

# PSTAT160A Stochastic Processes

## Section 5

**Authors:** Denisse Alejandra Escobar Parra, John Inston

**Date:** November 4, 2025

---

### Problem 1 - Dobrow 3.29

Consider the Markov Chain (MC) with transition probability matrix

$$P = \begin{bmatrix} 0.6 & 0.2 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.3 & 0.7 & 0 & 0 & 0 \\ 0 & 0.3 & 0.4 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0.9 \\ 0 & 0.2 & 0 & 0 & 0 & 0.8 & 0 \end{bmatrix}.$$

Identify the communication classes. Classify the states as recurrent or transient, and determine the period of each state.

### Problem 2 - Dobrow 3.46

Given a MC with transition matrix  $P$  and stationary distribution  $\pi$ , the time reversal is a Markov chain with transition matrix  $\tilde{P}$  defined by

$$\tilde{P}_{i,j} = \frac{\pi_j P_{j,i}}{\pi_i},$$

for all  $i, j \in \mathcal{S}$ .

1. Show that a Markov chain with transition matrix  $P$  is reversible if and only if  $P = \tilde{P}$ .
2. Show that the time reversal Markov chain has the same stationary distribution as the original one.

### Problem 3 - Dobrow 3.52

The board for a modified Snakes and Ladder game is shown in Figure 1 below. The game is played with a tetrahedron (4-sided) die.

1. Find the expected length of the game.
2. Assume that the player is on square 6. Find the probability that they will find themselves at square 3 before finishing the game.

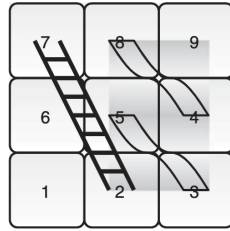


Figure 1: Modified snakes and ladders.

#### Problem 4 - Dobrow 3.57

In repeated coin flips consider the set of all 3-element patterns

$$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}.$$

Which patterns take the longest time, on average, to appear in repeated sampling? Which take the shortest?

#### Problem 5 - Dobrow 3.64

The evolution of forest ecosystems in the United States and Canada is studied in Strigul et al. (2012) using Markov chains. Five-year changes in the state of the forest soil are modeled with a 12-state Markov chain. The transition matrix can be found in the R-script file `forest.R`. About how many years does it take for the ecosystem to move from state 1 to state 12?