Problem Set #[2 - 1]

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Problem 1 The method of successive approximations yields that

$$x = [-0.89552239, 13.34328358, 45.64179104]^T$$

Code used in solving this problem is included on the repository under succ_approx.py. Using solvers to find the solution is equivalent to finding $x : (A - I)x = -b$. This system yields the same solution.

Problem 2 Since $w \in \mathbb{R}^+$, which is a complete metric space, we must show that $T(x) : \mathbb{R} \to \mathbb{R}$ with $T(x) = c(1-\beta) + \beta \sum_{k=1}^{K} \max\{w_k, x\} p_k$ is a contraction mapping. Note

$$|T(x) - T(y)| = \left| c(1-\beta) + \beta \sum \max\{w_k, x\} p_k - c(1-\beta) - \beta \sum \max\{w_k, y\} p_k \right| =$$
$$\beta \left| \sum \max w_k, x\} p_k - \sum \max\{w_k, y p_k \right| \leq \beta \left| \sum [\max\{w_k, x\} - \max\{w_k, y\}] p_k \right| \leq$$
$$\beta \sum |\max\{w_k, x\} - \max\{w_k, y\}| \, p_k \leq \beta \sum |x - y| \, p_k \leq \beta \sum \rho(x, y) p_k = \beta \rho(x, y)$$

Taking the supremum yields $\rho(T(x), T(y)) \leq \beta \rho(x, y)$, as desired. Thus T is a contraction mapping and has a unique fixed point. So the problem has a solution which is guaranteed to be unique. Furthermore, since it is a contraction mapping, you can solve it by selecting a starting point and using successive approximations until you arrive at the solution.

Problem 3 The plot is found below. Note that the reservation wage is increasing with compensation values. This makes sense because higher compensations values cause increased utility to unemployment, and deincentivize prospective employees from accepting a job with lower pay.

Figure 1: Reservation wage values