AlgePRO

Lesson 4

CUBE OF BINOMIALS

MATH 8 - QUARTER 1

Cube of a Binomial

To cube a binomial, multiply the binomial by itself twice. The cube of a binomial can take on two cases:

Case 1: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Case 2: $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

In both scenarios, the cube of a binomial contains four terms. The first and last terms are always the cubes of the individual terms in the binomial (a^3 and b^3). The middle two terms involve the product of both terms of the binomial, with coefficients of 3.

However, in Case 1 $(a + b)^3$, all terms are positive, while in Case 2 $(a - b)^3$, the terms containing odd powers of b are negative. For these cases, the pattern yields a four-term polynomial that represents the cube of the original binomial.

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Case 1: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Case 2: $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Remember:

1. The first term is always a^3 in both cases.
2. The second term is $3a^2b$ in Case 1, but $-3a^2b$ in Case 2.
3. The third term is $3ab^2$ in both cases.
4. The last term is b^3 in Case 1, but $-b^3$ in Case 2.

This pattern allows us to quickly expand the cube of any binomial without having to multiply it out step by step each time.

Cube of a Binomial Formula

We can also use the **FOIL method** and **distributive property** to multiply binomials:

$$(a + b)^3 = (a + b)(a + b)(a + b)$$

Last

First

$$= (a + b)(a + b)(a + b)$$

Inner

Outer

$$= (a^2 + ab + ab + b^2)(a + b)$$

$$= (a^2 + 2ab + b^2)(a + b)$$

$$= a^3 + a^2b + 2a^2b + 2ab^2 + ab^2 + b^3$$

After combining like terms, the result is:

$$= \mathbf{a^3 + 3a^2b + 3ab^2 + b^3}$$

Cube of a Binomial Formula

We can also use the **FOIL method** and **distributive property** to multiply binomials:

Examples:

1. $(x + 2)^3$

$$(x + 2)^3 = (x + 2)(x + 2)(x + 2)$$

Last

First

$$= (x + 2)(x + 2)(x + 2)$$

Inner

Outer

$$= (x^2 + 2x + 2x + 4)(x + 2)$$

$$= (x^2 + 4x + 4)(x + 2)$$

$$= x^3 + 4x^2 + 4x + 2x^2 + 8x + 8$$

$$\boxed{= x^3 + 6x^2 + 12x + 8}$$

2. $(2x - 1)^3$

$$(2x - 1)^3 = (2x - 1)(2x - 1)(2x - 1)$$

Last

First

$$= (2x - 1)(2x - 1)(2x - 1)$$

Inner

Outer

$$= (4x^2 - 2x - 2x + 1)(2x - 1)$$

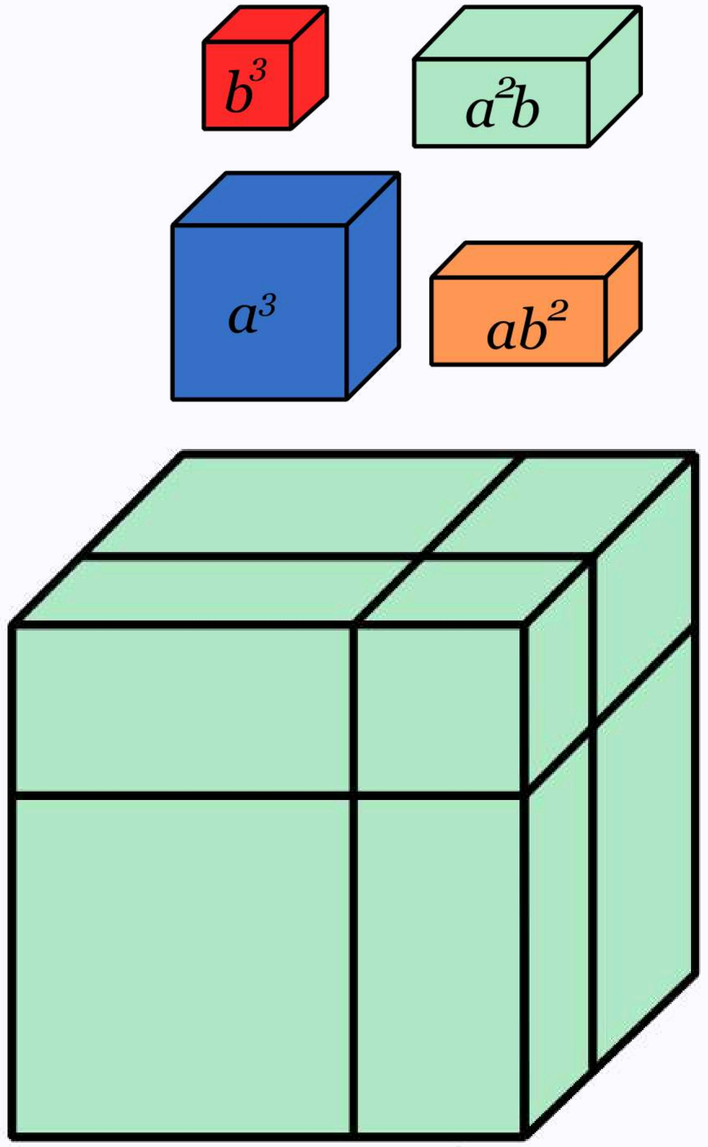
$$= (4x^2 - 4x + 1)(2x - 1)$$

$$= 8x^3 - 8x^2 + 2x - 4x^2 + 4x - 1$$

$$\boxed{= 8x^3 - 12x^2 + 6x - 1}$$

Geometrical Representation

The cube of a binomial $(a + b)^3$ can be visualized as a large cube with side length $(a + b)$. This cube can be decomposed into smaller cubes and rectangular prisms, each corresponding to a term in the algebraic expansion.

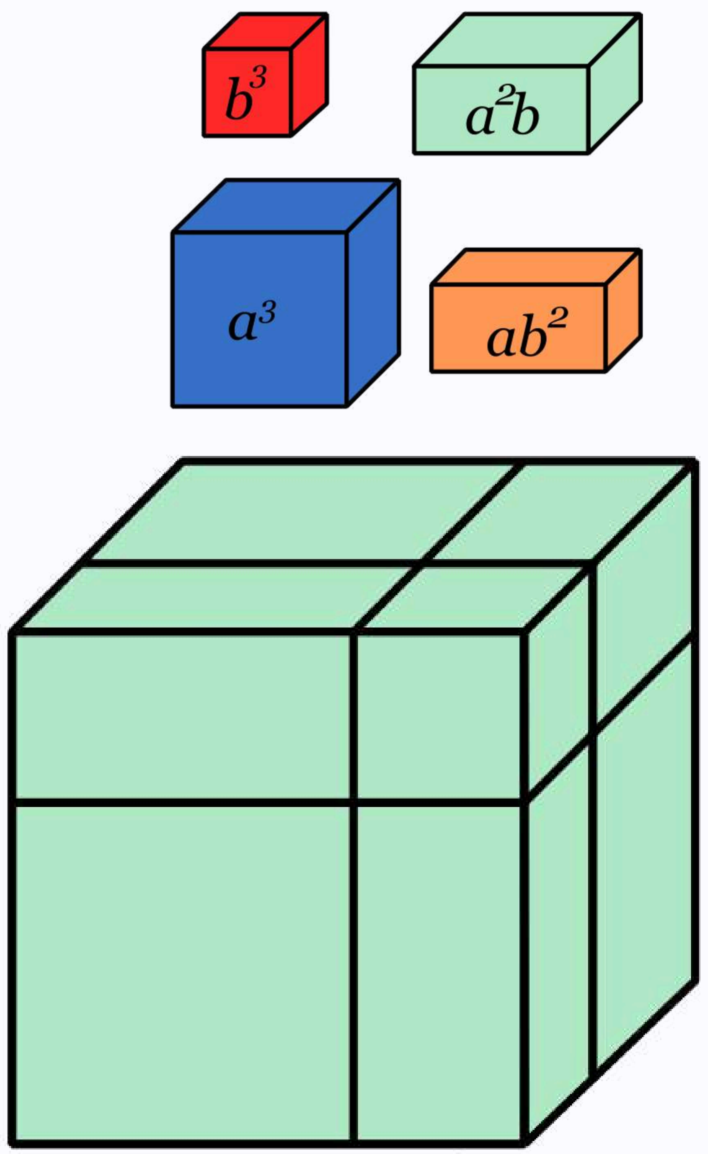


1. Large cube (a^3):
   - This represents the first term of the expansion.
   - It has dimensions $a \times a \times a$.

2. Three identical rectangular prisms ($3a^2b$):
   - These represent the second term of the expansion.
   - Each has dimensions $a \times a \times b$.
   - There are three of these because they can be placed on three faces of the large cube.

Geometrical Representation

The cube of a binomial $(a + b)^3$ can be visualized as a large cube with side length $(a + b)$. This cube can be decomposed into smaller cubes and rectangular prisms, each corresponding to a term in the algebraic expansion.



2. Three identical rectangular prisms ($3ab^2$):
    - These represent the third term of the expansion.
    - Each has dimensions $a \times b \times b$.
    - Again, there are three of these, fitting along three edges of the cube.

3. Small cube (b^3):
    - This represents the last term of the expansion.
    - It has dimensions $b \times b \times b$.
    - It fits in the corner where the three smaller rectangular prisms meet.

Worked Examples

Let us look at the following worked examples.

Direction: Expand the following terms.

1. $(3y + 2)^3$

Remember:

1. The first term is always a^3 in both cases.
2. The second term is $3a^2b$ in Case 1, but $-3a^2b$ in Case 2.
3. The third term is $3ab^2$ in both cases.
4. The last term is b^3 in Case 1, but $-b^3$ in Case 2.

Solution: Here, $a = 3y$ and $b = 2$, so

1. $a^3 = (3y)^3 = 27y^3$
2. $3a^2b = 3(3y)^2(2) = 54y^2$
3. $3ab^2 = 3(3y)(2)^2 = 36y$
4. $b^3 = 2^3 = 8$

Answer: Then, we now have $\mathbf{27y^3 + 54y^2 + 36y + 8}$

Worked Examples

Let us look at the following worked examples.

Direction: Expand the following terms.

1. $(3y + 2)^3$

Solution:

Using Distributive Property of Multiplication

$$(3y + 2)^3 = (3y + 2)(3y + 2)(3y + 2)$$

Last

First

$$= (3y + 2)(3y + 2)(3y + 2)$$

Inner

Outer

$$= (9y^2 + 6y + 6y + 4)(3y + 2)$$

$$= (9y^2 + 12y + 4)(3y + 2)$$

$$= 27y^3 + 36y^2 + 12y + 18y^2 + 24y + 8$$

Answer: $\boxed{27y^3 + 54y^2 + 36y + 8}$

Worked Examples

Let us look at the following worked examples.

Direction: Expand the following terms.

2. $(x - 5)^3$

Remember:

1. The first term is always a^3 in both cases.
2. The second term is $3a^2b$ in Case 1, but $-3a^2b$ in Case 2.
3. The third term is $3ab^2$ in both cases.
4. The last term is b^3 in Case 1, but $-b^3$ in Case 2.

Solution: Here, $a = x$ and $b = 5$, so

1. $a^3 = x^3$
2. $-3a^2b = -3x^2(5) = -15x^2$
3. $3ab^2 = 3x(5)^2 = 75x$
4. $-b^3 = -(5)^3 = -125$

Answer: Then, we now have $\mathbf{x^3 - 15x^2 + 75x - 125}$

Worked Examples

Let us look at the following worked examples.

Direction: Expand the following terms.

2. $(x - 5)^3$

Solution:

Using Distributive Property of Multiplication

$$(x - 5)^3 = (x - 5)(x - 5)(x - 5)$$

Last

First

$$= (x - 5)(x - 5)(x - 5)$$

Inner

Outer

$$= (x^2 - 5x - 5x + 25)(x - 5)$$

$$= (x^2 - 10x + 25)(x - 5)$$

$$= x^3 - 10x^2 + 25x - 5x^2 + 50x - 125$$

Answer: $\boxed{x^3 - 15x^2 + 75x - 125}$