AlgePRO

Lesson 3

THE PRODUCT OF A SUM AND DIFFERENCE OF THE SAME TWO TERMS

MATH 8 - QUARTER 1

Recap on the FOIL Method

As we have seen before, we can use the **FOIL** method to multiply binomials. Now let us refresh our memory! The formula for the **FOIL** method is as follows:

Examples:

1. $(x - 1)(x + 1)$

Last

First

$(x - 1)(x + 1)$

Inner

Outer

First: x^2
Outer: x
Inner: $-x$
Last: -1

Then, we have $x^2 + x - x - 1$.

By combining like terms, we now have $\mathbf{x^2 - 1}$.

2. $(x - 4)^2$

Last

First

$(x - 4)(x - 4)$

Inner

Outer

First: x^2
Outer: $-4x$
Inner: $-4x$
Last: 16

Then, we have $x^2 - 4x - 4x + 16$.

By combining like terms, we now have $\mathbf{x^2 - 8x + 16}$.

The Product of a Sum and Difference of the Same Two Terms

The product of the sum and difference of the same two terms is always the difference of two squares and is denoted by,

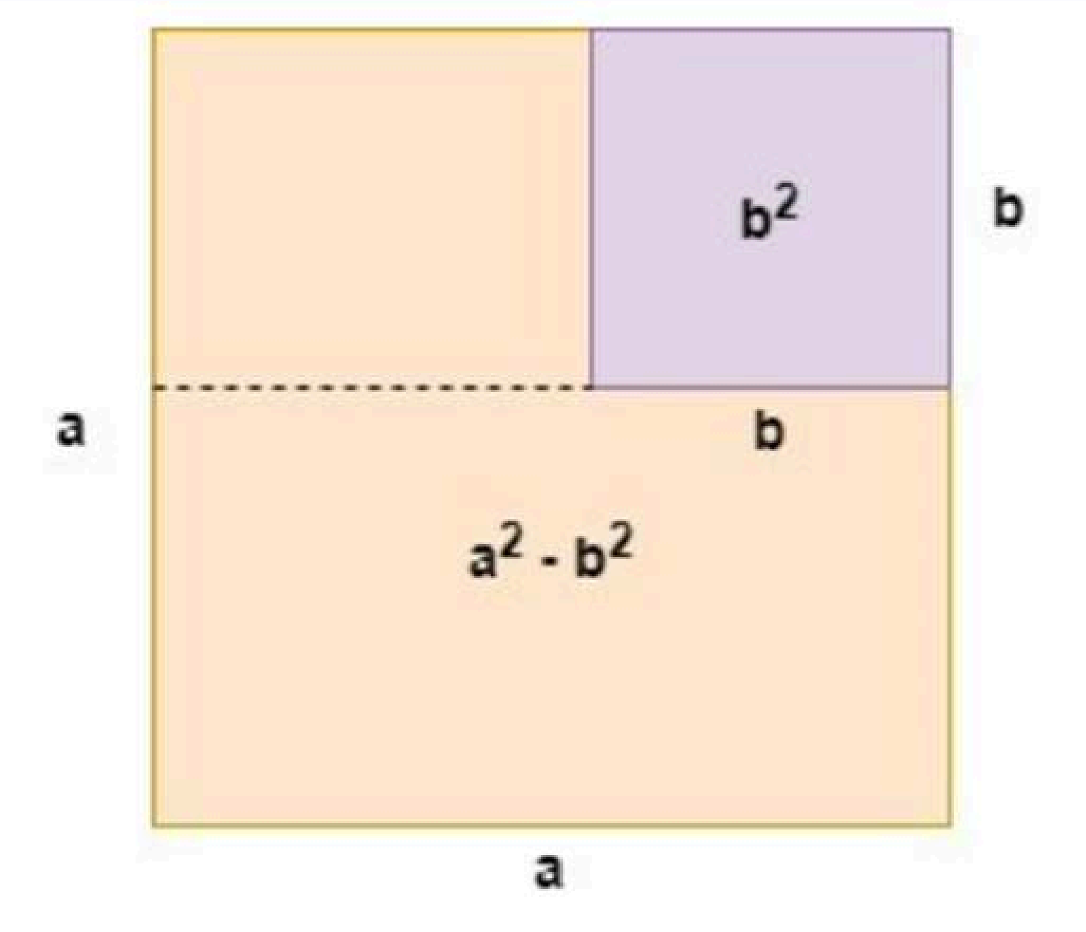
$$(a + b)(a - b) = a^2 - b^2$$

where a and b are the first and second terms, respectively.

It is the first term squared minus the second term squared. Thus, this resulting binomial is called a difference of squares.

Geometrical Representation

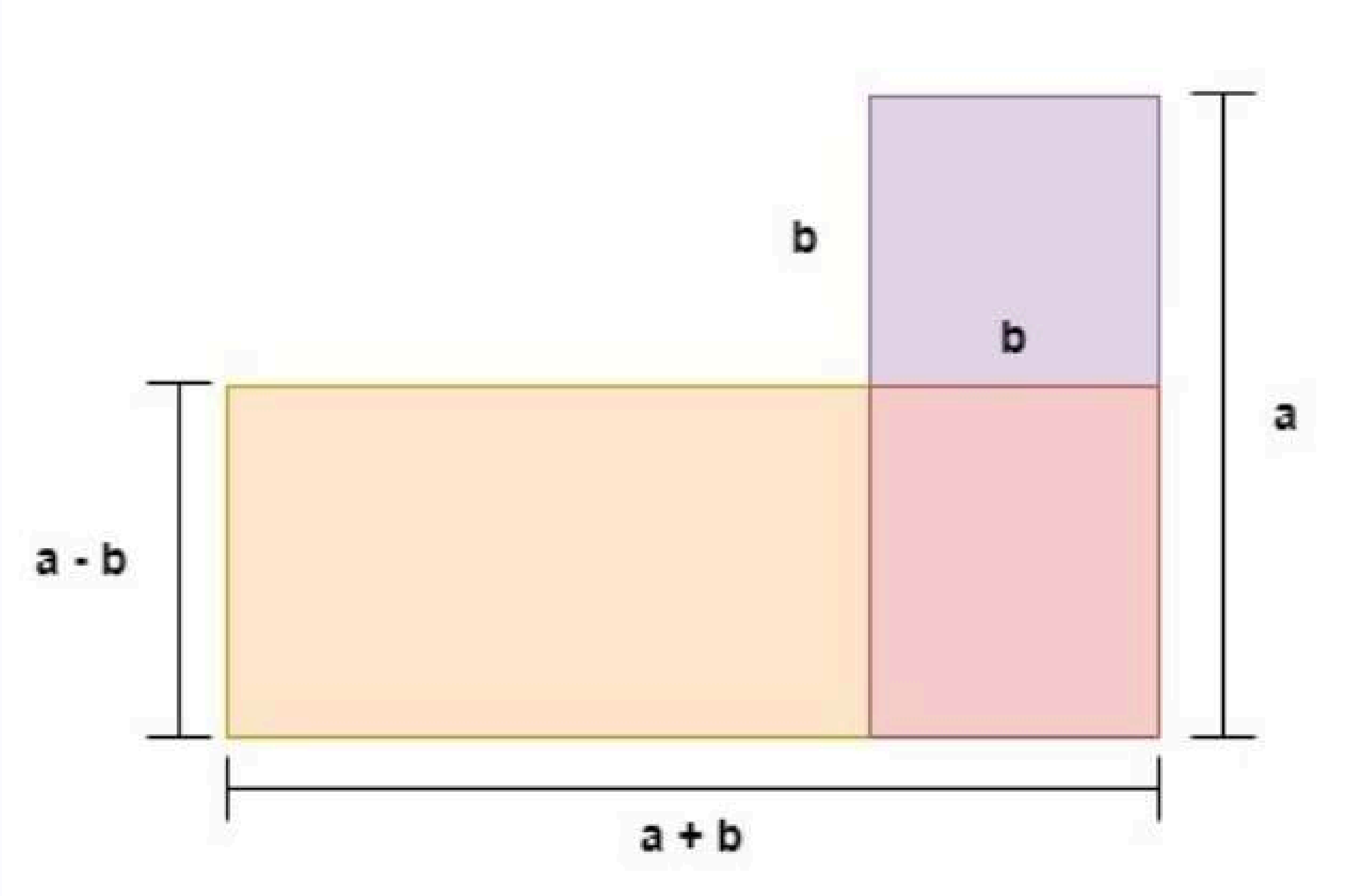
To help us understand what this means, let us try to visualize this pattern geometrically. Observe the square below.



Here, the large square has an area of a^2 and the purple square has an area of b^2. Thus, the area of the orange region is the area of the large square minus the area of the purple square, a^2-b^2. We have thus proven the right-hand side of the sum and difference formula.

To show the result of the left-hand side, we first draw a dotted horizontal line as shown in the diagram above. This creates a segment that separates the orange region into two rectangles. We will then move this new segment to the left-hand side as displayed below. For a clearer view, this new segment is represented by the colour red.

Geometrical Representation



We now have to find the area of the orange region and the area of the red region. Here, we see that the horizontal length of these two regions is $a + b$ while the vertical length is $a - b$. Thus, the combined area of the orange and red rectangle is $(a + b)(a - b)$, which satisfies the left-hand side of the sum and difference formula.

Worked Examples

Let us look at the following worked examples.

Direction: Expand the following terms.

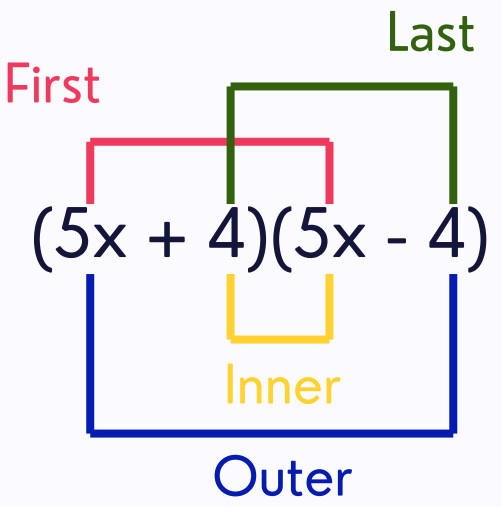
1. $(5x + 4)(5x - 4)$

Solution: Here, $a = 5x$ and $b = 4$, so

$(5x + 4)(5x - 4) = a^2 - b^2 = (5x)^2 - (4)^2$

Answer: $\mathbf{25x^2 - 16}$

Using FOIL Method



First: $25x^2$
Outer: $-20x$
Inner: $20x$
Last: -16

Then, we have $25x^2 - 20x + 20x - 16$.

By combining like terms, we now have $\mathbf{25x^2 - 16}$.

Worked Examples

Let us look at the following worked examples.

Direction: Expand the following terms.

2. $(3x + 7)(3x - 7)$

Solution: Here, $a = 3x$ and $b = 7$, so

$(3x + 7)(3x - 7) = a^2 - b^2 = (3x)^2 - (7)^2$

Answer: $\mathbf{9x^2 - 49}$

Using FOIL Method

Last

First

$(3x + 7)(3x - 7)$

Inner

Outer

First: $9x^2$
Outer: $-21x$
Inner: $21x$
Last: -49

Then, we have $9x^2 - 21x + 21x - 49$.

By combining like terms, we now have $\mathbf{9x^2 - 49}$.