LADR Done Right

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Chapter 1

Inner Product Spaces

1.A Inner Products and Norms

Exercise 5 Suppose $T \in \mathcal{L}(V)$ is such that $\|Tv\| \leq \|v\|$ for every $v \in V$. Prove that $T - \sqrt{2}I$ is invertible.

Proof. Suppose V is finite-dimensional. Suppose $u \in \operatorname{null}(T - \sqrt{2}I)$. Then we have $Tu = \sqrt{2}u$, and hence $\|Tu\| = \left\|\sqrt{2}u\right\| = \sqrt{2}\|u\|$. Since $\|tv\| \leq \|v\|$, this implies that $u = 0$. ■

Exercise 6 Suppose $u, v \in \mathcal{L}(V)$. Prove that $\langle u, v \rangle = 0$ if and only if

$$\|u\| \leq \|u + av\|$$

for all $a \in \mathbf{F}$.

Solution. ■

Exercise 8 Suppose $u, v \in V$ and $\|u\| = \|v\| = 1$ and $\langle u, v \rangle = 1$. Prove that $u = v$.

Proof. Note that

$$\|u - v\|^2 = \langle u - v, u - v \rangle = \|u\|^2 - \langle u, v \rangle - \langle v, u \rangle + \|v\|^2 = 0.$$

Hence $u - v = 0$ by definiteness. ■