

Discussion of Sesia et al. 2018 and the Knockoff Framework

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1. ON THE PROBLEM

The authors of Sesia et al. (2018) set out to design a procedure for variable selection with provable statistical guarantees. The *knockoff* algorithm proposed by Sesia et al. (2018), provably controls the *FDR* of conditionally independent variables. Denoting with x and y the predictor and outcome variables, respectively. The *false detection proportion*, a.k.a. the *false selection proportion*, is defined as V/R where R is the number of variables selected, and V is the number of such variables where $y|x_{-j}$ is independent of x_j . The knockoff algorithm of Sesia et al. (2018) provably control the $FDR := \mathbb{E}[FDP]$, at some user selected magnitude.

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Crucially for our comments: (1) The *FDR* is an expectation with respect to variability in x and y , i.e., a *random design* guarantee. (2) The procedure is *model free*, or *non-parametric* in that nothing is assumed on the parametric form of $y|x$. (3) The proofs assume full knowledge of F_x , i.e., the joint distribution of predictors, marginalized over y . (4) The method aims at good variable selection, not prediction.

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The method of Sesia et al. (2018) is similar in flavor to Barber & Candès (2015), but Sesia et al. (2018) is quite more general: (1) Barber & Candès (2015) assume a linear generative model, so that the null is simply $H_j : \beta_j = 0$. (2) Barber & Candès (2015) crucially assume $n > p$, and Gaussian distributed errors.

2. ON THE METHOD

The fundamental idea of the method is to generate variables that have all the properties of the original x_j , only that they are conditionally uncorrelated to y . These are termed *knockoff* variables. The method then proceeds to compute a test statistic that captures the difference in the strength of the dependence of the knockoff, and the original variable. This statistic is then compared to its resampling distribution: the distribution over resampled knockoffs.

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2.1. Other Variable Selection Methods

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2.2. Other Knockoffs

2.3. Permutation Testing and Symmetries

3. FUTURE RESEARCH

4. ON THE HYPOTHESES

5. ON THE PROBLEM SETUP

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REFERENCES

- 40 BARBER, R. F. & CANDÈS, E. J. (2015). Controlling the false discovery rate via knockoffs. *The Annals of Statistics* **43**, 2055–2085. 00123.
- SESA, M., SABATTI, C. & CANDÈS, E. J. (2018). Gene hunting with hidden markov model knockoffs. *Biometrika* 00000.

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