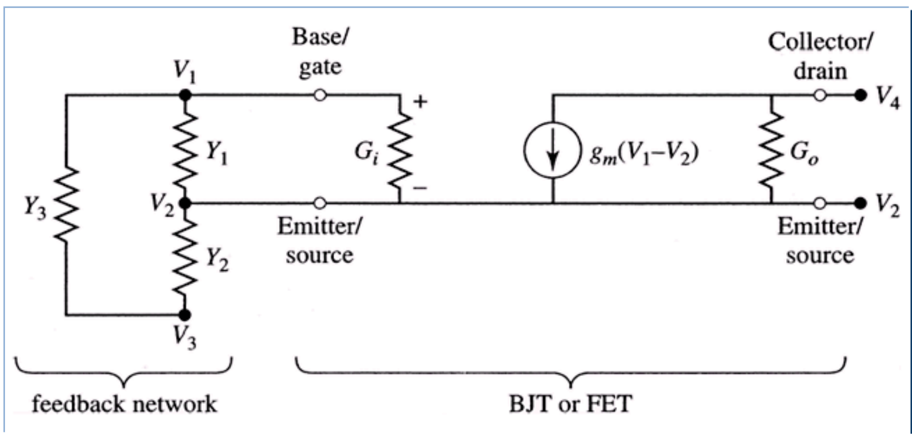
Two Port Oscillator

- Added to the active element are:
  - a feedback network N_F in order to make the device conditionally stable.
  - a terminating network N_T to fulfill the condition that the device is unstable.
  - a load network N_L to match the device to the specified load R_L

Two Port Oscillator

General circuit for a transistor oscillator.

- The transistor may be either a bipolar junction transistor or a field effect transistor.
- This circuit can be used for common emitter/source, base/gate, or collector/drain configurations by grounding either V_2, V_1, or V_4, respectively.
- Feedback is provided by connecting node V_3 to V_4.



Two Port Oscillator

Design of two port oscillator

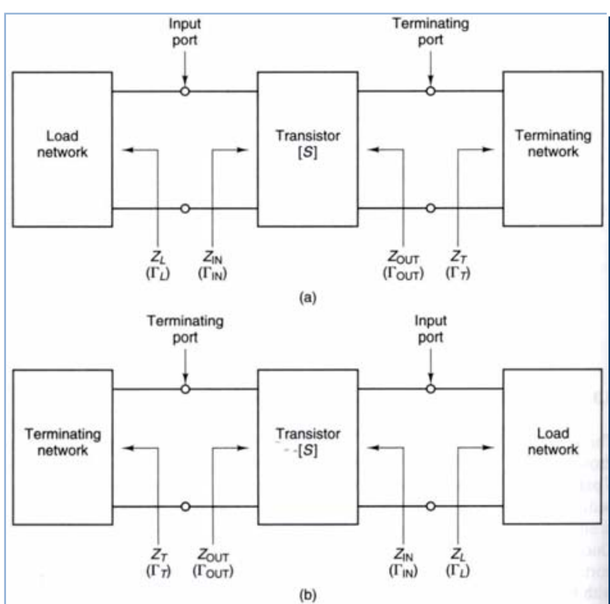


Figure 5.3.1 Two-port oscillator model.

Two Port Oscillator

Design steps

1 Choose a device which is conditionally stable.

2 Check the stability factor k and if necessary, add positive feedback to make k < 1.

3 Draw the stability circle for port 1 and choose Z_T such that the device is unstable.

4 Calculate $Z_{out} = R_{out} + j\,X_{out}$.

5 Design the load network N_L such that

$$Z_L = -\frac{R_{\text{out}}}{3} - jX_{\text{out}}$$

Example (4)

- A 2 GHz oscillator is to be designed using a common-base bipolar transistor. the S parameters of the transistor where measured at 10 GHz in a 50 Ω system and found to be

  $S11 = 0.94\angle 174$ $\quad$ $S12 = 0.013 \angle 98$

  $S21 = 1.90 \angle -28$ $\quad$ $S22 = 1.01 \angle -17$

- The stability factor k was calculated to have a value of -0.084. Although k < 0 , a lower stability factor should be used for oscillator design.
- A feedback inductor of 0.5 nH is added to the base terminal and the resulting S parameters

  $S11 = 1.04 \angle 173$ $\quad$ $S12 = 0.043 \angle 153$

  $S21 = 2.0 \angle -30$ $\quad$ $S22 = 1.05 \angle -1$

- The stability factor for these parameters is –0.8344 which drives the device more into the unstable region.

Example (4)

- Next we calculate the centers and radii of the stability circles.
- For the source stability circle

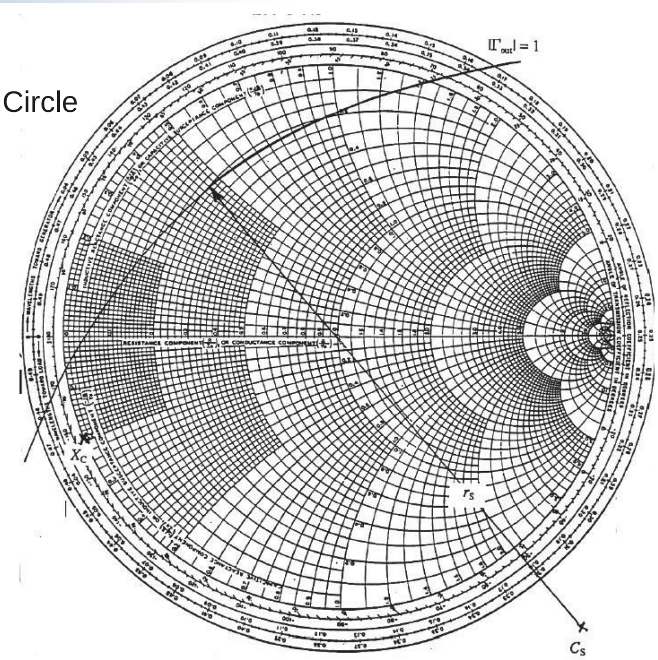
  $r_S = 2.1$ and $C_S = 1.38 \angle -51$.
- For the load stability circle

  $r_L = 1.39$ and $C_L = 0.782 \angle -112$

  We use either circles depending whether the terminating network NT is at port 1 or port 2.
- Since our topology assumes the terminating network to be at port 1, we use the source stability circle.
- From the chart, since |S22| >1, the inside of the circle is unstable

Example (4)

Source Stability Circle



Example (4)

- From the chart we see that a normalized capacitive impedance of -j 0.159 will make the device unstable. Thus we need a capacitor whose reactance is 50x0.159 = 7.95 S at 2 GHz.

$$C = \frac{7.95}{2\pi \times 2 \times 10^9} = 10 \quad \text{pF}$$

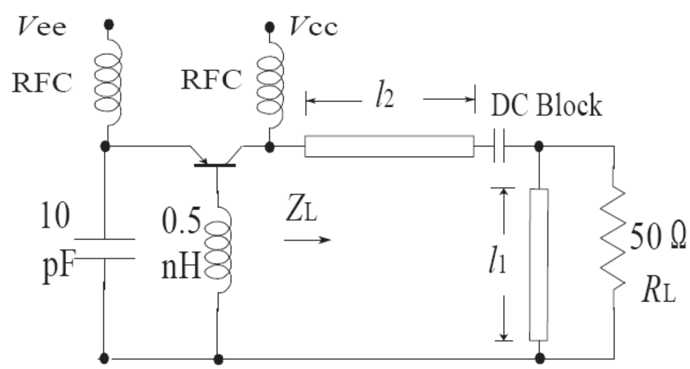
- With this capacitor connected at port 1, the output impedance was calculated to be Zout = -477 + j 58 Ω
- The required load impedance is then given by

$$Z_L = -\frac{R_{out}}{3} - jX_{out} = \frac{477}{3} - j58 \quad \Omega$$

- And the normalize load impedance is 3.18 – j1.16

We have now all the information to design the oscillator.

Example (4)



Circuit design for the transistor oscillator

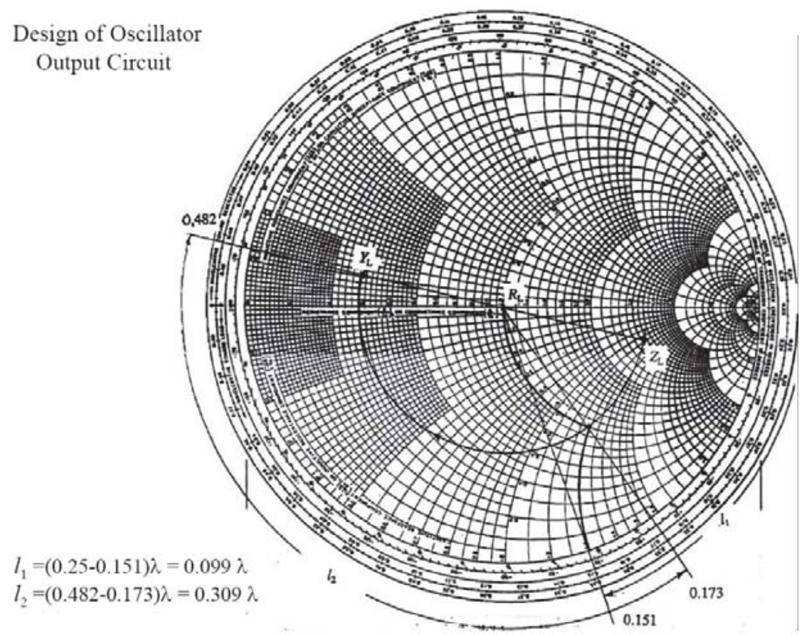
Example (4)

- In the above circuit, the RFC (RF chokes) must have very high reactances at the operating frequency. The DC block on the other hand must have a very low reactance. The transmission line and the short circuited stub form a matching network to transform the 50 Ω load to the required impedance Z_L.
- What remains is to calculate the lengths l1 and l2 of the transmission line and stub. From the Smith chart

  $l1 = (0.25 – 0.151)\ \lambda = 0.099\ \lambda$

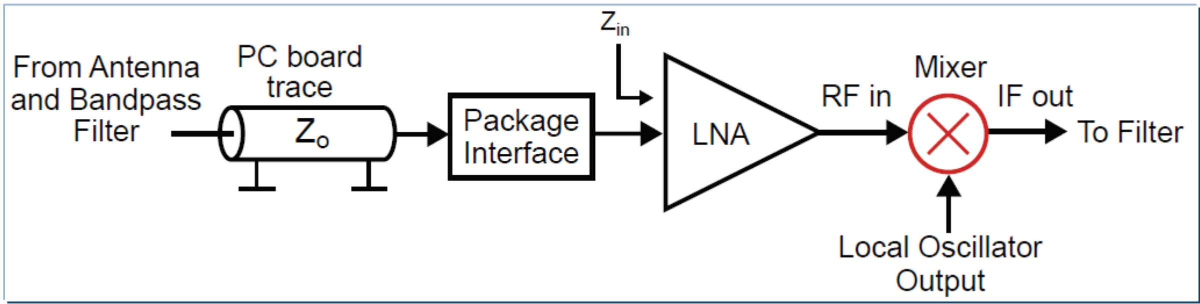
  $l2 = (0.482 – 0.173)\ \lambda = 0.309\ \lambda$
- After obtaining the lengths, the transmission lines can be designed in microstrip form for example.

Example (4)



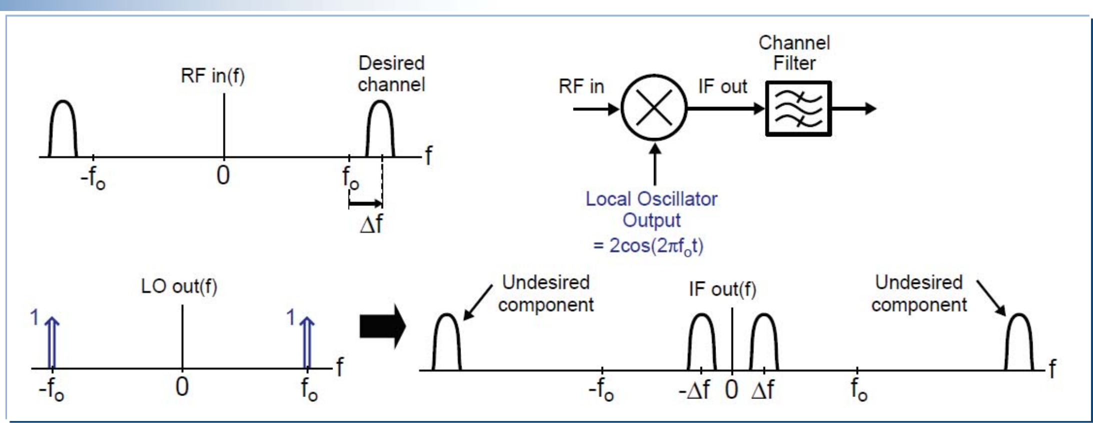
Mixer in Wireless Systems

- A Mixer uses two input signals at two distinct frequencies to create new frequencies at the output.



Heterodyne receiver system

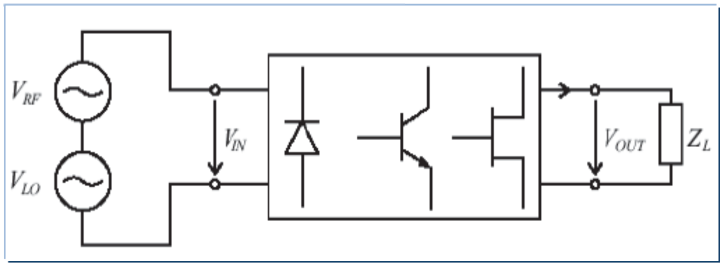
Mixer in Wireless Systems

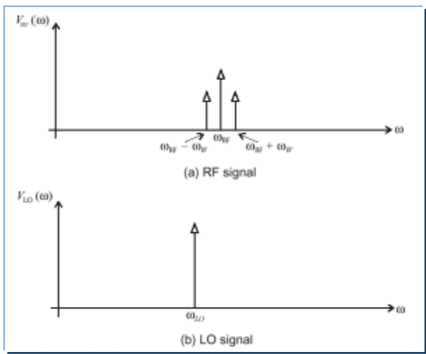


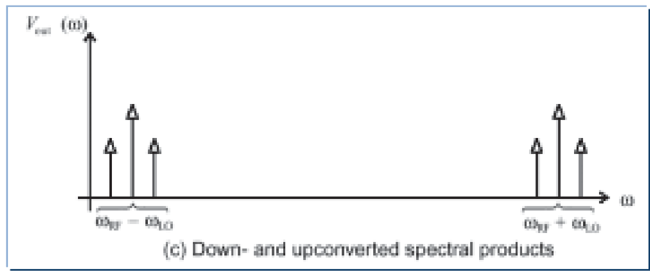
- RF spectrum converted to a lower IF center frequency
  - IF stands for intermediate frequency
    - IF frequency is nonzero : heterodyne or low IF receiver
    - IF frequency is zero : homodyne receiver
  - Use a filter at the IF output to remove undesired high frequency components

Introduction

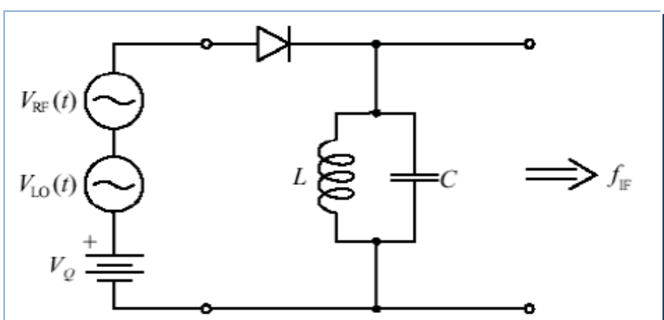
- Mixing Process Spectrum



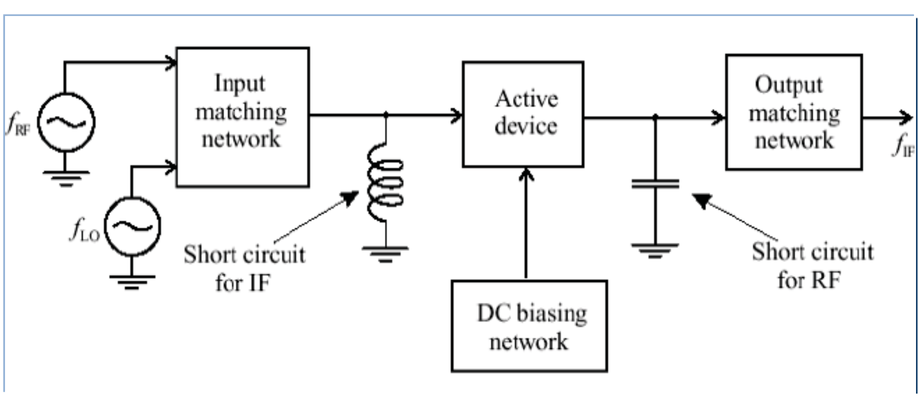




Simple Diode and FET Mixers



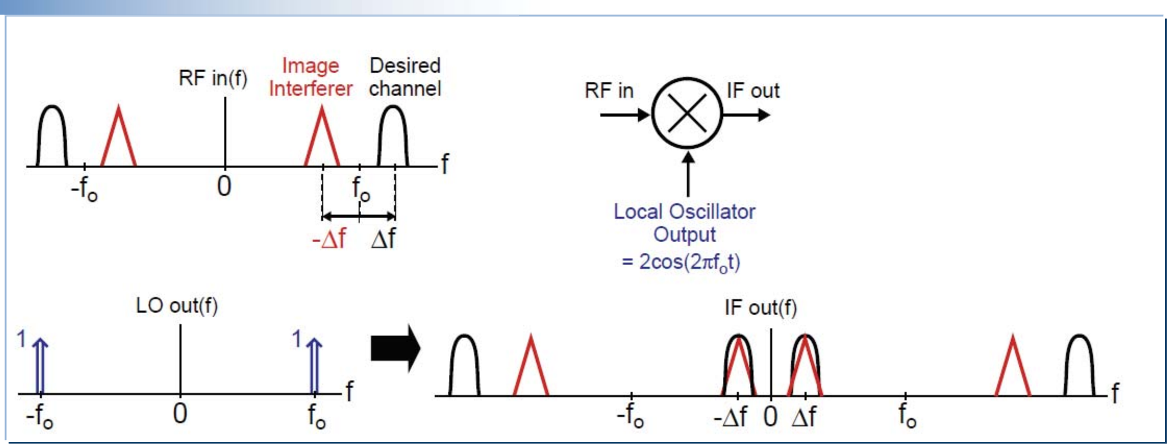
(a) Diode mixer



Design Issues

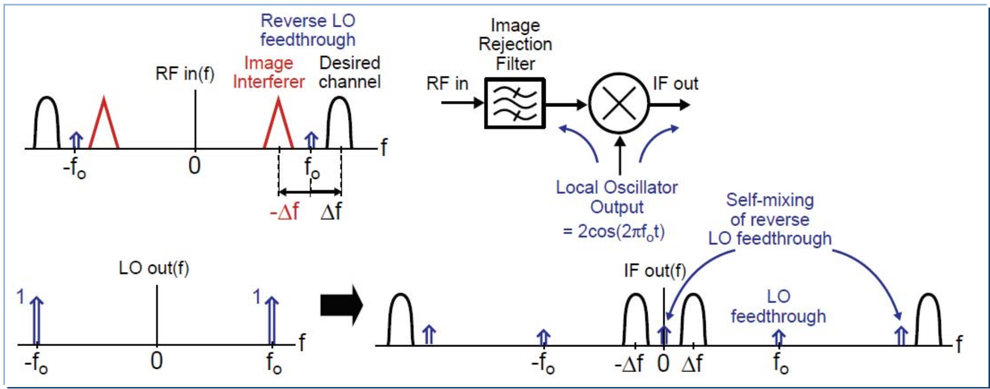
- **Noise Figure : impacts receiver sensitivity**
- **Linearity : impacts receiver blocking performance**
- **Conversion gain: lowers noise impact of following stages**
- **Power : want low power dissipation**
- **Isolation : want to minimize interaction between the RF, IF, and LO ports**
- **Sensitivity to process/temp variations**

Image Problem



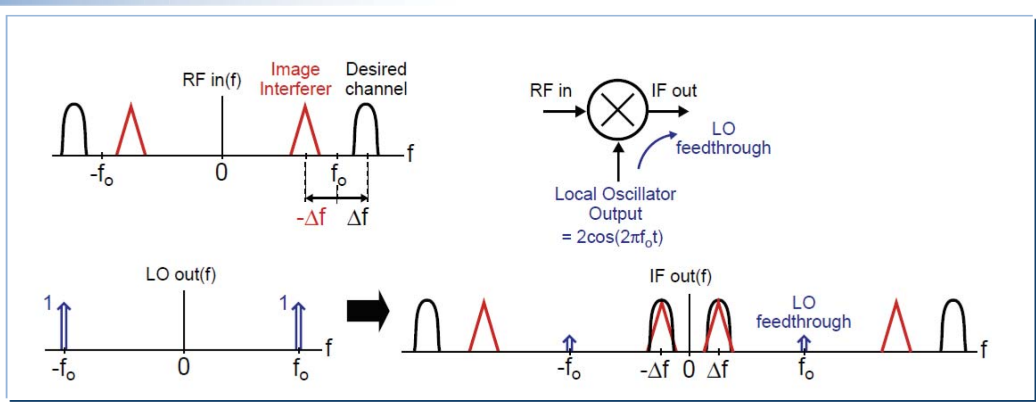
- When the IF frequency is nonzero, there is an image band for a given desired channel band - Frequency content in image band will combine with that of the desired channel at the IF output
- The impact of the image interference cannot be removed through filtering at the IF output!

Image Problem



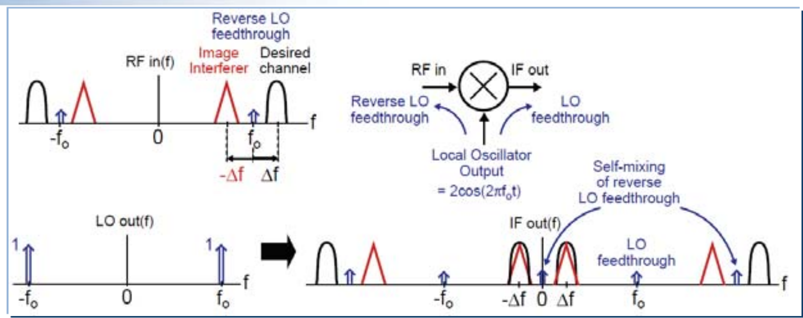
- An image reject filter can be used before the mixer to prevent the image content from aliasing into the desired channel at the IF output
- Issue – must have a high IF frequency
  - Filter bandwidth must be large enough to pass all channels
  - Filter Q cannot be arbitrarily large (low IF requires high Q)

LO feed-through



- will occur from the LO port to IF output port due to parasitic capacitance, power supply coupling, etc.
- Often significant since LO output much higher than RF signal
  - If large, can potentially desensitize the receiver due to the extra dynamic range consumed at the IF output
  - If small, can generally be removed by filter at IF output

Reverse LO feed-through



- will occur from the LO port to RF input port due to parasitic capacitance, etc.
  – If large, and LNA doesn’t provide adequate isolation, then LO energy can leak out of antenna and violate emission standards for radio
  – Must insure that isolate to antenna is adequate
- LO component in the RF input can pass back through the mixer and be modulated by the LO signal

  - DC and 2fo component created at IF output

  - Of no consequence for a heterodyne system, but can cause problems for homodyne systems (i.e., zero IF)