LESSON 4: Cube of Binomials

Learning Objectives

At the end of the lesson, you will be able to:

1. Understand the concept of cubing a binomial.
2. Learn to apply the formula for solving the cube of a binomial.
3. Utilize the concept to find the product of polynomials using the cube of a binomial.

The **Cube of a Binomial** refers to the result obtained by raising a binomial expression to the **power of 3**. This process involves multiplying the binomial by itself twice and expanding the expression, resulting in a trinomial. The general form of the cube of binomial, $(a+b)^3$, is expressed as $a^3+3a^2b+3ab^2+b^3$, showcasing the coefficients derived from the expansion. Understanding the cube of a binomial is fundamental in algebraic expressions and polynomial manipulations.

Cube of Binomial Formula

The formula for the cube of binomial $a+b$ and $a-b$ is given by:

$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$
$$(a-b)^3 = a^3-3a^2b+3ab^2-b^3$$

Derivation of $(a + b)^3$ using the distributive property method

$$\begin{aligned}
(a+b)^3 &= (a+b)(a+b)(a+b) \\
&= (a^2+2ab+b^2)(a+b) \\
&= a(a^2+2ab+b^2)+b(a^2+2ab+b^2) \\
&= a^3+2a^2b+ab^2+a^2b+2ab^2+b^3 \\
&= a^3+3a^2b+3ab^2+b^3
\end{aligned}$$

Derivation of $(a - b)^3$ using the distributive property method

$$\begin{aligned}
(a-b)^3 &= (a-b)(a-b)(a-b) \\
&= (a^2-2ab+b^2)(a-b) \\
&= a(a^2-2ab+b^2)-b(a^2-2ab+b^2) \\
&= a^3-2a^2b+ab^2-a^2b+2ab^2-b^3 \\
&= a^3-3a^2b+3ab^2-b^3
\end{aligned}$$

Steps to Solve the Cube of Binomial

1. Identify the Binomial. Suppose we have the binomial (a + b).
2. Cube the Binomial. Use the formula $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$ to expand the cube of the binomial.
3. Apply the Binomial Cube Formula. Substitute the values of a and b into the expanded expression.
4. Combine like terms and simplify the expression.

Example 1: Find the cube of the binomial $(x + 2)$.

Solution:

To find the cube of the binomial $(x + 2)$*, we'll apply the formula for the cube of a binomial, which is:*

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Here, $a = x$ *and* $b = 2$*.*

Substituting these values into the formula, we get:

$$\begin{aligned}(x + 2)^3 &= x^3 + 3x^2(2) + 3x(2)^2 + (2)^3 \\ &= x^3 + 6x^2 + 12x + 8\end{aligned}$$

Therefore, the cube of the binomial $(\mathrm{x} + 2)$ is $\mathrm{x}^3 + 6\mathrm{x}^2 + 12\mathrm{x} + 8$.

Example 2: Calculate the cube of the binomial $(3y - 4)$.

Solution:

To calculate the cube of the binomial $(3y - 4)$*, we'll use the formula for the cube of a binomial:*

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Here, $a = 3y$ *and* $b = 4$*.*

Substituting these values into the formula, we get:

$$\begin{aligned}(3y - 4)^3 &= (3y)^3 - 3(3y)^2(4) + 3(3y)(4)^2 - (4)^3 \\ &= 27y^3 - 3(9y^2)(4) + 3(3y)(16) - 64 \\ &= 27y^3 - 108y^2 + 144y - 64\end{aligned}$$

Therefore, the cube of the binomial $3\mathrm{y} - 4$ is $27\mathrm{y}^3 - 108\mathrm{y}^2 + 144\mathrm{y} - 64$.

Example 3: Determine the value of $(2a - 1)^3$.

Solution:

Using the formula for the cube of a binomial:

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Here, $a = 2a$ *and* $b = -1$*.*

Substituting these values into the formula, we get:

$$\begin{aligned}(2a - 1)^3 &= (2a)^3 + 3(2a)^2(-1) + 3(2a)(-1)^2 + (-1)^3 \\ &= (2a)^3 + 3(4a^2)(-1) + 3(2a)(1) + (-1) \\ &= 8a^3 - 12a^2 + 6a - 1\end{aligned}$$

Therefore, the value of $(2\mathrm{a} - 1)^3$ is $8\mathrm{a}^3 - 12\mathrm{a}^2 + 6\mathrm{a} - 1$.

SUMMARY

- Cubing a binomial involves raising a binomial to the third power, which means multiplying the binomial by itself three times.
- The cube of a binomial is denoted by:

  $$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
  $$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

  where a and b, are the first and second terms of the binomial, respectively.