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Course: CS590-A Algorithms

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Description: Homework 1 Algorithms

Problem 1

Definition 1. *$a \mid b$ ("a divides b") if and only if there exists some integer k such that $b = ak$. Equivalently, $a \mid b$ if and only if b has a remainder of 0 when divided by a (see question 2).*

1. Using the formal definition of divisibility above, prove that there exists positive integers a, b, and c such that $a \mid bc$, but $a \nmid b$ and $a \nmid c$.

   Theorem 1. *The Division Algorithm. For any integers a and b where $b \neq 0$, there exist a unique pair of integers q and r such that $a = qb + r$ and $0 \leq r \leq b$. The integers q and r are known as the quotient and remainder of $a \div b$, respectively.*

2. Using the formal definition of the remainder above, prove that if n and m are positive integers such that n has a remainder of r when divided by m and $r < \sqrt{m}$, n^2 has a remainder of r^2 when divided by m.

3. Use the formal definition of Big-Oh to prove that if $f(n) = n^x + an^y$, where a, x, and y are positive integers such that $x > y$, $f(n) = O(n^x)$.

4. Use the formal definition of Big-Omega to prove that if $f_1(n)$, $f_2(n)$, $g_1(n)$, and $g_2(n)$ are functions such that $f_1(n) = \Omega(g_1(n))$ and $f_2(n) = \Omega(g_2(n))$, $f_1(n) + f_2(n) = \Omega(\max(g_1(n), g_2(n)))$.