Comparison of estimators in quantum tomography

John Scott

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Introduction

- **The original motivation:** converting classical signal processing to quantum signal processing (originality: 100%)
- **The plan for project B:** time varying quantum tomography (originality: 80%)
- **Step one of the plan:** look into estimators for steady state quantum tomography (originality: 20%)
- **What I actually did:** look into estimators for quantum state tomography with one qubit (originality: 0%)

What is quantum state tomography

State estimation. You need many copies of the state because you need to be able to make repeated measurements and build up statistics.

Using expectation values to estimate the density matrix

The simplest way to do tomography is to just compute a density matrix from expectation values:

$$\rho = \frac{1}{2}(I + aX + bY + cZ) = \frac{1}{2}\begin{bmatrix} 1+c & a-ib \\ a+ib & 1-c \end{bmatrix} \qquad (1)$$

The matrix will come out Hermitian and trace 1, but might not have positive eigenvalues (so it isn't 'physical'). This problem gets worse depending on how pure the state is.

Quality of the esimate (loss functions): distance measures and fidelity

Pete's paper has three: the trace distance, the Hilbert-Schmidt distance, and the infidelity:

$$trace(A, B) = \frac{1}{2} Tr|A - B| \tag{2}$$

$$HS(A, B) = \frac{1}{\sqrt{2}} \left[tr(A - B)^2 \right]^{\frac{1}{2}} \tag{3}$$

$$infidelity(A, B) = 1 - \left[tr \sqrt{A^{\frac{1}{2}} B A^{\frac{1}{2}}} \right]^2 \tag{4}$$

Quality of the linear estimate

Put a fancy graph here

Extended norm minimisation: producing a physical density matrix

If you want physical estimates you can try to obtain them somehow. The simplest method is to take the linear estimate and project it onto the space of physical states. That's called extended norm minimisation (cite Pete and co.) There's some fancy confidence interval for the estimate which I haven't got working yet.

Doing ENM

You need to find an optimiser which will minimise the L2 distance between your (fixed) linear estimate and a (varying) physical density matrix. Parametrizing the space of physical density matrices can be done by writing (cite whichever paper it came from)

$$T = \begin{bmatrix} w & x + iy \\ 0 & z \end{bmatrix} \tag{5}$$

$$\rho = T^{\dagger} T \tag{6}$$

Then ρ is positive and hermitian. The trace 1 condition is given by requiring that

$$w^2 + x^2 + y^2 + z^2 = 1 \tag{7}$$

Other methods: maximum likelihood

You can also use maximum likelihood to get physical estimates.
But I didn't.