

PSTAT160A Stochastic Processes

Section 6

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Problem 1 - Dobrow 4.3

Let $X \sim \text{Poisson}(\lambda)$ and $Y \sim \text{Poisson}(\mu)$. Assume that X and Y are independent. Use PGFs to find the distribution of $X + Y$.

Problem 2 - Dobrow 4.4

If X has a *Negative Binomial Distribution* with parameters r and p , then X can be written as the sum of r i.i.d. *Geometric* random variables with parameter p . Use this fact to find the PGF of X . Then, use the PGF to find the mean and variance of the negative binomial distribution.

Problem 3 - Dobrow 4.12

A branching process has offspring distribution $\alpha = (\frac{1}{4}, \frac{1}{4}, \frac{1}{2})$. Find the following:

1. μ ;
2. $G(s)$;
3. The extinction probability;
4. $G_2(s)$; and
5. $\mathbb{P}(Z_2 = 0)$.

Problem 4 - Dobrow 4.18

Consider a branching process with offspring distribution

$$\alpha = (p^2, 2p(1-p), (1-p)^2); \quad 0 < p < 1.$$

The offspring distribution is binomial with parameters 2 and $1 - p$. Find the extinction probability η .

Problem 5 - Dobrow 4.30

R: Simulate the branching process in Problem 3 (Dobrow 4.12). Use your simulation to estimate the extinction probability e .