LESSON 2: Square of a Trinomial

Learning Objectives

At the end of the lesson, you will be able to:

1. Execute the method to find the square of a trinomial using the distributive property and binomial expansion methods.
2. Identify and create perfect square trinomials from given binomials.
3. Factorize perfect square trinomials into their binomial components.

TRINOMIAL

A **trinomial** refers to a mathematical expression or equation that consists of three terms. These terms are algebraic expressions or variables combined using addition and subtraction operations. Trinomials can take various forms, such as quadratic trinomials, where the highest power of the variable is squared, or cubic trinomials, where the highest power is cubed.

The general form of a trinomial is often expressed as $ax^2 + bx + c$, where a, b, and c represent coefficients, and x is the variable. Some examples of trinomials are:

- $2x^2 + 5x - 3$
- $-4y^2 - 2y + 7$
- $3a^2 - 6a + 1$

SQUARING A TRINOMIAL

Squaring a trinomial involves multiplying a trinomial by itself. A trinomial is an algebraic expression with three terms, typically of the form a + b + c where a, b, and c represent constants or variables.

Methods of Squaring a Trinomial

- Distributive Property Method
- Binomial Expansion Method

A. Distributive Property Method

Example: Find the square of trinomial $(\mathrm{ax}^2 + \mathrm{bx} + \mathrm{c})$.

Step 1: Apply distributive property.

$$= (ax^2 + bx + c)(ax^2 + bx + c)$$

Distribute each term in the first trinomial to every term in the second trinomial:

$$= ax^2(ax^2) + ax^2(bx) + ax^2(c) + bx(ax^2) + bx(bx) + bx(c) + c(ax^2) + c(bx) + c(c)$$

Step 2: Simplify and Combine Like Terms

$$= a^2x^4 + abx^3 + acx^2 + abx^3 + b^2x^2 + bcx + acx^2 + bcx + c^2$$

$$= a^2x^4 + 2abx^3 + 2acx^2 + b^2x^2 + 2bcx + c^2$$

Therefore, the expanded form of $(ax^2 + bx + c)^2$ is $a^2x^4 + 2abx^3 + 2acx^2 + b^2x^2 + 2bcx + c^2$.

B. Binomial Expansion Method

Another method involves using the binomial expansion formula.

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

Example: Find the square of trinomial $(a^2 + 2b + 3)$.

If we compare the above trinomial with a general trinomial in one variable, we can say that:

$$a = a^2, b = 2b, c = 3$$

Step 1: Square each term individually: Find a^2, b^2, and c^2.

$$a^2 = (a^2)^2 = a^4$$
$$b^2 = (2b)^2 = 4b^2$$
$$c^2 = (3)^2 = 9$$

Step 2: Compute the products of pairs of terms. Calculate $2ab$, $2ac$, and $2bc$.

$$2ab \ = \ 2(a^2)(2b) = 4a^2b$$
$$2ac \ = \ 2(a^2)(3) = 6a^2$$
$$2bc \ = \ 2(2b)(3) = 12b$$

Step 3: Add all the results together. Sum the squares of the individual terms and the products of the pairs of terms.

From step 1: $a^4 + 4b^2 + 9$

From step 2: $4a^2b + 6a^2 + 12b$

$$= a^4 + 4a^2b + 6a^2 + 4b^2 + 12b + 9$$

Therefore, the complete square of the trinomial $(a^2 + 2b + 3)^2$ is $a^4 + 4a^2b + 6a^2 + 4b^2 + 12b + 9$.

PERFECT SQUARE TRINOMIAL

Perfect square trinomial exists in two forms. i.e. $(a^2 + 2ab + b^2)$ or $(a^2 - 2ab + b^2)$. Now let's look into the steps to find the perfect square trinomial from the given binomial and vice versa.

Perfect Square Trinomial Formula

We can determine whether the given trinomial is a perfect square trinomial or not by a simple formula. Let's consider a trinomial $ax^2 + bx + c$ where x is a variable and a, b, c are constants then the given trinomial is a perfect square trinomial if and only if it satisfies the below condition

$$b^2 - 4ac \; = \; 0$$

a, b, c are constants.

Steps to find perfect square trinomial from binomial

So, the first term in the trinomial is the square of the first term in the binomial. The second term is twice the product of two terms in the binomial. The third term in the trinomial is the square of the second term in the binomial. If the binomial has a positive sign, then all the terms in the trinomial formed by squaring the binomial are positive. If the binomial has a negative sign, then the second term in the perfect squared trinomial will have a negative sign.

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

Here a and b are first and second terms in a binomial.

Below are the basic steps that are needed to be followed to find the perfect square trinomial from the binomial,

1. Find the square of the first term of a binomial.
2. Multiply the first term and second term of the binomial with
3. Find the square of the second term of a binomial.
4. Sum up all three terms obtained in the above steps.

Steps to Factorize the Perfect Square Trinomial

1. Write the perfect square trinomial in the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$. where the first & third terms are perfect squares, one being variable and the other being constant.
2. Also, the middle term is twice the product of the first and third terms.
3. If the middle term has a positive sign, then factors of a trinomial are $(a+b)(a+b)$. Else $(a-b)(a-b)$.

$$a^2 + 2ab + b^2 \; = (a+b)^2$$
$$a^2 - 2ab + b^2 \; = (a-b)^2$$

Sample Problems

Problem 1: Check if the given trinomial $x^2 + 4x + 4$ is a perfect square trinomial or not.

Solution:

On comparing a given trinomial with $ax^2 + bx + \mathrm{c}$

$$a = 1, b = 4, c = 4$$
$$\begin{aligned} b^2 - 4ac \; &= \; (4)^2 - 4(1)(4) \\ &= 16 - 16 \\ &= 0 \end{aligned}$$

$b^2 - 4ac \; = \; 0$, therefore, the given trinomial is a perfect square trinomial.

Problem 2: Check if the given trinomial $x^2 + 3x - 2$ is a perfect square trinomial or not.

Solution:

On comparing given trinomial with $ax^2 + bx + \mathrm{c}$

$$a = 1, b = 3, c = -2$$

$$\begin{aligned} b^2 - 4ac &= (3)^2 - 4(1)(-2) \\ &= 9 - 8 \\ &= 1 \end{aligned}$$

$b^2 - 4ac \neq 0$, therefore, the given trinomial is **not** a perfect square trinomial.

Problem 3: Find the factors of the given perfect square trinomial $x^2 - 6x + 9$.

Solution:

The trinomial $x^2 - 6x + 9$ can be rewritten as $x^2 - 2(3)x + 3^2$ which is of form $a^2 - 2ab + b^2$so factors are $(a - b)(a - b)$.

where $a = x, b = 3$

So, the factors of the given perfect square trinomial are $(x - 3)(x - 3)$.

Problem 4: Find the perfect square trinomial for the binomial $(2x - 1)$.

Solution:

The perfect square trinomial for a binomial of form $(a - b)$ is $a^2 - 2ab + b^2$.

From the given polynomial

$$a = 2x, b = 1$$

$$\begin{aligned} a^2 - 2ab + b^2 &= (2x)^2 - 2(1)(2x) + (1)^2 \\ &= 4x^2 - 4x + 1 \end{aligned}$$

Therefore, $4x^2 - 4x + 1$ is the perfect square trinomial for the given binomial.

SUMMARY

- A **trinomial** is a polynomial with three terms, usually expressed as $ax^2 + bx + c$. These terms can be combinations of constants or variables connected by addition or subtraction.
- **Squaring a Trinomial** involves multiplying the trinomial by itself.
- There are two methods to square a trinomial:
    a. **Distributive Property Method.** Expand by distributing each term of the trinomial across every other term and then simplify.
    b. **Binomial Expansion Method**. Use the formula $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$ to expand and simplify the expression.

- A **<u>perfect square trinomial</u>** can be recognized by the formula $ax^2 + bx + c$ where $b^2 - 4ac = 0$. It is recognizable as $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$ for positive and negative middle terms, respectively.