PSTAT160A Stochastic Processes
Section 2

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Problem 1 - Dobrow Q1.9

Assume that X is uniformly distributed on $\{1, 2, 3, 4\}$. If $X = x$, then Y is uniformly distributed on $\{1, \dots, x\}$. Find

(a) $\mathbb{P}(Y = 2 \mid X = 2)$

(b) $\mathbb{P}(Y = 2)$

(c) $\mathbb{P}(X = 2 \mid Y = 2)$

(d) $\mathbb{P}(X = 2)$

(e) $\mathbb{P}(X = 2, Y = 2)$

Problem 2 - Dobrow Q1.14

Random variables X and Y have joint density function

$$f(x, y) = 4e^{-2x}, \quad \text{for } 0 < y < x < \infty.$$

(a) Find the conditional density of X given $Y = y$.

(b) Find the conditional density of Y given $X = x$. Describe the conditional distribution.

Problem 3 - Dobrow Q1.18

From the definition of conditional expectation given an event, show that

$$\mathbb{E}(I_B \mid A) = \mathbb{P}(B \mid A)$$

Problem 4 - Dobrow Q1.33

R: Cards are drawn from a standard deck, with replacement, until an ace appears. Simulate the mean and variance of the number of cards required.