AlgePRO

Lesson 5

SPECIAL CASE ON THE PRODUCT OF BINOMIAL AND TRINOMIAL

MATH 8 - QUARTER 1

Special Case on the Product of Binomial and Trinomial

In our previous lessons, we've explored the properties of binomials and trinomials, including how to square and cube binomials. Today, we'll discover a fascinating special case that occurs when we multiply certain binomials and trinomials.

This special case is not only interesting mathematically, but it also provides us with a powerful shortcut for solving particular types of algebraic problems. By recognizing this pattern, we can quickly simplify expressions that might otherwise require lengthy calculations.
As we delve into this lesson, keep in mind how this special case relates to our earlier work with binomials and trinomials. You'll see that mathematics often reveals elegant patterns that connect different concepts in surprising ways.

Remember:

When multiplying a binomial by a trinomial, there are two special cases that result in a sum or difference of cubes

Special Case 1: $(x + a)(x^2 - ax + a^2) = x^3 + a^3$

Special Case 2: $(x - a)(x^2 + ax + a^2) = x^3 - a^3$

Special Case Formula

When multiplying a binomial by a trinomial, we can use the distributive property or the FOIL method extended to three terms:

Special Case 1: $(x + a)(x^2 - ax + a^2) = x^3 + a^3$

This special case is particularly useful and worth memorizing. It can be derived as follows:

1. Distribute the binomial $(x + a)$ to each term of the trinomial $(x^2 - ax + a^2)$
2. Simplify and combine like terms.

Let's break it down step-by-step:

Step 1: Distribution

$$(x + a)(x^2 - ax + a^2) = x(x^2) + x(-ax) + x(a^2) + a(x^2) + a(-ax) + a(a^2)$$

Special Case Formula

Special Case 1: $(x + a)(x^2 - ax + a^2) = x^3 + a^3$

Let's break it down step-by-step:

Step 2: Simplification

$= x(x^2) + x(-ax) + x(a^2) + a(x^2) + a(-ax) + a(a^2)$

$= x^3 - ax^2 + xa^2 + ax^2 - a^2x + a^3$

Step 3: Combining like terms

$= x^3 + (-ax^2 + ax^2) + (xa^2 - a^2x) + a^3$

$= x^3 + 0 + 0 + a^3$

$= x^3 + a^3$

Special Case Formula

Now let's break down Special Case 2:

Special Case 2: $(x - a)(x^2 + ax + a^2) = x^3 - a^3$

Step 1: Distribution

$$(x - a)(x^2 + ax + a^2) = x(x^2) + x(ax) + x(a^2) - a(x^2) - a(ax) - a(a^2)$$

Step 2: Simplification

$$= x(x^2) + x(ax) + x(a^2) - a(x^2) - a(ax) - a(a^2)$$

$$= x^3 + ax^2 + xa^2 - ax^2 - a^2x - a^3$$

Step 3: Combining like terms

$$= x^3 + (ax^2 - ax^2) + (xa^2 - a^2x) - a^3$$

$$= x^3 + 0 + 0 - a^3$$

$$= x^3 - a^3$$

Worked Examples

Let us look at the following worked examples.

Direction: Expand the following terms.

1. $(y + 2)(y^2 - 2y + 4)$

Remember:

When multiplying a binomial by a trinomial, there are two special cases that result in a sum or difference of cubes.

Special Case 1: $(x + a)(x^2 - ax + a^2) = x^3 + a^3$

Special Case 2: $(x - a)(x^2 + ax + a^2) = x^3 - a^3$

Solution: This matches Special Case 1 with $x = y$ and $a = 2$

Using the formula: $(x + a)(x^2 - ax + a^2) = x^3 + a^3$

$(y + 2)(y^2 - 2y + 4) = y^3 + 2^3$

Answer: Therefore, the answer is $\mathbf{y^3 + 8}$.

Worked Examples

Let us look at the following worked examples.

Direction: Expand the following terms.

1. $(y + 2)(y^2 - 2y + 4)$

Solution:

Using Distributive Property of Multiplication

$$(y + 2)(y^2 - 2y + 4) = y(y^2) + y(-2y) + y(4) + 2(y^2) + 2(-2y) + 2(4)$$

$$= y^3 - 2y^2 + 4y + 2y^2 - 4y + 8$$

$$= y^3 + 8$$

Answer: $\boxed{y^3 + 8}$

Worked Examples

Let us look at the following worked examples.

Direction: Expand the following terms.

2. $(x - 3)(x^2 + 3x + 9)$

Remember:

When multiplying a binomial by a trinomial, there are two special cases that result in a sum or difference of cubes.

Special Case 1: $(x + a)(x^2 - ax + a^2) = x^3 + a^3$

Special Case 2: $(x - a)(x^2 + ax + a^2) = x^3 - a^3$

Solution: This matches Special Case 2 with $x = x$ and $a = 3$

Using the formula: $(x - a)(x^2 + ax + a^2) = x^3 - a^3$

$(x - 3)(x^2 + 3x + 9) = x^3 - 3^3$

Answer: Therefore, the answer is $\mathbf{x^3 - 27}$.

Worked Examples

Let us look at the following worked examples.

Direction: Expand the following terms.

2. $(x - 3)(x^2 + 3x + 9)$

Solution:

Using Distributive Property of Multiplication

$$(x - 3)(x^2 + 3x + 9) = x(x^2) + x(3x) + x(9) + [(-3)(x^2)] + [(-3)(3x)] + [(-3)(9)]$$

$$= x^3 + 3x^2 + 9x - 3x^2 - 9x - 27$$

$$= x^3 - 27$$

Answer: $\boxed{x^3 - 27}$