AlgePRO

Lesson 1

SQUARE OF BINOMIALS

MATH 8 - QUARTER 1

Square of a Binomial

To square a binomial, multiply the binomial by itself. The square of a binomial can take on two cases:

Case 1: $(a + b)^2 = a^2 + 2ab + b^2$

Case 2: $(a - b)^2 = a^2 - 2ab + b^2$

In both scenarios, the middle term is twice the product of the terms in the binomial. However, in Case 1, we have a positive term while in Case 2, we have a negative term. For these cases, the pattern yields a perfect square trinomial.

Remember:

The square of a binomial is always the sum of:

1. The first term squared $\mathbf{(a)^2}$,
2. 2 times the product of the first and second terms **(2ab)**, and
3. the second term squared $\mathbf{(b)^2}$.

FOIL Method

We can also use the **FOIL** method to multiply binomials. The formula for the **FOIL** method is as follows:

Examples:

1. $(x - 1)(x + 1)$

First
Last

$(x - 1)(x + 1)$

Inner
Outer

First: x^2
Outer: x
Inner: $-x$
Last: -1

Then, we have $x^2 + x - x - 1$.

By combining like terms, we now have $\mathbf{x^2 - 1}$.

2. $(x - 4)^2$

First
Last

$(x - 4)(x - 4)$

Inner
Outer

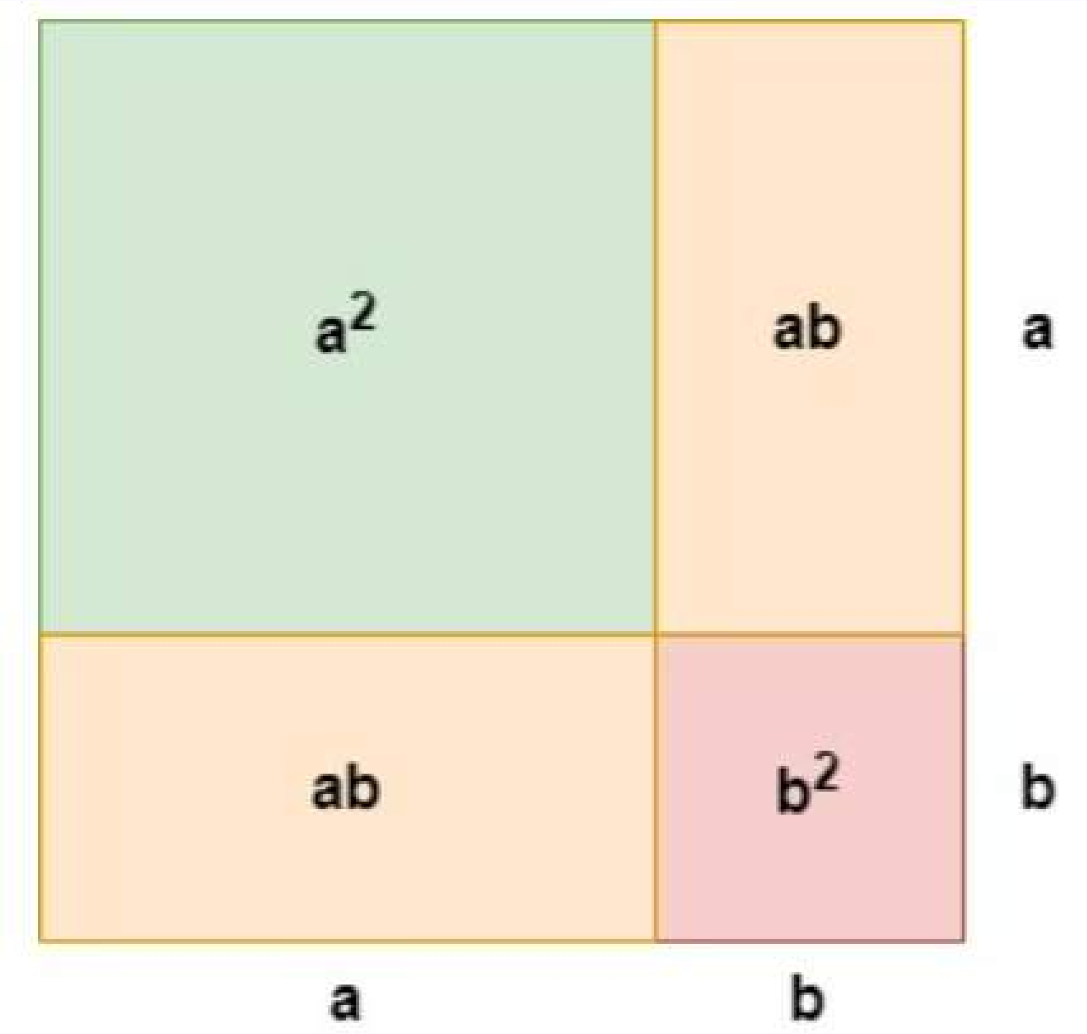
First: x^2
Outer: $-4x$
Inner: $-4x$
Last: 16

Then, we have $x^2 - 4x - 4x + 16$.

By combining like terms, we now have $\mathbf{x^2 - 8x + 16}$.

Geometrical Representation

So what does this mean geometrically speaking? For clarity, let us define the following terms.



Definition

A perfect square is an integer that is a square of an integer. For instance, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$ and so on.

A trinomial is a polynomial of three terms.

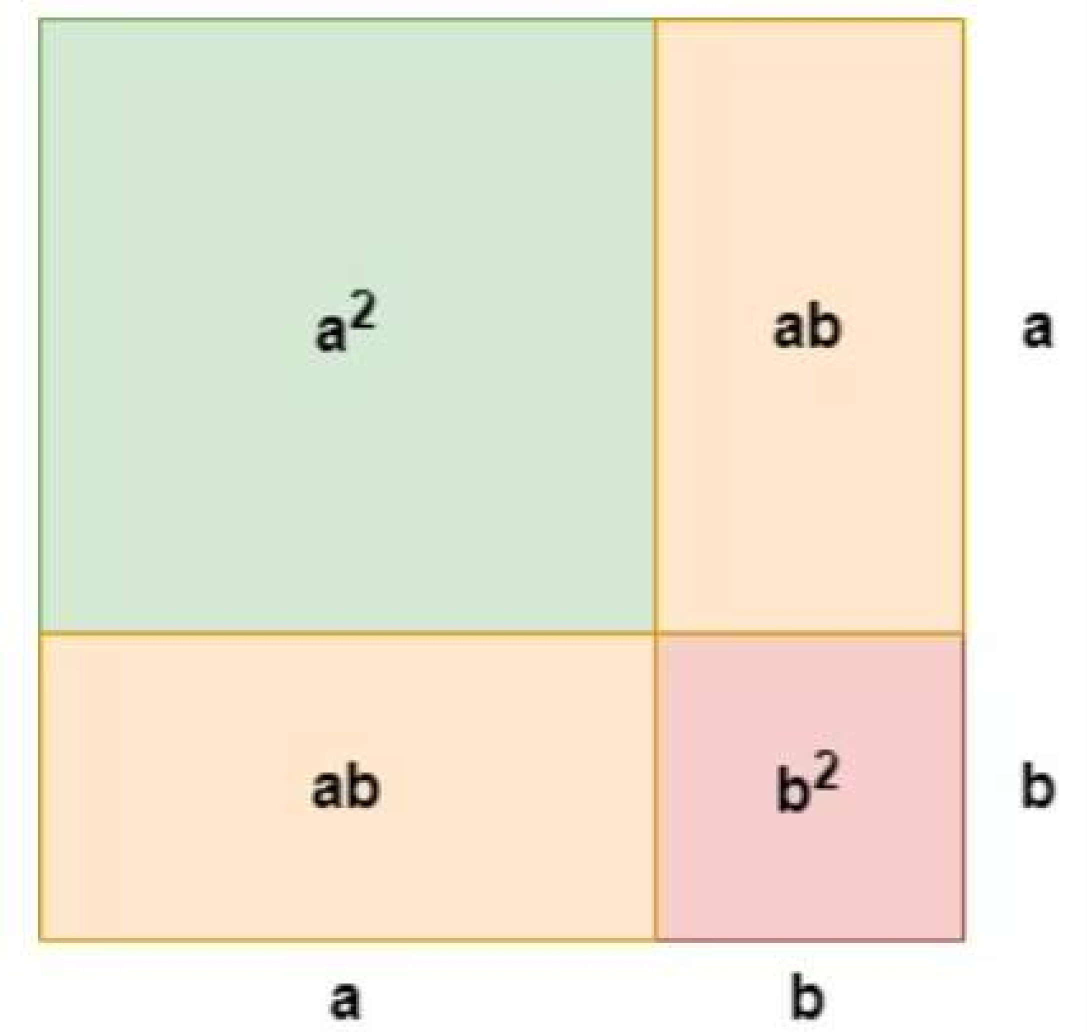
A binomial is a polynomial of two terms.

A perfect square trinomial is a trinomial that can be written as a square of a binomial such as the two cases above.

Geometrical Representation

Looking at the diagram below, we can see that the large square has an area of $(a + b)(a + b) = (a + b)^2$. This is made up of the area of the red and green squares and the area of the two identical orange rectangles.

Adding these up, we obtain $a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$. Thus, we have deduced the first case.



The same concept follows in the second case. In this case, we only want the area of the green square and the area of the red square which is $a^2 + b^2$.

Here, we are essentially subtracting the area of the two identical orange rectangles from the area of the green and red squares as:

$$a^2 - ab - ab + b^2 = a^2 - 2ab + b^2.$$

Worked Examples

Let us look at the following worked examples.

Direction: Expand the following terms.

1. $(x + 2)^2$

Remember:

The square of a binomial is always the sum of:

1. The first term squared $(a)^2$,
2. 2 times the product of the first and second terms $(2ab)$, and
3. the second term squared $(b)^2$.

Solution: Here, $a = x$ and $b = 2$, so

1. $a^2 = (x)^2 = x^2$
2. $2ab = 2(x)(2) = 4x$
3. $b^2 = (2)^2 = 4$

Answer: Then, we now have $x^2 + 4x + 4$.

Worked Examples

Let us look at the following worked examples.

Direction: Expand the following terms.

Using FOIL Method

1. $(x + 2)^2$

Last

First

$$(x + 2)(x + 2)$$

Inner

Outer

First: x^2
Outer: $2x$
Inner: $2x$
Last: 4

Then, we have $x^2 + 2x + 2x + 4$.

By combining like terms, we now have $\mathbf{x^2 + 4x + 4}$.

Worked Examples

Let us look at the following worked examples.

Direction: Expand the following terms.

2. $(4x - 3)^2$

Remember:

The square of a binomial is always the sum of:

1. The first term squared **$(a)^2$**,
2. 2 times the product of the first and second terms **(2ab)**, and
3. the second term squared **$(b)^2$**.

Solution: Here, $a = 4x$ and $b = -3$, so

1. $a^2 = (4x)^2 = 16x^2$
2. $2ab = 2(4x)(-3) = -24x$
3. $b^2 = (-3)^2 = 9$

Answer: Then, we now have **$16x^2 - 24x + 9$**.

Worked Examples

Let us look at the following worked examples.

Direction: Expand the following terms.

Using FOIL Method

1. $(4x - 3)^2$

Last

First

$(4x - 3)(4x - 3)$

Inner

Outer

First: $16x^2$
Outer: $-12x$
Inner: $-12x$
Last: 9

Then, we have $16x^2 - 12x - 12x + 9$.

By combining like terms, we now have $\mathbf{16x^2 - 24x + 9}$.