LESSON 1: Square of Binomials

Learning Objectives

At the end of the lesson, you will be able to:

1. Define a binomial and recognize examples of binomials.
2. Explain and apply the FOIL method to square a binomial.
3. Use the special rule to find the square of a binomial.
4. Identify the resulting perfect square trinomial from squaring a binomial.

BINOMIAL

A **binomial** is an algebraic expression that contains two different terms connected by addition or subtraction. In other words, a binomial is a two-term polynomial that presents the sum or subtraction in the following form:

$$ax^m + bx^n$$

Where x is a placeholder for variables represented by any letters, *n* and *m* are exponents, and *a* and *b* are coefficients.

Some examples of binomials are:

- $2x^2 + 3y^2$
- $x^2 + 1$
- $x^3 - 2\mathrm{x}$
- $2m + n$

SQUARING A BINOMIAL

The **square of a binomial** is a type of special product where you always end up with a **perfect square trinomial** as the answer. There are two primary methods to find the square of a binomial:

1. FOIL method
2. Special Rule

FOIL Method

The **FOIL method** is used to multiply two binomials together. FOIL is an acronym indicating multiplying terms in a binomial in a specific order,

- "**F**" is for "**first**," indicating to multiply the first term in each binomial together.
- "**O**" stands for "**Outer**." The outer terms are the two farthest away from each other. Here, the first term is in the first binomial, and the last term is in the second binomial.
- "**I**" is for "**Inner**." The two inner terms are the last term in the first binomial and the first term in the second binomial.
- "**L**" is for "**Last**." The last terms in each of the binomials are multiplied together.

After multiplying all the terms, add them up and combine them like terms.

To apply the FOIL method, one must expand the square to the shape of the product of a binomial by itself:

$$(a+b)^2 = (a+b)(a+b)$$
$$(a-b)^2 = (a-b)(a-b)$$

Now, let's use the FOIL method on these examples:

Example 1: $(q-3)^2$

Solution:

$$(q-3)^2 = (q-3)(q-3)$$

- **First**: Multiply the first term in each binomial.
  $$(q)(q) = q^2$$

- **Outer**: Multiply the two terms on the outsides. That is the first term in the first binomial and the last term in the second binomial.
  $$(q)(-3) = -3q$$

- **Inner**: Multiply the two innermost terms, which are the last term in the first binomial and the first term in the second binomial.
  $$(-3)(q) = -3q$$

- **Last**: Multiply the last term of each binomial.
  $$(-3)(-3) = 9$$

Add up the terms: $q^2 - 3q - 3q + 9$

Combine like terms: $q^2 - 6q + 9$

Therefore, the square of binomial $(q-3)$ is $\boldsymbol{q^2 - 6q + 9}$.

Example 2: $(x^3 - 2\mathrm{x})^2$

Solution:

$$(x^3 - 2\mathrm{x})^2 = (x^3 - 2\mathrm{x})(x^3 - 2\mathrm{x})$$

- **First**: Multiply the first term in each binomial.
  $$(x^3)(x^3) = x^6$$

- **Outer**: Multiply the two terms on the outsides. That is the first term in the first binomial and the last term in the second binomial.
  $$(x^3)(-2\mathrm{x}) = -2x^4$$

- **Inner**: Multiply the two innermost terms, which are the last term in the first binomial and the first term in the second binomial.

$$(-2\mathrm{x})(x^3) = -2x^4$$

- **Last**: Multiply the last term of each binomial.

$$(-2\mathrm{x})(-2\mathrm{x}) = 4x^2$$

Add up the terms: $x^6 - 2x^4 - 2x^4 + 4x^2$
Combine like terms: $x^6 - 4x^4 + 4x^2$

Therefore, the square of binomial $x^3 - 2\mathrm{x}$ is $\boldsymbol{x^6 - 4x^4 + 4x^2}$

Special Rule
To find the square of a binomial of the form $(a \pm b)^2$ using the special rule:

1. Square the first term: $\boldsymbol{a^2}$
2. Twice the product of the two terms: $\pm\boldsymbol{2ab}$.
3. Square the second term: $\boldsymbol{b^2}$
4. Sum up all three terms obtained in the above steps: $\boldsymbol{a^2 \pm 2ab + b^2}$

Thus, the square of a binomial is given by:

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

This rule can be used directly to find the square of a binomial without having to multiply the terms using the FOIL method.

Now let's look at the examples below and find the product using our special rule.
Example 1: Square the binomial $(a + 5)$.
First term: a
Second term: 5

Step 1: Square the first term.

$$(a)^2 = a^2$$

Step 2: Twice the product of the two terms.

$$2[(a)(5)] = 10a$$

Step 3: Square the second term.

$$(5)^2 = 25$$

Step 4: Sum up all three terms obtained in the above steps:

$$a^2 + 10a + 25$$

Therefore, the square of binomial $(a + 5)$ is $\boldsymbol{a^2 + 10a + 25.}$

Example 2: Simplify $(3x + 4y)^2$.
First term: $3x$
Second term: $4y$

Step 1: Square the first term.

$$(3x)^2 = 9x^2$$

Step 2: Twice the product of the two terms.

$$2[(3x)(4y)] = 24xy$$

Step 3: Square the second term.

$$(4y)^2 = 16y^2$$

Step 4: Sum up all three terms obtained in the above steps:

$$9x^2 + 24xy + 16y^2$$

Therefore, the square of binomial $(3x + 4y)$ is $9x^2 + 24xy + 16y^2$.

By understanding and using these methods, you can efficiently square binomials and recognize the resulting trinomials.

SUMMARY

- A **binomial** is a polynomial that has two terms separated by either a plus sign or a minus sign. A product will result in three terms.
- The square of a binomial results in a **perfect square trinomial**. There are two primary methods to find the square of a binomial:
  1. FOIL method
  2. Special Rule
- The **FOIL method** is used to multiply two binomials together. FOIL is an acronym indicating multiplying terms in a binomial in a specific order,
  - First terms,
  - Outer terms,
  - Inner terms, and
  - Last terms.
- The **special rule** uses a direct formula:

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$