PSTAT160A Stochastic Processes
Section 4

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Problem 1 - Dobrow Q2.1

A Markov chain has transition matrix

$$P := \begin{bmatrix} 0.1 & 0.3 & 0.6 \\ 0 & 0.4 & 0.6 \\ 0.3 & 0.2 & 0.5 \end{bmatrix},$$

with initial distribution $\alpha = \begin{bmatrix} 0.2 & 0.3 & 0.5 \end{bmatrix}$. Find the following:

1. $\mathbb{P}(X_7 = 3 | X_6 = 2)$,
2. $\mathbb{P}(X_9 = 2 | X_1 = 2, X_5 = 1, X_7 = 3)$,
3. $\mathbb{P}(X_0 | X_1 = 1)$,
4. $\mathbb{E}[X_2]$.

Problem 2 - Dobrow Q2.2

Let $X_0, X_1, \ldots$ be a Markov chain with transition matrix

$$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix},$$

and initial distribution $\alpha = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$. Find the following:

1. $\mathbb{P}(X_2 = 1 | X_1 = 3)$,
2. $\mathbb{P}(X_1 = 3, X_2 = 1)$,
3. $\mathbb{P}(X_1 = 3 | X_2 = 1)$,
4. $\mathbb{P}(X_9 = 1 | X_1 = 3, X_4 = 1, X_7 = 2)$.

Problem 3 - Dobrow Q2.4

For the general two-state chain with transition matrix

$$P = \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix},$$

and initial distribution $\alpha = \begin{bmatrix} \alpha_1 & \alpha_2 \end{bmatrix}$ find the following:

1. the two-step transition matrix
2. the distribution X_1.

Problem 4 - Dobrow Q2.6

A tetrahedron die has four faces labelled 1, 2, 3 and 4. In repeated independent rolls of the die $R_0, R_1, \ldots$, let $X_n = \max(R_0, \ldots, R_n)$ be the maximum value after $n + 1$ rolls, for $n \geq 0$:

1. Give an intuitive argument for why $X_0, X_1, \ldots$ is a Markov chain, and exhibit the transition matrix.
2. Find $\mathbb{P}(X_3 \geq 3)$.

Problem 5 - Dobrow Q2.7

Let $X_0, X_1, \ldots$ be a Markov chain with transition matrix P. Let $Y_n = X_{3n}$, for $n = 0, 1, 2, \ldots$. Show that $Y_0, Y_1, \ldots$ is a Markov chain and exhibit its transition matrix.